RESEARCH PAPER: 1991-8

THE DIFFERENTIAL DEMAND MODEL WITH HABIT PERSISTANCE

BY

Mark G. Brown

Jonq-Ying Lee

Senior Research Economists

FLORIDA DEPARTMENT OF CITRUS

Economic and Market Research Department

P.O. Box 110249

Gainesville, Florida 32611-2049 USA

Phone: 352-392-1874

Fax: 352-392-8634

Email: mgbrown@ufl.edu

www.floridajuice.com

The Differential Demand Model with Habit Persistence

Mark G. Brown
and
Jonq-Ying Lee*

*Mark G. Brown and Jonq-Ying Lee are Research Economists, Florida Department of Citrus. and Adjunct Associate Professor and Adjunct Professor, respectively, Department of Food and Resource Economics, University of Florida, Gainesville, Florida 32611.

The Differential Demand Model with Habit Persistence

Abstract

The differential demand system or Rotterdam model is extended to include lagged consumption through translation parameters, providing an alternative to simply adding constant terms to the model to allow for trends in consumption and changes in tastes. Application of the model to four broadly defined groups of goods indicates significant lag effects, resulting in differences in short-run and long-run income and price responses.

Key words: dynamic differential demand system, translation, lagged consumption.

The Differential Demand Model with Habit Persistence

The differential demand model or Rotterdam model, developed by Theil (1965) and Barten (1966), provides a first-order approximation of true demand. Recent analyses by Barnett, Byron and Mountain show the approximation is comparable to other popular flexible functional forms. To allow for trends in consumption and changes in tastes, a constant term is sometimes included in the Rotterdam demand specification (e.g., Theil, 1976; Barten, 1969; Deaton; Deaton and Muellbauer). The latter, of course, is a rough approximation. In other demand models, a common approach to allow for the impact of past consumption, in both single-equation and system specifications, has been to include lagged consumption in the model. In a demand system with n goods, inclusion of lagged consumption of each good results in n^2 additional responses to consider. A parsimonious approach to model the latter is through translating (Gorman; Pollak and Wales, 1980, 1981). Translation involves adding fixed quantity levels, referred to as translation parameters, to the direct utility maximization problem or, equivalently, fixed costs to the expenditure function. Sometimes, the fixed quantity levels are also referred to as subsistence quantities but, in general, are parameters that indicate preferences (in fact, the translation parameters might even be negative (Solari, Phlips, Jackson)). The demand impacts of lagged consumption can be specified through the translation parameters (over time, or across individuals or households, the translation parameters need not be fixed but may vary).

In this paper, the impact of lagged consumption is examined in the differential version of the translation model. In the next section, the differential demand model is extended to include lagged consumption through translation parameters. The extension

includes specifications for the long-run demand responses. For illustration purposes, the model is then applied to data on the demand for four broadly defined goods including food. The final section includes some concluding comments.

Model

In this section, the translation model is briefly reviewed and then approximated using the differential approach. The analysis includes development of long-run demand elasticities.

The consumer choice problem for translation can be written as

(1)
$$\text{maximize} \quad u = u\left(q_1^*, \ldots, q_n^*\right)$$
$$\text{subject to} \quad \sum p_i q_i^* = x^*$$

where subscript i indicates a particular good; $q_i^* = q_i - \gamma_i$, q_i being quantity and γ_i being the translation parameter; p_i is the price; and $x^* = x - \sum p_i \gamma_i$, x being income. The indirect utility function and expenditure or cost function for (1) are $u = \psi\left(p_1, \ldots, p_n, x^*\right)$, and $x = \sum p_i \gamma_i + c\left(p_1, \ldots, p_n, u\right)$, respectively. In (1), γ_i is sometimes referred to as a subsistence level and x^* as supernumerary income. In the cost function, fixed costs $\sum p_i \gamma_i$ are added to a general cost specification.

The demand equations for (1) can be written as

(2)
$$q_i = \gamma_i + q_i^*\left(p_1, \ldots, p_n, x^*\right).$$

To obtain the Rotterdam model for the translation specification of demand, first totally differentiate (2), i.e.,

$$d\,q_i = d\,\gamma_i - \frac{\partial q_i}{\partial x}\sum_j p_j\,d\,\gamma_j + \sum_j \frac{\partial q_i}{\partial p_j}\,d\,p_j + \frac{\partial q_i}{\partial x}\,d\,x \tag{3}$$

where $\frac{\partial q_i}{\partial p_j} = \frac{\partial q_i^*}{\partial p_j} - \frac{\partial q_i^*}{\partial x}\gamma_j$.

The Slutsky equation shows that $\frac{\partial q_i}{\partial p_j} = s_{ij} - q_j\frac{\partial q_i}{\partial x}$ where s_{ij} is the substitution effect (equivalently, $\frac{\partial q_i^*}{\partial p_j} = s_{ij} - \frac{\partial q_i^*}{\partial x}q_j^*$; given $\frac{\partial q_i^*}{\partial x} = \frac{\partial q_i}{\partial x}$ and $\frac{\partial q_i}{\partial p_j} = \frac{\partial q_i^*}{\partial p_j} - \frac{\partial q_i^*}{\partial x}\gamma_j$, $\frac{\partial q_i}{\partial p_j} = s_{ij} - \frac{\partial q_i}{\partial x}\left(q_j^* + \gamma_j\right) = s_{ij} - \frac{\partial q_i}{\partial x}q_j$). Substituting the latter expression for $\frac{\partial q_i}{\partial p_j}$ in (3), multiplying both sides of the equation by $\frac{p_i}{x}$, and noting $d\,q_i = q_i\,d\log q_i$ and $d\,p_i = p_i\,d\log p_i$, results in the Rotterdam model with translation

$$\omega_i\,d\log q_i = \frac{p_i}{x}\,d\,\gamma_i - \mu_i\sum_j \frac{p_j}{x}\,d\,\gamma_j + \mu_i\,d\log Q + \sum \pi_{ij}\,d\log p_j, \tag{4}$$

where $\omega_i = \frac{p_i q_i}{x}$, the budget share for the good; $\mu_i = p_i \frac{\partial q_i}{\partial x}$, the marginal propensity to consume out of income (MPC); $d \log Q = \sum \omega_j d \log q_j = d \log x - \sum \omega_j d \log p_j$; the Divisia volume index in differential form; and $\pi_{ij} = \frac{p_i p_j}{x} s_{ij}$, the Slutsky coefficient (the Divisia volume index relationship can be straightforwardly obtained by differentiation of the budget constraint). The difference between the usual Rotterdam model and specification (4) is the first two terms on the right side of (4) involving changes in the translation parameters. The first term is a direct effect due to a change in the translation parameter for the good in question, while the second term is an indirect income effect due to a change in supernumerary income caused by the overall change in the translation parameters. Changes in the translation parameters alone leave the budget unchanged with the direct and indirect effects resulting in a re-allocation of income.

The Slutsky coefficient can further be written as (Barten, 1964)

(5)
$$\pi_{ij} = v_{ij} - \phi \mu_i \mu_j$$

$$v_{ij} = \frac{p_i p_j}{x} \lambda u^{ij}$$

$$\phi = \left(\frac{\partial \lambda}{\partial x} \frac{x}{\lambda} \right)^{-1}$$

where $\lambda = \frac{\partial u}{\partial x}$, the marginal utility of income, and u^{ij} is the ij^{th} element of the matrix

$\left[\frac{\partial^2 u}{\partial q_i \, \partial q_j}\right]^{-1}$, the inverse of the Hessian matrix for the utility maximization problem. The

term ϕ is referred to as the income flexibility, while v_{ij} and $-\phi \, \mu_i \, \mu_j$ are the Slutsky coefficient terms corresponding to the specific and general substitution effects, $\lambda \, u^{ij}$ and

$-\frac{\lambda}{\frac{\partial \lambda}{\partial x}} \frac{\partial q_i}{\partial x} \frac{\partial q_j}{\partial x}$, respectively.

In analyzing food demand and similarly broadly defined goods, the assumption of separability is frequently made. In the present study, the assumption of preference independence or strong separability is made with the analysis focusing on four broadly defined goods--services, food, other nondurables (hereafter referred to simply as nondurables), and durables. In this case, the Hessian matrix and its inverse are diagonal with $u^{ij} = 0$ for $i \neq j$, and the model described by (4) and (5) simplifies to

$$\omega_i \, d \log q_i = \frac{p_i}{x} \, d \, \gamma_i - \mu_i \sum_j \frac{p_j}{x} \, d \, \gamma_j + \mu_i \, d \log Q + \phi \, \mu_i \left(d \log p_i - \sum_j \mu_j \, d \log p_j \right), \tag{6}$$

where $\pi_{ij} = \phi\ \mu_i\ \Delta_{ij} - \phi\ \mu_i\ \mu_j$, $\Delta_{ij} = 1$ if $i = j$, else $\Delta_{ij} = 0$. (To obtain (6), note $\sum_j \pi_{ij} =$

$\sum_j (v_{ij} - \phi\ \mu_i\ \mu_j) = 0$ or $\sum_j v_{ij} = \phi\ \mu_i$, based on homogeneity and adding-up; hence, for

preference independence $v_{ii} = \phi\ \mu_i$.)

The effects of past consumption can be introduced in the model by letting the translation parameter γ_i depend on lagged consumption. In this study, the change in the translation parameter is assumed to be roughly proportional to the change in lagged consumption, i.e.,

$$\frac{p_{i,t}}{x_t}\ d\ \gamma_{i,t} = a_i\ \omega_{i,t-1}\ d \log q_{i,t-1} \tag{7}$$

where subscript t has been added to indicate time, and a_i is a constant.

Pollak and Wales (1969), as well as others (e.g., Phlips, and Johnson, Hassan and Green), have similarly modeled the effects of lagged consumption through the translation parameter in the linear expenditure system (LES). As the underlying preferences for the LES are strongly separable, the differential model indicated by (6) and (7) and the LES with lagged consumption describe the same preferences. The translation approach, however, is more general than implied by the latter specifications; specifically, the general translation model does not restrict the price response. In differential form, the general model is described by specifications (4) and (7). The choice of the separable model in this study is based on the nature of the data as previously indicated.

The model defined by (6) and (7), or in general by (4) and (7), can be written in matrix notation as

(8) $$Y_t = L\, Y_{t-1} + U\left(X_t - W_t'\, P_t\right) + \Pi\, P_t$$

where $Y_t = [\omega_{it}\ d \log q_{it}]$,

$U = [\mu_i]$,

$L = \hat{A} - U\, A'$ with $A = [a_i]$ and $\hat{A} = \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix}$, the diagonal of A,

$X_t = d \log x_t$,

$W_t = [\omega_{it}]$,

$P_t = [d \log p_{it}]$,

$\Pi = [\Pi_{ij}]$.

The short-run demand responses are indicated by U, Π and A. The long-run demand responses are determined by successive substitution, following a procedure suggested by Theil (1971) for determining total impacts. For convenience, set Y_{t-1} and P_t to zero in (8) so that $Y_t = U\, X_t$. In all subsequent periods ($t + 1, \ldots$), set P_t and X_t to zero, so that $Y_{t+1} = L\, Y_t = L\, U\, X_t$, $Y_{t+2} = L\, Y_{t+1} = L^2\, Y_t = L^2\, U\, X_t \ldots$. The total impact is then

(9) $$\sum_{k=t}^{\infty} Y_k = S\, U\, X_t$$

$$S = I + L + L^2 + L^3 + \ldots ,$$

where I is the nxn identity matrix. Provided all the latent roots of L are less than one in absolute value, S converges to $(I - L)^{-1}$. The latter can be verified by noting $S - L\,S = I$ or $S(I - L) = I$, so that $S = (I - L)^{-1}$.

Similarly, for convenience set Y_{t+1} and X_t to zero in (8), so that $Y_t = (\Pi - U\,W')P_t$. Again setting P_t and X_t to zero in subsequent periods results in

$$\sum_{k=t}^{\infty} Y_k = (I - L)^{-1}(\Pi - U\,W')P_t. \tag{10}$$

Expressions (9) and (10) indicate the long-run income and price responses are $(I - L)^{-1}\,U$ and $(I - L)^{-1}\,(\Pi - U\,W')$, respectively.

The short-run elasticities for (8) are

the income elasticity: $e_i = \dfrac{\mu_i}{\omega_i}$,

the compensated price elasticities: $e_{ij}^{*} = \dfrac{\Pi_{ij}}{\omega_i}$,

and the uncompensated price elasticities: $e_{ij} = e_{ij}^{*} - \omega_j\,e_i$.

As the latter indicates, estimation of price and income elasticities in the Rotterdam model involves division by the budget shares. Long-run elasticities in this study are similarly estimated as $(I - L)^{-1}$ times the short-run elasticities. In the long run, the budget shares may change for a discrete income or price change; the long-run elasticity estimates here treat the changes in the budget share as negligible.

The basic restrictions of demand require--(1) adding-up: $\sum \mu_i = 1$ and $\sum_i \Pi_{ij} = 0$; (2) homogeneity: $\sum_j \Pi_{ij} = 0$; and (3) symmetry: $\Pi_{ij} = \Pi_{ji}$. The translation specification does not impose additional restrictions other than $q_i \geq \gamma_i$.

For estimation, ω_{it}, $d \log q_{it}$ and $d \log p_{it}$ can be approximated by $\frac{\omega_{i,t} + \omega_{i,t-1}}{2}$, $\log \frac{p_{i,t}}{p_{i,t-1}}$, and $\log \frac{q_{i,t}}{q_{i,t-1}}$, respectively.

Application

For illustration purposes, the model developed in the previous section was applied to U.S. Department of Commerce data on personal consumption expenditures for services, food, nondurables and durables.[1] The data are annual and the sample runs from 1929 through 1989. The expenditure data are measured in both actual and real (1982 = 100)

---

[1]The U.S. Department of Commerce product categories include the following goods.

(1) Services: housing, housing operation, transportation, medical care, and other.

(2) Food: food purchased for off-premise consumption, purchased meals and beverages, food furnished employees, and food produced and consumed on farms.

(3) Nondurables: clothing and shoes, gasoline and oil, fuel oil and coal, and other.

(4) Durables: motor vehicles and parts, furniture and household equipment, and other.

For a more detailed description of goods included in product categories, see pages 106-112 in "The National Income and Product Accounts of the United States, 1929-82," U.S. Department of Commerce.

dollars. Implicit prices were obtained by dividing actual expenditure by real expenditure for each of the four expenditure groups. Quantities were measured by real expenditures. U.S. Department of Commerce data on the U.S. population were used to put demand on a per capita basis. Treatment of data in this study follows the approach taken by Johnson, Hassan and Green in analyzing similar data for Canada.

As the data add-up by construction--income in the model is total consumer expenditure on the four expenditure categories--the error covariance matrix is singular and the equation for durables was excluded (Barten, 1969). The errors across equations were assumed to be contemporaneously correlated, and the full information maximum likelihood procedure was used to estimate the model with homogeneity and symmetry imposed.

Maximum likelihood estimates for model (8), with translation parameters dependent on changes in lagged consumption, are shown in Table 1. The R^2's for services, food, nondurables and durables were .79, .82, .55 and .71, respectively, indicating reasonably good fits for specifications in first differences (note that as the four demand equations are estimated jointly as a system, the R^2's have not been maximized (Barten and Bettendorf). Autocorrelation did not seem to be a major problem--estimation of the model assuming first-order autocorrelation yielded similar results.[2]

---

[2]The Durbin-Watson (DW) statistics for services, food, nondurables and durables were 2.47, 2.42, 1.58 and 1.66, respectively. For demand systems obeying the adding-up property, the DW statistic is not entirely appropriate as a measure of autocorrelation (Bewley)--for instance, if each equation is subject to first-order autocorrelation, with no assumed autocorrelation across equations, the autocorrelation parameter ρ should be the same in each equation (Berndt and Savin), and the DW test should reach the same conclusion across equations. In this case, the DW statistic only provides a guideline. Estimation of model (8) assuming first-order autocorrelation (each equation has the same ρ) yielded an estimate of -.26 for ρ with an asymptotic t statistic of -1.68. The estimates for the other parameters were roughly the same as when autocorrelation was not assumed ($\rho = 0$), except for the estimates for the translation parameters which were slightly higher for the autocorrelation model.

All coefficient estimates in Table 1 are about twice or greater in size than their corresponding standard error estimates, except the coefficient for the lagged durable consumption variable. As the lagged consumption variables may be capturing both inventory and habit persistence effects, the result for durables suggests inventory and habit effects balance out (Houthakker and Taylor; Sexauer; Tilley). The lagged results for services, food, and nondurables suggest dominance of habit effects; the results are interpreted as indicating dominance of habit effects over inventory effects when the change in the lagged variable positively affects the subsistence or translation term, and vice versa. Habit persistence effects appear to be strongest for services, with the coefficient estimate for the lag nearly three times the size of the coefficient estimate for the lag for food and about twice the size of the coefficient estimate for the lag for nondurables. The estimates for the MPC's for services, food and nondurables were roughly the same at about .2, while the MPC estimate for durables was at .37. The estimate of the income flexibility was negative at -.19 (an increase in income decreases the marginal utility of money).

Uncompensated short-run and long-run income and price elasticity estimates for the model are shown in Table 2. The elasticities are estimated at sample mean budget shares. Long-run income and price responses previously discussed ($(I - L)^{-1} U$ and $(I - L)^{-1} (\Pi - U W')$) can be obtained by multiplying the long-run elasticity estimates in Table 2 by the mean budget shares noted in the table. Of the 40 short-run and long-run elasticity estimates, all but six were more than twice their corresponding standard error estimates. The six elasticity estimates that were not significant were the cross-price elasticities with respect to the durable price for the equations for services, food and nondurables. The latter result stems from the finding that durables have a relatively high MPC but low average

propensity to consume (APC) or budget share with the compensated price effect and income effect balancing out ($e_{ij} = (-\phi\ \mu_i\ \mu_j\ -\ \omega_j\ \mu_i)\ /\omega_i$). The MPC for durables is larger than its APC, and hence its short-run income elasticity exceeds unity, at a value of 3.0; for food, the MPC and APC are approximately the same, resulting in an income elasticity of 1.0; while for nondurables and services, the MPC's are less than their corresponding APC's, resulting in income elasticities of .8 and .5, respectively.

All short-run own-price elasticity estimates are negative and less than one in absolute value, indicating inelastic demands. Durable goods have the highest own-price elasticity estimate at -.7. The own-price elasticity estimate for food was at -.3; the estimates for nondurables and services were slightly higher in absolute value. The short-run cross-price elasticity estimates were all negative, indicating gross complementary relationships, and less than one in absolute value, except for the elasticity for durables with respect to the price of services, which was at -1.1.

In the long run, the income and price elasticity estimates for services increase in absolute value in comparison to the short-run estimates, while the estimates for the other categories decrease, except those for own-price changes. The result is due to the relatively strong lagged effect for services. Over time, the translation results favor services, reducing the income and cross-price responses for the other goods. For services, the long-run income and own-price elasticity estimates increase to .9 and -.5, respectively. The long-run cross-price elasticity estimates for services, as well as the other goods, are not greatly different than the corresponding short-run estimates. The long-run income elasticity estimates for food and nondurables are slightly less than their corresponding short-run estimates; on the other hand, for durables the long-run income elasticity decreases to 2.2. The long-run own-

price elasticities for food, nondurables and durables are slightly larger than the corresponding short-run estimates.

Overall, the model appears to perform reasonably well. The results for the translation terms indicate significant consumption trends for services, food and nondurables. The consumption trend for services was particularly strong and dominated the model. Accounting for the lagged effects, the price and income responses (in absolute value) increase in the long run for services; while, for the other goods, the long-run effects decrease or are roughly unchanged.

Concluding Comments

The differential approximation of the translation model of demand with translation terms dependent on lagged consumption offers an alternative to simply adding time trend constants to the Rotterdam model. Analysis of four broadly defined goods indicates significant lagged effects which, along with the income effects and the initial short-run income and price responses, determine the long-run income and price responses. The differential model provides an approximation of demand comparable to other flexible functional forms. Translation and other extensions of the differential model that relax the assumption of constancy of the model coefficients offer additional flexibility. In a recent study by Theil et al., the MPC was specified as a varying parameter, equal to the value of the APC at each point in the sample plus a constant. The latter study also discusses other extensions allowing the basic parameters of the Rotterdam model to be functions of income and prices. As both the present study and the Theil et al. study suggest, the Rotterdam model might be made even more realistic by choosing appropriate parameter specifications.

Table 1. Maximum likelihood estimates for the Rotterdam model with translation parameters dependent on changes in lagged consumption.[a]

Product Group	Parameter			
	MPC μ_i		Translation Lag a_i	
Services	0.222	(0.026)[b]	0.593	(0.086)
Food	0.214	(0.015)	0.218	(0.115)
Nondurables	0.192	(0.026)	0.283	(0.123)
Durables	0.372	(0.041)	0.121	(0.128)
	Income Flexibility ϕ			
	-0.189	(0.094)		

[a] Model defined by equation (8).

[b] Asymptotic standard errors in parentheses.

Table 2. Uncompensated short-run and long-run elasticity estimates for the Rotterdam model with translation parameters dependent on changes in lagged consumption.[a]

Product Group	Type of Response	Elasticity[b]				
		Income	Price			
			Services	Food	Nondurables	Durables
Services	SR[c]	0.536 (0.063)[e]	-0.301 (0.041)	-0.093 (0.018)	-0.114 (0.019)	-0.029 (0.023)
	LR[d]	0.873 (0.124)	-0.520 (0.082)	-0.141 (0.036)	-0.173 (0.039)	-0.039 (0.043)
Food	SR	1.007 (0.069)	-0.374 (0.036)	-0.364 (0.073)	-0.214 (0.025)	-0.054 (0.041)
	LR	0.854 (0.095)	-0.265 (0.059)	-0.381 (0.099)	-0.170 (0.031)	-0.038 (0.039)
Nondurables	SR	0.769 (0.106)	-0.285 (0.046)	-0.133 (0.027)	-0.309 (0.061)	-0.041 (0.033)
	LR	0.710 (0.142)	-0.220 (0.069)	-0.114 (0.033)	-0.344 (0.087)	-0.032 (0.035)
Durables	SR	3.003 (0.330)	-1.114 (0.115)	-0.518 (0.060)	-0.639 (0.073)	-0.729 (0.212)
	LR	2.245 (0.439)	-0.696 (0.161)	-0.362 (0.069)	-0.445 (0.091)	-0.742 (0.275)

[a] Model defined by equation (8).

[b] Evaluated at budget share sample means: .413 for Services, .213 for Food, .249 for Nondurables, and .125 for Durables.

[c] Short-run.

[d] Long-run.

[e] Asymptotic standard errors in parentheses.

References

Barnett, W. A. "On the Flexibility of the Rotterdam Model: A First Empirical Look." *Eur. Econ. Rev.* 24(1984):285-89.

Barten, A. P. "Consumer Demand Functions Under Conditions of Almost Additive Preferences." *Econometrica* 32(1964):1-38.

Barten, A. P. "Theorie en empire van een volledig stelsel van vraagvergelijkingen." Doctoral Dissertation, University of Rotterdam. 1966.

Barten, A. P. "Maximum Likelihood Estimation of a Complete System of Demand Equations." *Eur. Econ. Rev.* 1(1969):7-73.

Barten, A. P. and L. G. Bettendorf. "Price Formation of Fish." *Eur. Econ. Rev.* 33(1989): 1509-25.

Berndt, E. R. and N. E. Savin. "Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances." *Econometrica* 43(1975):937-56.

Bewley, R. *Allocation Models: Specification, Estimation and Applications*. Cambridge, MA: Ballinger Publishing Co. 1986.

Byron, R. P. "On the Flexibility of the Rotterdam Model." *Eur. Econ. Rev.* 24(1984):273-83.

Deaton, A. S. "The Analysis of Consumer Demand in the United Kingdom, 1900-1970." *Econometrica* 42(1974):341-67.

Deaton, A. and J. Muellbauer. *Economics and Consumer Behavior*. Cambridge, MA: Cambridge University Press. 1980.

Gorman, W. M. "Tricks with Utility Functions," in M. J. Artis and A. R. Nobay (eds.), *Essays in Economic Analysis*. Cambridge: Cambridge University Press. 1970.

Houthakker, H. and L. D. Taylor. *Consumer Demand in the United States, 1929-1970*. Cambridge, MA: Harvard University Press. 1970.

Jackson, L. R. "Hierarchic Demand and the Engel Curve." *Review of Economics and Statistics* LXVI(1984):8-15.

Johnson, S. R., Z. A. Hassan, and R. D. Green. *Demand System Estimation: Methods and Applications*. Ames: The Iowa State University Press. 1984.

Mountain, D. C. "The Rotterdam Model: An Approximation in Variable Space." *Econometrica* 56(1988):477-84.

Pollak, R. A. and T. J. Wales. "Estimation of the Linear Expenditure System." *Econometrica* 37(1969):611-28.

Pollak, R. A. and T. J. Wales. "Comparison of the Quadratic Expenditure System and Translog Demand Systems with Alternative Specifications of Demographic Effects." *Econometrica* 48(1980):595-611.

Pollak, R. A. and T. J. Wales. "Demographic Variables in Demand Analysis." *Econometrica* 49(1981):1533-51.

Sexauer, B. "The Role of Habits and Stocks in Consumer Expenditure." *Quarterly J. Econ.* 91(1979):127-42.

Solari, L. *Thérie des choix et fonctions de consummation semi-agrégées*. Geneva: Librarie Droz. 1971.

Theil, H. "The Information Approach to Demand Analysis." *Econometrica* 33(1965):67-87.

Theil, H. *Principles of Econometrics*. New York: John Wiley & Sons, Inc. 1971.

Theil, H. *Theory and Measurement of Consumer Demand*, Vol. I, Vol. II. Amsterdam: North Holland. 1975, 1976.

Theil, H., Ching-Fan Chung and J. L. Seale, Jr. *International Evidence on Consumption Patterns*. Greenwich, CT: JAI Press, Inc. 1989.

Tilley, D. S. "Importance of Understanding Consumption Dynamics in Market Recovery Periods." *So. J. Agr. Econ.* 2(1979):41-6.

United States Department of Commerce/Bureau of Economic Analysis. "The National Income and Product Accounts of the United States, 1929-82." "Survey of Current Business," September 1990; "Business Statistics, 1961-88: A Supplement to the Survey of Current Business." Washington, D.C.