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UTILIZATION OF RELATIVE LAND ALLOCATION IN THE CALIBRATION OF AGRICULTURE SUPPLY MODELS BY THE PMP APPROACH

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Abstract:

This poster proposes a new procedure in agriculture supply modelling by the positive mathematical programming (PMP) approach. This approach is now widely used in last CAP reform simulations. However, simulation behaviour and performances of PMP procedures depend of the way parameters of the non linear total cost function in the objective function are recovered. We propose a new specification of the total cost function where land is explicitly considered as a fixed input. By using relative parts of land of the different activities this new PMP procedure permits to better capture production behaviour when economic conditions. It also permits to avoid a drawback of the early procedures concerning marginal activities.

Key words : Positive Mathematical Programming, CAP

JEL Code: Q10, Q18

Introduction

Positive mathematical programming (PMP) approach formulated by Richard Howitt (1995) uses crop allocations and costs observed in a base year to generate non linear programming models exactly calibrated on this base year situation. This calibration advantage which is hardly obtained in classical programming models added to the simplicity to implement made PMP approach widely used in programming models built for common agricultural policy reforms simulations ¹. However these models are relatively recent and simulation behaviour assessments and methodology improvements are still needed.

PMP approach combines mathematical programming and econometrics frameworks. A non linear objective function is redefined with a cost or yield function specification. This function have to describe the production behaviour so it conditions the ability of the resulting programming model to correctly respond to changes in economics conditions. As few statistical information is available to specify and estimate these econometric functions a dilemma appears. A choice of a very simple specification to limit the number of parameters to estimate leads to the risk of a poor production behaviour representation and a choice of more flexible specification leads to insoluble identification problems².

In standard PMP procedures cost (or yield) functions for each land-use activity are estimated separately from each other. Activity interactions are only considered in the programming model via fixed resource constraints. Yet, land fixity has a direct influence on total cost through crop rotations and other constraints which are not considered in the programming model.

In this poster, we consider the land fixity directly in total cost specification through a restricted cost function. The proposed specification exploits the information on parts of land used for each crop in total land. This specification is very simple and close to quadratic function with a diagonal matrix, but parameters for each activity are determined via the other activities. This permits to avoid linear cost function for marginal activities which is an important drawback in standard PMP procedures. A numerical example is given to illustrate the differences between the proposed procedure and some other standard ones.

PMP procedure with land restricted cost

The idea of positive mathematical programming calibration procedure is generally exposed in two phases. In *phase 1*, a linear programming problem is stated for the base year including flexibility constraints which limit production to the observed levels figures:

¹ Among programming models using PMP approach one can cite: CAPRI, MOREA, SEPALE...

² See Heckeles and Britz (2005) for a review of various materializations of the PMP approach.

$$\text{Max}_x Z = (p.r)' x - c' x \quad (1)$$

Subject to:

$$A x \leq b \quad (\lambda)$$

$$x \leq \bar{x} + \varepsilon \quad (\gamma)$$

$$x \geq 0$$

where x is the vector of production levels of the N activities, p the vector of product prices, r the vector of yields, c the vector of accounting cost per unit, A the matrix of coefficients in resource constraints, b the vector of available resource levels, \bar{x} the vector of observed activity levels, λ the vector of dual variables associated with the resource constraints and γ the vector of dual variables associated with the calibration constraints. ε is a vector of small positive numbers included to avoid programming degeneracy problem.

The classical interpretation of the constraints of the dual values of model (1) says that variable marginal cost at the observed production levels is equal to $\gamma + c$. *Phase 2* of PMP exploits this interpretation to specify a non-linear objective function by replacing the total variable cost by a function C satisfying this interpretation:

$$\frac{\partial C(\bar{x})}{\partial x} = c + \gamma \quad (2)$$

Any functional form satisfying the right curvature properties of the cost function (convex in activity levels) could be used. Quadratic form is generally used for its computational simplicity. However, condition (2) is not sufficient to uniquely determine the parameters. Different solutions were proposed in the literature. They consist to reduce some parameters in the cost function specification or to use prior information and specific econometric criterion for underdetermined problem³. One of the standard solution consists to remove cross terms and consider a diagonal quadratic matrix. Even if cross terms are removed Heckeli and Britz (2005) show that this function is relevant for the supply response of each product as fixed allocable inputs (resource constraints) still link all production activities with each other. For an activity i , the total variable cost at level x_i is

$$C(x)_i = (d_i + q_i x_i) x_i \quad \text{for } i=1, \dots, N \quad (3)$$

d_i and q_i are unknown parameters. We assume that the observed accounting cost corresponds to the average cost at \bar{x}

$$c_i = d_i + q_i \bar{x}_i \quad \text{for } i=1, \dots, N \quad (4)$$

Equation (2) and (4) uniquely determine d and q parameters

$$d_i = \bar{c}_i - \gamma_i \quad ; \quad q_i = \frac{\gamma_i}{\bar{x}_i} \quad i=1, \dots, N \quad (5)$$

³ Heckelei and Britz (2005) give a short overview of principal methods.

This solution gives poor simulation results as shown for instance in Heckeley and Britz (2000). It has also a drawback in determining the cost function for marginal activities where the flexibility constraint is not bending ($\gamma_m=0$). Marginal costs of these activities are constant and that is not compatible with the PMP principle. As a consequence the least profitable activities are wrongly advantaged in the simulations. Ad hoc solutions considers that the opportunity cost of a marginal activity is a prior percentage of the dual value of one resource constraint. For more flexibility in the simulations one can consider total cost function of degree tree

$$C(x)_i = (d_i + q_i x_i^2) x_i \quad \text{for} \quad i=1, \dots, N \quad (6)$$

Parameters are then

$$d_i = \bar{c}_i - \gamma_i/2 \quad ; \quad q_i = \frac{\gamma_i}{2 \bar{x}_i^2} \quad i=1, \dots, N \quad (7)$$

In this case marginal cost functions are quadratic with sharper tangents at the observed activity levels. Marginal gross margins according to activity levels are reduced especially for activities using less fixed resource. This advantage improves simulation behaviours according to the last specification but it still not clearly justified by production or economic considerations.

Agriculture supply modelling by PMP approach is generally used in a short term perspective where at least land input is unchanged. However, most total variable cost specifications are unrestricted cost functions. Resource constraints are only considered in the programming problem. Yet variable total cost is influenced by land fixity through agronomic constraints. A change in crop rotation could lead to agronomic problems and so to an extra cost to maintain yields. In an assessment of simulation behaviour of different PMP procedures, Gocht (2005) suggests more investigations in the PMP modelling structure and simulation results.

In an attempt to improve the variable total cost specification in capturing production behaviour, we suggest to directly introduce land input as an argument in our cost function specification. If one note S the fixed land the following specification is proposed

$$C(x)_i = (d_i + q_i x_i/S) x_i \quad \text{for} \quad i=1, \dots, N \quad (8)$$

Average cost at the observed allocation is

$$c_i = d_i + q_i \bar{x}_i/S \quad \text{for} \quad i=1, \dots, N \quad (9)$$

As all lands are used in the production, the proposed function is equivalent to a quadratic function and follows the right properties of a cost function. Contrary to formulation

(3) activity average cost don't increase according to the absolute surface level but to its relative level in total land fix input. This relation could be justified by land quality as suggested by Howitt (1995) but also by agronomic considerations. In the extreme case of monoculture more costs are needed to maintain the yields due to agronomic conditions of production.

Using relations (2) and (7) parameters q and q are calculated as

$$q = S^2 \begin{pmatrix} \bar{x}_1(S-\bar{x}_1) & -\bar{x}_2^2 & \dots & -\bar{x}_N^2 \\ -\bar{x}_1^2 & \bar{x}_2(S-\bar{x}_2) & \dots & -\bar{x}_N^2 \\ \vdots & \vdots & \ddots & \vdots \\ -\bar{x}_1^2 & -\bar{x}_2^2 & \dots & \bar{x}_N(S-\bar{x}_N) \end{pmatrix}^{-1} \gamma$$

and

$$d_i = c_i - q_i \bar{x}_i / S \quad \text{for } i=1, \dots, N \quad (10)$$

Interactions between activities is effective in our total cost specification as parameters are determined from dual values and observed allocation levels of all crops. Also the problem of marginal activities is no longer posed in this formulation.

To give more flexibility to the former specification one can consider a quadratic term in the average cost

$$C(x)_i = (d_i + q_i (x_i/S)^2) x_i \quad \text{for } i=1, \dots, N \quad (11)$$

parameters are then calculated as

$$q = S^3/2 \begin{pmatrix} \bar{x}_1^2(S-\bar{x}_1) & -\bar{x}_2^3 & \dots & -\bar{x}_N^3 \\ -\bar{x}_1^3 & \bar{x}_2^2(S-\bar{x}_2) & \dots & -\bar{x}_N^3 \\ \vdots & \vdots & \ddots & \vdots \\ -\bar{x}_1^3 & -\bar{x}_2^3 & \dots & \bar{x}_N^2(S-\bar{x}_N) \end{pmatrix}^{-1} \gamma$$

and

(12)

$$d_i = c_i - d_i \left(\frac{\bar{x}_i}{S} \right)^2 \quad \text{for } i=1, \dots, N$$

An illustrative example

To illustrate the PMP procedures exposed above we consider a numerical example of a farm of 100 hectares producing wheat, other cereals and rape seed. This numerical example is a simplified representation of an average farm in a field crop orientation region of France. The observed acreage allocation and production conditions in a base year are

	Wheat	Other cereals	Rape seed	Set aside
Surface (ha)	55	25	10	10
yield (t/ha)	8.2	6.4	2.9	
price (€/t)	104	117	184	
Variable cost (€/ha)	379	366	204	100

We consider two scenarios to illustrate and compare simulation performances of PMP procedures presented above:

Scenario 1 = 20% wheat price increase with all things being equal.

Scenario 2 = 20 % rape seed price increase with all things being equal.

These two scenarios are then simulated using the PMP procedures bases on the following cost function specifications:

$$M1 : C(x)_i = (d_i + q_i x_i) x_i \quad i=1, \dots, N$$

$$M2 : C(x)_i = (d_i + q_i x_i^2) x_i \quad i=1, \dots, N$$

$$M3 : C(x)_i = (d_i + q_i x_i/S) x_i \quad i=1, \dots, N$$

$$M4 : C(x)_i = (d_i + q_i (x_i/S)^2) x_i \quad i=1, \dots, N$$

The objective is not to prove the superiority of one procedure but to show the gaps that could appear in simulation results and the unlikelihood results of some procedures. Simulation results are resumed in the table below.

	Scenario 1			Scenario2		
	Wheat (ha)	Other cereals (ha)	Rape seed (ha)	Wheat (ha)	Other cereals (ha)	Rape seed (ha)
Base year	55,0	25,0	10,0	55.0	25.0	10.0
M1	63.6 (15.6)	19.5 (-22.0)	6.9 (-30.6)	53.1 (-3.5)	23.7 (-5.2)	13.2 (32.1)
M2	60.8 (10.5)	21.3 (-14.7)	7.9 (-21.2)	53.8 (-2.1)	24.2 (-3.2)	12.0 (19.8)
M3	60.8 (10.5)	21.1 (-15.5)	8.1 (-19.0)	53.8 (-2.1)	24.3 (-2.9)	11.9 (18.9)
M4	57.9 (5.2)	23.0 (-7.9)	9.1 (-1.0)	54.5 (-1.0)	24.7 (-1.2)	10.9 (8.5)

(*) figures in brackets indicate evolutions in percentage.

Results show a great contrasts in different procedure responses to price changes. Note that cost function parameters for rape seed are not determined directly in M1 and M2 procedures. Marginal cost of rape seed at the observed level is put -a priori- at 20% of land opportunity cost to avoid constant marginal cost. This hypothesis is not necessary in M3 and M4 procedures.

Effect of price increase are accentuated in procedures M1 and M2 comparatively to M2 and M3. However the mathematical form of the cost function has a great impact. Quadratic average cost function form in procedure M4 leads to half price elasticity in comparison with an affine average cost function form in procedure M3. Assessment of M3 and M4 procedures needs more investigations with ex-post simulations.

Conclusion

Information in relative land allocation is exploited to specify total variable cost function in a new PMP procedure. This procedure aims to improve simulation behaviour of PMP approach and resolve the drawback concerning marginal activities. However more ex-post simulations are needed to prove the interest of this new procedure according to the early ones.

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