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QUASI SEPARABILITY: A DIFFERENTIAL DEMAND SYSTEM APPROACH

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Quasi Separability: A Differential Demand System Approach

Mark G. Brown and Jonq-Ying Lee*

Abstract

A differential demand system for quasi separability is developed, following the approach used by Theil et al. in developing the Working-PI time series model. The Slutsky coefficients vary with commodity budget shares and intergroup substitution depends on group compensated price effects.

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Quasi Separability: A Differential Demand System Approach

The Rotterdam model, developed by Barten and Theil, has been a popular model, frequently used in empirical demand studies. The differential approach of the Rotterdam model provides a first-order approximation to any true demand system, and the model appears to perform reasonably well in general (Barnett and Byron). To estimate the Rotterdam model, the marginal propensities to spend and Slutsky coefficients for the goods in the system are usually assumed to be constant. Of course, the latter coefficients vary depending on income and price levels and the underlying preference structure. For some types of preferences, (e.g., preferences underlying the linear expenditure system), the assumption of constant coefficients does not seem to introduce a large error (Byron). However, for other types of preferences, the constant coefficient assumption may be a concern.

Theil et al. have recently proposed a model that relaxes the constant coefficient assumption for income. Using the Working model to describe the income effect, Theil et al. extend the Rotterdam model by making the marginal propensity to spend on a good equal to the budget share for the good plus a constant term. In the resulting model, called the Working-PI time series model, the budget shares are treated as exogenous, and the constant terms of the marginal propensities along with the Slutsky coefficients are estimated.

In this paper, attention is focused on relaxing the assumption on constancy of the Slutsky coefficients. A differential demand model is developed under the assumption of quasi separability. For illustrative purposes, the model is used to estimate a disaggregated system of fruit juice demand equations. The separability assumption allows for reductions in the parameter space and provides a simple description of two- or multi-stage budgeting. Quasi separability also is convenient for introducing certain types of demographic effects. As an example, the effects of generic advertising are discussed.

The paper is organized as follows. In the next section, the differential model is developed under the assumption of quasi separability. An interpretation of the model is then provided, after which the effects of demographic variables and, in particular, generic advertising are considered. In the following section, an application of the model to fruit juice demand data is discussed. In the final section, some concluding comments are offered.

Quasi Separable Differential Model

Preferences are quasi separable if the cost function can be written as (Gorman)

$$x = c\{u,\ c_1(p_1,\ u),\ \ldots,\ c_r(p_r,\ u),\ \ldots\}, \tag{1}$$

where x is total expenditure or income; $c(\cdot)$ is the cost function; u is utility; $c_r(\cdot)$ is a function for group r (c_r is subsequently interpreted as a group price index); and p_r is a vector of prices for goods in the group. Both $c(\cdot)$ and $c_r(\cdot)$ are assumed to obey the general properties of a cost function (Deaton and Muellbauer).

Based on Shepard's lemma, the Hicksian demand equation for a good can be written as

$$q_{ri} = \frac{\partial c}{\partial c_r}\,\frac{\partial c_r}{\partial p_{ri}}, \tag{2}$$

where subscript ri indicates good i in group r. Given $c_r(\cdot)$ is homogeneous of degree one in prices p_r, group expenditure x_r can be written as

$$x_r = \frac{\partial c}{\partial c_r}\,c_r. \tag{3}$$

To obtain the Slutsky coefficients for the differential model, take the log of (2) and find the compensated price elasticities $e^*_{ri,sj} = \dfrac{\partial \log q_{ri}}{\partial \log p_{sj}}$, i.e.,

$$e^*_{ri,sj} = \left[\frac{\partial \log\left[\dfrac{\partial c}{\partial c_r}\right]}{\partial \log c_s}\right]\frac{\omega_{sj}}{\omega_s} + \left[\frac{\partial \log\left[\dfrac{\partial c_r}{\partial p_{ri}}\right]}{\partial \log p_{sj}}\right]\Delta_{r,s} \tag{4}$$

where $\dfrac{\omega_{sj}}{\omega_s} = \dfrac{\partial \log c_s}{\partial \log p_{sj}} = \dfrac{p_{sj}q_{sj}}{x_s}$, the intragroup budget share; $\omega_{sj} = \left[\dfrac{\partial \log c}{\partial \log c_s}\right]\left[\dfrac{\partial \log c_s}{\partial \log p_{sj}}\right] = \dfrac{p_{sj}\,q_{sj}}{x}$, the overall budget share; $\omega_s = \dfrac{\partial \log c}{\partial \log c_s} = \dfrac{x_s}{x}$, the group budget share; and $\Delta_{rs} = 1$ if $r = s$, else $\Delta_{rs} = 0$.

The Slutsky coefficients $\Pi_{ri,sj} = \omega_{ri}\,e^*_{ri,sj}$'s are then[1]

$$\Pi_{ri,sj} = \phi_{rs} \frac{\omega_{sj}}{\omega_s} \omega_{ri} + \gamma_{ri,sj} \omega_{ri} \Delta_{rs} \tag{5}$$

where $\phi_{rs} = \left[\frac{\partial \log \left[\frac{\partial c}{\partial c_r}\right]}{\partial \log c_s}\right]$ and $\gamma_{ri,sj} = \frac{\partial \log \left[\frac{\partial c_r}{\partial p_{ri}}\right]}{\partial \log p_{sj}}$. Substituting (5) into the absolute price version of the Rotterdam model (Theil) results in the quasi separable differential model

$$\omega_{ri}\, d \log q_{ri} = \beta_{ri}\, d \log Q + \sum_{sj} \left[\phi_{rs} \frac{\omega_{sj}}{\omega_s} \omega_{ri} + \gamma_{ri,sj} \omega_{ri} \Delta_{rs}\right] d \log p_{sj}, \tag{6}$$

where $\beta_{ri} = p_{ri} \frac{\partial q_{ri}}{\partial x}$ and $d \log Q = \sum_{ri} \omega_{ri}\, d \log q_{ri}$.

To estimate (6), the β's, ϕ's and γ's might be treated as constants.[2] Treating the budget shares as exogenous is problematic, since they appear on both sides of the demand specifications. In the Working P-I time series model, the exogenous assumption is maintained (Theil et al.). Alternatively, mean budget shares might be used in specifying the Slutsky coefficients as an approximation. Another possibility would be to use lagged budget share values in the Slutsky coefficients as is sometimes done in specifying Stone's price index when estimating the almost ideal demand system (e.g., Eales and Unnevehr).

Symmetry in model (6) requires $\phi_{rs}\, \omega_r = \phi_{sr}\, \omega_s$ for price changes between subgroups and $\gamma_{ri,rj}\, \omega_{ri} = \gamma_{rj,ri}\, \omega_{rj}$ for price changes within a subgroup; homogeneity requires $\sum_s \phi_{rs} = 0$ and $\sum_j \gamma_{ri,rj} = 0$ (since $c(\cdot)$ and $c_r(\cdot)$ are both homogeneous of degree one, $\frac{\partial c}{\partial c_r}$ and $\frac{\partial c_r}{\partial p_{ri}}$ are homogeneous of degree zero); and adding-up requires $\sum_{ri} \beta_{ri} = 1$ and $\sum_{ri} \Pi_{ri,sj} = 0$, which, of course, is satisfied automatically. A test for the appropriateness of the separability assumption is suggested by Pudney.

Model Interpretation

Quasi separability provides a simple description of two-stage budgeting (Deaton and Muellbauer). In the first stage, income is allocated to each group according to the group budget share $\omega_r = \frac{\partial \log c}{\partial \log c_r}$, while in the

second stage, income allocated to a group is allocated to specific goods in the group according to the intragroup budget shares $\frac{\partial \log c_r}{\partial \log p_{ri}}$'s. The definition of ω_r suggests the group function c_r can be viewed as a price index for the group. Similarly, the term $\frac{\partial c}{\partial c_r}$ can be viewed as a group quantity index based on the distance function (Deaton and Muellbauer). The product of the group price and quantity indices is group expenditure as shown in equation (3), and the budget constraint for the first stage allocation can be written as $x = \sum_r \frac{\partial c}{\partial c_r} c_r$, which is also an expression of Euler's theorem regarding homogeneity of c with respect to the group price indices.

Consider the term ϕ_{rs} as defined in (5). The latter interpretations suggest that ϕ_{rs} can be viewed as the elasticity of the quantity index for group r with respect to the price index for group s, i.e., a compensated price elasticity for the first stage allocation problem.

Likewise, γ in equation (5) provides information on intragroup substitution. Using (2),

$$\gamma_{ri,rj} = \frac{\partial \log \left[\frac{q_{ri}}{\left[\frac{\partial c}{\partial c_r} \right]} \right]}{\partial \log p_{rj}}$$

, which indicates the γ's are elasticities measuring the sensitivity of the quantity to group-quantity-index ratios with respect to intragroup prices.

Equation (5) shows a change in the price of a good in group s, regardless of the good, affects the demand of a good in group r through the same channel ϕ_{rs}, with the overall cross-group effect depending on the budget shares of the goods involved. In addition, if the goods are from the same group, there is an intragroup effect through $\gamma_{ri,sj}$.

In general, one might find the interpretations for the ϕ's and γ's in this section useful for determining whether the parameter constancy assumptions in equation (6) are reasonable in an empirical application.

Demographic Variables and Generic Advertising

Quasi separability can also be a source of restrictions for demographic effects. Consider the quasi separable cost function with demographic variables a

$$x = c\left(u,\ c_1(p_1,\ a,\ u)\ \ldots,\ c_r(p_r,\ a,\ u),\ \ldots\right) . \tag{7}$$

As specified, equation (7) is not very useful for empirical work as the number of different demographic effects to be estimated would probably be too large. However, the separability restrictions on prices suggest that demographic variables might be similarly restricted; for particular studies, restricting the demographic variables in each group function might be reasonable. For example, consider the impact of generic advertising. (Generic advertising has become an important promotional tool for commodity groups such as dairy products, beef, pork, citrus, etc.; as such, the cost function in this section might be defined for total food expenditure and the group functions for specific commodity groups.) Assume advertising for a group only affects the group's price index, i.e.,

$$x = c\left(u,\ c_1(p_1,\ a_1,\ u),\ \ldots,\ c_r(p_r,\ a_r,\ u),\ \ldots\right), \tag{8}$$

where a_r is the level of generic advertising for group r. (The impact of generic advertising may work in different ways; e.g., the impacts might be described by either scaling or translating (Lee et al.). Here the precise way in which advertising affects the group price indices is not considered.)

Assuming (8) is the underlying cost function, the log differentiation of demand equation (2) with respect to a_s or the elasticity of demand with respect to advertising $e_{ri,as}$ is

$$e_{ri,as} = \frac{\partial \log \left[\dfrac{\partial c}{\partial c_r}\right]}{\partial \log c_s} \cdot \frac{\partial \log c_s}{\partial \log a_s} + \frac{\partial \log \left[\dfrac{\partial c_r}{\partial p_{ri}}\right]}{\partial \log a_s} \Delta_{rs}. \tag{9}$$

Likewise, the additional term to be added to model (6) is

$$\sum_s \omega_{ri}\, e_{ri,as}\, d \log a_s = \sum_s \left(\phi_{rs}\, \Psi_s\, \omega_{ri} + \theta_{ri,s}\, \omega_{ri}\, \Delta_{rs}\right) d \log a_s, \tag{10}$$

where $\Psi_s = \dfrac{\partial \log c_s}{\partial \log a_s}$ and $\theta_{ri,s} = \dfrac{\partial \log \left[\dfrac{\partial c_r}{\partial p_{ri}}\right]}{\partial \log a_s}$; i.e., Ψ_s is the elasticity of the price index for group s with respect to advertising for the group, while $\theta_{ri,r}$ is the elasticity for the quantity to group-quantity-index ratio with respect to advertising. For estimation purposes, the Ψ's and θ's might be approximated as constants.

Whether treatment of demographic effects as suggested above is reasonable would depend on a case-by-case basis. Quasi separability, however, provides a simple means to introduce demographic variables; how the variables affect demand as well as the parameter constancy assumptions can be clearly understood.

Application

Model (6) was used to estimate a system of demand equations for juice products. The products analyzed can be viewed as comprising a subgroup within a larger system. Seven juice categories--frozen concentrated orange juice (FCOJ), chilled read-to-serve orange juice (COJ), canned single-strength orange juice (CSSOJ), frozen concentrated grapefruit juice (FCGJ), chilled ready-to-serve grapefruit juice (CGJ), canned single-strength grapefruit juice (CSSGJ), and all remaining 100% juice products (RJ)--were considered. The orange-juice (OJ) products (FCOJ, COJ and CSSOJ) were hypothesized to belong to one group and the grapefruit-juice (GJ) products (FCGJ, CGJ and CSSGJ) to another.

Scanner data for grocery stores with annual sales of 4 million dollars or greater, provided by Nielsen Marketing Research, were used to estimate the model. The data are comprised of 132 weekly observations for week ending November 14, 1987, to week ending May 19, 1990. The scanner data include single-strength-equivalent dollar and gallon sales for the different juices. Prices were calculated by dividing dollar sales by gallon sales. Data on the U.S. population from the Bureau of Census, U.S. Department of Commerce, were used to put demand on a per capita basis.

A test for quasi separability was first made following an approach suggested by Pudney. The Rotterdam model with homogeneity and symmetry imposed was chosen as the unrestricted model. Present interest focuses on quasi separability, and tests for other restrictions such as homogeneity and symmetry are not considered here.

The Rotterdam model for juices can be written as

$$\omega_i \, d \log q_i = \beta_i \, d \log Q + \sum_{j=1}^{6} \Pi_{ij} \left(d \log p_j - d \log p_7 \right), \qquad i = 1, \ldots, 6, \tag{11}$$

where $i = 1, \ldots, 7$ for FCOJ, COJ, CSSOJ, FCGJ, CGJ, CSSGJ and RJ, respectively. Homogeneity has been imposed by setting $\Pi_{i7} = -\sum_{j=1}^{6} \Pi_{ij}$, while symmetry requires $\Pi_{ij} = \Pi_{ji}$. For estimation purposes, $d \log z$ can be measured by $\log(z_t/z_{t-1})$ where t indicates time and z stands for q_i and p_i; the budget share ω_{it} can be measured by $\bar{\omega}_{it} = \frac{\omega_{it} + \omega_{it-1}}{2}$ (Theil).

Given OJ products belong to one group and GJ products belong to another, model (11) is quasi separable at sample mean budget share values if

$$\Pi_{ji} = \Pi_{ij} = \Pi_{14} \frac{\bar{\omega}_i \bar{v}_j}{\bar{\omega}_4 \bar{v}_4}, \qquad i = 1, 2, 3;\ j = 4, 5, 6, \tag{12}$$

where the bars over ω_i and v_j indicate sample mean values and $v_j = \dfrac{\omega_j}{\sum_{i=4}^{6} \omega_i}$, the budget share for the grapefruit-juice subgroup.

Restriction (12) was tested using an asymptotic chi-square test suggested by Gallant and Jorgenson--unrestricted model (11) was estimated using the seemingly unrelated regression (SUR) method and the model covariance matrix was output; the eight restrictions given by (12) were imposed and the model was re-estimated by the SUR method using the covariance matrix from the unrestricted model; the objective function value (the generalized mean square error) for the unrestricted model was subtracted from the objective function value for the restricted model; the latter difference times the number of observations is asymptotically distributed as a chi-square statistic with eight degrees of freedom under the null hypothesis. Estimates of the restricted and unrestricted models are given in Table 1. The chi-square statistic was 8.13 and indicates quasi separability can be accepted at the .10 level of significance. For the unrestricted model, all marginal propensity to spend (MPS) estimates were significant, while 13 out of 21 Slutsky coefficient estimates were significant. For the restricted model, all but four of the 19 coefficient estimates were significant with the same coefficients in the two models being insignificant.

The latter analysis considers quasi separability only at mean budget share values. To allow for quasi separability elsewhere, model (6) was also estimated. For juices, the model with homogeneity imposed can be written as

$$\begin{aligned} \omega_i\, d \log q_i = {} & \beta_i\, d \log Q + \phi_{aa}\, \omega_i\, d \log P_a + \phi_{ab}\, \omega_i\, d \log P_b + \gamma_{i1}\, \omega_i (d \log p_1 - d \log p_3) \\ & + \gamma_{i2}\, \omega_i (d \log p_2 - d \log p_3), \qquad i = 1, 2, 3, \end{aligned} \tag{13}$$

and

$$\omega_i \, d \log q_i = \beta_i \, d \log Q + \phi_{ba} \, \omega_i \, d \log P_a + \phi_{bo} \, \omega_i \, d \log P_b + \gamma_{i4} \, \omega_i (d \log p_4 - d \log p_6)$$
$$+ \gamma_{i5} \, \omega_i (d \log p_5 - d \log p_6), \qquad i = 4, 5, 6,$$

where $d \log P_a = \sum_{i=1}^{3} v_i (d \log p_i - d \log p_7)$, $v_i = \frac{\omega_i}{\omega_a}$, $\omega_a = \sum_{i=1}^{3} \omega_i$;

$d \log P_b = \sum_{i=4}^{6} v_i (d \log p_i - d \log p_7)$, $v_i = \frac{\omega_i}{\omega_b}$, $\omega_b = \sum_{i=4}^{6} \omega_i$ as defined before in equation (12). The variables $d \log P_a$ and $d \log P_b$ are divisia price indices (relative to the price of RJ) for orange juice and grapefruit juice, respectively. For products from different groups, cross-price effects are felt through group price indices $d \log P_a$ and $d \log P_b$. For products from the same group, an intragroup or specific effect is also involved. Symmetry in specification (13) is imposed by requiring

$$\phi_{ba} = \phi_{ab} \left[\frac{\omega_a}{\omega_b}\right]; \quad \gamma_{21} = \gamma_{12} \left[\frac{\omega_1}{\omega_2}\right]; \quad \gamma_{31} = -(\gamma_{11} + \gamma_{12}) \frac{\omega_1}{\omega_3}; \; \gamma_{32} = -\left[\gamma_{12} \left[\frac{\omega_1}{\omega_2}\right] + \gamma_{22}\right] \frac{\omega_2}{\omega_3}; \gamma_{54} = \gamma_{45} \frac{\omega_4}{\omega_5};$$

$\gamma_{64} = -(\gamma_{44} + \gamma_{45}) \frac{\omega_4}{\omega_6}$; and $\gamma_{65} = -\left[\gamma_{45} \left[\frac{\omega_4}{\omega_5}\right] + \gamma_{55}\right] \frac{\omega_5}{\omega_6}$. As evident from the latter relationships, symmetry can not be imposed consistently at all budget share values. In the present study, symmetry was imposed at mean budget share values.

Since treating the budget shares involved in the definition of the Slutsky coefficient as exogenous may result in an endogeneity problem, model (13) was estimated using lagged budget shares in the Slutsky terms. For comparison, model (13) was also estimated with current budget shares in the Slutsky terms. The results for the two specifications were basically the same with only minor differences. Estimates of the specification using lagged budget shares are shown in Table 2.

All coefficient estimates for the more flexible specification of quasi separability shown in Table 2 were significant. Each product was normal with a positive MPS. RJ had the highest MPS at .35, followed by COJ at .32 and FCOJ at .26; the estimates for the other products ranged from .01 to .03. (The estimate for RJ is determined from the adding-up condition.)

The own-compensated price elasticity estimates (ϕ's) for the groups were all negative--the largest elasticity was for GJ at -1.7, followed by RJ at -1.1 and OJ at -.6. All cross-compensated price elasticity estimates were positive, ranging from .1 (ϕ_{12}) to 1.1 (ϕ_{21}). The estimates indicate overall substitution relationships between groups. (Table 2 only shows estimates for ϕ_{11}, ϕ_{12} and ϕ_{22}; estimates for the other ϕ were obtained by using homogeneity and symmetry relationships, and sample mean budget shares in the case of symmetry; the same procedure was used to estimate the specific price parameters, the γ's, not shown in Table 2. For RJ, only the ϕ's are calculated--a specific price parameter is not calculated since RJ is the only product in the group.)

The intragroup or specific own-price parameters, which are elasticities measuring sensitivity of quantity to group-quantity-index ratios with respect to intragroup prices, were all negative. For the OJ group, CSSOJ had the largest specific own-price parameter at -1.6, followed by FCOJ at -.9 and COJ at -.6. For the GJ group, FCGJ had the largest specific own-price parameter at -1.8, followed by CSSGJ at -1.6 and CGJ at -1.1. All specific cross-price effects were positive, ranging from .02 (γ_{23}) to 1.31 (γ_{65}). The positive parameter estimates indicate intragroup substitution.

The results of this section illustrate how quasi separability can be used to analyze disaggregated product demands. Separability restrictions are usually imposed when commodity groups are broadly defined. However, when studying large numbers of disaggregated products, reduction in the parameter space may also be necessary for estimation. Quasi separability offers a straightforward and easily understood approach to reduce the number of parameters.

Concluding Comments

The number of different demand specifications that might be considered for empirical analysis is quite large (Lewbel). Choosing a smaller set of specifications to examine closer and test is often based on the restrictive nature of the system. Quasi separability offers a simple description of two-stage budgeting and, when modeled using the differential approach, offers a moderate degree of flexibility. The approach suggested by Theil et al., which makes the Rotterdam model income parameters depend on the budget shares, can similarly be used to make the Slutsky coefficients for the quasi separability model depend on the budgets share. The differential quasi separable modeling approach also provides a simple means to introduce demographic variables. For some

studies, the simplicity may not be appropriate; however, for other studies, such as the study of the effects of generic advertising on demand, the approach may adequately depict the demand situation. Researchers will continue to search for "appropriate" demand specifications. The differential quasi separable model may be an alternative to consider for some problems.

Table 1. Seemingly unrelated regression estimates of Rotterdam model with and without quasi separability imposed at sample budget share means.[a]

Parameter[b]	Unrestricted Model[c]		Restricted Model[d]	
	Estimate	Approximate Standard Error	Estimate	Standard Error
β_1	0.261*	(0.009)[e]	0.262*	(0.009)
β_2	0.316*	(0.013)	0.315*	(0.013)
β_3	0.010*	(0.002)	0.010*	(0.002)
β_4	0.008*	(0.001)	0.008*	(0.001)
β_5	0.027*	(0.002)	0.027*	(0.002)
β_6	0.021*	(0.002)	0.021*	(0.002)
Π_{11}	-0.287*	(0.026)	-0.289*	(0.025)
Π_{12}	0.095*	(0.020)	0.110*	(0.018)
Π_{13}	0.007	(0.005)	0.005	(0.004)
Π_{14}	0.004	(0.003)	0.003*	(4×10^{-4})
Π_{15}	0.015*	(0.005)		
Π_{16}	0.013*	(0.004)		
Π_{22}	-0.322*	(0.030)	-0.347*	(0.025)
Π_{23}	0.002	(0.004)	0.004	(0.003)
Π_{24}	0.003	(0.003)		
Π_{25}	0.018*	(0.004)		
Π_{26}	0.008*	(0.004)		
Π_{33}	-0.018*	(0.004)	-0.019*	(0.003)
Π_{34}	-0.001	(0.003)		
Π_{35}	0.001	(0.003)		
Π_{36}	0.005*	(0.002)		
Π_{44}	-0.012*	(0.004)	-0.013*	(0.003)
Π_{45}	0.001	(0.002)	0.001	(0.002)
Π_{46}	0.001	(0.002)	0.002	(0.002)
Π_{55}	-0.059*	(0.004)	-0.060*	(0.003)
Π_{56}	0.007*	(0.002)	0.006*	(0.002)
Π_{66}	-0.040*	(0.002)	-0.041*	(0.002)
Objective * N[f]	534.1650		542.2965	

[a]Sample budget share means were .229 for FCOJ, .366 for COJ, .010 for CSSOJ, .006 for FCGJ, .027 for CGJ, .017 for CSSGJ, and .344 for RJ; with rounding error.

[b]As defined in specification (11).

[c]Without quasi separability.

[d]With quasi separability.

[e]Asymptotic standard errors are given in parentheses.

[f]Generalized mean square error times number of observations (131).

*Indicates significance at the α = .05 level.

Table 2. Seemingly unrelated regression estimates of differential quasi separable model.

Parameter[a]	Estimate	Approximate Standard Error
β_1	0.264*	(0.009)[b]
β_2	0.318*	(0.013)
β_3	0.010*	(0.002)
β_4	0.008*	(4×10^{-4})
β_5	0.026*	(0.002)
β_6	0.021*	(0.002)
ϕ_{11}	-0.647*	(0.049)
ϕ_{12}	0.088*	(0.013)
ϕ_{22}	-1.705*	(0.144)
γ_{11}	-0.909*	(0.074)
γ_{12}	0.866*	(0.073)
γ_{22}	-0.559*	(0.047)
γ_{44}	-1.798*	(0.143)
γ_{45}	1.052*	(0.121)
γ_{55}	-1.062*	(0.052)

[a]As defined in specification (13).

[b]Asymptotic standard error.

*Indicates significance at the α = .05 level.

Footnotes

1. As equation (5) indicates, the Slutsky coefficients for quasi separability depend on commodity budget shares. In contrast, the Slutsky coefficients for the usual Rotterdam model depend on commodity marginal propensities; i.e., $\Pi_{ri,sj} = v_{ri,sj} - \phi\ \beta_{ri}\ \beta_{sj}$ where $v_{ri,sj} = (p_{ri}p_{sj}/x)\ \lambda\ u^{ri,sj}$, $\lambda = \partial u/\partial x$, $u^{ri,sj}$ is the ri,sj^{th} element of the Hessian matrix $\left[\dfrac{\partial u}{\partial q_{ri}\ \partial q_{sj}}\right]$; $\phi = (\partial \log \lambda/\partial \log x)^{-1}$; and β_{ri} and β_{sj} are marginal propensities as defined in equation (6). For block additivity, an assumption often made in empirical studies, $v_{ri,sj} = 0$ for $r \neq s$. Both the terms $v_{ri,sj}$ and $\gamma_{ri,sj}\ \omega_{ri}$ (in equation (5)) indicate specific intragroup effects. The terms $-\phi\ \beta_{ri}\ \beta_{sj}$ and $\phi_{rs}\ \dfrac{\omega_{si}}{\omega_s}\ \omega_{rj}$ (in equation (5)) indicate general effects for intergroup (and intragroup) price changes. Given the Hessian matrix is negative definite, ϕ is negative and the term $-\phi\ \beta_{ri}\ \beta_{sj}$ is positive if both goods are normal (inferior). For quasi separability, the general effect is positive if ϕ_{rs} is positive. Subsequently, ϕ_{rs} is interpreted as a group compensated price elasticity involving group quantity and price indices. Hence, if the two groups are substitutes, in the Hicksian sense, the general intergroup term is positive.

2. The marginal propensity β_{ri} can also be expressed in terms of cost elasticities. From (1), obtain

$$\frac{\partial \log x}{\partial \log u} = \frac{\partial \log c}{\partial \log u} + \sum_s \left[\frac{\partial \log c}{\partial \log c_s}\right]\left[\frac{\partial \log c_s}{\partial \log u}\right];$$

hence,

$$\frac{\partial \log u}{\partial \log x} = \frac{1}{\dfrac{\partial \log c}{\partial \log u} + \sum_s \omega_s \left[\dfrac{\partial \log c_s}{\partial \log u}\right]}.$$

From (2), obtain $$\frac{\partial \log q_{ri}}{\partial \log u} = \frac{\partial \log \left[\dfrac{\partial c_r}{\partial p_{ri}}\right]}{\partial \log u} + \frac{\partial \log \left[\dfrac{\partial c}{\partial c_r}\right]}{\partial \log u} + \sum_s \frac{\partial \log \left[\dfrac{\partial c}{\partial c_r}\right]}{\partial \log c_s}\ \frac{\partial \log c_s}{\partial \log u}.$$

By definition, $\beta_{ri} = \omega_{ri} \dfrac{\partial \log q_{ri}}{\partial \log x} = \omega_{ri} \dfrac{\partial \log q_{ri}}{\partial \log u} \dfrac{\partial \log u}{\partial \log x}$;

hence, $$\beta_{ri} = \omega_{ri} \frac{\dfrac{\partial \log \left[\dfrac{\partial c_r}{\partial p_{ri}}\right]}{\partial \log u} + \dfrac{\partial \log \left[\dfrac{\partial c}{\partial c_r}\right]}{\partial \log u} + \sum_s \phi_{rs}\, \omega_s \dfrac{\partial \log c_s}{\partial \log u}}{\dfrac{\partial \log c}{\partial \log u} + \sum_s \omega_s \dfrac{\partial \log c_s}{\partial \log u}}.$$

If the cost function elasticities with respect to utility u are treated as constants, the expression for β_{ri} would vary with respect to ω_{ri} and ω_s. Treating β_{ri} as a constant is an approximation.

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