Chance constrained mathematical programming was used to compare the effects of two different crop yield relationships on the economic cost to irrigation farmers while maintaining an instream flow requirement. Results showed that the Stewart-Hagan programming model underestimated the economic cost and overestimated potential return flows. Water policies based on this model will lead to the implementation of socially unacceptable water allocation policies that may even apply more pressure on stream flows. Meaningful water policies could only be advanced by a thorough understanding of the economic and hydrological consequences of alternative water allocation policies through the integration of economic and hydrological models at catchment level.

1 INTRODUCTION

The past decade has seen a significant growth in environmental awareness in the community and increasing endorsement by governments of ecological sustainable development (Pigram, 1992 and Walmsley & Davies, 1991). Water is vital to the conservation of living resources and therefore the issue of how much water should be allocated for environmental management is indeed an extremely important one (Walmsley & Davies, 1991). Serious problems

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remain, however, with regard to the implementation of water allocation policies providing for environmental needs. Significant trade-offs can be gained by integrating water for environmental management and irrigation while economic and social costs are involved. Quantification of the potential trade-offs will generate important information that may help to resolve the social debate about water for environmental management (Johnson, Adams and Perry, 1991).

Failure to supply policy makers with the correct information may promote water policies with unintended consequences and may even exacerbate problems being addressed (Whittlesey & Huffaker, 1995). Grové & Oosthuizen (1997) developed a chance constrained mathematical programming (CCP) model to quantify the economic cost to irrigation farming of maintaining an instream flow requirement (IFR) under conditions of stochastic water supply at increasing probability levels. The CCP model used the Stewart-Hagan full season (SHF) crop yield relationship to optimise water use at farm level when water availability varied on a monthly basis. The SHF relationship ignored the effects of water deficits on crop yield when occurring in different crop growth stages. Final yields are, however, determined by both the timing (crop growth stages) and magnitude of water deficits (Doorenbos & Kassam, 1979). De Jager (1994) used simulation to show that the Stewart multiplicative (SM) crop yield relationship was the most accurate method of quantifying the effects of water deficits on crop yield. Using the SM equation in programming models to optimise water utilisation is complicated due to its non-linearity. De Jager (1997) attempted a random substitute method in the IRROPT model to optimise water use at farm level. The IRROPT model gave unsatisfactory results since it did not take the opportunity cost of water into account and larger areas could not be irrigated with water saved by applying deficit irrigation.

The main objective of this paper is to compare the CCP model (Grové & Oosthuizen, 1997) using the SHF crop yield relationship to optimise water use under conditions of stochastic water supply, and a newly developed model making use of the SM relationship. More specifically these two models were compared in terms of:

• the economic cost to irrigation farming of maintaining an instream flow requirement under conditions of stochastic water supply at increasing probability levels; and

• river withdrawals and potential return flows.

2. PROCEDURES
The procedures used to calculate deterministic equivalents for the chance constraints will be discussed first. This will ensure a thorough understanding of the method used to calculate the economic cost of maintaining an IFR at increasing probability levels under conditions of stochastic water supply. Thereafter the two crop yield relationships will be discussed in detail. The mathematical programming models were constructed in GAMS (Brooke, Kendrick & Meeraus, 1992) and solved with GAMS/MINOS (Murtagh & Saunders, 1987).

2.1 Deterministic equivalents taking instream flow requirements into account

The general procedure for calculating deterministic equivalents for a CCP model involves the quantification of the variability in terms of a probability distribution, in this case stream flow variability. Let \( x \) be variable stream flows and \( F(x) \) its cumulative probability function. Then for any stream flow level \( F(x) \) will generate the cumulative probability, \( p \), where \( 0 \leq p \leq 1 \). Thus the probability of realising stream flow of \( x \) or less is \( p \) and the probability of realising stream flow of \( x \) or more is \( 1-p \). Using the inverse transformation method (Rae, 1994) \( F(x) = p \) can be written as \( x = f(p) \). This equation will generate a deterministic equivalent of \( x \) that can be substituted into the right hand side of the programming model for any given level of \( p \). When the cumulative probability distribution \( F(x) \) corresponds to certain well-known theoretical probability distributions it is possible to define the function \( f(p) \) precisely. Making use of the power of GAMS the function \( f(p) \) can be built into the programming model.

Historical weather data was used in ACRU (Schulze, 1995) to simulate total stream flow variability on a monthly basis over a period of 31 years. Using BestFit \( ^\circledR \) (Palisade Corporation, 1995) Kolmogorov-Smirnov and Anderson-Darling test results indicated the Weibull distribution, the parameters of which have been obtained by means of MLE (Maximum Likelihood Estimators), to be the best statistical function for \( F(x) \).

The Weibull cumulative probability distribution is given by (Palisade Corporation, 1995:4-3):

\[
F(x) = 1 - \exp \left( -\left( \frac{x}{\beta} \right)^{\alpha} \right) \quad (1)
\]

Using inverse transformation equation (1) can be written as:
Equation (2) was programmed into GAMS to determine deterministic equivalents of total water available for allocation at a specified probability level. Water was allocated according to the principle of riparianity by multiplying the deterministic equivalent with the farmers’ proportionate share of total water. The IFR was subtracted from the farmers’ water allocation to ensure that there was enough water in the river to maintain the IFR at a probability of 1-p. The higher the probability level of maintaining the IFR the less water was available for irrigation resulting in economic cost. Whenever the IFR was greater than the farmers’ water allocation, stream flow levels was too low to maintain the IFR at the specified probability.

2.2 Optimising water use at farm level

To optimise water use at farm level one needs to know the impact of water shortages on final yields. The response of crop yield to water supply was modelled with the SM and SHF equations using crop yield response factors ($ky$) which relates relative yield decrease ($1-Ya/Ym$) to relative evapotranspiration deficit ($1-ETa/ETm$). The GAMS equations used to quantify the effects of water shortages on total wheat yield for the two yield relationships were:

a) Stewart multiplicative (SM model)

$$Ya = Qw \times Ym \times \prod_{g=1}^{4} \left( 1 - ky_g \left( 1 - \frac{\sum_{r=1}^{6} ETa_{tg}}{Qw \times \sum_{r=1}^{6} ETm_{tg}} \right) \right)$$  \hspace{1cm} (3)

b) Stewart- Hagan full season (SHF model)

$$Ya = Qw \times Ym \times \left( 1 - ky \left( 1 - \frac{\sum_{r=1}^{6} ETa_r}{Qw \times \sum_{r=1}^{6} ETm_r} \right) \right)$$  \hspace{1cm} (4)

with variables:
hectares planted to wheat; \( Qw > 0.001 \) to prevent division by zero

estimated total wheat yield

actual water consumption by the crop

parameters:

- \( Ym \) maximum potential wheat yield per hectare
- \( ky \) crop yield response factor
- \( ETm \) maximum potential water consumptively used by 1ha wheat (no water deficits)

set identifiers:

- \( g \) growth stages
- \( t \) months

The SM model (Equation (3)) calculated the relative evapotranspiration deficit \( (1-ETa/ETm) \) for each growth stage and combined the effects of water stress in different growth stages in a multiplicative way making it a non-linear model. Thus the reduction in final yield was more than proportional if the crop was deficit irrigated in more than one crop growth stage. Since the CCP model was a monthly model a weighed \( ky \) factor was used if more than one crop growth stage applied to a specific month. The \( ky \) factors for both models were taken from Doorenbos & Kassam (1979).

The SHF model (Equation (4)) ignored the timing of water deficits since the relative evapotranspiration deficit \( (1-ETa/ETm) \) was calculated over the entire growing season. The crop was, however, not allowed to be deficit irrigated below 20% of full consumption \( (ETm) \) of each month. This was done to prevent excessive water deficits during early crop growth stages that might kill the crop before it reached maturity.

As both the models did not relate total water applied to final yield as in crop water production functions, synthetic irrigation activities (Grové, 1997 and Willis, 1993) were used to account for application efficiencies.

### 3. RESULTS

Results are presented for a representative farm in Winterton with a scheduled area of 170 ha of which 50 ha were utilised in a normal year (baseline) for the production of wheat in winter. It was assumed that irrigation farmers have to
share proportionally in the cost of maintaining the IFR. The effects of the two yield models on the economic cost to irrigation farming of maintaining an IFR of 578 mm. ha, water withdrawals and return flows were calculated as deviations from the baseline situation while the differences between the models are shown in the last part of Table 1.

3.1 Economic efficiency

It can be seen from Table 1 that the economic cost to farmers increases with both models as the probability of maintaining the instream flow requirement of 578 mm. ha. increases, with the SM model deviating the most from the baseline at all the probability levels. Although wheat was only deficit irrigated in August with the SM model at a probability level of 50%, whereas deficit irrigation was applied in July, August and September at the same probability level with the SHF model, 12 ha less wheat was irrigated with the SM model resulting in a gross margin reduction of R27,250. As the probability of maintaining the IFR increases, the difference between the two models decreases. At a probability level of 80% the economic cost to farmers deviates only R4,650 more from the baseline with SM model. The differences in the results of the two models could be explained by the way the effects of water stress were handled.

At a probability level of 80% both models deficit irrigated wheat to the maximum allowed level of 20% in September leading to no changes in the areas irrigated, water abstracted and potential return flows. The yields of the SM model were, however, lower than that of the SHF model resulting in the R4,650 difference in gross margins. With the SM model the relative evapotranspiration deficit was calculated only for one of the crop growth stages. On the other hand the SHF model calculated the 20% consumptive use deficit in September over the sum of \( ET_m \) for the entire growing season. The ratio \( 1-ET_a/ET_m \) was thus much smaller for the SHF model leading to smaller decreases in final yields even though the \( ky \) factor for the SHF model was larger than the SM model.

Although there were no differences between the amount of water withdrawn by the two models at all probability levels in September and October, the magnitude of consumptive use deficits differed. With the SM model wheat
Table 1: Economic cost to irrigation farmers in the Winterton area and hydrological impact of two alternative methods of quantifying the effects of water deficits on crop yield, taking into account the instream flow requirement of 578 mm.ha at increasing levels of probability.

<table>
<thead>
<tr>
<th>Probability ( (1-p) )</th>
<th>G-Margin ( G )</th>
<th>Area ( (\text{ha}) )</th>
<th>Water withdrawn ( (\text{mm. ha}) )</th>
<th>Potential return flows ( (\text{mm. ha}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>July</td>
<td>August</td>
</tr>
<tr>
<td><strong>BASELINE</strong></td>
<td>No</td>
<td>197185</td>
<td>50</td>
<td>7639</td>
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<td><strong>SHF MODEL</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stewart-Hogan full season</td>
<td>0.50</td>
<td>-19503</td>
<td>4</td>
<td>-1233</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
<td>-66978</td>
<td>-10</td>
<td>-2629</td>
</tr>
<tr>
<td></td>
<td>0.70</td>
<td>-119218</td>
<td>-28</td>
<td>-4304</td>
</tr>
<tr>
<td></td>
<td>0.80</td>
<td>-166429</td>
<td>-42</td>
<td>-6358</td>
</tr>
<tr>
<td><strong>SM MODEL</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stewart multiplicative</td>
<td>0.50</td>
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<td>-1247</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
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<td>-22</td>
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</tr>
<tr>
<td></td>
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<tr>
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</tr>
<tr>
<td>0.80</td>
<td>-4560</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

1 Probability of maintaining the instream flow requirement.
2 Gross Margin.
3 A negative sign indicates that the SM model deviates more from the baseline than the SHF model and a positive sign the opposite.

--- Indicates months of deficit irrigation.
was always deficit irrigated in the month in which stream flow was most limiting and only to the extent that further deficit irrigation in that specific month would have caused water deficits in other months. Therefore smaller areas were irrigated with the SM model. Due to the multiplicative way in which water deficits were accounted for in the SM model water deficits in more than one growth stage would have led to more than proportional reductions in final yield making it economically inefficient to deficit irrigate wheat in more than one crop growth stage. Since the SHF model ignored the effects of water deficits in different crop growth stages it was economically efficient to apply deficit irrigation to the maximum in the month of the growing season in which stream flow was most limiting. At a probability level of 50% the crop was deficit irrigated to the maximum during August. The water saved in this manner was used to irrigate 4 ha more than the baseline, leading to water shortages and thus the need to apply deficit irrigation to some extent during July and September.

3.2 Hydrological impact

In Table 1 only the months of the growing season in which deficit irrigation took place are reported since the deviations in the other months could be explained completely by changes in irrigated areas.

August and September were the months in which water supplies were most limiting. All the available water in those months was therefore used, which led to no differences in the amounts of water withdrawn in these two months. In July the amount of water withdrawn with the SM model deviated more from the baseline than the SHF model at all probability levels, except for a 80% probability. The difference at a probability level of 50% was only 14 mm ha even though 12 ha less wheat was irrigated with the SM model. The reason being the magnitude of deficit irrigation applied with the SHF model at that probability level.

During the growing season potential return flows decreased less from the baseline at all the probability levels with the SM model even though less hectares of wheat was irrigated. The probability levels and months in which potential return flows decreased less from the baseline corresponded to the months and probability levels where the SHF model allowed deficit irrigation resulting in high water application efficiencies and thus few losses. With the SM model deficit irrigation was applied in only one month, hence low application efficiencies in the other months. There was thus a trade-off between more hectares with high efficiencies (SHF model) and less hectares with low efficiencies (SM model). In July at a probability level of 70% potential
return flows of the SM model decreased 64 mm/ha more from the baseline than the SHF model. The reason being the fact that no deficit irrigation was applied by either the two models and 2 ha less wheat was irrigated with the SM model.

4. CONCLUSIONS AND POLICY IMPLICATIONS

The results showed that the Stewart multiplicative (SM) model, when compared to the Stewart-Hagan full season (SHF) model has a significant effect on the economic cost to irrigation farming and the hydrology in terms of reduced return flows. It is concluded that water allocation policies based on the results of these two models and the actual outcome of the implemented policies would also differ substantially.

The SHF model did not take the timing and magnitude of water deficits on crop yield into account, resulting in the underestimation of the economic cost to irrigation farming and overestimation of potential return flows. Water allocation policies providing for environmental flows based on this model would have led to the implementation of policies that maintain the IFR at socially unacceptable probability levels resulting in a net social loss to the community. Stream flows may even come under more pressure due to the overestimation of the contribution of potential return flows. Water allocation policies to provide for environmental needs should be based on the SM since this model would lead to policies that maintain the IFR at more acceptable probability levels, resulting in economic gains to society.

Meaningful water policies can only be advanced by a thorough understanding and correct quantification of the economic and hydrological consequences of alternative water allocation policies. Better understanding of these interactions can only be gained through the integration of economic and hydrological models at catchment level.

ACKNOWLEDGEMENTS

1. Financial assistance provided by the Water Research Commission (WRC) is gratefully acknowledged. The views of the authors do not necessarily reflect those of the WRC.

2. The authors wish to thank the Computing Centre for Water Research (CCWR), University of Natal, for their assistance and the utilisation of computer facilities.
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