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MEASURING AND DECOMPOSING AGRICULTURAL PRODUCTIVITY AND PROFITABILITY CHANGE[1]

by

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Abstract: The total factor productivity (TFP) of a multiple-output multiple-input firm can be defined as the ratio of an aggregate output to an aggregate input. With this definition, index numbers that measure changes in TFP can be expressed as the ratio of an output quantity index to an input quantity index. This paper uses the term *multiplicatively complete* to describe TFP index numbers that are constructed in this way. O'Donnell (2008) shows that, irrespective of the returns to scale and/or scope properties of the production technology, *all* multiplicatively complete TFP index numbers can be decomposed into widely-used measures of technical change and technical efficiency change, as well as unambiguous measures of scale and mix efficiency change. Members of the class of multiplicatively complete TFP index numbers include the Fisher, Tornquist and Moorsteen-Bjurek indexes, but not the popular Malmquist index of Caves, Christensen and Diewert (1982a). This paper uses data envelopment analysis (DEA) to compute and decompose Moorsteen-Bjurek indexes of world agricultural TFP change for the period 1970-2001. In a DEA model that prohibits technical regress, only two countries are found to maximize TFP during the study period: Nepal from 1970 to 1995, and Thailand for several years in the late 1990s. The paper explains how changes in the agricultural terms of trade have drawn other larger agricultural producers away from TFP-maximizing input-output points. The annual rate of technical progress in global agriculture is estimated to be less than 1% per annum.

1. INTRODUCTION

Improvements in agricultural productivity are a fundamental pre-condition for economic development. Sachs (2008, p.26) puts it in very simple terms: "when agricultural productivity is low – so that the typical family farm basically feeds itself, with only a small surplus to trade with urban dwellers – most of the population must be engaged in food production in order to subsist. It is only when agricultural productivity is very high – so that a farm family can feed many urban residents – that a significant share of the population can reside in urban areas and be engaged in manufacturing and services". In more general terms, when agricultural productivity is high, land, labour, capital and other resources can be released from food production to expand the non-agricultural

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sectors of the economy. Valuable resources such as land and water can also be preserved for environmental use. In coming decades, if populations continue to grow and natural resource stocks continue to be depleted, growth in agricultural productivity will become increasingly important for maintaining the environment and improving standards of living.

Effective public policy in this area requires identification of the main drivers of productivity growth. In agriculture, two of the main drivers are technical progress and technical efficiency improvement. Technical progress usually refers to the expansion in the production possibilities set that comes about through increased knowledge, while technical efficiency improvement essentially refers to increases in output-input ratios made possible by eliminating mistakes in the production process. Public policies designed to improve agricultural productivity can be targeted at these different components. Policies to improve productivity through technical progress include, for example, any policies leading to investment in scientific research and development. Examples of complementary policies designed to increase productivity through improvements in technical efficiency include education, training and extension programs. By carefully defining the various components of agricultural productivity change, this paper provides insights into the ways different public policy options can promote or retard growth. Included among these are public policies that alter the agricultural terms of trade.

To empirically measure the components of productivity growth, we first need a precise definition of productivity, and then a productivity index number formula that is consistent with this definition. In the case of multiple-output multiple-input firms, total factor productivity (TFP) can be defined as the ratio of an aggregate output to an aggregate input. This definition underpins the seminal work of Jorgenson and Grilliches (1967) and is the definition used throughout this paper. With this definition, index numbers that measure changes in TFP can be expressed as the ratio of an output quantity index to an input quantity index (a measure of output growth divided by a measure of input growth). I use the term *multiplicatively complete*[1] to refer to index numbers constructed in this way. The class of multiplicatively complete TFP indexes includes the well-known Paasche, Laspeyres, Fisher and Tornquist TFP indexes[2]. However, the popular Malmquist TFP index of Caves, et al. (1982a) is incomplete. There also exists a class of *additively complete* TFP index numbers, and this class includes indexes of the type proposed by Bennet (1920). Indexes that are not complete are said to be *incomplete*, with the implication that they are generally biased measures of TFP change.

Multiplicatively-complete TFP index numbers are important for two reasons. First, O'Donnell (2008) has used an aggregate quantity-price framework to demonstrate that profitability can be decomposed into the product of a multiplicatively-complete TFP index number and an index measuring the change in the terms of trade. Second, he shows that *all* multiplicatively complete TFP index numbers can be further decomposed into an unambiguous measure of technical change and several recognizable measures of efficiency change. Among the efficiency change components are input- and output-oriented measures of technical, scale and mix efficiency change. The technical and scale efficiency change components are the familiar components discussed in the literature, but the mix efficiency change component appears to be new. In the output-oriented case, mix efficiency is a measure of the maximum increase in TFP that is technically feasible when outputs can vary but inputs are held fixed. The difference between output-oriented measures of technical and mix efficiency is that the former preserves the output mix while the latter allows outputs to vary freely. Input- and output-oriented measures of mix efficiency are closely related to well-known measures of allocative efficiency. However, the two sets of measures are generally distinct, coinciding if and only if aggregate quantities are computed using price-weighted linear aggregator functions.

The structure of the paper is as follows. Section 2 demonstrates that the Moorsteen-Bjurek TFP index is consistent with the most basic definition of TFP as the ratio of an aggregate output to an aggregate input. Section 3 defines measures of technical, scale and mix efficiency in terms of these aggregate outputs and inputs, and demonstrates that the Moorsteen-Bjurek index can be written as the product of a measure of technical change and these efficiency components. Section 4 outlines the DEA programs required to compute and decompose the Morrsteen-Bjurek index. Section 5 applies the methodology to FAO data on the agricultural inputs and outputs of 88 countries from 1970 to 2001. A similar FAO data set was also used by Coelli and Rao (2005) to investigate agricultural TFP growth. The paper is concluded in Section 6.

2. DISTANCE-BASED TFP INDEXES

With a view to eventually computing and decomposing distance-based Malmquist and Moorsteen-Bjurek indexes, it is convenient to start with a distance-based representation of the production technology. The Shephard (1953) input and output distance functions can be written:

(2.1) $$D_I^t(x,q) = \max_{\rho}\{\rho > 0 : (x/\rho, q) \in T^t\} \quad \text{and}$$ (input distance function)

(2.2) $$D_O^t(x,q) = \min_{\delta}\{\delta > 0 : (x, q/\delta) \in T^t\}$$ (output distance function)

where $q \in \Re_+^J$ and $x \in \Re_+^K$ denote vectors of outputs and inputs, and T^t denotes the period-*t* production possibilities set. The input distance function measures the largest radial contraction of the input vector that is technically feasible while holding the output vector fixed, while the output distance function measures the inverse of the largest radial expansion of the output vector that is possible while holding the input vector fixed. The output distance is the Farrell (1957) output-oriented measure of technical efficiency, while the input distance is the inverse of the Farrell (1957) input-oriented measure. Irrespective of the returns to scale or scope properties of the technology, the output and input distance functions are linearly homogeneous and non-decreasing in output quantities and input quantities respectively.

Let $q_{nt} \in \Re_+^M$ and $x_{nt} \in \Re_+^K$ denote the observed output and input vectors of firm *n* in period *t*. O'Donnell (2008) motivates distance-based input quantity indexes by observing that the ratios $D_I^t(x_{nt}, q_{nt}) / D_I^t(x_{ms}, q_{nt})$ and $D_I^s(x_{nt}, q_{ms}) / D_I^s(x_{ms}, q_{ms})$ only depart from unity as x_{nt} departs from x_{ms}. This makes them natural measures of input quantity change. The Malmquist input quantity index suggested by Caves, et al. (1982a, p.1397) is the geometric average of these two ratios:

(2.3) $$X_{ms,nt}^M = \left(\frac{D_I^t(x_{nt}, q_{nt}) D_I^s(x_{nt}, q_{ms})}{D_I^t(x_{ms}, q_{nt}) D_I^s(x_{ms}, q_{ms})} \right)^{1/2}$$ (Malmquist input index)

where the ordering of the subscripting on $X_{ms,nt}^M$ means that the index measures the difference between the inputs of firm *n* in period *t* and the inputs of firm *m* in period *s* using the latter as the base. The corresponding output index is:

(2.4) $$Q_{ms,nt}^M = \left(\frac{D_O^t(x_{nt}, q_{nt}) D_O^s(x_{ms}, q_{nt})}{D_O^t(x_{nt}, q_{ms}) D_O^s(x_{ms}, q_{ms})} \right)^{1/2}$$ (Malmquist output index)

The monotonicity and homogeneity properties of the two distance functions ensure that these indexes satisfy basic axioms of index number theory, including monotonicity, homogeneity, identity and proportionality.

The input and output distance functions can be used to not only construct input and output quantity indexes, but also to construct TPF indexes. Three distance-based TFP indexes are:

(2.5) $$TFP_{ms,nt}^{MB} = \left(\frac{D_O^t(x_{nt}, q_{nt}) D_O^s(x_{ms}, q_{nt})}{D_O^t(x_{nt}, q_{ms}) D_O^s(x_{ms}, q_{ms})} \right)^{1/2} \left(\frac{D_I^t(x_{ms}, q_{nt}) D_I^s(x_{ms}, q_{ms})}{D_I^t(x_{nt}, q_{nt}) D_I^s(x_{nt}, q_{ms})} \right)^{1/2}$$ (Moorsteen-Bjurek)

(2.6) $$TFP_{ms,nt}^{IM} = \left(\frac{D_I^t(x_{ms}, q_{ms}) D_I^s(x_{ms}, q_{ms})}{D_I^t(x_{nt}, q_{nt}) D_I^s(x_{nt}, q_{nt})} \right)^{1/2}$$ and (input-oriented Malmquist)

(2.7) $$TFP_{ms,nt}^{OM} = \left(\frac{D_O^t(x_{nt}, q_{nt}) D_O^s(x_{nt}, q_{nt})}{D_O^t(x_{ms}, q_{ms}) D_O^s(x_{ms}, q_{ms})} \right)^{1/2}$$ (output-oriented Malmquist)

The Moorsteen-Bjurek TFP index is the ratio of the output quantity index and the input quantity index given by (2.3) and (2.4), and is compatible with the idea that TFP growth is simply the excess of output growth over input growth. The input- and output-oriented TFP indexes given by (2.6) and (2.7) are the geometric averages of period-*t* and period-*s* input- and output-oriented Malmquist TFP indexes proposed by Caves, et al. (1982a, p.1402). These Malmquist TFP indexes (and their geometric average) cannot in general be expressed as the ratio of an output quantity index to an input quantity index. Thus, they are multiplicatively incomplete.

The popularity of Malmquist indexes derives partly from the fact that they can be decomposed. For example, Fare, Grosskopf, Norris and Zhang (1994b, p.71) rewrite the input-oriented index as:

$$(2.8) \qquad TFP_{ms,nt}^{IM} = \frac{D_I^s(x_{ms}, q_{ms})}{D_I^t(x_{nt}, q_{nt})} \times \left(\frac{D_I^t(x_{nt}, q_{nt}) D_I^t(x_{ms}, q_{ms})}{D_I^s(x_{nt}, q_{nt}) D_I^s(x_{ms}, q_{ms})} \right)^{1/2}$$

The first term on the right-hand is the ratio of input-oriented technical efficiency measures defined in Section 3 below – it is unambiguously a measure of pure input-oriented technical efficiency change. The second term is interpreted by Fare, et al. (1994b, p.71) as a measure of technical change.

The ability to write a TFP index as the ratio of an output index to an input index is a necessary but not sufficient condition for multiplicative completeness. It must also be possible to write the output and input indexes themselves as ratios of aggregate outputs and inputs. Fortunately, most, if not all, common output and input quantity indexes can be written in this way. Indeed, the entire economic theory approach to index number construction is based on the idea that index numbers can be derived from particular aggregator functions, and that the properties of different index number formulas are directly related to the properties of the different aggregator functions on which they are based – examples of different types of aggregator functions can be found in Diewert (1976) and Caves, Christensen and Diewert (1982b). O'Donnell (2008) observes that the aggregator functions underpinning (2.3) and (2.4) are

$$(2.9) \qquad X(x_k) = \left(D_I^t(x_k, q_{nt}) D_I^s(x_k, q_{ms}) \right)^{1/2} \equiv X_k \qquad \text{and}$$

$$(2.10) \qquad Q(q_k) = \left(D_O^t(x_{nt}, q_k) D_O^s(x_{ms}, q_k) \right)^{1/2} \equiv Q_k$$

for $k = ms, nt$. This means that the index number formulas given by equations (2.3) to (2.5) can be written in the more compact form:

$$(2.11) \qquad X_{ms,nt}^{M} = \frac{X_{nt}}{X_{ms}} \qquad \text{(Malmquist input index)}$$

$$(2.12) \qquad Q_{ms,nt}^{M} = \frac{Q_{nt}}{Q_{ms}} \qquad \text{and} \qquad \text{(Malmquist output index)}$$

$$(2.13) \qquad TFP_{ms,nt}^{BM} = \frac{Q_{ms,nt}^{M}}{X_{ms,nt}^{M}} = \frac{Q_{nt} / X_{nt}}{Q_{ms} / X_{ms}} = \frac{TFP_{nt}}{TFP_{ms}} \qquad \text{(Moorsteen-Bjurek)}$$

where $TFP_{nt} \equiv Q_{nt} / X_{nt}$ denotes the TFP of firm *n* in period *t*. Equations (2.11) to (2.13) reveal that the Moorsteen-Bjurek index is consistent with the most basic definition of TFP as the ratio of an aggregate output to an aggregate input. That is, the Moorsteen-Bjurek index is multiplicatively complete.

Associated with the aggregate quantities Q_{nt} and X_{nt} aggregate prices P_{nt} and W_{nt} with the properties $P_{nt} Q_{nt} = p_{nt}' q_{nt} \equiv R_{nt}$ and $W_{nt} X_{nt} = w_{nt}' x_{nt} \equiv C_{nt}$. These properties are known as product rules and are trivially satisfied. Importantly, they allow us to write profitability change as the product of a TFP index and a measure of the change in the terms of trade:

$$\Gamma_{ms,nt} \equiv \frac{\Gamma_{nt}}{\Gamma_{ms}} = \frac{R_{nt}/C_{nt}}{R_{ms}/C_{ms}} = \frac{P_{nt}/W_{nt}}{P_{ms}/W_{ms}} \times TFP_{ms,nt} \quad (2.14)$$

where $\Gamma_{nt} \equiv R_{nt}/C_{nt}$ denotes profitability. Re-arranging (2.14) yields

$$TFP_{ms,nt} = \frac{R_{ms,nt}/P_{ms,nt}}{C_{ms,nt}/W_{ms,nt}} \quad (2.15)$$

where $R_{ms,nt} = R_{nt}/R_{ms}$, $C_{ms,nt} = C_{nt}/C_{ms}$, $P_{ms,nt} = P_{nt}/P_{ms}$ and $W_{ms,nt} = W_{nt}/W_{ms}$ are revenue, cost and output and input price indexes respectively. Thus, TFP change can be measured as the ratio of an implicit output quantity index to an implicit input quantity index.

3. DECOMPOSING MULTIPLICATIVELY-COMPLETE TFP INDEXES

O'Donnell (2008) uses an aggregate quantity-price framework to demonstrate that all multiplicatively-complete TFP indexes can be decomposed into a measure of technical change and several measures of efficiency change. The demonstration is aided greatly by the ability to depict the TFP of a multiple-input multiple-output firm in two-dimensional aggregate quantity space. The basic idea is illustrated in Figure 1. In this figure, the TFP of firm *n* in period *t* is given by the slope of the ray passing through the origin and point A, while the TFP of firm *m* in period *s* is given by the slope of the ray passing through the origin and point Z. Let lower-case *a* and *z* denote the angles between the horizontal axis and the rays passing through points A and Z. Then the TFP index that measures the change in TFP between the two firms can be compactly written $TFP_{ms,nt} = \tan a / \tan z$. This ability to write a multiplicatively-complete TFP index as the ratio of (tangent) functions of angles in aggregate quantity space is used by O'Donnell (2008) to conceptualise several alternative decompositions of TFP change. For example, let *e* denote the angle between the horizontal axis and the ray passing through the origin and *any* non-negative point E. Then it is clear from Figure 1 that the change in TFP between the two firms can be decomposed as $TFP_{ms,nt} = \tan a / \tan z = (\tan a / \tan e)(\tan e / \tan z)$.

Within this framework, a potentially infinite number of points E can be used to effect a decomposition of a multiplicatively-complete TFP index. O'Donnell (2008) focuses only on those points that feature in common definitions of input- and output-oriented measures of efficiency. Expressed in terms of aggregate quantities, the efficiency measures that feature in an output-oriented decomposition of TFP change are:

$$OTE_{nt} = \frac{Q_{nt}}{\bar{Q}_{nt}} \quad (3.1)$$ (output-oriented technical efficiency)

$$OSE_{t} = \frac{\bar{Q}_{nt}/X_{nt}}{\tilde{Q}_{nt}/\tilde{X}_{nt}} \quad (3.2)$$ (output-oriented scale efficiency)

$$OME_{nt} = \frac{\bar{Q}_{nt}}{\hat{Q}_{nt}} \quad (3.3)$$ (output-oriented mix efficiency)

$$ROSE_{nt} = \frac{\hat{Q}_{nt}/X_{nt}}{Q_t^*/X_t^*} \quad (3.4)$$ (residual output-oriented scale efficiency)

$$RME_{nt} = \frac{\tilde{Q}_{nt}/\tilde{X}_{nt}}{Q_{nt}^*/X_{nt}^*} \quad (3.5)$$ (residual mix efficiency)

(3.6) $$TFPE_{nt} = \frac{Q_{nt} / X_{nt}}{Q_{nt}^{*} / X_{nt}^{*}}$$ (TFP efficiency)

where $\bar{Q}_{nt} \equiv Q_{nt} / D_O^t(x_{nt}, q_{nt})$ is the maximum aggregate output that is technically feasible when x_{nt} is used to produce a scalar multiple of q_{nt}; $\hat{Q}_{nt}$ is the maximum aggregate output that is feasible when using x_{nt} to produce any output vector; $\tilde{Q}_t$ and $\tilde{X}_t$ are the aggregate output and aggregate input obtained when we maximize TFP subject to the constraint that the output and input vectors are scalar multiples of q_{nt} and x_{nt} respectively; and Q_t^* and X_t^* are the aggregate output and aggregate input at the point where TFP is unconditionally maximized. The maximum TFP that is possible using the production technology available in period t is $TFP_t^* \equiv Q_t^* / X_t^*$.

The technical efficiency measure defined by (3.1) is the measure proposed by Farrell (1957), while the scale efficiency measure is defined by, for example, Balk (2001). The output-oriented mix efficiency measure (3.3) does not appear to have been previously defined in the literature, although it is closely related to a measure of revenue-allocative efficiency first proposed by Farrell (1957). To motivate this unfamiliar measure, O'Donnell (2008) considers the two-output case where the aggregator function is linear: $Q(q_{nt}) = \alpha_1 q_{1nt} + \alpha_2 q_{2nt}$. Figure 2 depicts this special case in output space. In this figure, the curved line passing through points V and C is the familiar production frontier representing all technically-efficient output combinations that can be produced using x_{nt}. The dashed line passing through point A is an iso-output line that maps all output combinations that have the same aggregate output as at point A. If the output mix and the input vector are held fixed, then aggregate output and TFP are maximized by radially expanding outputs to point C. However, if restrictions on the output mix are relaxed, aggregate output and TFP are maximized by moving around the frontier to point V. The ratio of the distance 0A to the distance 0C in Figure 2 is the output-oriented measure of technical efficiency defined in equation (3.1): $OTE_{nt} = Q_{nt} / \bar{Q}_{nt} = \|A\| \|C\|$. The ratio of the distance 0H to the distance 0V is the output-oriented measure of mix efficiency: $OME_{nt} = \bar{Q}_{nt} / \hat{Q}_{nt} = \|H\| \|V\|$. Output-oriented mix efficiency is simply a measure of the increase in TFP that comes about by relaxing restrictions on output mix. In the same way that scale efficiency is a measure of the potential productivity gains that can be achieved through economies of scale, mix efficiency is a measure of the potential gains that can be achieved through economies of scope.

To obtain further insights into the relationships between aggregate quantities and measures of efficiency, O'Donnell (2008) maps input-output combinations into aggregate quantity space. Figure 3 presents such a mapping for the input-output combinations represented by points A, C and V in Figure 2. In Figure 3, the curve passing through points C and D represents the frontier of a *restricted* production possibilities set. The set is restricted insofar as it only contains input and output vectors that can be written as scalar multiples of x_{nt} and q_{nt}. When these mix restrictions are relaxed, the firm has access to the expanded (or unrestricted) production possibilities set bounded by the curve passing through points V and E. Output-oriented technical efficiency measures the proportionate increase in TFP as the firm moves from point A to point C, output-oriented mix efficiency measures the increase in TFP as the firm moves from C to V, while output-oriented scale efficiency measures the increase in TFP as the firm moves from C to D.

In Figure 3, point D represents the input-output combination that maximizes TFP when the input and output mixes are held fixed. For this reason, point D is known as the point of *mix-invariant optimal scale* (MIOS). When mix restrictions are relaxed, the input-output combination that maximizes TFP is the combination represented by point E. For this reason, point E is known as the point of *maximum productivity* (MP). The residual output-oriented scale efficiency measure defined by (3.4) measures the proportionate increase in TFP that takes place as the firm moves from point V to point E. O'Donnell (2008) uses the term *scale* for this measure because "any movement around an unrestricted production frontier is a movement from one mix-efficient point to another, so any improvement in TFP is essentially a scale effect ... [He also uses] the term *residual* because, even though all the points on the unrestricted frontier are mix-efficient, they may nevertheless have different input and output mixes ... Thus, what is essentially a measure of scale efficiency may contain a residual mix effect" (p.20). Another important residual measure of efficiency is residual mix efficiency, defined by (3.5). This is a measure of the proportionate increase in TFP that occurs as the firm moves from point D to point E. The term residual is also used here because, although the move from point D on the mix-restricted frontier to point E on the unrestricted frontier is primarily a mix effect, it may also involve a change in scale. Residual mix efficiency can also be viewed as the component of TFP change that remains after accounting for pure technical efficiency and pure scale efficiency effects.

Finally, the measure of TFP efficiency defined by (3.6) measures the proportionate increase in TFP as the firm moves all the way from point A to point E. Figure 3 illustrates two of many pathways from A to E, and therefore illustrates two of many decompositions of TFP efficiency: in terms of angles,

$$(3.7)\qquad TFPE_{nt} = \frac{\tan a}{\tan e} = \left(\frac{\tan a}{\tan c}\times\frac{\tan c}{\tan v}\times\frac{\tan v}{\tan e}\right) = \left(\frac{\tan a}{\tan c}\times\frac{\tan c}{\tan d}\times\frac{\tan d}{\tan e}\right)$$

or, in terms of aggregate quantities,

$$(3.8)\qquad TFPE_{nt} = \frac{Q_{nt}/X_{nt}}{Q^*_{nt}/X^*_{nt}} = \left(\frac{Q_{nt}}{\bar{Q}_{nt}}\times\frac{\bar{Q}_{nt}}{\hat{Q}_{nt}}\times\frac{\hat{Q}_{nt}/X_{nt}}{Q^*_t/X^*_t}\right) = \left(\frac{Q_{nt}}{\bar{Q}_{nt}}\times\frac{\bar{Q}_{nt}/X_{nt}}{\tilde{Q}_{nt}/\tilde{X}_{nt}}\times\frac{\tilde{Q}_{nt}/\tilde{X}_{nt}}{Q^*_{nt}/X^*_{nt}}\right)$$

or, in terms of measures of efficiency,

$$(3.9)\qquad TFPE_{nt} = \frac{TFP_{nt}}{TFP^*_t} = \left(OTE_{nt}\times OME_{nt}\times ROSE_{nt}\right) = \left(OTE_{nt}\times OSE_{nt}\times RME_{nt}\right)$$

These decompositions provide a basis for an output-oriented decomposition of a multiplicatively complete TFP index. The easiest way to see this is to rewrite (3.9) as:

$$(3.10)\qquad TFP_{nt} = TFP^*_t\times\left(OTE_{nt}\times OME_{nt}\times ROSE_{nt}\right) = TFP^*_t\times\left(OTE_{nt}\times OSE_{nt}\times RME_{nt}\right)$$

A similar equation holds for firm *m* in period *s*. Thus,

$$(3.11)\qquad TFP_{ms,nt} = \frac{TFP_{nt}}{TFP_{ms}} = \left(\frac{TFP^*_t}{TFP^*_s}\right)\times\left(\frac{OTE_{nt}}{OTE_{ms}}\times\frac{OME_{nt}}{OME_{ms}}\times\frac{ROSE_{nt}}{ROSE_{ms}}\right) = \left(\frac{TFP^*_t}{TFP^*_s}\right)\times\left(\frac{OTE_{nt}}{OTE_{ms}}\times\frac{OSE_{nt}}{OSE_{ms}}\times\frac{RME_{nt}}{RME_{ms}}\right)$$

The first term in parentheses on the right-hand side of equation (3.11) measures the difference between the maximum TFP possible using the technology available in period *t* and the maximum TFP possible using the technology available in period *s*. Thus, it is a natural measure of technical change. The economy/industry experiences technical progress or regress as TFP^*_t/TFP^*_s is greater than or less than 1. The other ratios on the right-hand side of (3.11) are obvious measures of technical efficiency change, (residual) mix efficiency change, and (residual) scale efficiency change. O'Donnell (2008) derives the input-oriented counterparts to equations (3.7) to (3.11) and demonstrates that the input- and output-oriented measures of technical change are plausibly identical. For the sake of completeness, the input-oriented counterpart to (3.11) is given by:

$$(3.12)\qquad TFP_{ms,nt} = \frac{TFP_{nt}}{TFP_{ms}} = \left(\frac{TFP^*_t}{TFP^*_s}\right)\times\left(\frac{ITE_{nt}}{ITE_{ms}}\times\frac{IME_{nt}}{IME_{ms}}\times\frac{RISE_{nt}}{RISE_{ms}}\right) = \left(\frac{TFP^*_t}{TFP^*_s}\right)\times\left(\frac{ITE_{nt}}{ITE_{ms}}\times\frac{ISE_{nt}}{ISE_{ms}}\times\frac{RME_{nt}}{RME_{ms}}\right)$$

where

$$(3.13)\qquad ITE_{nt} = \frac{\bar{X}_{nt}}{X_{nt}} \qquad \text{(input-oriented technical efficiency)}$$

$$(3.14)\qquad ISE_{nt} = \frac{Q_{nt}/\bar{X}_{nt}}{\tilde{Q}_{nt}/\tilde{X}_{nt}} \qquad \text{(input-oriented scale efficiency)}$$

(3.15) $IME_{nt} = \frac{\hat{X}_{nt}}{\bar{X}_{nt}}$ (input-oriented mix efficiency)

(3.16) $RISE_{nt} = \frac{Q_{nt} / \hat{X}_{nt}}{Q^*_{nt} / X^*_{nt}}$ (residual input-oriented scale efficiency)

and RME_{nt} is the measure of residual mix efficiency defined by equation (3.5). In equations (3.13) to (3.16), $\bar{X}_{nt} \equiv X_{nt} / D^t_I(x_{nt}, q_{nt})$ is the minimum aggregate input that is possible when using a scalar multiple of x_{nt} to produce q_{nt}, while $\hat{X}_{nt}$ is the minimum aggregate input that is possible when using *any* input vector to produce q_{nt}.

4. USING DEA TO COMPUTE AND DECOMPOSE TFP INDEXES

In principle, any multiplicatively-complete TFP index can be decomposed using the framework outlined in Section 3. This section presents the DEA problems needed to compute and decompose the Moorsteen-Bjurek index. I selected this index from among the class of multiplicatively-complete indexes primarily because it is a distance-based index and DEA methodology for estimating distances is relatively straightforward. A second reason is that it is closely related to the multiplicatively-incomplete Malmquist index that has for some time been the index number of choice in the productivity decomposition literature. For purposes of comparison, this section also presents the DEA problems used by Fare, Grosskopf, Lindgren and Roos (1994a) to compute and decompose that index.

The DEA Approach

Input- and output-oriented DEA models are both underpinned by the assumption that the production frontier is locally linear. In the input-oriented case, local linearity means that for any input vectors in the neighbourhood of x_{nt} the production frontier takes the linear form

(4.1) $\mu' q_{nt} = \alpha + \upsilon' x_{nt}$

where μ and υ are non-negative and α is unsigned. The fact that α is unsigned means the technology potentially exhibits variable returns to scale (VRS): if $\alpha < 0$ the technology exhibits local increasing returns to scale (IRS); if $\alpha \geq 0$ it exhibits local non-increasing returns to scale (NIRS); if $\alpha > 0$ it exhibits local decreasing returns to scale (DRS); and if $\alpha = 0$ it exhibits local constant returns to scale (CRS). In the output-oriented case, local linearity means that for output vectors in the neighbourhood of q_{nt} the production frontier takes the form

(4.2) $\eta' q_{nt} = \beta + \phi' x_{nt}$

where η and ϕ are non-negative and the intercept β is again unsigned to allow for variable returns to scale. Different notation is used for the parameters in (4.1) and (4.2) to make it clear that they are defined with reference to possibly different neighbourhoods.

Associated with the (local) frontiers (4.1) and (4.2) are the (local) input and output distance functions:

(4.3) $D^t_I(x_{nt}, q_{nt}) = \frac{\upsilon' x_{nt}}{\mu' q_{nt} - \alpha} \geq 1$ and

(4.4) $D^t_O(x_{nt}, q_{nt}) = \frac{\eta' q_{nt}}{\beta + \phi' x_{nt}} \leq 1$

DEA involves selecting values of the unknown parameters to minimize the value of the input distance function (4.3) and/or maximise the value of the output distance function (4.4).

A word of caution is in order concerning notation. Strictly speaking, the unknown parameters in equations (4.1) to (4.4) should also have firm and time subscripts to indicate that these relationships only hold for observations in the neighbourhoods of x_{nt} (in the input-oriented case) and q_{nt} (in the output-oriented case). Different functions (i.e., different parameters) may be relevant in the neighbourhoods of other input and output vectors x_{ms} and q_{ms}. I have chosen to suppress these subscripts, partly for notational simplicity, but mainly for consistency with the way DEA problems are presented in the efficiency literature. However, it needs to be remembered that these parameters may change from point to point, so the ratios on the right-hand sides of (4.3) and (4.4) cannot be blindly substituted into equations (2.9) and (2.10) in order to identify the aggregate inputs and outputs corresponding to different input and output vectors.

Primal Problems

The primal input-oriented DEA problem involves selecting values of μ, υ and α in order to minimise $D_I^t(x_{nt}, q_{nt})$ (or, equivalently, choosing parameters to maximize its inverse). Aside from the non-negativity restrictions on μ and υ, the only constraints on the parameters are that they must satisfy $D_I^t(x_{ir}, q_{ir}) \geq 1$ for $i = 1,..., N$ and $r = 1,...,t$. Imposing these constraints at these particular data points (i.e., at the input-output choices of all firms in all periods up to and including period *t*) implicitly prohibits technical regress; if technical regress is to be permitted then the constraints should only be imposed for $i = 1,..., N$ and $r = t$ (all firms in period *t* only). Irrespective of the number of points at which the constraints are imposed, there are infinitely many solutions to the resulting minimisation problem[3]. A common method of identifying a unique solution is to set $\upsilon' x_{nt} = 1$. With this normalizing constraint, the input-oriented DEA linear program (LP) for firm *n* in period *t* takes the form:

$$\text{(4.5a)} \qquad D_I^t(x_{nt}, q_{nt})^{-1} = \max_{\alpha,\mu,\upsilon} \quad \mu' q_{nt} - \alpha$$

$$\text{(4.5b)} \qquad \text{s.t.} \quad \mu' q_{ir} - \upsilon' x_{ir} - \alpha \leq 0 \quad \text{for } i = 1,..., N \text{ and } r = 1,...,t$$

$$\text{(4.5c)} \qquad \upsilon' x_{nt} = 1$$

$$\text{(4.5d)} \qquad \mu, \upsilon \geq 0$$

Primal output-oriented DEA problems involve selecting values of η, κ and β in order to maximise $D_O^t(x_{nt}, q_{nt})$. In the case where technical regress is prohibited, the unknown parameters are constrained so that $D_O^t(x_{ir}, q_{ir}) \leq 1$ for $i = 1,..., N$ and $r = 1,...,t$. A local solution can be identified using the normalisation $\eta' q_{nt} = 1$, in which case the output-oriented analogue of LP (4.5) is

$$\text{(4.6a)} \qquad D_O^t(x_{nt}, q_{nt})^{-1} = \min_{\alpha,\mu,\upsilon} \quad \beta + \phi' x_{nt}$$

$$\text{(4.6b)} \qquad \text{s.t.} \quad -\eta' q_{ir} + \phi' x_{ir} + \beta \geq 0 \quad \text{for } i = 1,..., N \text{ and } r = 1,...,t$$

$$\text{(4.6c)} \qquad \eta' q_{nt} = 1$$

$$\text{(4.6d)} \qquad \eta, \phi \geq 0$$

Problems (4.5) and (4.6) can be solved using standard LP software packages. However, sometimes it is more convenient and enlightening to obtain solutions after rewriting the problem in an alternative, dual, form.

Dual Problems

Every normal primal linear program has a dual form with the property that if the primal and the dual LPs both have feasible solutions then the optimized values of the two objective functions are equal. The dual form of the normal maximisation LP (4.5), for example, is

$$(4.7a) \qquad D_I^t(x_{nt}, q_{nt})^{-1} = \min_{\rho,\theta} \quad \rho$$

$$(4.7b) \qquad \text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \geq q_{nt}$$

$$(4.7c) \qquad \rho x_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \geq 0$$

$$(4.7d) \qquad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1$$

$$(4.7e) \qquad \rho, \theta_{ir} \geq 0 \text{ for } i = 1,...,N \text{ and } r = 1,...,t.$$

If the production possibilities set is convex then this dual input-oriented problem has a very simple interpretation. Convexity of the production possibilities set means that any convex combination[4] of observed data points, such as the double-summations in (4.7b) and (4.7c), are technically feasible output and input levels. This particular LP seeks to scale down the input vector while holding the output vector fixed. Here, the role of the constraints (4.7b) and (4.7c) is to ensure that the observed output and scaled-down input vectors are technically feasible. The constraint (4.7d) holds with strict equality because α in the primal problem (4.5) was unsigned to allow for variable returns to scale. If $\alpha \geq 0$ (non-increasing returns to scale) then the constraint (4.7d) becomes:

$$(4.8) \qquad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} \leq 1$$

If $\alpha_t = 0$ (constant returns to scale) then the constraint (4.7d) is absent from the dual problem altogether. Irrespective of the form of the returns to scale constraint, a basic feasible solution (BFS) to this problem is $\rho = 1$ and $\theta_{ir} = I(i = n, r = t)$, where $I(.)$ is an indicator function that takes the value 1 if the argument is true and 0 otherwise.

The dual form of the output-oriented problem (4.6) has a similar structure:

$$(4.9a) \qquad D_O^t(x_{nt}, q_{nt})^{-1} = \max_{\lambda,\theta} \quad \lambda$$

$$(4.9b) \qquad \text{s.t.} \quad \lambda q_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \leq 0$$

$$(4.9c) \qquad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \leq x_{nt}$$

$$(4.9d) \qquad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1$$

$$(4.9e) \qquad \lambda, \theta_{ir} \geq 0 \text{ for } i = 1,...,N \text{ and } r = 1,...,t.$$

This particular LP seeks to scale up the output vector while holding the input vector fixed. The constraints (4.9b) and (4.9c) ensure the observed input and scaled-up output vector are technically feasible, while the constraint (4.9d) allows for variable returns to scale. Again, to allow for NIRS we simply replace (4.9d) with (4.8), and to allow for CRS we omit (4.9d) altogether. A BFS is $\lambda = 1$ and $\theta_{ir} = I(i = n, r = t)$.

Several other LP problems are needed to compute and decompose the Malmquist and Moorsteen-Bjurek TFP indexes given by equations (2.5) to (2.7). To avoid repetition, the remainder of this section focuses on input-oriented problems. Corresponding output-oriented LPs are presented in the appendix.

Additional LPs for Computing and Decomposing the Input-Oriented Malmquist Index

Either of the dual LPs given by (4.7) or (4.9) can be used to identify the production technology (production frontier), but this alone does not allow us to compute and decompose Malmquist TFP indexes. In the input-oriented case, that involves solving the following additional LPs[5]:

$$(4.10a)\quad D_I^t(x_{ms},q_{ms})^{-1} = \min_{\rho,\theta} \quad \rho$$

$$(4.10b)\quad \text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \geq q_{ms}$$

$$(4.10c)\quad \rho x_{ms} - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \geq 0$$

$$(4.10d)\quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1$$

$$(4.10e)\quad \rho,\theta_{ir} \geq 0 \text{ for } i=1,...,N \text{ and } r=1,...,t.$$

and

$$(4.11a)\quad D_I^s(x_{nt},q_{nt})^{-1} = \min_{\rho,\theta} \quad \rho$$

$$(4.11b)\quad \text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}q_{ir} \geq q_{nt}$$

$$(4.11c)\quad \rho x_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}x_{ir} \geq 0$$

$$(4.11d)\quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1$$

$$(4.11e)\quad \rho,\theta_{ir} \geq 0 \text{ for } i=1,...,N \text{ and } r=1,...,s.$$

These problems do not always have a solution, even under the assumption of constant returns to scale. For example, problem (4.10) is always infeasible if technical regress is permitted and no firms in period *t* produce a positive amount of an output that is produced by firm *m* in period *s*. In that case, irrespective of the returns to scale assumption, there are no values of θ_{ir} that can satisfy constraint (4.10b). If solutions to problems (4.10) and (4.11) are available then it is straightforward to compute the index and implement the Fare, et al. (1994b, p.71) decomposition given by equation (2.8)

Additional LPs for Computing and Decomposing the Moorsteen-Bjurek Index

Computing (but not decomposing) the Moorsteen-Bjurek TFP index involves solving two slightly different, and also possibly infeasible, input-oriented problems:

$$(4.12a)\quad D_I^t(x_{ms},q_{nt})^{-1} = \min_{\rho,\theta} \quad \rho$$

$$(4.12b)\quad \text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \geq q_{nt}$$

$$(4.12c)\quad \rho x_{ms} - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \geq 0$$

$$(4.12d)\quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1$$

$$(4.12e)\quad \rho,\theta_{ir} \geq 0 \text{ for } i=1,...,N \text{ and } r=1,...,t.$$

and

$$(4.13a)\quad D_I^s(x_{nt},q_{ms})^{-1} = \min_{\rho,\theta} \rho$$

$$(4.13b)\quad \text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}q_{ir} \geq q_{ms}$$

$$(4.13c)\quad \rho x_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}x_{ir} \geq 0$$

$$(4.13d)\quad \sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir} = 1$$

$$(4.13e)\quad \rho,\theta_{ir} \geq 0 \text{ for } i=1,...,N \text{ and } r=1,...,s.$$

Decomposing the index into the components identified by O'Donnell (2008) then involves solving a further two LPs. To motivate these additional LPs it is convenient to rewrite the dual problem (4.7) in the alternative form:

$$(4.14a)\quad \bar{X}_{nt}/X_{nt} = \min_{\rho,\theta,v_{nt}} (\iota_K' x_{nt})^{-1}(\iota_K' v_{nt})$$

$$(4.14b)\quad \text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \geq q_{nt}$$

$$(4.14c)\quad v_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \geq 0$$

$$(4.14d)\quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1$$

$$(4.14e)\quad v_{nt} - \rho_t x_{nt} = 0$$

$$(4.14f)\quad \rho, v_{nt},\theta_{ir} \geq 0 \text{ for } i=1,...,N \text{ and } r=1,...,t$$

where ι_K denotes a $K \times 1$ vector of ones, and $\bar{X}_{nt}$ and X_{nt} are the aggregate inputs used in Section 3 to define input-oriented technical efficiency: $ITE_{nt} = \bar{X}_{nt}/X_{nt} = D_I^t(x_{nt},q_{nt})^{-1}$.. To see that the two problems (4.7) and (4.14) are equivalent, simply substitute the equality constraint (4.14e) into both the inequality constraint (4.14c) and the objective function (4.14a). The fact that $\iota_K' x_{nt}$ is a known scalar means the objective function is still linear in the decision variables, meaning the problem (4.14) is still a linear program. This formulation is useful because the constraint (4.14e) makes it explicit that input-oriented technical efficiency involves holding the input mix fixed. Input-oriented mix efficiency measures the improvement in TFP when this constraint is relaxed. When (4.14e) is relaxed, the dual input-oriented DEA problem simply becomes:

$$(4.15a)\quad \hat{X}_{nt}/X_{nt} = \min_{\theta,v} (\iota_K' x_{nt})^{-1}(\iota_K' v)$$

$$(4.15b)\quad \text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \geq q_{nt}$$

$$(4.15c)\quad v - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \geq 0$$

$$(4.15d)\quad \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1$$

$$(4.15e)\quad v,\theta_{ir} \geq 0 \text{ for } i=1,...,N \text{ and } r=1,...,t$$

where $\hat{X}_{nt}$ is the maximum aggregate output that is possible holding the output vector fixed, and $\hat{X}_{nt}/X_{nt} = IME_{nt} \times ITE_{nt}$ is the product of the input-oriented measures of mix and technical efficiency introduced in Section 3. A basic feasible solution to this problem is $v = x_{nt}$ and $\theta_{ir} = I(i=n, r=t)$. At this BFS the value of the objective function is one, implying that the minimized value of the objective function (a measure of the product of technical and mix efficiency) lies in the unit interval.

Observe that problem (4.15) allows the input vector to be chosen freely while holding the output vector fixed. A closely-related LP that relaxes all constraints on both the output and input vectors is given by

$$(4.16a) \qquad TFP_t^* = \max_{\theta_{ir}, z, v} \quad (\iota_K' z)$$

$$(4.16b) \qquad \text{s.t.} \quad z - \sum_{i=1}^{N} \sum_{r=1}^{t} \theta_{ir} q_{ir} \leq 0$$

$$(4.16c) \qquad \sum_{i=1}^{N} \sum_{r=1}^{t} \theta_{ir} x_{ir} - v \leq 0$$

$$(4.16d) \qquad \iota_K' v = 1$$

$$(4.16e) \qquad z, v, \theta_{ir} \geq 0 \text{ for } i = 1,...,N \text{ and } r = 1,...,t$$

where $TFP_t^* = Q_{nt}^* / X_{nt}^*$ denotes the maximum TFP that is possible using the technology available in period *t* (see Section 3). The constraints (4.16b) and (4.16c) ensure that the input and output vectors are technically feasible, while the normalizing constraint (4.16d) identifies a unique solution to the problem in much the same way that constraint (4.5c) identified a unique solution to problem (4.5). A BFS is $z = 0_J$, $v = \iota_K \times K^{-1}$ and $\theta_{ir} = 0$ for all $i = 1,...,N$ and $r = 1,...,t$, where 0_J denotes a $J \times 1$ vector of zeros. At this BFS the value of the objective function is zero, implying $TFP_t^* \geq 0$.

Solutions to the LPs given by (4.5), (4.12) to (4.16) and their output-oriented counterparts are all that is required to decompose the Moorsteen-Bjurek index into the measures of efficiency change defined by O'Donnell (2008). They are also sufficient to separately identify the *levels* of technical efficiency, scale efficiency and mix efficiency for all observed input-output combinations:

$$(4.17) \qquad ITE_{nt} = D_I^t(x_{nt}, q_{nt})^{-1}$$

$$(4.18) \qquad ISE_{nt} = D_I^t(x_{nt}, q_{nt}) / H_I^t(x_{nt}, q_{nt}) \qquad \text{and}$$

$$(4.19) \qquad IME_{nt} = D_I^t(x_{nt}, q_{nt}) \times (\hat{X}_{nt} / X_{nt})$$

where $H_I^t(x_{nt}, q_{nt})$ is the input-distance under the assumption of constant returns to scale.

5. EMPIRICAL EXAMPLE

Coelli and Rao (2005) use DEA to compute and decompose output-oriented Malmquist TFP indexes of agricultural productivity change for 93 countries from 1980 to 2000 permit technical regress and assume the technology exhibits constant returns to scale. This section computes and decomposes Moorsteen-Bjurek TFP indexes using the methodology described in Sections 3 and 4. Technical regress is prohibited and the technology is assumed to exhibit variable returns to scale.

Data

The data was originally sourced from the FAO[6] and comprises observations on the agricultural inputs and outputs of 88 countries for the period 1970 to 2001. Thus, the panel is longer in time-series and slightly shorter in cross-section than the panel used in the Coelli and Rao (2005) study. Data was available for two outputs (crops and animals) and five inputs (land, labour livestock, tractors and fertilizer). For details concerning the construction of these variables, see Coelli and Rao (2005, p.121-122). Some descriptive statistics are reported in Table 1. Observe from Table 1 that all variables have been normalized to have unit means.

Efficiency Scores

Output- and input-oriented measures of technical, scale and mix efficiency for a subset of countries in a subset of years are reported in Table 2. Methods for identifying *levels* of residual mix and scale efficiency are currently unavailable, so this table only reports pure technical, scale and mix efficiency scores. Before examining the numbers more closely, three points should be made. First, the technical and scale efficiency measures reported in this table are the measures that have been used by efficiency and productivity analysts for decades – it is only the measures of mix efficiency that are new. Second, the technology has been estimated using DEA, which makes no allowance for statistical noise, so any measurement errors in the data will be manifest in these estimates of efficiency. Third, like the vast majority of efficiency studies appearing in the literature, the technology has been specified in a way that does not take proper account of risk, and there is strong evidence that this failure can lead to downwardly biased estimates of technical efficiency – see O'Donnell, Chambers and Quiggin (2006).

With this in mind, observe that, in 1970 and 1971, Australian farmers were technically and scale efficient, and were producing a productivity-maximizing combination of outputs. However, they were input-mix inefficient – then, as now, Australian agricultural production was characterised by large land to labour and land to capital ratios. Also observe that New Zealand agriculture was highly inefficient in 1975 and in 1979. There was a large increase in recorded output-oriented technical efficiency in 1976, and an equally large fall in that measure in 1979. These changes give rise to a pattern of TFP change that is slightly more exaggerated than the pattern observed in other estimates of NZ agricultural TFP – see Forbes and Johnson (2001). Finally, observe that Zimbabwe was technically-, scale- and mix-efficient in 1999. However, the efficiency measures reported in Table 2 are not exhaustive, and residual measures of efficiency for Zimbabwe are less than one.

The Components of TFP Change

The efficiency estimates reported in Table 2 have been used to compute the estimates of TFP and efficiency change reported in Table 3. All values reported in this table are non-cumulative firm-specific index numbers, which means they measure year-on-year changes for individual countries. Among other things, the estimates reported in Table 3 indicate that productivity in Australian agriculture was pretty much the same in 1971 as it had been in 1970, despite the fact that there had been significant technical progress – at that time there was a 10.5% increase in the maximum TFP possible using the available technology. The frontier moved, Australia stood still, and this was measured as a fall in efficiency. Table 3 also reveals that NZ agricultural TFP was 2.2 times higher in 1971 than it had been in 1970, mainly due to the large increase in output-oriented technical efficiency evident in Table 2. The magnitude of this increase was matched only by the magnitude of the fall a couple of years later – the maximum and minimum values reported at the bottom of Table 3 reveal that the large increases and decreases in NZ TFP during the 1970s were, in fact the largest increases and decreases in TFP recorded in the sample. Finally, the fastest rate of technical progress was 10.5%, snd that occurred in the first year of the sample. The average rate of technical change across the entire sample was 0.9% per annum, slightly less than the 1.1% reported by Coelli and Rao (2005) using a Malmquist approach and allowing for technical regress.

A clearer picture of the nature of TFP change in Australia, New Zealand and the USA is provided in Figures 5 to 7. These graphs are cumulative measures of the components of TFP change:. The dotted lines are the estimated indexes of technical change, while the solid black lines are the TFP indexes. Only these two lines are indexes, and they are normalized to take the value 1 in 1970. The remaining lines are levels of pure technical, scale and mix efficiency. Importantly, these pure efficiency effects and are not exhaustive – there are some residual scale and mix efficiency indexes that do not appear on these graphs.

Observe from Figure 5 that the index of technical change is non-decreasing, in line with the decision to rule out technical regress. TFP in Australian agriculture fell behind world's best practice in periods 6 through 20 (1975 to 1990) due to technical inefficiency. Governments can improve technical efficiency through education and extension programs, and it would be interesting to see what was happening to agricultural extension services during the late 1980s. By the end of the sample period, Australian agricultural TFP was only 10% higher than it had been in 1970. This represents an annual average increase in TFP of only 0.3%, only a fraction of the rate estimated broadacre agriculture Mullen, Scobie and Crean (2008). This suggests that TFP growth in broadacre agriculture has outpaced TFP growth in agriculture more generally.

Figure 6 shows that, with the exception of a couple of years in the mid-1970s, NZ farmers have been technically inefficient for most of the sample period. This graph suggests raises questions concerning the quality of the data in the late 1970s. Smoother estimates of TFP change would almost certainly be obtained if the technology was estimated using a methodology that accounted for statistical noise.

Finally, Figure 7 reveals that the rate of TFP growth in US agriculture has been twice the rate of technical progress. All pure efficiency scores have been quite high, implying that this improvement in TFP has been driven by residual scale and mix effects. Indeed, the average annual rate of growth in residual input scale efficiency was 2.7%. Interestingly, this number is fairly close to the TFP growth rate estimated by Coelli and Rao (2005). This is not surprising – those authors used a constant-returns-to-scale Malmquist methodology that does not properly account for the effects of either mix or scale.

Peers

Within this framework it is straightforward to compute peers and targets for each country in each time period. Of most interest is the TFP-maximizing country in each time period. Table 4 lists these countries and the estimated maximum TFP. Maximum possible TFP was approximately 35%% higher at the end of the sample period than it was at the start of the sample period. Observe that only two countries are represented in Table 4: Nepal and Thailand.

Profitability

Changes in the agricultural terms of trade may have drawn large agricultural producers away from TFP-maximizing input-output points. The importance of the terms of trade is illustrated in Figure 4, which depicts a measure of profit efficiency in aggregate output space. In Figure 4, the dashed lines passing through points A, E and K are isoprofit lines with the same slope W_t / P_t but different intercepts. Among other things, Figure 4 illustrates that i) Firm A maximizes profit at the technically-efficient but scale-inefficient point K, ii) the profit-maximizing point of production does not necessarily coincide with the TFP-maximizing point of production, iii) the profit efficiency of Firm A can be decomposed into a component measuring the change in profits as Firm A moves to maximize TFP (at point E), and a further change in profits as Firm A moves to maximize profits (at point K), and iv) the profit- and TFP-maximizing points coincide (at point E) if and only if maximum TFP (the slope of the ray through point E) equals the inverse of the terms of trade (the slope of the isoprofit line) (i.e., if $Q_t^* / X_t^* = W_t / P_t = \tan e$).

6. CONCLUSION

One of the main aims of this paper has been to demonstrate that decomposition of the Moorsteen-Bjurek index is empirically feasible using large real-world data sets. Estimates of TFP and efficiency change were obtained using DEA programs that prohibited technical regress but allowed for variable returns to scale. A problem with DEA methodology is that it makes no allowance for statistical noise. Some of the large variations in the estimates reported in this study may be due to noise, but they may also arise from failure to properly account for the risky nature of the production process. There is scope to use existing econometric estimators and more flexible representations of the technology to obtain even more reliable estimates of agricultural TFP and TFP change. In summary:

- there is a large class of TFP indexes that can be decomposed into measures of technical change, technical efficiency change, mix efficiency change, and scale efficiency change.
- Improvements in technical efficiency will lead to increases in both TFP and profitability. Policies designed to improve technical efficiency include education and extension programs.

- Except in very special cases, maximising TFP reduces profitability.
- Changes in the terms of trade can be expected to induce changes in production patterns and measures of TFP change.
- Restrictions on input and output choices may prevent countries from maximizing TFP and/or profitability. Restrictions may take the form of absolute resource constraints (e.g., land) and government regulations (e.g., restrictions on the production of GM food).

APPENDIX

Output-oriented problems that correspond to the input oriented problems (3.10) to (3.13) and (3.15) in the main text are:

(A.10a) $D_O^t(x_{ms}, q_{ms})^{-1} = \max_{\lambda,\theta} \quad \lambda$

(A.10b) s.t. $\lambda q_{ms} - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \leq 0$

(A.10c) $\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \leq x_{ms}$

(A.10d) $\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1$

(A.10e) $\lambda, \theta_{ir} \geq 0$ for $i = 1,..., N$ and $r = 1,..., t$.

(A.11a) $D_O^s(x_{nt}, q_{nt})^{-1} = \max_{\lambda,\theta} \quad \lambda$

(A.11b) s.t. $\lambda q_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}q_{ir} \leq 0$

(A.11c) $\sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}x_{ir} \leq x_{nt}$

(A.11d) $\sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir} = 1$

(A.11e) $\lambda, \theta_{ir} \geq 0$ for $i = 1,..., N$ and $r = 1,..., s$.

(A.12a) $D_O^t(x_{nt}, q_{ms})^{-1} = \max_{\lambda,\theta} \quad \lambda$

(A.12b) s.t. $\lambda q_{ms} - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \leq 0$

(A.12c) $\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \leq x_{nt}$

(A.12d) $\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1$

(A.12e) $\lambda, \theta_{ir} \geq 0$ for $i = 1,..., N$ and $r = 1,..., t$.

(A.13a) $D_O^s(x_{ms}, q_{nt})^{-1} = \max_{\lambda,\theta} \quad \lambda$

(A.13b) s.t. $\lambda q_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}q_{ir} \leq 0$

(A.13c) $\sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}x_{ir} \leq x_{ms}$

(A.13d) $\sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir} = 1$

(A.13e) $\lambda, \theta_{ir} \geq 0$ for $i = 1,..., N$ and $r = 1,..., s$.

(A.15a) $$\hat{Q}_{nt} / Q_{nt} = \max_{\theta, z_{nt}} \quad (\iota_K' q_{nt})^{-1} (\iota_K' z_{nt})$$

(A.15b) $$\text{s.t.} \quad z_{nt} - \sum_{i=1}^{N} \sum_{r=1}^{t} \theta_{ir} q_{ir} \leq 0$$

(A.15c) $$\sum_{i=1}^{N} \sum_{r=1}^{t} \theta_{ir} x_{ir} \leq x_{nt}$$

(A.15d) $$\sum_{i=1}^{N} \sum_{r=1}^{t} \theta_{ir} = 1$$

(A.15e) $$z_{nt}, \theta_{ir} \geq 0 \text{ for } i = 1, ..., N \text{ and } r = 1, ..., t.$$

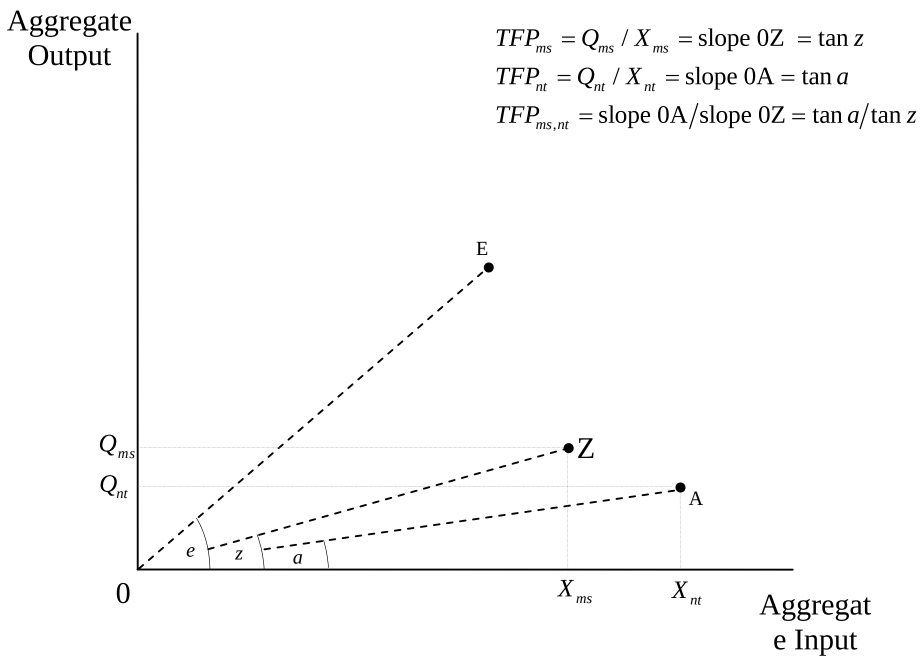

Figure 1. Total Factor Productivity Change

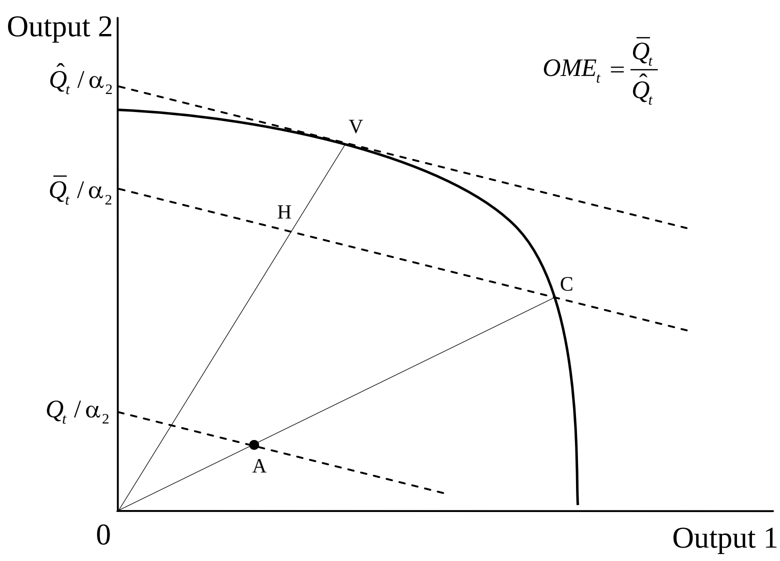

Figure 2. Output-Oriented Mix Efficiency for a Two Output Firm

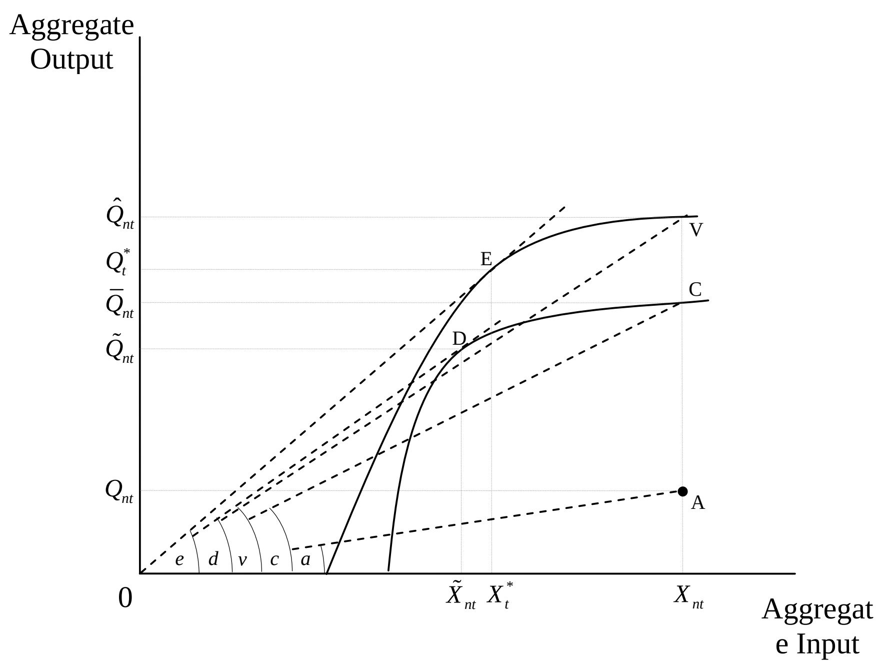

Figure 3. Output-Oriented Decompositions of TFP Efficiency

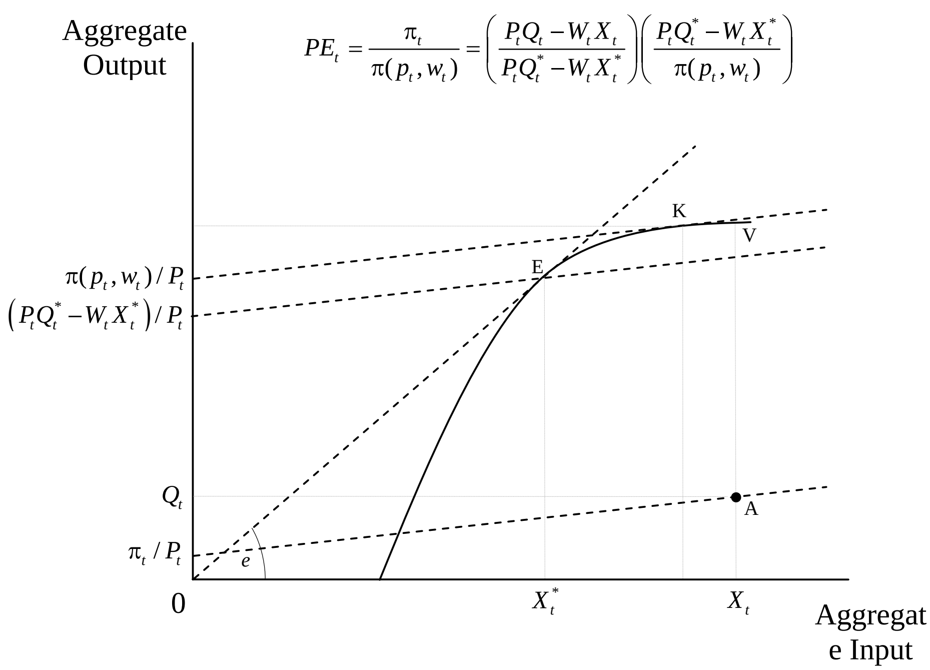

Figure 4. Profit Efficiency for a Multiple-Input Multiple-Output Firm

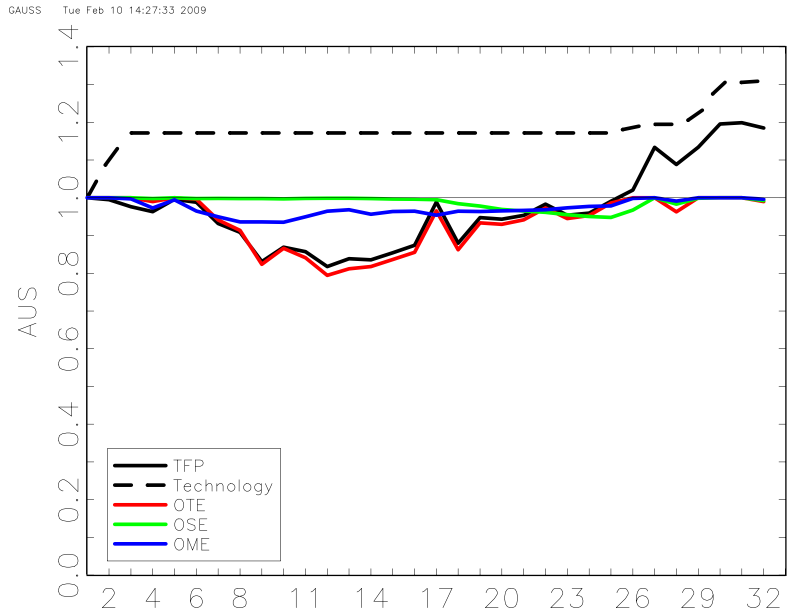

Figure 5. Components of TFP Change: Australia, 1970-2001

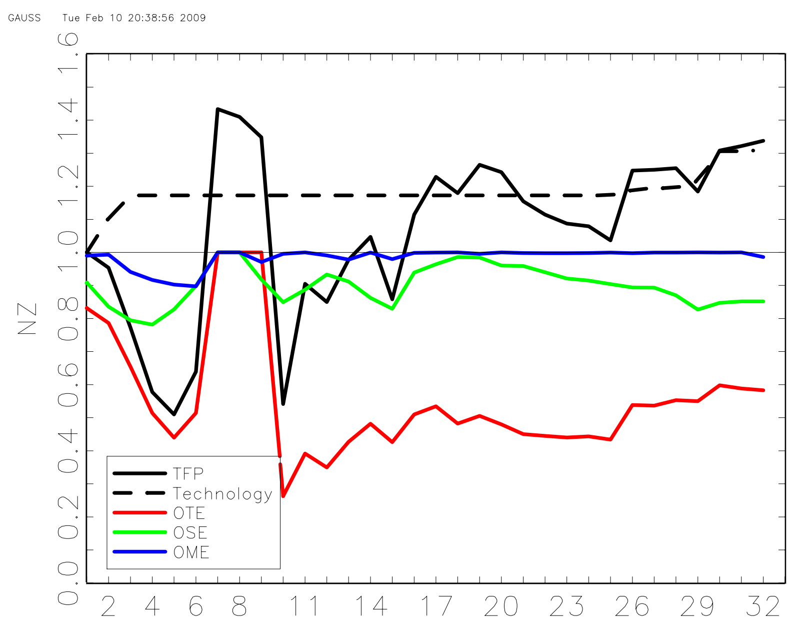

Figure 6. Components of TFP Change: New Zealand, 1970-2001

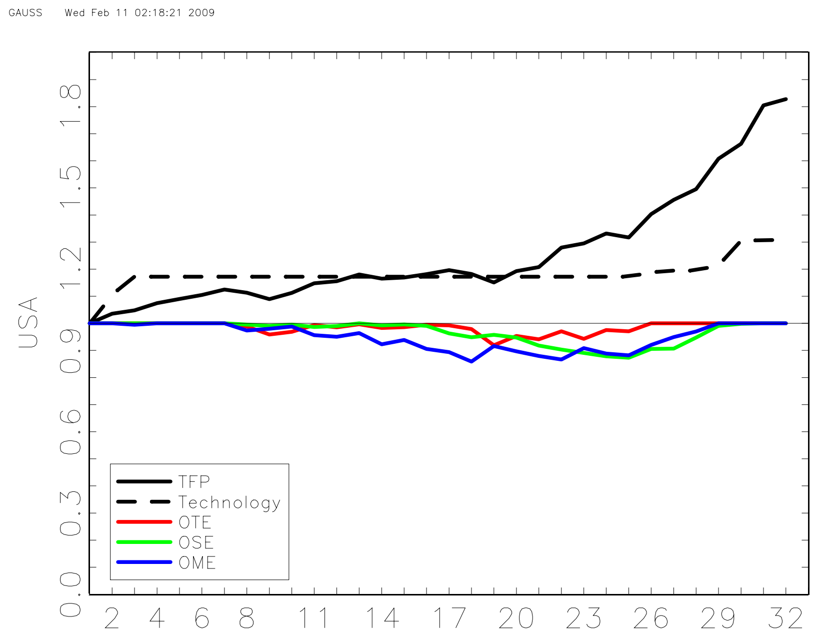

Figure 7. Components of TFP Change: United States, 1970-2001

Table 1. DESCRIPTIVE STATISTICS

	Mean	SD	Min	Max
CROP_OUT	1.000	2.695	0.003852	32.24
ANIMAL_O	1.000	2.604	0.009128	31.37
AREA	1.000	2.063	0.01215	12.84
LABOUR	1.000	4.420	0.005800	42.97
LIVESTOC	1.000	2.401	0.01012	20.77
TRACTORS	1.000	2.830	1.406e-005	24.69
FERTILIZ	1.000	3.142	8.273e-005	33.44
CROP_OUT per AREA	2.068	2.671	0.001237	20.25
CROP_OUT per LABOUR	4.132	6.932	0.1424	52.22
CROP_OUT per LIVESTOC	1.446	1.859	0.008842	13.29
CROP_OUT per TRACTORS	29.29	201.5	0.05071	5382.
CROP_OUT per FERTILIZ	17.90	82.66	0.09166	2098.
ANIMAL_O per AREA	2.764	4.900	0.009106	39.07
ANIMAL_O per LABOUR	10.92	22.69	0.04081	142.4
ANIMAL_O per LIVESTOC	1.282	1.248	0.1186	7.868
ANIMAL_O per TRACTORS	12.68	38.53	0.1931	718.6
ANIMAL_O per FERTILIZ	8.555	27.37	0.1420	529.8
CROP_OUT per ANIMAL_O	1.834	2.115	0.01802	14.17
AREA per LABOUR	8.908	33.26	0.06434	306.3
AREA per LIVESTOC	1.794	4.195	0.1080	59.78
AREA per TRACTORS	54.42	222.2	0.01228	3138.
AREA per FERTILIZ	38.00	137.5	0.05208	2528.
LABOUR per LIVESTOC	1.190	1.581	0.01088	9.832
LABOUR per TRACTORS	59.00	390.1	0.009331	1.110e+004
LABOUR per FERTILIZ	33.71	153.5	0.01353	2894.
LIVESTOC per TRACTORS	35.66	150.2	0.03641	3631.
LIVESTOC per FERTILIZ	23.39	91.15	0.05846	1900.
TRACTORS per FERTILIZ	1.746	4.188	0.0005408	98.15

Table 2. MEASURES OF TECHNICAL, SCALE and MIX EFFICIENCY

Obs	Year	Country	OTE (D)	OSE (E)	OME (F)	ITE (H)	ISE (I)	IME (J)
2	1970	ANG	0.5011	0.9777	0.9992	0.4901	0.9997	0.2210
3	1970	ARG	1.000	1.000	1.000	1.000	1.000	0.5178
4	1970	AUS	1.000	1.000	1.000	1.000	1.000	0.3464
5	1970	AUT	0.9046	0.9274	0.9849	0.9903	0.8472	0.09643
90	1971	ANG	0.4719	0.9522	0.9933	0.4594	0.9781	0.2267
91	1971	ARG	1.000	1.000	1.000	1.000	1.000	0.5137
92	1971	AUS	1.000	1.000	1.000	1.000	1.000	0.3327
93	1971	AUT	0.8175	0.9624	0.9199	0.9533	0.8253	0.09910
497	1975	NZL	0.5145	0.8985	0.8978	0.5577	0.8288	0.5354
585	1976	NZL	1.000	1.000	1.000	1.000	1.000	0.3256
673	1977	NZL	1.000	1.000	1.000	1.000	1.000	0.3388
761	1978	NZL	1.000	0.9179	0.9712	1.000	0.9179	0.3452
849	1979	NZL	0.2631	0.8492	0.9958	0.5277	0.4234	0.6716
2724	1999	USA	1.000	1.000	1.000	1.000	1.000	0.5393
2725	1999	URU	0.8424	0.7193	0.9483	0.7036	0.8613	0.8755
2726	1999	VEN	1.000	0.9878	0.9908	1.000	0.9878	0.4218
2727	1999	VIE	1.000	1.000	1.000	1.000	1.000	0.8697
2728	1999	ZIM	1.000	1.000	1.000	1.000	1.000	1.000
2812	2000	USA	1.000	1.000	1.000	1.000	1.000	0.5468
2813	2000	URU	0.8049	0.7150	0.9267	0.6530	0.8813	0.8525
2814	2000	VEN	1.000	1.000	1.000	1.000	1.000	0.4274
2815	2000	VIE	1.000	1.000	1.000	1.000	1.000	0.9281
2816	2000	ZIM	0.9787	0.9858	0.9675	0.9891	0.9754	0.9338
Mean			0.8037	0.8518	0.9494	0.7917	0.8647	0.5214
Minim			0.1854	0.2959	0.4907	0.2007	0.2959	0.06205
Maxim			1.000	1.000	1.000	1.000	1.000	1.000

Table 3. OUTPUT-ORIENTED DECOMPOSITION OF MOORSTEEN-BJUREK TFP INDEX

Obs	Year	Ctry	TFP Index = (A)	Tech Change x (B)	Eff Change (C)	dOTE (D)	dOSE (E)	dOME (F)	dROSE (G)
90	1971	ANG	0.9462	1.105	0.8561	0.9418	0.9738	0.9942	0.9144
91	1971	ARG	0.9258	1.105	0.8377	1.000	1.000	1.000	0.8377
92	1971	AUS	0.9951	1.105	0.9003	1.000	1.000	1.000	0.9003
93	1971	AUT	0.9610	1.105	0.8695	0.9037	1.038	0.9340	1.030
:	:	:	:	:	:	:	:	:	:
497	1975	NZL	1.253	1.000	1.253	1.171	1.085	0.9949	1.075
585	1976	NZL	2.244	1.000	2.244	1.944	1.113	1.114	1.036
673	1977	NZL	0.9838	1.000	0.9838	1.000	1.000	1.000	0.9838
761	1978	NZL	0.9562	1.000	0.9562	1.000	0.9179	0.9712	0.9846
849	1979	NZL	0.4017	1.000	0.4017	0.2631	0.9252	1.025	1.489
937	1980	NZL	1.671	1.000	1.671	1.488	1.045	1.004	1.118
:	:	:	:	:	:	:	:	:	:
2812	2000	USA	1.013	1.003	1.010	1.000	1.000	1.000	1.010
2813	2000	URU	0.9686	1.003	0.9659	0.9555	0.9939	0.9773	1.034
2814	2000	VEN	1.008	1.003	1.005	1.000	1.012	1.009	0.9961
2815	2000	VIE	1.002	1.003	0.9993	1.000	1.000	1.000	0.9993
2816	2000	ZIM	0.9957	1.003	0.9928	0.9787	0.9858	0.9675	1.049
Mean			1.013	1.009	1.004	0.9992	0.9992	0.9993	1.006
Minim			0.4017	1.000	0.4017	0.2631	0.5625	0.5878	0.5919
Maxim			2.244	1.105	2.244	1.944	1.864	1.696	1.681

Table 4. TFP-MAXIMIZING COUNTRIES

Year	Country	TFP
1970	NEP	1.032
1971	NEP	1.140
1972	NEP	1.209
1973	NEP	1.209
1974	NEP	1.209
1975	NEP	1.209
1976	NEP	1.209
1977	NEP	1.209
1978	NEP	1.209
1979	NEP	1.209
1980	NEP	1.209
1981	NEP	1.209
1982	NEP	1.209
1983	NEP	1.209
1984	NEP	1.209
1985	NEP	1.209
1986	NEP	1.209
1987	NEP	1.209
1988	NEP	1.209
1989	NEP	1.209
1990	NEP	1.209
1991	NEP	1.209
1992	NEP	1.209
1993	NEP	1.209
1994	NEP	1.209
1995	NEP	1.226
1996	THA	1.232
1997	THA	1.232
1998	THA	1.254
1999	THA	1.347
2000	THA	1.347
2001	NEP	1.351

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[1] O'Donnell (2008) uses the term *multiplicatively admissible*, with the harsh implication that any index number that violates this property is inadmissible as a TFP index. However, TFP indexes that violate this property can still be used provided we recognize that they overlook one or more components of TFP change. The term *multiplicatively complete* carries with it the more palatable implication that any index number that violates this property is simply incomplete.

[2] There also exists a class of *additively complete* TFP index numbers. Bennet (1920) -type TFP indexes are members of this class.

[3] To see this, simply observe from the structure of the input distance function (4.3) that if $(\alpha^*, \mu^*, \upsilon^*)$ minimises $D_I^t(x_{nt}, q_{nt})$ then $(\lambda\alpha^*, \lambda\mu^*, \lambda\upsilon^*)$ also minimizes $D_I^t(x_{nt}, q_{nt})$ for all $\lambda > 0$.

[4] A *convex combination* is a linear combination of points where all coefficients are non-negative and sum to 1. A convex combination of two points lies on the straight line segment connecting those two points.

[5] Computing the input-oriented Malmquist index also involves computing $D_I^s(x_{ms}, q_{ms})$. This distance measure is simply the value of $D_I^t(x_{nt}, q_{nt})$ for the base firm in the base period. Thus, it can be computed using LP (4.7).

[6] I am grateful to Tim Coelli and Prasada Rao for providing the data.