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**Analysis of Selected Vegetables and Fruits in U.S. Market:
An Application of Demand Systems**

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Abstract

Demand analyses have been known to be quite sensitive to the chosen functional forms. Since no one specification fits all data the best, researchers have been preoccupied with finding ways to select among various functional forms. This study addresses this concern by proposing a formulation which obviates the need to choose among various functional forms. The approach is tested using four functional forms of direct demand system (RDS, CBS, AIDS, and NBR) and four functional forms of inverse demand system (RIDS, Laitinen-Theil, AIIDS, and the hybrid RIDS-AIIDS) on wholesale data for selected vegetables and fruits.

Introduction

Several studies in the past have considered the issue of how to choose among popular functional forms when conducting demand analyses. Parks (1969) used the average information inaccuracy concept. A relatively high average inaccuracy is taken to be an indicator of less satisfactory behavior. Deaton (1978) applied a non-nested test to compare demand systems with the same dependent variables. However, this procedure is not suitable when comparing models with different dependent variables as in the case of comparing the Almost Ideal Demand System with the Rotterdam Demand System. Barten (1993) developed a method that can deal with non-nested models with different dependent variables. Briefly, the method starts with a hypothetical general model as a matrix weighted linear combination of two or more basic models. A solution is found for one of the dependent variables, followed by estimating consistently the transformed matrix weights associated with the other models. Next, statistical tests are carried out on the matrix weights to determine whether they are significantly different from zero. This matrix

weighted linear combination can be considered as a synthetic demand allocation system which under appropriate restrictions yields the different forms of the demand system. The synthetic model can therefore be used to statistically test which model best fits a particular data set. A drawback in applying this procedure is that it is necessary to impose a set of restrictions for the purpose of estimating. For example, the differentials need to be replaced by finite first differences and the budget shares by their moving averages. This affects the coefficients of the demand system which are functions of the moving average of the budget share. As a result, each functional form produces a different set of elasticities. In order to address this problem, we propose a formulation which greatly simplifies the process.

Accordingly, the primary objective of this research is to propose a formulation that obviates the need to choose among the popular functional forms when conducting demand analysis. Specifically, our goal will be to show that when the proposed system is adopted the elasticities across the various functional forms are the same. Four functional forms of the direct demand system and four functional forms of the inverse demand system will be investigated using data on selected fruits and vegetables. A secondary objective of the paper is to analyze and discuss the computed elasticities obtained from the direct and inverse demand systems.

The paper proceeds as follows. The next section commences with a brief discussion of the rationale for our proposed formulation, followed by the detailed derivations of the various functional forms for both direct and inverse demand models utilizing our suggested framework. Then, we discuss the data and econometric methods employed in our analysis. Subsequently, the results from our empirical application are discussed. In the final section, we conclude with a few brief remarks.

Theoretical Framework

The income and compensated price elasticities can be calculated from the coefficients of the direct demand system while the scale and compensated quantity elasticities can be computed from the coefficients of the inverse demand system. Theoretically, every functional form of the direct or inverse demand system should have the same price and income elasticities. However as mentioned earlier this situation does not hold when utilizing many of the common functional forms. In this section, we show how it is possible to obtain such a result for each functional form of the direct demand system by working with the Marshallian demand function and the cost function. Similar analyses are done for each functional form of the inverse demand system by utilizing the distance function.

Direct Demand System

We begin by first considering the case of the Rotterdam Demand System (RDS). A system of the direct demand relationships can be found by working with the utility maximization problem. Under the utility maximization problem, the Marshallian demand function demonstrates a unique set of optimal quantities which maximize the utility function subject to the budget constraint for any set of given positive prices and income. Follow Theil (1965), by working with the derivative of the Marshallian demand function, we obtained the logarithmic version of the Rotterdam Demand System, RDS:

$$(1) \quad d(\ln q_i) = \eta_i d(\ln Q) + \sum_j \varepsilon_{ij} d(\ln p_j), \quad i, j = 1, \dots, n,$$

where $d(\ln q_i)$ is the derivative of logarithmic of variable q_i , q_i is the positive quantities bought and consumed of good i , p_i is the price of good i , η_i is the income (budget, wealth, total expenditure) elasticity of demand for commodity i , ε_{ij} is the Slutsky or compensated price elasticity of good i and good j , $d(\ln Q)$ is the Divisia volume index, where $d(\ln Q) = \sum_i w_i d(\ln q_i)$,

w_i is the budget share, where $w_i = p_i q_i / m$, and m is the total budget of the consumer's allocation, where $m = \sum_i p_i q_i$.

For the purpose of the econometric model development, the form of the data forces us to work with finite change, and for this purpose we introduce the following explicit time series notation. Write t for any finite period say, a week, which takes the values $1, \dots, T$, where T is the total number of weeks for which the relevant data are available. Let the derivative operator d be the log-change operator; that is, if x is any variable, x_t is its value in time t , then:

$$(2) \quad d(\ln x_t) = \Delta(\ln x_t) = \ln x_t - \ln x_{t-1} = \ln (x_t / x_{t-1}).$$

Rewriting equation (1) by replacing q_i with q_{it} , p_i with p_{it} and $d(\ln Q)$ with $d(\ln Q_t)$ where $d(\ln Q_t) = \sum_i w_{it}^* d(\ln q_{it})$, and $w_{it}^* = (w_{it} + w_{it-1}) / 2$, we get the econometric model development for the logarithmic version of the RDS model:

$$(3) \quad d(\ln q_{it}) = \eta_i d(\ln Q_t) + \sum_j \varepsilon_{ij} d(\ln p_{jt}) + v_{it},$$

where v_{it} is the disturbance for the demand equation of good i in time t .

In order to satisfy the neoclassical restrictions, which include adding-up, homogeneity and symmetry restrictions, we premultiply both sides of the logarithmic version of the demand system by the budget share. For the purpose of the econometric model development, we proposed a new formulation by premultiplying both sides of the logarithmic version of the demand system by the mean of the budget share, \bar{w}_i , where $\bar{w}_i = \sum_t w_{it} / T$, instead of using the moving average of the budget share, w_{it}^* . Then the econometric model development for the RDS model can be written as:

$$\bar{w}_i d(\ln q_{it}) = \bar{w}_i \eta_i d(\ln Q_t) + \sum_j \bar{w}_i \varepsilon_{ij} d(\ln p_{jt}) + v_{it} \quad \text{or more conveniently as}$$

$$(4) \quad \bar{w}_i d(\ln q_{it}) = c_i d(\ln Q_t) + \sum_j c_{ij} d(\ln p_{jt}) + v_{it},$$

where $c_i = \bar{w}_i \eta_i$, and $c_{ij} = \bar{w}_i \varepsilon_{ij}$.

Second, we will consider the Almost Ideal Demand System commonly referred to as the AIDS model. Following Deaton and Muellbauer (1980), the AIDS model can be obtained from the cost or expenditure function which defines the minimum expenditure necessary to attain a specific utility level at given prices. The AIDS cost function is written as:

$$(5) \quad w_i = \alpha_i + \sum_j s_{ij} \ln p_j + \beta_i \ln (m / P),$$

The logarithmic version of the of the AIDS model is obtained by adding $d(\ln p_i) - d(\ln P) - d(\ln Q)$, on both sides of the logarithmic version of the RDS model, equation (1), resulting in the following equation:

$$(6) \quad d(\ln w_i) = (\eta_i - 1) d(\ln Q) + \sum_j (\varepsilon_{ij} + \delta_{ij} - w_j) d(\ln p_j),$$

where $d(\ln w_i) = d(\ln q_i) + d(\ln p_i) - d(\ln P) - d(\ln Q)$, $d(\ln P)$ is the Divisia price index, where $d(\ln P) = \sum_i w_i d(\ln p_i)$, and $\delta_{ij} = 1$ if $i = j$, else $\delta_{ij} = 0$.

In order to get the econometric model development for the logarithmic version of the AIDS model, we rewrite equation (6), by replacing w_i with w_{it} , $d(\ln Q)$ with $d(\ln Q_t)$, and p_i with p_{it} , resulting in the following equation:

$$(7) \quad d(\ln w_{it}) = (\eta_i - 1) d(\ln Q_t) + \sum_j (\varepsilon_{ij} + \delta_{ij} - w_{jt}^*) d(\ln p_{jt}) + v_{it}.$$

In order to satisfy the neoclassical restrictions, we premultiply both sides of equation (7) by \bar{w}_i , then the econometric model development for the AIDS model can be written as:

$$\begin{aligned} dw_{it} &= (\bar{w}_i \eta_i - \bar{w}_i) d(\ln Q_t) + \sum_j (\bar{w}_i \varepsilon_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i w_{jt}^*) d(\ln p_{jt}) + v_{it} \quad \text{or} \\ dw_{it} + \sum_j (\bar{w}_i w_{jt}^*) d(\ln p_{jt}) - \sum_j (\bar{w}_i \bar{w}_j) d(\ln p_{jt}) \\ &= (\bar{w}_i \eta_i - \bar{w}_i) d(\ln Q_t) + \sum_j (\bar{w}_i \varepsilon_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i \bar{w}_j) d(\ln p_{jt}) + v_{it} \quad \text{or} \\ (8) \quad dw_{it} + \bar{w}_i [d(\ln P_t) - d(\ln P_t^*)] &= \beta_i d(\ln Q_t) + \sum_j s_{ij} d(\ln p_{jt}) + v_{it}, \end{aligned}$$

where $dw_{it} = \bar{w}_i d(\ln w_{it}) = \bar{w}_i [d(\ln q_{it}) + d(\ln p_{it}) - d(\ln P_t) - d(\ln Q_t)]$, $d(\ln P_t) = \sum_i w_{it}^* d(\ln p_{it})$, $d(\ln P_t^*) = \sum_i \bar{w}_i d(\ln p_{it})$, $\beta_i = \bar{w}_i \eta_i - \bar{w}_i$, and $s_{ij} = \bar{w}_i \varepsilon_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i \bar{w}_j$.

Third, we will now consider the Dutch Central Bureau of Statistics Demand System (CBS). In 1985, Keller and van Driel created the CBS model which is a hybrid of the AIDS model and the RDS model. This system has the AIDS income coefficients and the RDS price coefficients. By subtracting $d(\ln Q)$ from both sides of the logarithmic version of the RDS model, equation (1), we get the logarithmic version of the CBS model:

$$(9) \quad d(\ln q_i) - d(\ln Q) = (\eta_i - 1) d(\ln Q) + \sum_j \varepsilon_{ij} d(\ln p_j).$$

Rewriting equation (9), by replacing q_i with q_{it} , $d(\ln Q)$ with $d(\ln Q_t)$, and p_i with p_{it} , we get the econometric model development for the logarithmic version of the CBS model:

$$(10) \quad d(\ln q_{it}) - d(\ln Q_t) = (\eta_i - 1) d(\ln Q_t) + \sum_j \varepsilon_{ij} d(\ln p_{jt}) + v_{it}.$$

In order to satisfy the neoclassical restrictions, we premultiply both sides of equation (10) by \bar{w}_i , then the econometric model development for the CBS model can be written as:

$$\bar{w}_i [d(\ln q_{it}) - d(\ln Q_t)] = (\bar{w}_i \eta_i - \bar{w}_i) d(\ln Q_t) + \sum_j \bar{w}_i \varepsilon_{ij} d(\ln p_{jt}) + v_{it} \quad \text{or}$$

$$(11) \quad \bar{w}_i [d(\ln q_{it}) - d(\ln Q_t)] = \beta_i d(\ln Q_t) + \sum_j c_{ij} d(\ln p_{jt}) + v_{it}.$$

Fourth, we will consider the National Bureau of Research Demand System (NBR). In 1994, Neves considered another hybrid of the AIDS model and the RDS model. The NBR model has the RDS income coefficients and the AIDS price coefficients. The logarithmic version of the NBR model is obtained by adding $d(\ln p_i) - d(\ln P)$ to both sides of the logarithmic version of the RDS model, equation (1), resulting in the following equation:

$$(12) \quad d(\ln q_i) + d(\ln p_i) - d(\ln P) = \eta_i d(\ln Q) + \sum_j (\varepsilon_{ij} + \delta_{ij} - w_j) d(\ln p_j).$$

Rewriting equation (12), by replacing q_i with q_{it} , $d(\ln P)$ with $d(\ln P_t)$, $d(\ln Q)$ with $d(\ln Q_t)$, and p_i with p_{it} , we get the econometric model development for the logarithmic version of the NBR model:

$$(13) \quad d(\ln q_{it}) + d(\ln p_{it}) - d(\ln P_t) = \eta_i d(\ln Q_t) + \sum_j (\varepsilon_{ij} + \delta_{ij} - w_{jt}^*) d(\ln p_{jt}) + v_{it}.$$

In order to satisfy the neoclassical restrictions, we premultiply both sides of equation (13) by \bar{w}_i , then the econometric model development for the NBR model can be written as:

$$\begin{aligned} & \bar{w}_i [d(\ln q_{it}) + d(\ln p_{it}) - d(\ln P_t)] \\ &= \bar{w}_i \eta_i d(\ln Q_t) + \sum_j (\bar{w}_i \varepsilon_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i w_{jt}^*) d(\ln p_{jt}) + v_{it} \quad \text{or} \\ & \bar{w}_i [d(\ln q_{it}) + d(\ln p_{it}) - d(\ln P_t)] + \sum_j (\bar{w}_i w_{jt}^*) d(\ln p_{jt}) - \sum_j (\bar{w}_i \bar{w}_j) d(\ln p_{jt}) \\ &= \bar{w}_i \eta_i d(\ln Q_t) + \sum_j (\bar{w}_i \varepsilon_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i \bar{w}_j) d(\ln p_{jt}) + v_{it} \quad \text{or} \\ (14) \quad & \bar{w}_i [d(\ln q_{it}) + d(\ln p_{it}) - d(\ln P_t)^*] = c_i d(\ln Q_t) + \sum_j s_{ij} d(\ln p_{jt}) + v_{it}. \end{aligned}$$

The above derivations show quite clearly that in all cases each of the coefficients of the selected functional forms is a function of the mean of the budget share, \bar{w}_i . As a result, the income and compensated price elasticities are unchanged across all the functional forms and can be calculated by using the following equations:

$$(15) \quad \eta_i = c_i / \bar{w}_i = (\beta_i / \bar{w}_i) + 1 \quad \text{for the income elasticity,}$$

$$(16) \quad \varepsilon_{ij} = c_{ij} / \bar{w}_i = (s_{ij} / \bar{w}_i) + \bar{w}_j - \delta_{ij} \quad \text{for the compensated price elasticity.}$$

The uncompensated price elasticity can be calculated by using the Slutsky equation:

$$(17) \quad \mu_{ij} = \varepsilon_{ij} - \eta_i \bar{w}_j,$$

where μ_{ij} is the uncompensated price elasticity of good i and good j .

Inverse Demand System

Following Barten and Bettendorf (1989), a system of compensated inverse demand relationships can be found by working with the distance function which is dual to the demand function from the utility maximization problem. The distance function indicates the minimum expenditure necessary to attain a specific utility level, u , at given quality, q , and it can be written as $g(u, q)$. Start by first considering the Rotterdam Inverse Demand System commonly referred to as the RIDS model. We can derive the RIDS model by working with the distance function. By differentiating the distance function with respect to quantity, we get the compensated inverse demands express prices as a function of the quantities and specific utility level. By totally differentiating the system of compensated inverse demand relationships, we obtain the logarithmic version of the RIDS model:

$$(18) \quad d(\ln \pi_i) = \zeta_i d(\ln Q) + \sum_j \xi_{ij} d(\ln q_j),$$

where π_i is the normalized price of good i , ζ_i is the scale elasticity of good i , and ξ_{ij} is the compensated quantity elasticity of good i and j .

Rewriting equation (18), by replacing π_i with π_{it} , $d(\ln Q)$ with $d(\ln Q_t)$, and q_i with q_{it} , we get the econometric model development for the logarithmic version of the RIDS model:

$$(19) \quad d(\ln \pi_{it}) = \zeta_i d(\ln Q_t) + \sum_j \xi_{ij} d(\ln q_{jt}) + v_{it}.$$

In order to satisfy the neoclassical restrictions, we premultiply both sides of equation (19) by \bar{w}_i , then the econometric model development for the RIDS model can be written as:

$$\bar{w}_i d(\ln \pi_{it}) = \bar{w}_i \zeta_i d(\ln Q_t) + \sum_j \bar{w}_i \xi_{ij} d(\ln q_{jt}) + v_{it} \quad \text{or}$$

$$(20) \quad \bar{w}_i d(\ln \pi_{it}) = h_i d(\ln Q_t) + \sum_j h_{ij} d(\ln q_{jt}) + v_{it},$$

where $h_i = \bar{w}_i \zeta_i$, and $h_{ij} = \bar{w}_i \xi_{ij}$.

Second, we will now consider the Almost Ideal Inverse Demand System (AIIDS) which can be obtained by working with the distance function. The AIIDS model represents the budget shares as a function of quantities:

$$(21) \quad w_i = \alpha_i + \sum_j \gamma_{ij} (\ln q_j) + b_i \sum_i w_i (\ln q_i).$$

The logarithmic version of the AIIDS model is obtained by adding $d(\ln q_i)$ on both sides of the logarithmic version of the RIDS model, equation (18), resulting in the following equation:

$$(22) \quad d(\ln w_i) = d(\ln \pi_i) + d(\ln q_i) = (\zeta_i + 1) d(\ln Q) + \sum_j (\xi_{ij} + \delta_{ij} - w_j) d(\ln q_j).$$

Rewriting equation (22), by replacing w_i with w_{it} , $d(\ln Q)$ with $d(\ln Q_t)$, and q_i with q_{it} , we get the econometric model development for the logarithmic version of the AIIDS model:

$$(23) \quad d(\ln w_{it}) = (\zeta_i + 1) d(\ln Q_t) + \sum_j (\xi_{ij} + \delta_{ij} - w_{jt}^*) d(\ln q_{jt}) + v_{it}.$$

In order to satisfy the neoclassical restrictions, we premultiply both sides of equation (23) by \bar{w}_i , then the econometric model development for the AIIDS model can be written as:

$$\begin{aligned} \bar{w}_i d(\ln w_{it}) &= (\bar{w}_i \zeta_i + \bar{w}_i) d(\ln Q_t) + \sum_j (\bar{w}_i \xi_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i w_{jt}^*) d(\ln q_{jt}) + v_{it} \quad \text{or} \\ dw_{it} + \sum_j (\bar{w}_i w_{jt}^*) d(\ln q_{jt}) - \sum_j (\bar{w}_i \bar{w}_j) d(\ln q_{jt}) \\ &= (\bar{w}_i \zeta_i + \bar{w}_i) d(\ln Q_t) + \sum_j (\bar{w}_i \xi_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i \bar{w}_j) d(\ln q_{jt}) + v_{it} \quad \text{or} \\ (24) \quad \bar{w}_i [d(\ln \pi_{it}) + d(\ln q_{it}) + d(\ln Q_t) - d(\ln Q_t^*)] &= b_i d(\ln Q_t) + \sum_j \gamma_{ij} d(\ln q_{jt}) + v_{it}, \end{aligned}$$

where $dw_{it} = \bar{w}_i d(\ln w_{it}) = \bar{w}_i [d(\ln \pi_{it}) + d(\ln q_{it})]$, $d(\ln Q_t^*) = \sum_i \bar{w}_i d(\ln q_{it})$, $b_i = \bar{w}_i \zeta_i + \bar{w}_i$, and $\gamma_{ij} = \bar{w}_i \xi_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i \bar{w}_j$.

Third, we will now consider the Laitinen and Theil's Inverse Demand System (Laitinen-Theil). Following Laitinen and Theil (1979), by using Antonelli matrix, which is identical to the reciprocal Slutsky matrix under homothetic preferences, we can obtain the logarithmic version of the Laitinen-Theil:

$$(25) \quad d(\ln p_i) - d(\ln P) = d(\ln \pi_i) + d(\ln Q) = (\zeta_i + 1) d(\ln Q) + \sum_j \xi_{ij} d(\ln q_j).$$

This inverse demand model has the AIIDS scale coefficients and the RIDS quantity coefficients. We also get the logarithmic version of the Laitinen-Theil model by adding $d(\ln Q)$ to both sides of the logarithmic version of the RIDS model, equation (18).

Rewriting equation (25), by replacing π_i with π_{it} , $d(\ln Q)$ with $d(\ln Q_t)$, and q_i with q_{it} , we get the econometric model development for the logarithmic version of the Laitinen-Theil model:

$$(26) \quad d(\ln \pi_{it}) + d(\ln Q_t) = (\zeta_i + 1) d(\ln Q_t) + \sum_j \xi_{ij} d(\ln q_{jt}) + v_{it}.$$

In order to satisfy the neoclassical restrictions, we premultiply both sides of equation (26) by \bar{w}_i , then the econometric model development for the Laitinen-Theil model can be written as:

$$\bar{w}_i [d(\ln \pi_{it}) + d(\ln Q_t)] = (\bar{w}_i \zeta_i + \bar{w}_i) d(\ln Q_t) + \sum_j \bar{w}_i \xi_{ij} d(\ln q_{jt}) + v_{it} \quad \text{or}$$

$$(27) \quad \bar{w}_i d[\ln (p_{it}/P_t)] = b_i d(\ln Q_t) + \sum_j h_{ij} d(\ln q_{jt}) + v_{it},$$

where $\bar{w}_i d[\ln (p_{it}/P_t)] = \bar{w}_i [d(\ln p_{it}) - d(\ln P_t)] = \bar{w}_i [d(\ln \pi_{it}) + d(\ln Q_t)]$.

Fourth, we will now consider the Rotterdam Almost Ideal Inverse Demand System (RAIIDS) which has the RIDS scale effects and the AIIDS quantity effects. By adding $d(\ln q_i) - d(\ln Q)$ to both sides of the logarithmic version of the RIDS model, equation (18), we get the logarithmic version of the RAIIDS model:

$$(28) \quad d(\ln w_i) - d(\ln Q) = \zeta_i d(\ln Q) + \sum_j (\xi_{ij} + \delta_{ij} - w_j) d(\ln q_j).$$

Rewriting equation (28), by replacing w_i with w_{it} , $d(\ln Q)$ with $d(\ln Q_t)$, and q_i with q_{it} , we get the econometric model development for the logarithmic version of the RAIIDS model:

$$(29) \quad dw_{it} - d(\ln Q_t) = \zeta_i d(\ln Q_t) + \sum_j (\xi_{ij} + \delta_{ij} - w_{jt}^*) d(\ln q_{jt}) + v_{it}$$

In order to satisfy the neoclassical restrictions, we premultiply both sides of equation (29) by \bar{w}_i , then the econometric model development for the RAIIDS model can be written as:

$$\begin{aligned}
dw_{it} - \bar{w}_i d(\ln Q_t) &= \bar{w}_i \zeta_i d(\ln Q_t) + \sum_j (\bar{w}_i \xi_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i w_{jt}^*) d(\ln q_{jt}) + v_{it} \quad \text{or} \\
dw_{it} - \bar{w}_i d(\ln Q_t) + \sum_j (\bar{w}_i w_{jt}^*) d(\ln q_{jt}) - \sum_j (\bar{w}_i \bar{w}_j) d(\ln q_{jt}) \\
&= \bar{w}_i \zeta_i d(\ln Q_t) + \sum_j (\bar{w}_i \xi_{ij} + \bar{w}_i \delta_{ij} - \bar{w}_i \bar{w}_j) d(\ln q_{jt}) + v_{it} \quad \text{or} \\
(30) \quad dw_{it} - \bar{w}_i d(\ln Q_t^*) &= h_i d(\ln Q_t) + \sum_j \gamma_{ij} d(\ln q_{jt}) + v_{it}.
\end{aligned}$$

The scale and compensated quantity elasticity can be calculated from the coefficients of each functional form of the inverse demand system by using the following equations:

$$(31) \quad \zeta_i = h_i / \bar{w}_i = (b_i / \bar{w}_i) - 1 \quad \text{for the scale elasticity,}$$

$$(32) \quad \xi_{ij} = h_{ij} / \bar{w}_i = (\gamma_{ij} / \bar{w}_i) + \bar{w}_j - \delta_{ij} \quad \text{for the compensated quantity elasticity.}$$

The uncompensated quantity elasticity can be calculated by using the Antonelli equation:

$$(33) \quad \psi_{ij} = \xi_{ij} + \zeta_i \bar{w}_j,$$

where ψ_{ij} is the uncompensated quantity elasticity of good i and good j .

Data and Methods

Empirical application of the model was carried out for four ($n = 4$) selected fruits and vegetables utilizing weekly data covering the period the period 1994 to 1998. Weekly wholesale prices and quantity unloads were collected from the Market News Branch of the Fruit and Vegetable Division, Agricultural Marketing Service of the United States Department of Agriculture. In this study, there are 208 observations of quantities and prices for each commodity in each market, $T = 208$. The selected commodities are tomatoes, bell peppers, cucumbers, and strawberries. The markets include Atlanta, New York, Los Angeles, and Chicago.

In conducting our analysis the system of equations for each functional form is estimated by the seemingly unrelated regressions (SUR) method. In order to get the estimation with

homogeneity and symmetry condition, following Barten (1969), we employ the maximum likelihood estimation with homogeneity and symmetry constraints imposed. Mathematical and statistical software, GAUSS, was used to perform the estimation.

Empirical Results and Discussion

The results from Table 1 and 2 show that, by using the new formulation, the income coefficients in the RDS model are the same as the income coefficients in the NBR model and the income coefficients in the AIDS model are the same as the income coefficients in the CBS model. Consequently, the income elasticity calculated from the income coefficient is unchanged across these four functional forms. In addition, the price coefficients are the same between the RDS model and the CBS model, and between the AIDS model and the NBR model. The price elasticity calculated from the price coefficient is also the same across all functional forms.

The results from Table 3 and 4 show that, by using the new formulation, the RIDS has the same scale coefficients as the RAIIDS model and the AIIDS model has the same scale coefficients as the Laitinen-Theil model. The quantity coefficients are the same between the RIDS model and the Laitinen-Theil model, and between the AIIDS model and the RAIIDS model. Consequently, the scale elasticity, and the quantity elasticity are unchanged across all functional forms of the inverse demand system.

The results from Table 5 show that the log-likelihood value from the system using the mean of the budget share, \bar{w}_i , to multiply the logarithmic version of the demand system is higher than the log-likelihood value from the econometric demand model which replace every budget share in the system by its moving average, w_{it}^* (Theil, 1971). By multiplying the logarithmic version of the demand system by the mean of the budget share, each coefficient of the demand systems is a function of \bar{w}_i instead of a function of w_{it}^* . The log-likelihood value from the

system using the mean of the budget share is also the same across all functional forms. These empirical results support the theory that the elasticity should be the same across all functional forms.

The results from Table 6 show that all of the elasticities for all markets have the correct sign according to theory. Tomato has the highest absolute value of the own substitution elasticity for every market. In contrast, strawberry has the lowest absolute value of the own substitution elasticity for every market except the New York market. The scale and quantity elasticities of the inverse demand system are closer between the Atlanta and Los Angeles market and between the Chicago and New York market.

Conclusions

Demand analyses have been known to be quite sensitive to the chosen functional forms. Since no one specification fits all data the best, researchers have been preoccupied with finding ways to select among various functional forms. In this study we proposed a formulation which obviates the need to choose among various functional forms. In order to get the demand system that satisfy the neoclassical restrictions, we multiply the budget share to the logarithmic of demand system. From both a theoretical as well as empirical point of view our analysis suggests that it is important to use the mean of the budget share, \bar{w}_i , instead of the moving average of the budget share, w_{it}^* , to multiply the logarithmic of the demand system. By using the mean of the budget share to multiply the logarithmic of the demand system, we can obviate the need to choose among various functional forms.

References

- Barten, A.P. "Estimating Demand Equation." *Econometrica* 36, Issue 2:269-280, 1968.
- Barten, A.P. "Maximum Likelihood Estimation of a Complete System of Demand Equations." *European Economic Review* 1:7-73, 1969.
- Barten, A.P., and L.J. Bettendorf. "Price Formation of Fish: An Application of an Inverse Demand System." *European Economic Review* 33:1509-1525, 1989.
- Barten, A.P. "Consumer Allocation Models: Choice of Functional Form." *Empirical Economics* 18:129-158, 1993.
- Brown, M.G., J.Y. Lee, and J. Seale. "A Family of Inverse Demand Systems and Choice of Functional Form." *Empirical Economics* 20: 519-530, 1995.
- Deaton, A. "Specification and Testing in Applied Demand Analysis." *The Economic Journal* 88:524-536, 1978.
- Deaton, A., and J. Muellbauer. "An Almost Ideal Demand System." *American Economic Review* 70:312-326, 1980.
- Keller, W.J. and J. van Driel. "Differentiable Consumer Demand Systems." *European Economics Review* 27: 375-390, 1985.
- Laitinen, K., and H. Theil. "The Antonelli Matrix and the reciprocal Slutsky matrix." *Economics Letters* 3:153-157, 1979.
- Neves, P.D. "A Class of Differential Demand Systems." *Economics Letters* 44:83-86, 1994.
- Parks, R.W. "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms." *Econometrica* 37:629-650, 1969.
- Theil, H. "The Information Approach to Demand Analysis." *Econometrica* 33:67-87, 1965.
- Greene, W. H. *Econometric Analysis*. New Jersey: Prentice-Hall, Inc., 2000.

Mas-Colell, A., M.D. Whinston and J.R. Green. *Microeconomic Theory*. New York: Oxford University Press, 1995.

Theil, H. *Principles of Econometrics*. New York: John Wiley & Sons, Inc., 1971.

Table 1. The Estimation of the Coefficients of the Direct Demand System for Chicago and New York Market

RDS	Chicago				New York			
	Tomato	Bell Pepper	Cucum ber	Straw berry	Tomato	Bell Pepper	Cucum ber	Straw berry
c_i	0.453 (0.030)	0.201 (0.020)	0.275 (0.023)	0.065 (0.013)	0.608 (0.025)	0.061 (0.014)	0.190 (0.018)	0.137 (0.011)
c_{ii}	-0.030 (0.021)	-0.021 (0.012)	-0.024 (0.016)	-0.024 (0.007)	-0.052 (0.025)	-0.047 (0.014)	-0.061 (0.015)	-0.034 (0.010)
σ_i	0.081	0.053	0.064	0.036	0.117	0.065	0.083	0.052
R^2	0.539	0.353	0.413	0.129	0.748	0.124	0.386	0.450
CBS								
β_i	0.016 (0.030)	-0.003 (0.020)	0.015 (0.023)	-0.034 (0.013)	0.170 (0.025)	-0.135 (0.014)	-0.063 (0.018)	0.024 (0.011)
c_{ii}	-0.030 (0.021)	-0.021 (0.012)	-0.024 (0.016)	-0.024 (0.007)	-0.052 (0.025)	-0.047 (0.014)	-0.061 (0.015)	-0.034 (0.010)
σ_i	0.081	0.053	0.064	0.036	0.117	0.065	0.083	0.052
R^2	0.008	0.019	0.023	0.086	0.205	0.346	0.135	0.074
AIDS								
β_i	0.016 (0.030)	-0.003 (0.020)	0.015 (0.023)	-0.034 (0.013)	0.170 (0.025)	-0.135 (0.014)	-0.063 (0.018)	0.024 (0.011)
s_{ii}	0.216 (0.021)	0.141 (0.012)	0.169 (0.016)	0.065 (0.007)	0.194 (0.025)	0.110 (0.014)	0.128 (0.015)	0.066 (0.010)
σ_i	0.081	0.053	0.064	0.036	0.117	0.065	0.083	0.052
R^2	0.328	0.419	0.358	0.304	0.310	0.412	0.255	0.189
NBR								
c_i	0.453 (0.030)	0.201 (0.020)	0.275 (0.023)	0.065 (0.013)	0.608 (0.025)	0.061 (0.014)	0.190 (0.018)	0.137 (0.011)
s_{ii}	0.216 (0.021)	0.141 (0.012)	0.169 (0.016)	0.065 (0.007)	0.194 (0.025)	0.110 (0.014)	0.128 (0.015)	0.066 (0.010)
σ_i	0.081	0.053	0.064	0.036	0.117	0.065	0.083	0.052
R^2	0.605	0.533	0.569	0.385	0.747	0.318	0.488	0.486

Standard errors are shown in parentheses below the parameter estimates

Table 2. The Estimation of the Coefficients of the Direct Demand System for Atlanta and Los Angeles Market

RDS	Atlanta				Los Angeles			
	Tomato	Bell Pepper	Cucum ber	Straw berry	Tomato	Bell Pepper	Cucum ber	Straw berry
c_i	0.676 (0.024)	0.114 (0.016)	0.127 (0.015)	0.047 (0.011)	0.690 (0.022)	0.123 (0.014)	0.085 (0.011)	0.055 (0.009)
c_{ii}	-0.070 (0.015)	-0.027 (0.008)	-0.036 (0.010)	-0.018 (0.006)	-0.061 (0.012)	-0.018 (0.007)	-0.018 (0.006)	-0.008 (0.005)
σ_i	0.053	0.036	0.033	0.024	0.055	0.034	0.028	0.023
R^2	0.804	0.223	0.303	0.129	0.829	0.295	0.264	0.166
CBS								
β_i	0.111 (0.024)	-0.056 (0.016)	-0.056 (0.015)	-0.036 (0.011)	0.114 (0.022)	-0.059 (0.014)	-0.061 (0.011)	-0.040 (0.009)
c_{ii}	-0.070 (0.015)	-0.027 (0.008)	-0.036 (0.010)	-0.018 (0.006)	-0.061 (0.012)	-0.018 (0.007)	-0.018 (0.006)	-0.008 (0.005)
σ_i	0.053	0.036	0.033	0.024	0.055	0.034	0.028	0.023
R^2	0.187	0.114	0.153	0.090	0.202	0.126	0.215	0.112
AIDS								
β_i	0.111 (0.024)	-0.056 (0.016)	-0.056 (0.015)	-0.036 (0.011)	0.114 (0.022)	-0.059 (0.014)	-0.061 (0.011)	-0.040 (0.009)
s_{ii}	0.176 (0.015)	0.114 (0.008)	0.113 (0.010)	0.059 (0.006)	0.184 (0.012)	0.131 (0.007)	0.107 (0.006)	0.078 (0.005)
σ_i	0.053	0.036	0.033	0.024	0.055	0.034	0.028	0.023
R^2	0.427	0.428	0.468	0.403	0.526	0.622	0.631	0.563
NBR								
c_i	0.676 (0.024)	0.114 (0.016)	0.127 (0.015)	0.047 (0.011)	0.690 (0.022)	0.123 (0.014)	0.085 (0.011)	0.055 (0.009)
s_{ii}	0.176 (0.015)	0.114 (0.008)	0.113 (0.010)	0.059 (0.006)	0.184 (0.012)	0.131 (0.007)	0.107 (0.006)	0.078 (0.005)
σ_i	0.053	0.036	0.033	0.024	0.055	0.034	0.028	0.023
R^2	0.817	0.509	0.540	0.398	0.843	0.668	0.673	0.568

Standard errors are shown in parentheses below the parameter estimates

Table 3. The Estimation of the Coefficients of the Inverse Demand System for Chicago and New York Market

Chicago					New York			
RIDS	Tomato	Bell Pepper	Cucum ber	Straw berry	Tomato	Bell Pepper	Cucum ber	Straw berry
h_i	-0.449 (0.025)	-0.223 (0.019)	-0.232 (0.019)	-0.081 (0.012)	-0.458 (0.019)	-0.197 (0.013)	-0.239 (0.016)	-0.112 (0.009)
h_{ii}	-0.022 (0.014)	-0.019 (0.011)	-0.019 (0.011)	-0.019 (0.005)	-0.025 (0.012)	-0.028 (0.008)	-0.043 (0.010)	-0.016 (0.005)
σ_i	0.067	0.052	0.051	0.032	0.079	0.051	0.067	0.036
R^2	0.629	0.403	0.439	0.198	0.796	0.585	0.588	0.537
Laitinen -Theil								
b_i	-0.011 (0.025)	-0.018 (0.019)	0.028 (0.019)	0.018 (0.012)	-0.020 (0.019)	-0.001 (0.013)	0.014 (0.016)	0.001 (0.009)
h_{ii}	-0.022 (0.014)	-0.019 (0.011)	-0.019 (0.011)	-0.019 (0.005)	-0.025 (0.012)	-0.028 (0.008)	-0.043 (0.010)	-0.016 (0.005)
σ_i	0.067	0.052	0.051	0.032	0.079	0.051	0.067	0.036
R^2	0.023	0.014	0.016	0.079	0.033	0.056	0.095	0.059
AIIDS								
b_i	-0.011 (0.025)	-0.018 (0.019)	0.028 (0.019)	0.018 (0.012)	-0.020 (0.019)	-0.001 (0.013)	0.014 (0.016)	0.001 (0.009)
γ_{ii}	0.224 (0.014)	0.143 (0.011)	0.174 (0.011)	0.070 (0.005)	0.222 (0.012)	0.129 (0.008)	0.146 (0.010)	0.084 (0.005)
σ_i	0.067	0.052	0.051	0.032	0.079	0.051	0.067	0.036
R^2	0.521	0.474	0.565	0.476	0.657	0.657	0.497	0.602
RAIDS								
h_i	-0.449 (0.025)	-0.223 (0.019)	-0.232 (0.019)	-0.081 (0.012)	-0.458 (0.019)	-0.197 (0.013)	-0.239 (0.016)	-0.112 (0.009)
γ_{ii}	0.224 (0.014)	0.143 (0.011)	0.174 (0.011)	0.070 (0.005)	0.222 (0.012)	0.129 (0.008)	0.146 (0.010)	0.084 (0.005)
σ_i	0.067	0.052	0.051	0.032	0.079	0.051	0.067	0.036
R^2	0.722	0.610	0.660	0.574	0.754	0.846	0.756	0.683

Standard errors are shown in parentheses below the parameter estimates

Table 4. The Estimation of the Coefficients of the Inverse Demand System for Atlanta and Los Angeles Market

RIDS	Atlanta				Los Angeles			
	Tomato	Bell Pepper	Cucum ber	Straw berry	Tomato	Bell Pepper	Cucum ber	Straw berry
h_i	-0.543 (0.029)	-0.178 (0.018)	-0.191 (0.017)	-0.096 (0.011)	-0.522 (0.031)	-0.188 (0.021)	-0.152 (0.019)	-0.116 (0.013)
h_{ii}	-0.076 (0.018)	-0.041 (0.010)	-0.032 (0.010)	-0.011 (0.005)	-0.112 (0.021)	-0.038 (0.014)	-0.037 (0.012)	-0.018 (0.007)
σ_i	0.060	0.038	0.036	0.022	0.068	0.048	0.043	0.029
R^2	0.702	0.343	0.384	0.322	0.701	0.289	0.222	0.280
Laitinen -Theil								
b_i	0.022 (0.029)	-0.008 (0.018)	-0.009 (0.017)	-0.013 (0.011)	0.053 (0.031)	-0.006 (0.021)	-0.005 (0.019)	-0.021 (0.013)
h_{ii}	-0.076 (0.018)	-0.041 (0.010)	-0.032 (0.010)	-0.011 (0.005)	-0.112 (0.021)	-0.038 (0.014)	-0.037 (0.012)	-0.018 (0.007)
σ_i	0.060	0.038	0.036	0.022	0.068	0.048	0.043	0.029
R^2	0.062	0.128	0.025	0.049	0.134	0.044	0.066	0.055
AIIDS								
b_i	0.022 (0.029)	-0.008 (0.018)	-0.009 (0.017)	-0.013 (0.011)	0.053 (0.031)	-0.006 (0.021)	-0.005 (0.019)	-0.021 (0.013)
γ_{ii}	0.170 (0.018)	0.100 (0.010)	0.117 (0.010)	0.065 (0.005)	0.132 (0.021)	0.111 (0.014)	0.088 (0.012)	0.068 (0.007)
σ_i	0.060	0.038	0.036	0.022	0.068	0.048	0.043	0.029
R^2	0.308	0.330	0.388	0.484	0.235	0.219	0.183	0.323
RAIIDS								
h_i	-0.543 (0.029)	-0.178 (0.018)	-0.191 (0.017)	-0.096 (0.011)	-0.522 (0.031)	-0.188 (0.021)	-0.152 (0.019)	-0.116 (0.013)
γ_{ii}	0.170 (0.018)	0.100 (0.010)	0.117 (0.010)	0.065 (0.005)	0.132 (0.021)	0.111 (0.014)	0.088 (0.012)	0.068 (0.007)
σ_i	0.060	0.038	0.036	0.022	0.068	0.048	0.043	0.029
R^2	0.628	0.541	0.614	0.619	0.588	0.471	0.417	0.511

Standard errors are shown in parentheses below the parameter estimates

Table 5. The Log-likelihood Value (LV) of the Direct and Inverse Demand System by Using the Mean and Moving Average of the Budget Share

Direct Demand System	LV(\bar{w}_i)	LV(w_{it}^*)			
		RDS	CBS	AIDS	NBR
Atlanta	1297.388	1280.276	1292.382	1278.007	1254.068
Los Angeles	1350.614	1329.698	1339.734	1305.874	1279.178
Chicago	993.237	959.299	973.739	966.903	949.128
New York	812.169	786.229	807.706	794.366	773.513
Inverse Demand System	LV(\bar{w}_i)	LV(w_{it}^*)			
		RIDS	Laitinen-Theil	AIIDS	RAIIDS
Atlanta	1282.237	1254.501	1260.174	1237.487	1223.637
Los Angeles	1162.507	1122.705	1143.139	1124.520	1097.929
Chicago	1081.366	1033.537	1052.444	1034.765	1015.744
New York	1002.924	962.034	994.543	953.065	918.743

Table 6. The Elasticity of the Direct Demand and Inverse Demand System

Elasticity	Direct Demand			Inverse Demand		
	η_i	ε_{ii}	μ_{ii}	ζ_i	ξ_{ii}	ψ_{ii}
Atlanta						
Tomato	1.1972	-0.1247	-0.8002	-0.9617	-0.1352	-0.6778
Bell Pepper	0.6681	-0.1592	-0.2727	-1.0460	-0.2426	-0.4204
Cucumber	0.6945	-0.1969	-0.3235	-1.0485	-0.1754	-0.3665
Strawberry	0.5648	-0.2149	-0.2621	-1.1526	-0.1328	-0.2291
Los Angeles						
Tomato	1.1979	-0.1054	-0.7951	-0.9075	-0.1953	-0.7178
Bell Pepper	0.6749	-0.0986	-0.2221	-1.0302	-0.2086	-0.3971
Cucumber	0.5834	-0.1252	-0.2106	-1.0365	-0.2500	-0.4018
Strawberry	0.5801	-0.0859	-0.1410	-1.2194	-0.1915	-0.3073
Chicago						
Tomato	1.0364	-0.0677	-0.5211	-1.0259	-0.0502	-0.4990
Bell Pepper	0.9864	-0.1047	-0.3061	-1.0905	-0.0938	-0.3165
Cucumber	1.0591	-0.0912	-0.3666	-0.8920	-0.0726	-0.3045
Strawberry	0.6571	-0.2390	-0.3036	-0.8188	-0.1896	-0.2701
New York						
Tomato	1.3886	-0.1180	-0.7260	-1.0453	-0.0560	-0.5138
Bell Pepper	0.3120	-0.2406	-0.3017	-1.0040	-0.1447	-0.3416
Cucumber	0.7498	-0.2423	-0.4321	-0.9438	-0.1709	-0.4097
Strawberry	1.2116	-0.2992	-0.4361	-0.9910	-0.1451	-0.2571