Structuring Exotic Options Contracts on Water to Improve the Efficiency of Resource Allocation in the Water Spot Market

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Abstract

With the current drought in South-Eastern Australia highlighting the scarcity and value of inland Australia’s water resources, focus turns to how these resources can be allocated more efficiently. The first major step was taken almost a decade ago with the separation of land and water property rights allowing openly traded water markets. This study assesses the potential economic benefits that options contracts bring to the water market in the Murray Valley water market. Exotic call options are estimated using both Black-Scholes and skewness-and-kurtosis-amended Black-Scholes financial option pricing methods that are based on three years of data on water prices. While the presence of options would result in significant economic benefits in the more efficient trade of water on the open market for lower-value crops, there were mixed results from the attempt to price such options.

Key words: options, skewness-and-kurtosis-amended Black-Scholes model, water.

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1. Introduction

Water has long been recognised as one of inland Australia’s most scarce and valuable resources. It is now increasingly recognised that its efficient allocation is important so as to allow agriculture, the environment and urban populations to co-exist. The scarce and uncertain availability of water has been highlighted in recent times by the drought that has plagued South-Eastern Australia. The drought, which has significantly reduced the recharge of water stocks, coupled with population growing at a faster rate than the growth in water supply facilities, has created a distinct imbalance between supply and demand (WSAA 2005).

The relative scarcity of water in the Australian landscape has resulted in water playing a key role in public policy in the past decade, including the National Water Initiative and more recently the federal government’s $10 billion ‘Ten Point Plan’ which aims to upgrade water infrastructure and technology and address the over-allocation of water in the Murray Darling Basin (Sim 2007). The most important policy with respect to water in recent times was the decision by the Council of Australian Governments (COAG) in 1995 to introduce a water reform process. State governments introduced a system of water entitlements or allocations backed by the separation of water property rights from land title. This reform provided clear specification of entitlements in terms of ownership, volume, reliability, transferability and, if applicable, quality (Bjornlund and O'Callaghan 2003). By using a free market as the price discovery tool for water, supply and demand theoretically determine what water is worth and then direct that water to its highest-value enterprise.

The introduction of an openly traded water market in Australia has allowed this reallocation to happen to some extent, but the continuous intervention by the government in this market does limit its efficiency somewhat. A possible solution to this problem is the use of derivative securities, namely options contracts in the water market. The underlying idea of an options contract is to create a risk-sharing mechanism that, ex ante, provides a ‘fair price’ to both parties and ensures that the risk transferred between the two parties is mutually beneficial. Most studies of water options contracts have been undertaken on the more mature and openly traded water markets in the United States of America, with water options considered to bring positive economic outcomes.
Studies on options contracts in an Australian context are limited. The aim of this paper is to determine whether this mutually beneficial transfer of risk between parties can result in increased efficiency in the allocation of water, which will be measured by the level of economic benefit these contracts generate. In the second part of the study, an attempt is made to value a call options contract on water using a financial option pricing model. This is illustrated through a case study based on the temporary and permanent trade of water entitlement on the Murray River in South-Eastern Australia. The Murray River was selected because it has the largest volume of water traded on the open market (NSW DNR 2007). It is the most developed of the Australian water markets and most likely to benefit from the inclusion of options contracts in the marketplace.

2. Studies of Water Contracts

There is a substantial literature covering options contracts in the United States water market. Articles by Michelsen and Young (1993), Villinski (2003), Hansen, Howitt and Williams (2006), Ranjan, Gollehon and Aillery (2004) and Watters (1995) have influenced the direction of research in this study to determine the applicability of options contracts to Australian water markets. Further to the work done by these analysts, Page and Hafi (2007) and Hafi et al. (2005) examined water options contracts based specifically on Australian conditions. These seven articles are summarised in this section.

Michelsen and Young (1993) observed that the use of multiple exercise ‘exotic’ options contracts on temporary water rights are a least-cost means of providing drought insurance for urban water agencies. They developed six criteria to establish water supply options contracts based on a review of earlier literature (Randall 1981; Howe, Alexander and Moses 1982; Young 1972; Cox and Rubenstein 1985). The first criterion stipulates that the water supply must be reliable enough to provide water in drought years. Second, property rights must be definable and transferable for market exchange. Third, agricultural operations must be capable of being suspended while, fourth, both the buyer and seller must have realistic knowledge of water values and costs. Fifth, it must be possible to elicit a probability of exercise and, finally, total option costs must be less than the purchaser’s next most costly supply alternative. We address each of these criteria from an Australian perspective in a later section.
The model proposed by Michelsen and Young prices the option as the cost of the most likely supply alternative minus the cost of exercising the option, with a positive present value representing net economic benefit in using options contracts. This model has been applied to a case study exploring the transfer of water from rural to urban users; however, it is easily transferable to the scenario of evaluating transfer of water from lower-value irrigation to high-value horticultural use, which is the problem addressed in this study.

Michelsen and Young also suggested terms and provisions for water options contracts, fixing the exercise price at the value of the forgone agricultural production of the writer. Option water quantity, method and time of delivery all need to be specified. Finally, there should be right of first refusal that allows the seller to retain the option of selling their permanent entitlement before contract termination, but which also allows the option holder the opportunity to match the offered price for the water.

The major shortcoming of Michelsen and Young’s approach is their model’s failure to place a value on the option premium. They did, however, determine that there is a positive present value of option benefit for the transfer water from rural to urban use under the majority of their case-study scenarios; in other words, there is economic benefit in having these options to augment urban water supplies during drought.

Watters (1995) conducted another study based on the water transfer market by applying the commonly used financial derivative approaches of the binomial tree and Black-Scholes (BS) methods to value call options on water in southern California. He was able to conclude that call option premiums in the Rio Grande were not significantly mis-priced, suggesting that current options were written with a high probability of exercise values that in turn translate into high volatility in the option. Watters’ approach to studying whether current water options contracts are efficiently priced may be somewhat problematic in an Australian context due to limited water options contracts existing at present.

Villinski (2003) also set out to value a multiple exercise water options contract to augment water supplies during drought. But instead of using an economic benefit model like Michelsen and Young, she priced the option financially using dynamic programming built around the BS model after explaining that the BS method is an inadequate means of pricing an option in a thinly traded water market. This is because, according to Villinski, water markets violate a number of the base assumptions of the BS model.
including the presence of transaction costs, the presence of arbitrage opportunities, and non-normally distributed water market prices.

Although Villinski (2003) argues that dynamic programming is a more robust means of price water options contracts, it is still only as good as the data input. The model is still susceptible to the accuracy of its parameters, while the mean and volatility must represent the true path of the log price of water as in the BS model.

Ranjan et al. (2004) addressed the dynamics between spot market transactions and option markets, when conditions permit the market transfer of water. They concluded that spot and options markets serve the needs of both buyers and sellers in terms of smoothing fluctuations (reducing uncertainty) and sharing risks associated with water supply and demand. But despite the benefits that these markets offer, their development has been slow to date, to which the Australian context bears strong similarities.

Ranjan et al. (2004) commented that spot markets offer higher rewards for water sellers, but these sellers also bear larger risks due to the associated price fluctuations of these markets. This rings true of the spot market in Australia, where sellers of water are being offered much higher rewards in the current drought with market prices above $850/ML, but this price is subject to considerable volatility. Ranjan et al. (2004) described options markets as a means of hedging against water price fluctuations but observed that they still place substantial risks on farmers participating in these markets. Translating to an Australian context, this risk will present substantial resistance to participation in water markets, especially due to the large fluctuations in water supplies, thus slowing participation in both spot and option markets.

Hansen et al. (2006) used a simulation-optimisation approach to place a value on the transfer of water uncertainty from one party to another across several locations in California. They employed a mathematical programming framework to analyse whether increased trading among water agencies across time and space would result in significant gains from trade. The authors noted that, as the value of water increases, institutional mechanisms such as water markets evolve to improve its allocation. Noting the effectiveness of markets in improving the allocation for water to yield gains to trade (Howitt 1994), Hansen et al. (2006) suggested that more flexible institutional mechanisms such as options contracts would further improve the allocation of water. They thus provide an alternative additional storage construction to augment water supplies during times of drought.
Hansen et al. used the binomial option pricing model presented by Cox, Ross and Rubenstein (1979) based on a distribution created in a simulation–optimisation model to generate their value of the option. However, the price of their option using this method will converge towards the BS value as the number of iterations in the simulation increases, and since the number of iterations in the binomial tree would increase its accuracy the BS model could be substituted as the pricing model. Hansen et al. (2006) suggested that there needs to be further discussion on why previous theoretical calculations of option value have exceeded option premiums on existing bilateral contracts. Their suggested reasons include mechanisms other than the market to allocate water (e.g. storage), while avoiding transaction cost may be another reason.

In addition to the American-based literature on water options contracts, there are two papers in which options contracts are applied in an Australian context. First, Hafi et al. (2005) considered the use of multiple-exercise European options contracts to augment wet-season flows in Australian river systems in order to create a beneficial flooding event for natural vegetation. It is the opposite scenario to the drought option presented above, but helps to place water options contracts in an Australian setting.

Hafi et al. (2005) recognised that water transfers in Australia can be classified with respect to the permanence of trade, with permanent trade resulting in the transfer of a property right while temporary trade refers to the transfer of water allocation for any one year. They also acknowledged the disproportionate levels of risk that each of these transfer methods places on the parties involved in the transaction. Permanent transfer places large amounts of risk on the seller while temporary transfer places the risk on the buyer. Hafi et al. (2005) proposed options contracts as a potential mechanism to redistribute this risk more evenly, thereby facilitating the more efficient transfer of water.

Hafi et al. (2005) used the pricing model presented by Michelsen and Young (1993) to value the economic benefits associated with options contracts. This model was applied to a case study on the Murrumbidgee River using varying exercise prices, with the majority of scenarios presenting positive economic benefit. Their study shows that water options contracts can provide economic benefit in the allocation of scarce resources under Australian conditions. The major downfall with this approach once again is its failure to value the option premium, instead assuming that the two parties negotiate the value.
Page and Hafi (2007) presented results of their research into a drought option in Australian conditions whereby urban water supplies are augmented by options contracts on rural water. They argued that investments in excess supply capacity may be inefficient and are likely to be costly as a significant buffer of supply may be required in order to eliminate the effects of seasonal variability. Instead, they suggested that water options contracts may be a cost-effective means to provide a similar supply buffer for dry periods.

Once again, this model is based upon Michelsen and Young’s 1993 model which values the option as the discounted difference between the cost of option exercise and the cost of an alternative supply with all the associated problems of assigning a value to the option premium. They used their model to test a number of exercise values based on the opportunity cost to irrigators, with most exercise values providing a positive net present value, meaning the option is beneficial.

What can be taken from these two Australian examples is the fact that water options contracts do provide positive economic benefit in the form of a more efficient allocation of water. However, very little work has been done in applying a financial option pricing model to the water scenario in order to assign a premium to the option with respect to its intrinsic and time values. While it has been proven that there is economic benefit in transferring water from rural use to a higher-value urban use, is there also value in using options contracts to transfer water from lower-value broad-scale irrigation to higher-intensity horticultural industries? That is the question addressed in the remainder of the paper.

3. Water Option Contracts

Options contracts are financial instruments that derive their value from another underlying asset and give the holder the right but not the obligation to buy or sell that asset. Financial options are openly traded across exchanges internationally. They are based on equities, indexes, commodities and interest rate futures to name just a few. The options explored in this instance have a value that is based on the value of temporary water transfer values.

Options contracts come primarily in the two basic forms of calls and puts and can be American or European with respect to exercise. An American option allows the holder to exercise the option at any time before the specified maturity date. In contrast, a
European option can only be exercised on the specified date of maturity (expiry date). An American option is more expensive because of its greater probability of being exercised. A European call option is used in this study. Hafi et al. (2005) explained that options contracts on water are allocated for the temporary use of a renewable resource while leaving the permanent entitlement unchanged. They differ from options on stocks and commodities that result in the full transfer of the underlying asset. It is this transfer of temporary use of a renewable resource that creates the opportunity for an option to be exercised several times.

The second difference between water options and exchange-traded financial options is the trigger for exercise. Michelsen and Young (1993) stated that a drought-augmenting options contract on water depends on the quantity supplied, while a financial options exercise is based on the market price of the underlying asset. The interpretation is that the probability of exercise on water options contracts is based on the expected number of years with water shortage rather than being based explicitly on the asset’s value with respect to the exercise price.

These differences between water options and financial options lead to the option proposed in this study being termed as exotic under the classification by Hertzler (2004). The exotic option to be tested is a package of European options contracts with maturities spanning from one to five years, to give a multiple-exercise call options contract that is five years in length, or one to ten years, to give a multiple exercise call options contract of ten years in length.

Michelsen and Young (1993) proposed six conditions to establish water options contract markets. The first of these conditions is that water supply must be reliable enough to provide sufficient water for option use during drought years (Michelsen and Young 1993). This is because the writer is legally bound to deliver an agreed volume of water. In an Australian context this significantly restricts the type of water licence on which options can be written. Analysis of water volume allocations on the Murray River over the past three years from the NSW DNR (2007) shows that allocations for general-security and conveyance licences are extremely variable and therefore violate the reliability criterion. Only high-security licences exhibit reliable allocation of water and are therefore the only feasible licence class for water options.
The second criterion requires that individual property rights must be definable and transferable for market exchange (Michelsen and Young 1993). The water reform process introduced by COAG in 1995 enabled this criterion to be satisfied.

Next, Michelsen and Young (1993) specified that agricultural operations must be capable of being temporarily suspended. The approach followed in this study for options on water involves the transfer of water from broad-scale lower-value irrigators whose activities are largely based on annual crops to horticultural operators. As long as these growers are compensated for forgone profits on the water and any fixed overhead costs in their operations, there is no reason why agricultural operations in the model could not be suspended.

The fourth condition imposed by Michelsen and Young (1993) requires both the buyer and seller to have realistic knowledge of water use values and alternative supply costs. Because both parties in this model are irrigators, water is a significant input in their business model. Therefore, it is argued that since water plays such a large role in their businesses, both parties should have a realistic knowledge of values and costs in order that a fair price is set.

The probability and severity of drought (the expected probability of option exercise) must also be able to be estimated within acceptable limits of risk for both users (Michelsen and Young 1993). The unpredictability of future weather conditions represents a significant problem in fulfilling this requirement, especially in drought-prone south-eastern Australia. Page and Hafi (2007) attempted to account for this unpredictability by eliciting their probability value from historical records. This study takes the same approach, assuming a drought probability value elicited by Page and Hafi (2007) but using 36 years of data to 2005/2006 to simulate Murray River annual flows with an exercise trigger of 80 000 ML. Of course, with climate change the use of this ‘frequentist’ approach might not be an accurate guide for the future.

The final criterion placed on water markets by Michelsen and Young (1993) is that the total options contract costs, including both transaction costs and transport costs to the purchaser’s point of intake, must be less than the purchaser’s next most costly water supply. In other words, in order for an options contract to be feasible, the costs of exercising the option must be less than the closest logical alternative.
Two propositions are tested in this study. The first proposition concerns the potential economic benefits associated with the application of options contracts in Australian water supply conditions to improve the efficiency of allocation during periods of drought. The purpose is to ascertain whether these contracts provide positive economic benefits with respect to the efficient allocation of scarce water resources in the transfer of water from irrigator to irrigator. The second proposition to test is whether a financial options pricing model can be used to value the option premium within the bounds of this economic benefit.

4. Method

4.1 Modelling the net present value of options benefit (PVOB)

4.1.1 Defining the PVOB

The aim of the first hypothesis is to determine whether the existence of an option for water rights provides positive economic benefits in the transfer of irrigation water rights between farmers. The approach put forward by Michelsen and Young (1993) is followed, with the options contract valued as the difference in cost between alternative supplies of water and the cost of exercising the option. It is a method to value the option from a buyer’s perspective, by attempting to minimise the potential costs of augmenting water supply during periods of drought. This is a somewhat less traditional method of valuation than that of financial markets where it is the writer of an option who determines the value of the option based on intrinsic and time values in the market.

The model proposed by Michelsen and Young (1993) is designed to discover the present value of an options contract as a function of the cost of an alternative minus the expected exercise cost and appreciation of the alternative supply. This approach is denoted in the following formula:

\[
PVOB = \sum_{t=0}^{T} [(K_{t=0}r + M_t) - (E^tP_t)dt + [K_{t=0} - K_{t=0}(1 + a)^T]dt]
\]

where: \( PVOB = \) net present value of option benefits

\( t = \) year

\( T = \) contract termination year
\[ K_{t=0} = \text{capital cost of alternative supply at the beginning of the option term} \]

\[ r = \text{annual interest rate} \]

\[ M = \text{annual maintenance cost of the alternative} \]

\[ E = \text{exercise cost of option} \]

\[ P = \text{annual probability of option exercise (} 0 \leq P \leq 1) \]

\[ d_t = \text{discount factor for present value, } 1/(1 + r)^t \]

\[ a = \text{annual rate of appreciation of alternative supply.} \]

The above formula is decomposed into the PVOB, the cost of an alternative water source \((K_{t=0}r + M_t)\), the cost of exercising the option \((E^*P_t)\), and the forgone capital appreciation of the alternative source of supply \((K_{t=0} - K_{t=0}(1 + a)^T)\). For this model, the alternative source is selected to be the purchase of a high-security water entitlement of equivalent volume to that for which the options contract would be written. This is because it is considered to be a more cost-efficient means of increasing water entitlement than building additional infrastructure.

The PVOB indicates whether the option holds economic benefit. If it is positive, the options contract is a more cost-effective means of augmenting water supply during a drought. On the other hand, if the PVOB is negative it would be more beneficial for the buyer to pursue the alternative water source as it is cheaper than buying the options contract. This outcome would render the option worthless and the process of options trading uneconomic.

The cost of an alternative water supply can be decomposed into two factors. The first factor is denoted by the capital cost of owning the alternative for the option period \((K_{t=0}r)\), where \(K\) is the capital outlay to gain access to a megalitre (ML) of water and \(r\) is the interest rate that accounts for the opportunity cost of capital investment in this source of water. Effectively, this can be thought of as the interest cost of owning the asset given the entitlement is bought in the first year and sold for the same value at the end of hypothetical option period.

In the model, the capital outlay on an additional high-security allocation on the Murray River Regulated System has been valued at $1423/ML. This figure has been obtained by taking the weighted-average permanent transfer price per ML over the past three
years from the NSW Department of Natural Resources (DNR 2007). The reason for using three years of data is the extremely thin trade of high-security water licences over the period, with only one licence transferred so far in the current financial year (2007/08). The real risk-free interest rate assumed in this model is 4 per cent p.a., based on data on government bond yields in Australia over the past 15 years.

The second cost of alternative supply is its annual maintenance cost, M. Owning another licence has certain fixed costs associated with it, which for the purpose of the model have been assumed to rest on the owner of the licence. These fixed costs would include items such as government levies, asset maintenance and renewal funds. Their value, estimated by the NSW DPI (2007) at $14.02/ML, represents the total fixed water costs per hectare assuming 100 per cent allocation in the Murray River Regulated System.

The cost of exercise, denoted as \((E^*P)_t\) in equation (1), comprises the ‘exercise’ or ‘strike’ price \((E)\), which is paid to the writer of the option as compensation for the sale of their water entitlement, and the probability \((P)\) that the option will be exercised in any given year. The method for allocating a value for the exercise price for the option has been addressed in the previous literature. Michelsen and Young (2003) set \(E\) at the price at which farmers are willing to release water supplies or, in other words, a value that would fully compensate them for forgoing the benefits of using that water. Following this procedure, Page and Hafi (2007) valued \(E\) at the marginal value of high-security water at drought allocations. The interpretation of what \(E\) should be follows Just, Hueth and Schmitz (1982) who identified the extent to which writers of options should be compensated for forgone incomes as a result of sacrificing their water entitlement. It is assumed that the writer must be reimbursed for both the profit that would have been earned on the water and any capital and fixed costs that the writer is carrying whilst forfeiting access to that water. Following convention, net profit equals gross receipts minus variable costs minus fixed costs minus capital costs. A simple rearrangement of this formula gives net profit plus fixed costs plus capital costs equal to gross receipts minus variable costs. Since the gross margin for an activity is calculated as gross receipts minus variable costs, fair compensation is equal to the gross margin for the writer’s forgone water use. Gross margins per ML for rice, wheat, maize, soybeans, broccoli and potatoes (for processing) were used in the model as varying values of \(E\). The gross margins are estimated by stochastic simulation assuming triangular distributions in Simetar (Richardson, Schumann and Feldman 2006, p. 14) for crop
yields and prices, based on data published by NSW DPI (2007). Variable costs were assumed to be deterministic. A total of 10 000 samples were run and mean sample gross margins were calculated to give the exercise values presented in Table 1.

<table>
<thead>
<tr>
<th>Crop</th>
<th>GM/ML forgone ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-grain rice</td>
<td>128.97</td>
</tr>
<tr>
<td>Medium-grain rice</td>
<td>110.91</td>
</tr>
<tr>
<td>Maize</td>
<td>188.54</td>
</tr>
<tr>
<td>Soybeans</td>
<td>62.89</td>
</tr>
<tr>
<td>Wheat</td>
<td>159.44</td>
</tr>
<tr>
<td>Broccoli</td>
<td>463.43</td>
</tr>
<tr>
<td>Potatoes for processing</td>
<td>1279.03</td>
</tr>
</tbody>
</table>

Source: Adapted and updated data from NSW DPI (2007).

The previous literature was consulted to obtain a value for the probability that the option will be exercised in any year. The most applicable study of Australian conditions has been by Page and Hafi (2007). Assessing the feasibility of options contracts for rural to urban transfer, they estimated the annual probability of exercise as the expected number of shortages for Canberra over a ten-year period. The same P value of 0.3 is used as the basis for calculating the potential net economic benefit of water options contracts. This figure reflects urban demand for water, and not that of irrigators. Sensitivity analysis is undertaken with respect to the probability of exercise, and the results are reported below. It is sufficient to note at this point that the P value does not significantly influence the economic viability of the option.

The final component of the model is the forgone capital appreciation of the alternative source of water, \( K_{t=0}-K_{t=0}(1+a)^T \). The expected sale price of the licence in five or ten years time is subtracted from the initial purchase price to calculate the missed capital returns. This is an important factor to take into account as buying a permanent water licence is an investment in an asset with static to diminishing supply and growing demand; thus, the licence is expected to appreciate in value during the ownership
period. Hafi et al. (2005) illustrated this trend for the Murrumbidgee River Regulated System. Over the previous 14 years, general-security water licences in the system increased in value by an average of 5 per cent per year. Research on the Murray River Regulated System indicates that high-security licences have increased in value by 4.5 per cent per year over the past three years NSW DNR (2007). Countering this appreciation would be a commensurate increase in the opportunity cost of temporary water licences, which should be taken into account when calculating crop gross margins. Michelsen and Young (1993) point out that water rights are assumed to be non-depreciating assets, but structural alternatives with limited lifetimes must be depreciated and permanent water licences rely on such structures. In sum, there is no easy way to estimate changes in capital value so the approach followed in this study is to assume no capital appreciation or depreciation.

4.2 Options pricing

4.2.1 The Black-Scholes method

One problem with the model presented by Michelsen and Young (1993) is its failure to assign a definitive value to the premium a writer would demand as compensation for forgoing their rights to irrigation water. All that is suggested is that the purchaser and grantor must come to some agreement on this value, which could lead to market failure caused by differences in information available to each user. The second hypothesis tested in this paper aims to overcome this deficiency by applying a financial-based option model to assign a value to the intrinsic and time value of the option. If this method is successful, the option premium derived from pricing formula will be valued within the PVOB that was calculated when testing the values of options contracts.

There are two commonly used methods for valuing financial options, the binomial tree method and the BS method. According to Hull (2006), the binomial tree is a useful and popular means to price an option by constructing a decision tree mapping the different potential paths the optioned asset’s price may take. This method was developed by Cox, Ross and Rubeninstein (1979) and presents a simple discrete-time pricing formula valuing the option through arbitraging (Cox, Ross and Rubenstein 1979).

The BS model, developed by Black and Scholes (1973) and extended by Merton (1973), follows arguments similar to the no-arbitrage arguments used in the binomial tree
approach whereby a riskless portfolio is set up, consisting of positions in both the
derivative and asset markets (Hull 2006). In the absence of arbitrage opportunities, the
return from the portfolio must be the risk-free rate of interest, thus leading to the Black–
Scholes–Merton differential equation (Hull 2006). The rationale behind the selection of
the BS method to price the option in this study comes from Benninga and Wiener (1997)
who described the binomial option price as converging towards that of the BS price.

In deriving their model, Black and Scholes (1973) and Merton (1973) drew on seven
relatively restrictive assumptions:

1. The asset price is normally distributed following a geometric Brownian motion
   with constant volatility and drift.

2. The asset can be short sold with the full use of the proceeds permitted.

3. There are no transaction costs or taxes, with all securities being perfectly
divisible.

4. There are no dividends during the life of the derivative.

5. There are no riskless arbitrage opportunities.

6. Security trading is continuous.

7. The risk-free rate of interest, r, is constant and the same for all maturities.

Hull (2006) should be consulted for a comprehensive overview of the derivation of the
Black-Scholes-Merton differential equation. The formula derived by Black and Scholes
(1973) and Merton (1973) to price a European call option at time 0 on a non-dividend-
paying stock is:

\[
c = S_0N(d_1) - Ke^{-rTN(d_2)} \quad (2)
\]

and

\[
d_1 = \frac{\ln(S_0/K) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} \quad (3)
\]

\[
d_2 = d_1 - \sigma\sqrt{T} \quad (4)
\]

where

- c is the European call price
- \(S_0\) is the stock value at time zero
- K is the strike/exercise price
In explaining the terms of the above equations, details are given on the assumed figures used in the model. The European call price, represented by $c$, is the option premium that the writer would charge the purchaser in compensation for forgoing their entitlement to use the irrigation water.

Hull (2006) described $S_0$ as the stock value at the beginning of the option period. This concept is in theory extendable to the price of any asset; in this study, it is applied to water at the beginning of the option period. The value of water, $S_0$, in the model does not take the current (October 2007) price of temporary transfer water of more than $850/ML (NSW DNR 2007). This is because this price is considered abnormally high. If this price were to be used to estimate the exercise price in the above formula, the option would have an intrinsic value greater than the PVOB before time value is even considered, thus rendering the option worthless. Instead, a distribution of the price of water over the past three years is initially used. A second scenario is then considered using a lower volatility and based on a distribution of water prices prevailing over the two years from August 2004, which excludes the exceptionally high recent prices.

In the model, $K$ represents the exercise price of the option (Hull 2006). The test conducted in this section is whether the BS method can value the option within the PVOB generated in order to test the first hypothesis. If so, the exercise prices will remain in the range of values required to compensate the grantor of the option for forgone revenue and any fixed overhead costs for associated broad-scale irrigation crops grown in the Murray River region.

The risk-free rate of interest ($r$) is used to discount future cash flows. It is assumed in the model to be 4 per cent per annum, as employed in the estimation of PVOBs.

According to Hull (2006), $\sigma$ denotes the volatility of the given stock price and can either be estimated from historical price data or implied from the pricing of similar options. Since this is a proposal for a real option, which is rare in the Australian water market, it is difficult to find other options of the same nature to determine the implied volatility. Instead, volatility is initially calculated from 38 months of temporary price data from the NSW DNR (2007). Hull (2006) gives the formula to calculate volatility that is followed in
this study, with a slight modification in the annotation. The so-called ‘close-to-close’ formula is \( \sigma = s^* \sqrt{M} \), where \( s \) is the standard deviation of \( \ln(S_m/S_{m-1}) \), \( M \) is the number of months in year (12) given the use of monthly data, and \( S_m \) is the spot water price in month \( m \) \( (m = 2, \ldots, 38) \). The volatility calculated for temporary-transfer high-security water over the past three years varies closely around 1.0, or 100 per cent, depending on which months in this period are included (it is less than 1.0 when the exceptionally high prices of the final two months of the data set are dropped). This value seems unusually high when compared with the volatility of stocks; however, it must be remembered that in the current drought market water prices have more than tripled. In the second scenario mentioned above, a volatility of 0.5 is used as more representative of normal climatic conditions.

\( T \) signifies the time to maturity of the option. For the model estimated in this study, 5-year and 10-year option terms are used. The 5-year time period means that we are effectively packaging five separate European calls to calculate a value for this multiple exercise option. The first call has one year to maturity, the second two years to maturity, and so on to the fifth having five years to maturity. In the case of a 10-year option, there would be 10 separate European calls. Therefore, each contract has a separate premium attached and the total value of the option is the sum of all premiums.

Finally, \( N \) stands for the cumulative normal distribution function which is applied to \( d_1 \) and \( d_2 \) (Hull 2006). It is, in effect, the probability of the option expiring ‘in the money’ and therefore being exercised, given that the underlying returns are normally distributed. This implies that water prices have a log-normal distribution.

### 4.2.2 Modifying the Black-Scholes model for pricing water

Various attempts have been made to modify the BS model to calculate option prices to take into account the non-normality of distributions of \( d_1 \) and \( d_2 \) in equation (2) by including terms to account for the positively skewed distribution of water prices and its leptokurtic properties. Examples of this density-expansion approach cited by Corrado (2007) include, among others, Jarrow and Rudd (1982), Corrado and Su (1996), Heston (1993), Ane (1999), Jondeau and Rockinger (2000), and Chauveau and Gatfaoui (2002). The skewness-and-kurtosis-amended BS model, termed the SKABS model by Vähämaa (2003:10), was developed by Corrado and Su (1996). It is based on a Gram-Charlier expansion of the standard normal density function to adjust for skewness and kurtosis,
and was amended by Brown and Robinson (2002) for an errant minus sign. This amended version is used to provide the SKABS model using the formula (Vähämaa 2003:7):

\[ c = S_0 N(d) - Ke^{-rT}N(d - \sigma \sqrt{T}) + \mu_3 Q_3 + (\mu_4 - 3)Q_4 \]  

(5)

where

\[ Q_3 = \frac{1}{3!} S_0 \sigma \sqrt{T} [(2\sigma \sqrt{T} - d)n(d) + \sigma^2 TN(d)] \]  

(6)

\[ Q_4 = \frac{1}{4!} S_0 \sigma \sqrt{T} [(d^2 - 1 - 3\sigma \sqrt{T}(d - \sigma \sqrt{T}))n(d) + \sigma^3 T^{3/2}N(d)] \]  

(7)

\[ d = (\ln(S_0/K) + (r + \sigma^2/2)T)/\sigma \sqrt{T} \]  

(8)

\( n(.) \) is the standard normal density function.

\( \mu_3 Q_3 \) and \( (\mu_4 - 3)Q_4 \) measure the effects of skewness and kurtosis, respectively, on the option price.

Other notation is as previously defined. The calculation of \( d \) is equivalent to the calculation of \( d_1 \) and \( d_2 \) in the Black-Scholes formula. Vähämaa (2003:7) explained that this formula ‘is particularly convenient from a hedging point of view since it yields closed form solutions for the hedge ratios’. One potential problem is the absence of a ‘hidden’ martingale no-arbitrage restriction on the expected future asset price, of the type recently outlined by Corrado (2007). Corrado (2007:528) introduced a hidden restriction ‘via a reduction in parameter space for Gram-Charlier expansions calibrating the expansion’. But his application of the restriction in an example appears to have had minimal effects on fitted call option prices.

4.2.3 Stochastic simulation

The stochastic simulations of the non-normally distributed water prices were made for two scenarios (\( \sigma = 1.0 \) and \( \sigma = 0.5 \), and with different distributions of water prices) by employing the kernel density estimated random variable in Simetar to obtain 10 000 samples of option prices. This simulation procedure adopts ‘Parzen type kernel density estimators [Gaussian type] to evaluate a smoothed value that represents a point on the cumulative distribution function (CDF)’ (Richardson, Schumann and Feldman (2006:19). The kernel density estimated random variable was chosen among other sample-based
distributions and the standard log-normal distribution that is assumed for underlying asset prices in the standard BS model. Selection was based on a comparison of CDFs using a scalar measure to compare the difference between the actual and fitted CDFs by calculating ‘the sum of squared differences between two CDFs with an added penalty for differences in the tails’ (Richardson et al. 2006:37). The distribution for the second scenario when $\sigma = 0.5$ is assumed to mirror that for water prices in the two-year period from August 2004. Stochastic independence is assumed between enterprise gross margins and the spot price of water in both scenarios.

The proportions of in-the-money options (that is, with the water spot price greater than the exercise price) for contracts at time zero in scenario 1 range from 4 per cent to 51 per cent, with only soybeans showing a substantial proportion. Those for scenario 2 are considerably higher because spot water prices are on average much higher; they range from 34 per cent for maize and wheat to 69 per cent for soybeans.

5. Results and Discussion

5.1 Estimation of PVOBs

Results for the estimation of the value of options contracts for the two selected contract durations are presented in Table 2. They show that the PVOBs on irrigation water are positive for both the 5-year and 10-year contract lengths for crops with lower gross margins per ML. This set of results shows two things. First, as the exercise price increases, the PVOBs of options contracts for the transfer of water diminish. Second, options contracts facilitating transfer from low-value water uses to higher-value uses provide positive PVOBs while those from one high-value use to another high-value use are of little or no economic benefit.

Another significant result from this analysis is the increased PVOB of a longer option period. In total benefit terms, a 10-year option provides greater economic benefit than one with a 5-year duration, which is logical because a long-term option provides the buyer with greater flexibility. But it places more risk on the grantor and, in turn, potentially a greater option premium. Due to the discounting of future benefits, a 5-year option provides greater PVOBs on a yearly basis. Since the shorter term would mean less risk for the option grantor, the premium would be lower than that of the 10-year
option while on a yearly basis the option would provide greater PVOB. Therefore, it appears that the 5-year option would be of more economic benefit than the 10 year option.

<table>
<thead>
<tr>
<th>Crop</th>
<th>Exercise price ($/ML)</th>
<th>PVOB 5-year contract ($/ML)</th>
<th>PVOB 10-year contract ($/ML)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soybeans</td>
<td>62.89</td>
<td>289.51</td>
<td>496.57</td>
</tr>
<tr>
<td>Wheat</td>
<td>159.44</td>
<td>124.08</td>
<td>212.70</td>
</tr>
<tr>
<td>Medium-grain rice</td>
<td>110.91</td>
<td>206.57</td>
<td>354.29</td>
</tr>
<tr>
<td>Long-grain rice</td>
<td>128.97</td>
<td>186.71</td>
<td>316.98</td>
</tr>
<tr>
<td>Maize</td>
<td>188.54</td>
<td>73.96</td>
<td>126.73</td>
</tr>
<tr>
<td>Broccoli</td>
<td>463.43</td>
<td>-410.17</td>
<td>-1798.69</td>
</tr>
<tr>
<td>Potatoes</td>
<td>1279.03</td>
<td>-1309.70</td>
<td>-3086.00</td>
</tr>
</tbody>
</table>

Table 2 Present Values of Options Contracts for Differing Contract Duration

Through the use of Michelsen and Young’s (1993) model, we are able to determine whether there is economic benefit in options contracts. But it does not calculate the premium that the writer should charge to compensate for the risk of granting the option. All that can be elicited is that the premium should be within the bounds of the net present economic benefit. Therefore, for a wheat grower granting a 5-year call option the premium should be between $0/ML/year and $49/ML/year. Page and Hafi (2007) suggested a solution to this vague result: negotiation between the two counterparties. But there is large potential for market failure due to information asymmetry between the two parties, and it is believed that a quantitative method for assessing the option’s intrinsic and time value is necessary. Through the second hypothesis, an attempt is made to provide a fair value for an option premium by applying the BS model to water temporary transfer values.

5.2 Estimation of water option prices

The BS model is used in two ways to calculate water options contract prices for two water price scenarios. The scenarios were based on the Murray River temporary water licence transfers over the 38 months, August 2004 to September 2007 (with unquoted July prices interpolated). Water prices varied widely over this period, with lower prices in
the first two years, punctuated by the occasional spike, and extremely high prices prevailing during the final year. The first scenario covered all three years while the second scenario was based on prices prevailing in the first two years of this period. The second scenario therefore omits the very high water prices of the past year and has much lower price volatility.

Estimation of option prices is undertaken using a stochastic simulation in which 10,000 samples are obtained from a sample-based cumulative distribution function of the water price data. Results presented for the 5-year and 10-year options show that the relatively high volatility and unusually high prices for water transfer in recent years are the main causes of the negative net values. Price volatility based on the past three years of Murray River temporary water licence transfers was calculated at around 100 per cent, whereas in all of the examples used by Hull (2006) the volatility figure used was less than 40 per cent. The high volatility is likely to be because either the extremely high recent prices in the water market make writing an options contract very unattractive or a number of the restrictive assumptions of the model are broken that make it unable to price these options adequately. We examine first the violation of assumptions and later assess whether the option value is regularly greater than the PVOB in periods of lower and less volatile prices.

The primary shortcoming of BS pricing water options is the violation of the assumption that the ‘underlying asset’s price follows a geometric Brownian process’ (Villinski 2003) whereby changes in an asset’s price follow a continuous-time, non-stationary, Markov process, and prices are normally distributed (Villinski 2003). To satisfy the conditions of the Markov property, only today’s prices can be relevant in predicting tomorrow’s prices (Villinski 2003). However, the process of formation of water prices is significantly more complex as they fluctuate seasonally and in response to weather forecasts, thus violating this assumption of the BS model.

Clearly, the temporary transfer prices of irrigation water over the past three years, on which the model is based, are not normally distributed. This is illustrated in Figure 1, where the distribution is strongly positively skewed, with a large tail to higher prices representing the impact on water values of current drought conditions. Although it represents the water market situation in only three years, such a highly skewed distribution is likely to be structural in the market, with regular demand for water in most years interspersed with occasional spikes in prices caused by drought conditions. It is
partly leptokurtic, in that the right-hand side of the distribution has a thick tail, and partly platykurtic, in that the left-hand side of the distribution has hardly any tail.

The second assumption breached by water is the non-existence of transaction costs. Colby (1990) argues that water exchanges worldwide are subject to regulation or constraint in some form, and this intervention imposes costs within the marketplace due to greater uncertainty for both buyers and sellers.

![Figure 1 Distribution of prices of Murray River temporary water licence transfers, August 2004 to September 2007.](image)

Finally, the assumption of no arbitrage is violated. Villinski (2003) argues that in thinly traded markets such as water markets, arbitrage opportunities arise for investors to buy at one price and sell immediately for a higher one. This is true of the Murray River water market. On 18 September 2007, for example, an investor could have bought water at $800/ML and sold it for $850/ML on the same day when only 18 trades were registered.

Due to the violation of the above three assumptions, there is doubt over the BS method as a suitable model for pricing irrigation water options in the Murray River water market. It could be argued that the presence of transaction costs and arbitrage opportunities are unlikely to be major sources of imperfection. But the egregious violation of the
assumption of normal distributions of \( d_1 \) and \( d_2 \) in the BS formula (equation (2)) is of concern. The results of the alternative SKABS model to accommodate these non-normal distributions are presented below.

5.2.2 Results of stochastic simulations

Tables 3 and 4 contain the mean sample option prices, proportions of samples having option prices within PVOB bounds and option prices as a proportion of PVOBs for the stochastic simulations of the 5-year and 10-year option contracts for the two scenarios using the SKABS model. The two higher-value crops, broccoli and potatoes for processing, are excluded from the list of crops because they have negative estimated PVOBs. As reported above, the first scenario assumes an implied volatility of 0.5 and less extreme distribution of water prices and the second scenario assumes an implied volatility of 1.0 with an extremely positively skewed distribution of water prices.

<table>
<thead>
<tr>
<th>Table 3 Results of Stochastic Simulations Based on Data for Scenario 1 for 5-Year and 10-Year Options: SKABS Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean sample option price ($/ML)</strong></td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td><strong>5-year options:</strong></td>
</tr>
<tr>
<td>Soybeans</td>
</tr>
<tr>
<td>Wheat</td>
</tr>
<tr>
<td>Medium-grain rice</td>
</tr>
<tr>
<td>Long-grain rice</td>
</tr>
<tr>
<td>Maize</td>
</tr>
<tr>
<td><strong>10-year options:</strong></td>
</tr>
<tr>
<td>Soybeans</td>
</tr>
<tr>
<td>Wheat</td>
</tr>
<tr>
<td>Medium-grain rice</td>
</tr>
<tr>
<td>Long-grain rice</td>
</tr>
<tr>
<td>Maize</td>
</tr>
</tbody>
</table>
Table 4 Results of Stochastic Simulations Based on Data for Scenario 2 for 5-Year and 10-Year Options: SKABS Model

<table>
<thead>
<tr>
<th></th>
<th>Mean sample option price ($/ML)</th>
<th>Proportion of option prices within PVOB bounds (%)</th>
<th>Option price as a proportion of PVOB (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5-year options:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soybeans</td>
<td>476.22</td>
<td>51.6</td>
<td>164.5</td>
</tr>
<tr>
<td>Wheat</td>
<td>460.94</td>
<td>45.7</td>
<td>371.5</td>
</tr>
<tr>
<td>Medium-grain rice</td>
<td>467.97</td>
<td>45.9</td>
<td>226.5</td>
</tr>
<tr>
<td>Long-grain rice</td>
<td>465.24</td>
<td>45.6</td>
<td>249.2</td>
</tr>
<tr>
<td>Maize</td>
<td>457.13</td>
<td>45.6</td>
<td>618.1</td>
</tr>
<tr>
<td><strong>10-year options:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soybeans</td>
<td>772.62</td>
<td>51.8</td>
<td>155.6</td>
</tr>
<tr>
<td>Wheat</td>
<td>764.34</td>
<td>45.7</td>
<td>359.3</td>
</tr>
<tr>
<td>Medium-grain rice</td>
<td>768.21</td>
<td>45.9</td>
<td>216.8</td>
</tr>
<tr>
<td>Long-grain rice</td>
<td>766.71</td>
<td>45.6</td>
<td>241.9</td>
</tr>
<tr>
<td>Maize</td>
<td>762.21</td>
<td>45.6</td>
<td>601.4</td>
</tr>
</tbody>
</table>

The 5-year mean sample option prices in Table 3 for the first scenario range from $66/ML to $82/ML while results for the 10-year options for the first scenario follow a similar pattern, ranging from $165/ML to $177/ML. Most sample means of option prices are substantially less than the PVOBs. The proportion of option price to PVOB is lowest for the 5-year soybeans options contract, at 28.3 per cent, and highest for the 10-year maize options contract, at 130 per cent. The proportions of option prices within the PVOB bounds range between 70 per cent and 83 per cent and are similar for both sets of option contracts. Even at the lowest proportion, taking out an options contract appears to be expensive.

The SKABS results reported for Scenario 2 in Table 4 show that the sample mean option prices are now well above the PVOBs, in contrast to results for Scenario 1. Only around one-half of the sampled option prices fall within the PVOB bounds, with proportions
consistent across crops and between the two contract periods. Mean sample options prices are multiples of PVOBs, making options trading an unattractive proposition.

Figure 2 shows the surplus of PVOBs over the estimated option prices for the volatility range from 0.1 to 0.7. Once a volatility index of 0.7 is reached, all sample mean option prices exceed the respective PVOBs except for soybeans.

![Figure 2 Surplus of PVOBs over option prices in the SKABS model.](image)

The results using the SKABS method in water markets not subject to current levels of drought stresses can yield a pricing result within the economic benefit generated by the option. It is concluded, therefore, that the major factor behind the lack of opportunities for irrigators to benefit from buying a call option contract in water is the current level of extraordinarily high water prices and concomitant high levels of price volatility. Were prices to return to levels prior to the past 12 months, and were price volatility not to be so great, prospects for developing a market for water options would improve greatly.

Results of the stochastic simulations show that it is not the assumption of normality of distributions of $d_1$ and $d_2$ in the BS method that causes option prices to exceed PVOBs in many samples. Proportions of samples where estimated option prices are less than
PVOBs are even lower in the SKABS models than in the BS models, due to the high rates of volatility of water prices and skewed nature of the water price distribution.

5.3 Implementation of water options contracts

After demonstrating that options contracts on water can provide positive economic benefit by increasing the allocation of scarce water resources, we now discuss how these contracts should be implemented within water markets. Page and Hafi (2005) suggest that an option market similar to that of the current spot market could be put in place for the trade of options, overseen by an independent government body or existing exchange such as the Sydney Futures Exchange.

Following the problems encountered with the SKABS method, however, setting up a trading market could encounter significant difficulty in the absence of an appropriate trigger mechanism for exercising an option and a pricing model that provides a definitive ‘fair value’ on the option premium. The trigger mechanism needs to include an appropriate criterion for water shortage, and take account of the seasonality in water supply and water demand for different crops, especially the different seasonal requirements of winter and summer crops.

The approach presented by Michelsen and Young (1993) does not specifically value the time and intrinsic values of the option and therefore each contract would need to be tendered out in order to price the premium. The result would be significant transaction costs inherent in market operations due the large burdens of information required in this process. A method suggested by Page and Hafi (2005) to get around this problem is the standardisation of terms and conditions for all options contracts, which would reduce transaction costs and, as a result, improve the gain from trade in these instruments. Each water call option would need to state: the exercise price of the option; date of maturity; total volume of water for which the contract is established; potential strike dates within the life of the option (set to the same date each year); option premium; and a clause to cover any governmental intervention affecting the water entitlement on which the option is based. By specifying each of the above, the burden of information gathering would be decreased thereby reducing the costs associated with each transaction for an options contract.

Michelsen and Young (1993) raised other considerations within the terms and conditions of such a contract. They suggested the insertion of a clause for the right of first refusal.
Should the option grantor decide to sell the water entitlement before the termination date of the contract, the holder of the option would have the right to match the offered price for the water right (Michelsen and Young 1993). Their second suggestion was the inclusion of an escalation clause in the agreement to protect sellers against the effects of inflation.

Another area of investigation is the application of barrier options to water markets. A barrier option is an option whose payoff depends on whether the underlying asset’s price reaches a certain level during a certain period of time (Hull 2006). Hull (2006) describes these options as knock–in and knock–out options whereby they either come into existence or cease to exist when the underlying asset price reaches a certain barrier. This method could be useful in reducing the risk associated with the large positive tail of water price distributions. By reducing this risk, the option valuation model may not overprice the premium on the option with respect the economic benefit they generate. A second benefit of a barrier option would be if the barrier were to apply to water allocation. For example, the option would terminate if allocations on licences were to drop below 80 per cent. This would reduce the risk of the option writer not being able to provide the contracted water, and thus reduce the risk premium they must be paid to grant such an option.

A further area for study is limiting the number of available option exercises to less than the total number of potential exercise dates. An example of this approach would be a 10-year multiple exercise call option that allows for a maximum of five exercises over the lifetime of the contract. Villinski (2004) proposed such a contract, to be valued through the use of dynamic programming. The benefit of such a contract would be the cap placed on the probability of exercise, which would result in a reduction in the risk premium demanded by the writer of the option. It could be another means of confining the value of the option premium to within the economic value of the contract. But Villinski (2004) found that the valuation of these contracts was not significantly sensitive to the probability of exercise and so this method could have limited success.

Although the stipulation of the above contract terms is a major step towards implementing call options on temporary transfer water allocations, there are still concerns about the efficient pricing of such an option. Until a model can accurately assign a quantitative value to the intrinsic and time value of water options in an
Australian context, they will not be efficiently priced and therefore cannot be properly implemented.

6. Conclusions

As resources become scarcer and rise in value, it becomes more important to allocate them to their most valuable use. Water is no exception to this maxim with its demand ever increasing and supply staying relatively constant. The institutional mechanisms involved in allocating these resources will have to adapt. In the mid-to-late-1990s, institutional innovation came in the form of a separation in land and water property rights that created the opportunity for water entitlements to be traded on the open market. In the years following this policy change, the water market has increased in popularity and now forms the primary means of price discovery for water transfers.

Although an open market has increased the allocative efficiency of water over the past decade, there is still room for instruments such as options contracts to increase the efficiency of resource allocation further through the redistribution of the risks borne by buyers and sellers in the open market. The beneficial influence on water resource allocation in the more highly developed water markets of the United States of America suggests progress is possible in the introduction of options contracts to facilitate trade in local water markets. In this study, similar results have been reported with water options contracts creating positive present economic benefit in the trade of water from lower-value broad-scale irrigation to greater-intensity high-return horticultural pursuits.

The method of valuing this benefit does have one major downfall, with doubts about its ability to value the option premium. The SKABS model was employed to take account of non-normality in distributions but had even less success than the BS model in pricing the options within the beneficial range when spot water prices were highly volatile. But both models successfully priced options within the range of PVOBs when the volatility of water prices was assumed to be in the low to moderate range. The usual caveat about such a set of results prevails: is it worth bothering about a form of protection that you can afford when you do not need it, but cannot afford when you do need it?
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Source: [http://www.abc.net.au/rural/vic/content/2006/s1834179.htm](http://www.abc.net.au/rural/vic/content/2006/s1834179.htm)


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