AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

Returns to Scale of Production Function: Pooled, Within and Between Quantile Regression Approach

Maya Reimbaeva
Dept. of Agribusiness and Applied Economics
NDSU, Fargo, ND 58108-6050
E-mail: maya.reimbaeva@ndsu.edu

Saleem Shaik[1]
504 Richard H Barry Hall
Dept. of Agribusiness and Applied Economics
NDSU, Fargo, ND 58108-6050
Phone: (701) 231 7459; Fax: (701) 231 7400
E-mail: Saleem.Shaik@ndsu.edu

Ashok K. Mishra
211 Agriculture Administration Building
Dept. of Agricultural Economics and Agribusiness
LSU, Baton Rouge, LA 70803
Phone: (225) 578 0262; Fax: (225) 578 2716
E-mail: Amishra@lsu.edu

Selected Paper prepared for presentation at the Southern Agricultural Economics Association Annual Meeting, Orlando, FL, February 6-9, 2010

Copyright 2010 by Reimbaeva, Shaik and Mishra. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided this copyright notice appears on all such copies.

[1] Corresponding author.

Returns to Scale of Production Function: Pooled, Within and Between Quantile Regression Approach

1. Introduction

In the last century, the United States, regional and state's agriculture have undergone an impressive transformation with much debate about changes in their farm economic structure. This paper examines the changes in input resource use in the production of crops and livestock and the relationship between the uses of inputs to produce outputs using a primal production framework. Apart from this functional relationship, there is a growing interest in how these relations (linear or non-linear) have evolved across cross-section units, over time and across quantiles due to changes in technology.

Changes in the input and output variables has been examined for the U.S. agriculture sector using the primal production function [Marschak, and Andrews (1944); Mundlak (1963); Hoch (1958, 1962); Zellner, Kmenta, and Dreze (1966); Schmidt (1988)], and the dual cost function [Nerlove (1963); Fuss, and McFadden (1978); Diewert (1974); McElroy (1987)] or the profit function [Weaver (1983); Lopez (1985); Dixon, Garcia, and Anderson (1987); Antle (1984)]. There is a widespread use of ordinary least square (OLS) in examining the changes in farm economic structure accounting for autocorrelation and heteroscedasticity, alternative functional forms, and estimation techniques. Most research involved estimation of the relationship between endogenous and exogenous variables at the mean. With the introduction of quantile regression (QR) methods by Koenker and Bassett (1978), the relationship between endogenous and exogenous variables can be estimated and examined at each quantile. In general, QR proves to be extremely useful whenever one is interested in focusing on particular

segments of the analyzed conditional distribution. QR has been developed and applied to cross-section data; here quantile regression is applied to cross-section time-series data to examine the shape across cross-section units, linear or non-linear relationship over time between the endogenous and exogenous variables. Recently, Marroquin and Shaik (2009) have estimated the production, restricted cost, and restricted profit functions using North Dakota agriculture sector time-series data from 1960-2004. They have applied to time-series data to examine the shape and the linear or non-linear relationship between the endogenous and exogenous variables in the estimation of the production, cost, and profit functions. Finally the difference between the traditional OLS and quantile regression results suggest a non-linear relationship between the endogenous and exogenous variables.

Since the theory related to panel QR has yet to be established, here the spatial and temporal variation is accounted with the use of between and within regression (Mundlak et al.) and extended to QR framework. This would allow the differentiation of the contribution of between time-series (TS) and cross-section (CS), and within to parameter coefficient at each quantile. Second, this methodology would allow the estimation of panel QR using traditional alternative panel estimation. As a step in this direction, the paper presents the pooled, between and within QR returns to scale estimates of a production function.

The rest of this paper is organized as follows: Second section presents the conceptual framework and data used in the empirical application. The third section focuses on the specific features of the empirical model and the results of the production, cost, and profit functions. Finally the conclusions are presented and scope for future research is proposed.

2. Conceptual framework

Past and current econometric estimates have focused on the estimation of the production function using the traditional time-series and panel procedures. Here, an extension to estimate the production function using QR is presented.

2.1. Pooled Production function

Production theory assumes that the relationship between multiple outputs and inputs is reflected by the concept of a *transformation function*. With some additional assumptions and aggregation of all outputs, the input-output relationship is often reduced to a *production function* (Fuss and McFadden, 1978). The production function represents the relation between nonallocable input vectors, $x = \left(x_1, x_1, ..., x_n\right) \in \Re_+^N$ used in the production of an output vector, $y = \left(y_1, y_1, ..., y_m\right) \in \Re_+^M$. Different functional forms can be applied in the context of agricultural production functions. This research uses the Cobb-Douglas function to represent the production function characterized as:

$$(1) \qquad y_{it} = f\left(x_{k,it} \mid \alpha\right) \;\; or \;\; y_{it} = A\sum_{k=1}^{K}\left(x_{k,it}^{\alpha_k}\right)$$

where k = 1…K is the number of inputs, cross-section *i*=1 … N and time-series *t*=1 … T.

Following Koenker and Bassett (1978), a single equation econometric model can be extended to quantile regression to examine the changes in coefficients across the distribution of endogenous model. The quantile regression provides parameter coefficients estimation for any quantile in the range of zero and one (0, 1) conditional on the exogenous variables. Following Koenker and Hallock (2001, p. 146) the QR for production function can be represented as:

$$\hat{\alpha}(\tau)=\min_{\alpha\in R^{p}}\sum_{t=1}^{T}\tau\left(y_{t}-x_{k,t}^{\prime}\alpha_{k}\right)^{2} \quad \text{for any quantile}, \tau\in(0,1)$$

(2) or

$$\hat{\alpha}(\tau)=\min_{\alpha\in R^{p}}\left[\sum_{t\in\{t:y_{t}\geq x_{k,t}^{\prime}\alpha_{k}\}}\tau\left|y_{t}-x_{k,t}^{\prime}\alpha_{k}\right|+\sum_{t\in\{t:y_{t}<x_{k,t}^{\prime}\alpha_{k}\}}(1-\tau)\left|y_{t}-x_{k,t}^{\prime}\alpha_{k}\right|\right]$$

The quantile regression as defined in equation 2 is used as the basis for the empirical model presented here:

(3) $$Q_{\tau}\left[y_{it}\mid x_{k,it}\right]=\alpha_{0,\tau}+\alpha_{k,\tau}x_{k,it}$$

where *y* is aggregate output, $Q_{\tau}\left[y\mid x_{k}\right]$ is the τth quantile of *y* conditional on covariate matrix, $\mathbf{X}_{k}$ that includes the quantities of capital, land, labor, materials, energy, and chemicals. The coefficient $\alpha_{k,\tau}$ represents the returns to covariates or inputs at the τth quantile.

2.2. Between Cross-section Production function

The Cobb-Douglas function to represent the between cross-section production function can be characterized as:

(4) $$\bar{y}_{i}=f\left(\bar{x}_{k,i}\mid\alpha\right) \quad or \quad \bar{y}_{i}=A\sum_{k=1}^{K}\left(\bar{x}_{k,i}^{\alpha_{k}}\right)$$

where $\bar{y}_{i}=\sum_{t=1}^{T}y_{it}/T$, k = 1…K is the number of inputs, and cross-section *i*=1 … N.

The quantile regression as defined in equation 4 is used as the basis for the empirical model presented here:

(5) $$Q_{\tau}\left[\bar{y}_{i}\mid\bar{x}_{k,i}\right]=\alpha_{0,\tau}+\alpha_{k,\tau}\bar{x}_{k,i}$$

2.3. Between Time-series Production function

The Cobb-Douglas function to represent the between time-series production function can be characterized as:

(6) $$\bar{y}_t = f\left(\bar{x}_{k,t} \mid \alpha\right) \;\; or \;\; \bar{y}_t = A\sum_{k=1}^{K}\left(\bar{x}_{k,t}^{\alpha_k}\right)$$

where $\bar{y}_t = \sum_{n=1}^{N} y_{it} / N$, k = 1…K is the number of inputs, and time-series t=1 … T.

The quantile regression as defined in equation 6 is used as the basis for the empirical model presented here:

(7) $$Q_\tau\left[\bar{y}_t \mid \bar{x}_{k,t}\right] = \alpha_{0,\tau} + \alpha_{k,\tau}\bar{x}_{k,t}$$

2.4. Within Cross-section and Time-series Production function

The Cobb-Douglas function to represent the within cross-section, time-series production function can be characterized as:

(8) $$\tilde{y}_t = f\left(\tilde{x}_{k,t} \mid \alpha\right) \;\; or \;\; \tilde{y}_t = A\sum_{k=1}^{K}\left(\tilde{x}_{k,t}^{\alpha_k}\right)$$

where $\tilde{y}_t = y_{it} - \bar{y}_i - \bar{y}_t + \bar{y}$ *and* $\bar{y} = \sum_{n=1}^{N}\sum_{t=1}^{T} y_{it} / NT$, k = 1…K is the number of inputs, and time-series t=1 … T.

The quantile regression as defined in equation 8 is used as the basis for the empirical model presented here:

(9) $$Q_\tau\left[\tilde{y}_t \mid \tilde{x}_{k,t}\right] = \alpha_{0,\tau} + \alpha_{k,\tau}\tilde{x}_{k,t}$$

3. Data and variables used in the analysis

The U.S. Department of Agriculture's Economic Research Service (ERS) constructs and publishes the state and aggregate production accounts for the farm sector[2]. The features of the state and national production accounts are consistent with gross output model of production and are well documented in Ball et al. (1999). Output is defined as gross production leaving the farm, as opposed to real value added. Price of land is based on hedonic regressions. Specifically the price of land in a state is regressed against land characteristics and location (state dummy). Prices of capital inputs are obtained on investment goods prices, taking into account the flow of capital services per unit of capital stock in each state (Ball et al, 2001). In the primal production function, physical input and output quantities are used in the estimation.

4. Empirical Model and Results

To measure the farm input and output change characterizing the U.S. agriculture from the time period 1960-2004, the pooled, between and within production function is estimated using QR. Second the time varying parameter coefficients estimated by the QR are also presented.

4.1 Production function

Empirical representation of the Hicks-neutral technical change of the production function as defined in equation 3 can be represented as:

$$\begin{aligned} \ln AO_QI_t = {} & \alpha_0 + \alpha_1 \ln cap_QI_t + \alpha_2 \ln land_QI_t \\ & + \alpha_3 \ln lab_QI_t + \alpha_4 \ln mat_QI_t \\ & + \alpha_5 \ln eng_QI_t + \alpha_6 \ln chem_QI_t + \alpha_T T \end{aligned} \tag{10}$$

[2] The data are available at the USDA/ERS website http://www.ers.usda.gov/data/agproductivity/.

where *AO_QI*, *Cap_QI*$_t$, *Land_QI*$_t$, *Lab_QI*$_t$, *Mat_QI*$_t$, *Eng_QI*$_t$ and *Chem_QI*$_t$ and *T* characterize aggregate output, capital, land, labor, aggregate materials, energy, chemicals, and technology, respectively. The parameter coefficient and the significance for each quantile ranging from 10 to 90 percent are presented. Because quantile regression presents snapshots at different points of a conditional distribution, they represent a parsimonious way of describing the whole distribution.

4.2 Results

The parameters obtained from the QR for the pooled, TS, CS and Within QR estimation expose statistical significance between the agricultural inputs and aggregate output for the period 1960-2004 using state-level data. Table 1 presents the average parameter coefficients of 6 input quantities and technology across nine quantiles. The rows in the table represents pooled, between cross-section, between time-series, and within cross-section time-series quantile. The results in the table are striking in several respects. The measurement of technology "year" is significant across all quantiles and in particular for pooled data and between time series (bottom block of table). One thing that stands out here is that the elasticity estimates are very close ranging from 0.9 percent for the first quantile to almost 1.3 percent for the 90th quantile, in the case of pooled data. Similar estimates are obtained for between time series production function quantile regressions.

The pooled production function estimates are significant for the first five quantiles, the fifth quantile representing the average regression. For example, an additional unit of capital increases output by about 6 percent in the first and third quantile, whereas on average (5th quantile) it only increases output by about 3 percent. On the other hand, when considering between times series production function results in table 1 show that capital significantly affects output in the second, eighth, and ninth quantile. Results indicate that a 1 percent increase in

capital increases output by more than, 8, 13, and 15 percent, in eighth and ninth quantile. Capital may have a significant impact in the higher quantiles because the farms with higher production tend to substitute more of capital for labor.

With regard to chemicals, results in table 1 show that increase in chemical inputs increase output. This is true across all quantiles and different time and space production function, in particular, pooled, between time-series, and within cross-section and time-series. The elasticity estimates are pretty consistent across all quantiles as well. For example, an additional unit of chemical increases output by 5 percent on average (5th quantile) to as high as 8 percent in the 8th and 9th quantile, for pooled data. On the other hand, such estimates are little higher for within cross-section and time-series, anywhere from 8 percent in the 9th quantile to 11 percent in 4th quantile. The impact of chemicals on aggregate output is however, significantly higher for between times-series estimates for all quantiles, with exception to first two quantiles. Elasticity estimates in table 1 indicate that an additional unit of chemical increase aggregate output by about 9 percent in the 3rd quantile and to as high as 14 percent in the 7th quantile, with an average increase in out of 12 percent (5th quantile).

Parameter estimates on energy input is similar to that of chemicals. For example, across all quantiles pooled and within cross-section and time-series table 1 shows that all energy has a significant effect on output. Results indicate that an additional unit of energy, on average (5th quantile) increase output by 10 percent. Interestingly the estimates are higher for output in the 1st quantile in the pooled, between times-series, and pooled and within cross-section and time-series types of production function estimated, ranging from 12-15 percent. Except of the 1st and 2nd quantile in the between time-series production function estimation, parameter estimates on labor input is significant across all quantiles for pooled, between time-series, and within cross-section

and time-series (table 1). A consistent and smaller estimate of labor on output is observed when the output is estimated through within cross-section and time-series model. Elasticity estimates range from 6.6 percent, on average, to about 9 percent in the 2nd quantile. However, these estimates are higher when output is estimated through pooled and between time-series model. Elasticity estimates range from 9 percent, on average, to about 16 percent in the 8th quantile.

Land and materials are other inputs used in production of agricultural output. Unfortunately, land elasticity is not significant for much of the quantiles and across pooled, and between production function estimated, with the exception of within cross-section and time-series. In some cases, for example, in between cross-section production function estimation are negative for some quantiles. This may indicate that land may have lost importance in modern agriculture, especially is the U.S. However, land has a significant effect on output when estimating output through pooled data. Specifically, elasticity estimates are significant for the 2nd to 8th quantile. Estimates indicate that a unit change in land increases output by 16.7 percent on average, to as little as 6 percent for output in the 2nd quantile. The parameter estimates are very consistent for the within cross-section and time-series. Elasticity estimates indicate that land increase out by 9 and 8 percent in the upper quantile (8th and 9th), at the lower end and about 16 percent on average, (5th quantile).

Finally, parameter in table 1 show that materials have a significant impact on output across all quantiles and various types of production function estimated. A surprising finding here is that the estimates are much bigger than any other input. Although the estimate very much consistent over the quantiles and across various types of production function estimated (between cross-section, pooled, between time-series, and within cross-section and time-series). Results in table 1 show that a unit change in materials increases output by 32, 43, 37, and 48 percent, on

average (5th quantile) when output is estimated between cross-section, pooled, between time-series, and within cross-section and time-series, respectively.

References

Antle, J.M (1984). The Structure of U.S. Agricultural Technology, 1910-78. *American Journal of Agricultural Economics*, 66 (4), 414-421.

Diewert, W.E., (1974). Applications of Duality Theory pp. 106-171 in M.D. Intriligator and D.A. Kendrick (ed.), *Frontiers of Quantitative Economics*, Vol. II, Amsterdam: North-Holland Publishing Company.

Dixon, B. L., Garcia, P., and Anderson, M. (1987). Usefulness of Pretests for Estimating Underlying Technologies Using Dual profit Functions. *International Economic Review*, 28(3), 623-633.

Evenson, R, E.; and Kislev, Y. (1975). Agricultural Research and Productivity. New Haven: Yale University Press.

Fuss M. and D. McFadden (eds.), *Production Economics: A Dual Approach to Theory and Applications.* Amsterdam: North Holland, 1978.

Hoch, I. (1958). Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function. *Econometrica*, 26 (4), 566-578.

_____(1962). Estimation of the Production Function Parameters Combining Time-Series and Cross-Section Data. *Econometrica*, 30 (1), 34-53.

Koenker, R., and Bassett, G. (1978). Regression quantiles. Econometrica, 46 (1), 33-50.

Koenker, R., and K.Hallock. (2001). Quantile regression. *Journal of Economic Perspectives*, 15(4), 143–156.

Lau, L. J. (1976). A Characterization of the Normalized Restricted Profit Function. *Journal of Economic Theory*, 12 (1), 131-63.

Lau, L. T., and Yotopoulos, P. A. (1989). The Metaproduction Function Approach to Technological Change in World Agriculture. *Journal of Development Economics*, 31 (2), 241-269.

Lopez, R. E. (1985). Structural Implications of a Class of Flexible Functional Forms for Profit Functions. *International Economic Review*, 26 (3), 593-601.

Marschak, J., and Andrews, Jr., W. H. (1944). Random simultaneous equations and the theory of production. *Econometrica*, 12 (3-4), 143-205.

McElroy, M. B. (1987). Additive General Error Models for Production, Cost, and Derived Demand or Share Systems. *The Journal of Political Economy*, 95 (4), 737-757.

McFadden, D. (1962). *Factor Substitution in the Economic Analysis of Production*, Ph.D.Dissertation, Univ. of Minnesota.

Mundlak, Y. (1963), Estimation of Production and Behavioral Functions from a Combination of cross-section and time-series data. In: C.F. Christ, Editor, *Measurement in Economics: Studies in Mathematical Economics and Econometrics (p. 138-166)*, Stanford, CA: Stanford University Press.

Mundlak, Yair, Donald F. Larson, and Rita Butzer, "Rethinking within and between Regressions: The Case of Agricultural Production Functions," *Annales d'Economie et de Statistique* 55-56 (1999), 475-501.

Nerlove, M. (1963). Returns to scale in electricity supply. In: C.F. Christ, Editor, *Measurement in Economics* Press *(p. 167–198):* Stanford, CA: Stanford University Press.

Schmidt, P. (1988). Estimation of a Fixed-effect Cobb-Douglas System using Panel Data. *Journal of Econometrics*. 37 (3), 361-380.

Weaver, R.D. **(**1983). Multiple Input, Multiple Output Production Choices and Technology in the U.S. Wheat Region. *American Journal of Agricultural Economics,* 65 (1), 45-56.

Zellner, A., Kmenta, J. and Dreze, J. (1966). Specification and Estimation of Cobb-Douglas Production Function Models. *Econometrica*, 34 (4), 784-795.

Table 1. Pooled, Between TS and CS, and Within Quantile Regression Production function results by Quantile, 1960 to 2004

		Quantile								
Model	Parameter	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
Between CS	Capital	0.350	0.283	0.178	0.154	0.092	0.153	-0.051	0.012	-0.094
POOL	Capital	**0.058**	**0.049**	**0.058**	**0.036**	**0.033**	0.031	0.030	0.004	0.024
Between TS	Capital	0.045	**0.084**	0.020	0.002	-0.006	0.004	0.016	**0.125**	**0.147**
WITHIN	Capital	0.067	0.021	0.028	0.032	0.024	0.043	0.042	0.074	0.078
Between CS	Chemicals	0.101	0.091	0.052	0.044	0.068	0.037	0.083	0.122	0.129
POOL	Chemicals	**0.062**	**0.047**	**0.050**	**0.050**	**0.049**	**0.065**	**0.073**	**0.084**	**0.083**
Between TS	Chemicals	0.036	0.038	**0.092**	**0.102**	**0.124**	**0.130**	**0.139**	**0.113**	**0.115**
WITHIN	Chemicals	**0.099**	**0.098**	**0.103**	**0.105**	**0.093**	**0.094**	**0.099**	**0.097**	**0.082**
Between CS	Energy	0.302	0.323	**0.366**	0.365	0.485	0.495	**0.756**	0.335	0.487
POOL	Energy	**0.119**	**0.085**	**0.098**	**0.118**	**0.104**	**0.141**	**0.149**	**0.169**	**0.134**
Between TS	Energy	**0.152**	0.105	0.046	0.011	-0.037	-0.017	0.082	**0.154**	**0.146**
WITHIN	Energy	**0.121**	**0.124**	**0.103**	**0.099**	**0.098**	**0.080**	**0.066**	**0.065**	**0.114**
Between CS	Labor	-0.031	-0.015	0.020	0.033	0.007	-0.050	-0.043	-0.008	-0.054
POOL	Labor	**0.100**	**0.123**	**0.105**	**0.098**	**0.092**	**0.078**	**0.059**	**0.059**	**0.049**
Between TS	Labor	0.034	0.056	**0.095**	**0.103**	**0.118**	**0.093**	**0.096**	**0.161**	**0.154**
WITHIN	Labor	**0.084**	**0.086**	**0.081**	**0.077**	**0.066**	**0.064**	**0.068**	**0.073**	**0.065**
Between CS	Land	-0.176	-0.144	-0.008	0.000	-0.073	0.072	-0.046	-0.114	-0.066
POOL	Land	0.007	**0.057**	**0.092**	**0.133**	**0.167**	**0.120**	**0.115**	**0.088**	0.050
Between TS	Land	0.004	0.048	0.001	-0.022	-0.038	-0.045	-0.096	-0.122	-0.118
WITHIN	Land	**0.112**	**0.148**	**0.148**	**0.151**	**0.158**	**0.139**	**0.131**	**0.090**	**0.075**
Between CS	Materials	0.276	**0.327**	**0.324**	**0.345**	**0.319**	**0.292**	0.253	**0.492**	**0.469**
POOL	Materials	**0.392**	**0.434**	**0.424**	**0.430**	**0.429**	**0.432**	**0.448**	**0.467**	**0.502**
Between TS	Materials	**0.286**	**0.269**	**0.303**	**0.359**	**0.373**	**0.384**	**0.388**	**0.397**	**0.439**
WITHIN	Materials	**0.491**	**0.491**	**0.476**	**0.472**	**0.480**	**0.474**	**0.476**	**0.481**	**0.453**
Between CS	RTS	0.821	0.865	0.931	0.940	0.898	0.999	0.952	0.839	0.871
POOL	RTS	0.746	0.805	0.837	0.876	0.887	0.878	0.885	0.884	0.856
Between TS	RTS	0.568	0.612	0.567	0.565	0.546	0.558	0.635	0.841	0.895
WITHIN	RTS	0.973	0.968	0.939	0.935	0.919	0.893	0.881	0.879	0.866
Between CS	Year									
POOL	Year	**0.009**	**0.010**	**0.011**	**0.011**	**0.012**	**0.012**	**0.012**	**0.013**	**0.013**
Between TS	Year	**0.011**	**0.012**	**0.011**	**0.011**	**0.011**	**0.010**	**0.009**	**0.012**	**0.012**
WITHIN	Year	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000