Risk, Insurance, and the Provision of Public Goods Under Uncertainty

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Introduction

• Interested in examining economies where agents face risk of loss with some probability
• One example is a flood-prone community
• There may exist a public good that can affect the probability that the agents in a community experience a loss
• Examples include levies, dams
Pareto efficient provision of Public goods

• Identifying and implementing Pareto efficient levels of public goods under uncertainty is challenging for several reasons:
  – Often agents have incentive to misrepresent their preferences for the public good to attempt to free-ride.
  – Unaware of any mechanism design work focused on the provision of public goods under uncertainty
  – Need complete markets (insurance) to achieve Pareto efficiency.
Complexity of Insurance

• Insurance markets are traditionally handled in economic theory with Arrow-Debreu securities, one security for each unique set of endowments in the economy.

• Problem: With certain kinds of risk, number of states of the economy can grow unmanageably large as the number of agents increase. Real-life insurance bears little resemblance to Arrow-Debreu securities.
Risk: Definitions

• Suppose there is an economy with N agents and that there are M possible states of nature each agent can experience. The number of states of the economy will depend on N, M and the type of risk that is presents.
Types of Risks

• Joint Risk: All agents experience the same state of the world.

• Graduated Risk: Agents live in an ordered environment, a river valley for example, and the loss each agent suffers can be no greater than the agents in a lower state in the ordered environment.

• Idiosyncratic Risk: Each agent can experience any of the possible states of world. This doesn’t have to be a completely independent process, risks can be correlated, but there must be some element of idiosyncratic or individual risk involved.
Computational Complexity

• Joint Risk: M states. Does not depend on number of agents, just the states of nature M.

• Graduate Risk: Defined recursively as

$$S(M,N) = \begin{cases} If(M == 2, Return[N + 1]) \\ Else Return[\sum_{i=1}^{M} S(M - 1, N - i)] \end{cases}$$

• Idiosyncratic Risk: N*M^N. Number of states of the economy grows exponentially with the number of agents
Limiting Behavior

• For graduated risk, the limiting behavior is such that the number of states of the economy increases in polynomial fashion with the order of the polynomial given by \( M-1 \). Thus, for \( M=2 \) the problem is linear. \( M=3 \) is quadratic etc.
Current practice

• Army Corp of Engineers is responsible for constructing most flood control projects
• Complex cost-benefit rules are used to determine which projects are undertaken.
• Flood insurance is provided via separate agency, part of the Federal Emergency Management Agency.
Desired alternative

• Would be useful if the insurance component of the problem and the public good component could be combined as the two decisions are related. Part of the demand for a levee could come from risk aversion. This would make flood an insurance and a levy substitutes. Insurer likely to have strong preferences over levee height as it affects premiums.
FEMA meets the Army Corp of Engineers

• What if we allowed a monopoly insurer to pick premiums for agents and allowed the insurer to provide the public good out money received from the premiums?

• Theory of the second best: with one market failure (public good) adding a second (market power) may be welfare improving
Rationale

• Insurer has strong incentive to provide and maintain the public good because it decreases the probability of having to pay out claims
• Only one price is necessary, so efficient with information
• Monopoly insurer can fund public good provision out of revenue from insurance premiums (ability to pay)
Single priced contracts

• We restrict the insurance company to selling single-priced contracts. That is, the premium an agent pays doesn’t not depend on the realizations experienced by other agents (not Arrow-Debreu). However, there is a risk that the insurance company will default and not pay your claim at all.
Default risk

• Tradeoffs with single priced contracts:
  – Limits number of prices in markets
  – Agents don’t need to trust or verify claims made by other agents, only required to trust insurance company

• Disadvantages
  – In a sufficiently bad year, no way to avoid default
  – Assume there is a government regulator who sets maximum probability of default. Insurance companies must make decisions to ensure probability of default doesn’t exceed threshold.
Simple example

- Insurer surprises agents with public good
- Future work to look at mechanism design
- To simplify mathematical model, assume agents face idiosyncratic risk
Problem details

• Logarithmic utility: \( u = \ln (c) \)
• Each agent has endowment of 1
• Loss of 0.5 with probability \( p(\delta) = .1-.05 \delta^{.5} \)
• Cost function is \( C(\delta) = 5 \delta^2 \)
• Limit on default risk, \( \alpha \), is the exogenous parameter
Agents’ problem

\[
\max_I (1 - \alpha)(p \cdot u(e - c \cdot I + I - d) + (1 - p)u(e - c \cdot I)) + \alpha(p \cdot u(e - c \cdot I - d) + (1 - p)u(e - c \cdot I))
\]

- Alpha is probability of default
- P is probability of loss
- C is insurance premium
- I is quantity of insurance purchased
- D is loss due to bad outcome occurring
Insurer’s problem

• Maximize profit subject to a default constraint
• With idiosyncratic risk, probability of paying out a given number of claims is governed by binomial distribution
• Breakeven point:

\[ N \times c \times I(c) - c(\delta) - X \times I(c) = 0 \]
Insurer’s problem cont

• Solvency constraint is given by

\[ 1 - \text{BinomialCDF}(N, p(\delta), X) \leq \alpha^* \]
Insurer’s problem cont

\[
\max_{c,\delta} \sum_{i=0}^{N} \left( \frac{N!}{i!(N-i)!} p(\delta)^i (1 - p(\delta))^{N-i} \right) \times N \times c \times I(c) - c(\delta) - i \times I(c)
\]

s.t.

\[
1 - \sum_{i=0}^{\text{Floor}\left(\frac{N \times c \times I(c) - c(\delta)}{I(c)}\right)} \frac{N!}{i!(N-i)!} p(\delta)^i (1 - p(\delta))^{N-i} \leq \alpha^*
\]
Public good as a function of $\alpha$
Utility as a function of $\alpha$
Expected profit as a function of $\alpha$
Conclusion

• Provision of public goods under uncertainty presents both mechanism design and computational challenges
• Only a limited literature exists in this area
• Numerical simulations indicate a tradeoff between solvency and utility.