An Experimental Investigation of Inter-temporal Risk Decision-making

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What do these issues have in common?

- **Teen Smoking and Obesity**
- **Buying a farm or a home**
- **Saving the rainforest or an endangered species**
- **Building in a Hurricane Zone or flood plain**
- **Investing in basic research**
- **Going to grad school**

- ***There are both risk and time dimensions to the problem***

Problems With Risk and No Time Dimensions?

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Futures Speculation

Problems With Risk and No Time Dimensions?

Futures Speculation

Bungee Jumping

Problems With Risk and No Time Dimensions?

Russian Roulette

Futures Speculation

Bungee Jumping

Risk Models

- Von Neumann and Morganstern Utility Function
  - Max $\sum_i U(W_j)\ P(W_j)$
  - where there are J ending wealth states and $P(W_j)$ is the probability of outcome W_j
  - Risk preferences reflected in the Pratt/Arrow coefficient U"/U'

Time is still ignored. Results are assumed to be obtained instantaneously

Investment Analysis

$$NPV = -\text{Investment} + \frac{\text{Net Cash Flow}_1}{1+i} + \frac{\text{Net Cash Flow}_2}{(1+i)^2} + \frac{\text{Net Cash Flow}_3}{(1+i)^3} + ...$$

NPV = Net Present Value
Investment = the initial investment
Net Cash flow = Income minus expenses associated with the investment
i = discount rate
N = length of planning horizon

A time dimension but not risk. Future cash flows are assumed certain

Discounted Expected Utility

- Probability – p
- Discrete time – t
- Discount rate 0< *β<1*
- *u[c(z^t)]* =Utility of consumption given state z^t

$$U(\{c(s_t)\}) = \sum_{t=0}^{\infty} \beta^t \sum_{z^t \in Z^t} p(z^t) u[c(z^t)] = E \sum_{t=0}^{\infty} \beta^t u(c_t),$$

Kreps-Porteus Preferences

- Constant Elasticity Aggregator

$$V[u, \mu(U)] = [(1-\beta)u^\rho + \beta\mu(U)^\rho]^{1/\rho}$$

- Constant elasticity certainty equivalent

$$\mu(U) = [E(U^\alpha)]^{1/\alpha}.$$

- If ρ=α this becomes discounted expected utility
- 1-α = coefficient of relative risk aversion
- 1/(1-ρ) = inter-temporal elasticity of substitution
- Prefer early resolution of uncertainty if α < ρ.

Experimental Design

- A within-subject design is used where each experimental subject participated in three treatments.
- The first set of tasks was involved participants making choices between riskless payoffs that occurred at different periods of time.
  - Harrison Lau, and Williams (2002)
- The second set of tasks involved the participants making choices between two risky outcomes that paid out in the same time period
  - Holt and Laury (2002)
- The third set of tasks combined the features of the first two sets of tasks and decision makers were confronted with choices between two risky outcomes paying out in differing time periods.
- To avoid an order effect, the order with which the sets of tasks related to the discounting, risk aversion, and mixed decision exercises were varied.

Details of the Nine Tasks

Task Set	Task	Outcomes	Timing of Option A	Timing of Option B
1	1	certain	1 week	13 weeks
1	2	certain	1 week	37 weeks
2	3	risky	present	present
2	4	risky	1 week	1 week
2	5	risky	13 weeks	13 weeks
2	6	risky	37 weeks	37 weeks
3	7	risky	1 week	13 weeks
3	8	risky	1 week	37 weeks
3	9	risky	13 weeks	37 weeks

Experiment to Elicit Discount Rate

Choices	Option A	Option B	Decision
1	Receive $10 in 1 week	Receive $10.00 in 13 weeks	
2	Receive $10 in 1 week	Receive $10.50 in 13 weeks	
3	Receive $10 in 1 week	Receive $11.00 in 13 weeks	
4	Receive $10 in 1 week	Receive $11.50 in 13 weeks	
5	Receive $10 in 1 week	Receive $12.00 in 13 weeks	
6	Receive $10 in 1 week	Receive $12.50 in 13 weeks	
7	Receive $10 in 1 week	Receive $13.00 in 13 weeks	
8	Receive $10 in 1 week	Receive $13.50 in 13 weeks	
9	Receive $10 in 1 week	Receive $14.00 in 13 weeks	
10	Receive $10 in 1 week	Receive $14.50 in 13 weeks	
11	Receive $10 in 1 week	Receive $15.00 in 13 weeks	
12	Receive $10 in 1 week	Receive $15.50 in 13 weeks	
13	Receive $10 in 1 week	Receive $16.00 in 13 weeks	
14	Receive $10 in 1 week	Receive $16.50 in 13 weeks	
15	Receive $10 in 1 week	Receive $17.00 in 13 weeks	

Holt & Laury Risk Preference Questionnaire

Decision	Option A			Option B	
1	10% chance of $10.00,	90% chance of $8.00		10% chance of $19.00,	90% chance of $1.00
2	20% chance of $10.00,	80% chance of $8.00		20% chance of $19.00,	80% chance of $1.00
3	30% chance of $10.00,	70% chance of $8.00		30% chance of $19.00,	70% chance of $1.00
4	40% chance of $10.00,	60% chance of $8.00		40% chance of $19.00,	60% chance of $1.00
5	50% chance of $10.00,	50% chance of $8.00		50% chance of $19.00,	50% chance of $1.00
6	60% chance of $10.00,	40% chance of $8.00		60% chance of $19.00,	40% chance of $1.00
7	70% chance of $10.00,	30% chance of $8.00		70% chance of $19.00,	30% chance of $1.00
8	80% chance of $10.00,	20% chance of $8.00		80% chance of $19.00,	20% chance of $1.00
9	90% chance of $10.00,	10% chance of $8.00		90% chance of $19.00,	10% chance of $1.00
10	100% chance of $10.00,	0% chance of $8.00		100% chance of $19.00,	0% chance of $1.00

Our Hybridized Technique

Decision	Option A		Option B		Which Option is Preferred?
1	10% chance of $10.00 in 1 week,	90% chance of $8.00 in 1 week	10% chance of $19.00 in 13 weeks	90% chance of $1.00 in 13 weeks	
2	20% chance of $10.00 in 1 week,	80% chance of $8.00 in 1 week	20% chance of $19.00, in 13 weeks	80% chance of $1.00 in 13 weeks	
3	30% chance of $10.00 in 1 week,	70% chance of $8.00 in 1 week	30% chance of $19.00, in 13 weeks	70% chance of $1.00 in 13 weeks	
4	40% chance of $10.00 in 1 week,	60% chance of $8.00 in 1 week	40% chance of $19.00, in 13 weeks	60% chance of $1.00 in 13 weeks	
5	50% chance of $10.00 in 1 week,	50% chance of $8.00 in 1 week	50% chance of $19.00, in 13 weeks	50% chance of $1.00 in 13 weeks	
6	60% chance of $10.00 in 1 week,	40% chance of $8.00 in 1 week	60% chance of $19.00, in 13 weeks	40% chance of $1.00 in 13 weeks	
7	70% chance of $10.00 in 1 week,	30% chance of $8.00 in 1 week	70% chance of $19.00, in 13 weeks	30% chance of $1.00 in 13 weeks	
8	80% chance of $10.00 in 1 week,	20% chance of $8.00 in 1 week	80% chance of $19.00, in 13 weeks	20% chance of $1.00 in 13 weeks	
9	90% chance of $10.00 in 1 week,	10% chance of $8.00 in 1 week	90% chance of $19.00, in 13 weeks	10% chance of $1.00 in 13 weeks	
10	100% chance of $10.00 in 1 week,	0% chance of $8.00 in 1 week	100% chance of $19.00, in 13 weeks	0% chance of $1.00 in 13 weeks	

Model Development

- CE of option A

$$\mu_A = \left[p_A^{low} \left(x_A^{low} \right)^{1-\alpha} + \left(1 - p_A^{low} \right) \left(x_A^{high} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$$

- Random Utility of A

$$U_{Aic} = \left(\frac{1}{1+d} \right)^{t_{Aic}} \left[\frac{\mu_{Aic}^{1-\rho}}{1-\rho} \right] + \varepsilon_{Ai} + \varepsilon_{Aic}$$

- Difference in Utility of Option A & B

$$\Delta U_{ic} = \left(\frac{1}{1+d} \right)^{t_{Aic}} \left[\frac{\mu_{Aic}^{1-\rho}}{1-\rho} \right] - \left(\frac{1}{1+d} \right)^{t_{Bic}} \left[\frac{\mu_{Bic}^{1-\rho}}{1-\rho} \right] + (\varepsilon_{Ai} - \varepsilon_{Bi})$$

Estimates of the CRRA Kreps-Porteus Model

Parameter	Description	Estimate	Standard Error
ρ	Curvature of utility across time	0.356^{**a}	0.014
α	Curvature of utility across risky outcomes	0.155^{**}	0.043
d	Discount rate	0.386^{**}	0.026
σ_i^2	Variance of individual-specific error	0.907^{**}	0.236

- The estimated value for the constant relative risk aversion parameter, α, was 0.155, implying a modest degree of risk aversion

- The estimated value of *ρ* is 0.356, which implies an inter-temporal elasticity of substitution of 1/0.356 = 2.81.

-The estimated annual discount rate of 0.39, while high compared to market interest rates, is well within the range of values estimated in previous literature.

-we can reject the hypothesis that *α* = *ρ* at the p<0.001 level of significance.

Estimates of the Power-Expo Kreps-Porteus Model

$$\frac{x^{1-\alpha}}{1-\alpha} \qquad \mu_A = \left[p_A^{low}\left(x_A^{low}\right)^{1-\alpha} + \left(1 - p_A^{low}\right)\left(x_A^{high}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$$

$$\frac{1-e^{-ax^{1-r}}}{1-a} \qquad \mu_A = \left(-\frac{\ln\left(p_A^{low}e^{-a\left(x_A^{low}\right)^{1-r}} + (1-p_A^{low})e^{-a\left(x_A^{high}\right)^{1-r}}\right)}{a}\right)^{\frac{1}{1-r}}$$

Kreps-Porteus with the power-expo form

Parameter	Description	Estimate	Standard Error
λ	Curvature of utility across time – 1	-1.534[**a]	0.495
γ	Curvature of utility across time – 2	0.777[**]	0.062
a	Curvature of utility across risky outcomes – 1	0.009	0.005
r	Curvature of utility across risky outcomes – 2	-0.773[**]	0.270
d	Discount rate	0.324[**]	0.067
σ_i^2	Variance of individual-specific error	1.348[**]	0.366

Two asterisks indicate the parameter is significantly different than zero at the 0.01 level.
Test that of the restriction of $\alpha = \lambda$ and $r = \gamma$ is reject at p=0.001.

Conclusions

- We were able to strongly reject the hypothesis that risk and time preferences were governed by a single parameter.
  - If DEU is rejected few of our behavioral models are robust
- The second specification considered a more flexible power-expo form for both the risk and time functions. Again, we were able to strongly reject the hypothesis of parameter equality across the risk and time functions.

Conclusions

- Results revealed a pattern of behavior, with increasing aversion to risk as outcomes increases and decreasing aversion to delay as outcomes increased.

- Our results reveal that as the dollar pay-offs increase, people increasingly prefer an early resolution of uncertainty.

- Ultimately, want to test if our results predicts real world behavior.