

A Recreation Optimization Model Based on the Travel Cost Method

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A recreation allocation model is developed which efficiently selects recreation areas and degree of development from an array of proposed and existing sites. The model does this by maximizing the difference between gross recreation benefits and travel, investment, management, and site-opportunity costs. The model presented uses the Travel Cost Method for estimating recreation benefits within an operations research framework. The model is applied to selection of potential wilderness areas in Colorado. This example is then extended to show the model's capability in budget analysis and in planning to meet recreation targets.

Recent literature on travel cost models has emphasized the importance of taking into account the presence of an existing site that a proposed site (being evaluated) will substitute (perfectly) for. For example, Cicchetti *et al.*, who applied an approach suggested earlier by Burt and Brewer, state:

Accordingly, any project or policy that results in a reduction in the travel time input t_i required in the production of services such as those provided by site i , may in turn be said to result in a reduction in price, P_i . One way in which this can be accomplished would be the development of a new site, close to the recreationist, which provides the same services. The strategy in any empirical application, . . . , is then to pick an existing site for which the new one can be assumed to (perfectly) substitute, and trace the effects of the price reduction through the system of derived demands . . . [for the existing sites]. (Cicchetti *et al.*, p. 1262)

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It should be noted that more recent literature has shown that the system of demand functions is not necessary. Only the own demand function for the single existing site is needed for a consumer surplus measure associated with a single price change (Just *et al.*; Hof and King). The important point is that if a new site substitutes (perfectly) for an old one, then the willingness to pay for the new site is the change in consumer surplus it creates under the demand curve for the old site. This should be distinguished from the total consumer surplus under a new demand function (for a new, independent commodity).

The Case of More Than One Proposed Site

In the case where there is more than one proposed site to be evaluated, the problem may be more complex. If the proposed sites could be regarded as new commodities that do not substitute for any other commodities, then they could theoretically be evaluated one at a time, in any order. On the other hand, if the proposed sites substitute for an existing site as discussed above, then the estimated value of any given proposed site would be affected by whether or not other (perfect substi-

tute) proposed sites are developed. So, the purely arbitrary order of evaluating proposed sites could seriously affect site development decisions. Also, it may be that the "optimal solution" would be partial development of some or all of the proposed sites rather than treating each site as an all-or-nothing project.

What is needed in the case of more than one proposed site perfectly substituting for an existing site is a method of *simultaneously* determining the levels and locations of recreation site development that will maximize net benefits. This would imply a cost-minimizing combination of travel from the potential origins and resource inputs from the proposed and existing sites. The purpose of this paper is to describe the structure of an operations research model that could accomplish this task in a regional recreation planning context.

The conventional means of calculating net benefits from a travel cost model is to deduct the cost of operating the new site, opportunity costs, and the amortized investment cost from the price-induced change in consumer surplus (see Burt and Brewer, p. 817). Another way of looking at this same net social value measurement is that travel costs, operating costs, opportunity costs, and investment costs are deducted from gross benefits measured by the "first stage" demand function.¹ Looking at the problem in this way, the development of a new site implies a change in the mix of inputs (sites and travel) which, in turn, leads to the price change and a change in net benefits. If the gain in consumer surplus created by the input real-

location exceeds the site costs (operating, opportunity, and investment), then the development of the proposed site represents an improvement in net social value.

This point of view suggests the type of model discussed in the next section. Recreators from different origins can potentially recreate at any of the proposed sites or at the existing site. Thus, on the supply side, the model is somewhat like a "transportation model," since it seeks an efficient set of origin-destination "deliveries." On the demand side, prices are determined endogenously with demand functions specified for different origins, so the model is somewhat like a "spatial equilibrium" model (see, for example, Martin). The solution to this model represents both an optimization and a means of logically evaluating more than one proposed site simultaneously.

A Recreation Allocation Model (RECAM)

The basic choice variables in this model are the amounts of recreation, measured here in recreation visitor days (RVD's), to be consumed at each existing and proposed site by individuals from each origin.² A mathematical depiction of the problem to be solved follows.

Maximize:

$$\sum_{j=1}^J \int_0^{E_j} D_j(E_j) dE_j - \sum_{j=1}^J \sum_{i=1}^I (T_{ij} + M_{ij} + C_{ij})R_{ij}$$

Subject to:

$$\sum_{i=1}^I R_{ij} = E_j; \quad j = 1, J$$

$$\sum_{j=1}^J R_{ij} \leq A_i; \quad i = 1, I$$

¹ The "first stage" demand curve is the direct regression of visits per capita against travel cost. The "second stage" demand curve is derived by determining visitation to be expected with a set of postulated "increments in costs facing individuals at each origin" (see Dwyer *et al.*, pp. 87-94). Burt and Brewer showed that for single sites, the sum of consumer surpluses from the "first stage" demand curves (across origins) is equivalent to the area under the "second stage" demand curve.

² For the remainder of this discussion, the word "origin" will be used to indicate either "zone" or "population center," whichever is more appropriate for the specific planning situation encountered.

where:

E_j = number of RVD's per year from origin j

D_j = "first stage" demand function for the existing site, and origin j

R_{ij} = the number of RVD's/year to site i (including the existing site and all proposed sites), from origin j

T_{ij} = the travel cost per R_{ij}

M_{ij} = the management cost per R_{ij}

C_{ij} = the opportunity cost per R_{ij}

A_i = the capacity in RVD's/year for site i

J = the number of origins

I = the number of sites (existing and proposed).

While each origin's per capita "first stage" demand function for the given existing site would typically be the same,³ each origin has its own demand function in this formulation. This is required because population levels and travel costs to the existing and proposed sites vary across origins. All of the origins' demand functions are independent.

Proper specification of the demand functions (D_j) would include all relevant cross price terms for imperfect substitute sites (currently existing). Following Burt and Brewer, the proposed sites are all assumed to substitute perfectly for one and only one existing site, with all other existing site prices held constant. Since these cross prices are held constant, they would enter the linear programming objective function in the same way as an intercept term would. Thus, for simplicity, they will be ignored in this paper. Since the model structure will use a linear programming solution procedure, the downward slope

of each origin's demand curve will be approximated in a piecewise fashion using a straightforward segmentation of the demand curves. One could also use the approach presented by Duloy and Norton.

The optimal solution to this problem may not be predictive of actual recreation use if recreators are not cost-minimizers or if use is allowed to exceed the site capacities. The actual implementation of the optimal solution in regional planning is thus left as a separate problem. As an evaluation model, the assumption of constrained optimizing behavior is implied in the solution.

Figure 1 depicts a linear programming structure for solving this type of problem with two proposed sites and two origins (Z_1 and Z_2). This simple example does not include the piecewise approximated demand functions. The first six columns are the basic choice variables—the number of RVD's to be consumed by each origin at each existing or proposed site. It should be clear that the different sites "enter solution" according to efficiency (net benefit maximization) criteria, thus avoiding arbitrary ordering of the evaluation.

The first row contains the travel cost for each of these choice variables and totals the travel costs into column 7. The second row contains the site management cost for each of these choice variables and totals the site management costs into column 8. Management costs might include amortized investment costs. The third row contains the opportunity cost of each of these choice variables and totals the opportunity costs into column 9. The opportunity costs would generally be the net value of commodities other than recreation that would be foregone because of recreational use of the land. Obviously, in some cases there would be no opportunity costs. It is expected that the site costs for a given RVD at a given site will be the same for all origins. Rows 4 and 5 total the number of RVD's for each site into columns 10 and 11. These totals will subsequently be

³ In fact, the per capita "first stage" demand functions are typically regressed across origins in the aggregate (zonal) travel cost model. In deriving the "second stage" demand function, the consumer surpluses from the different origins are added up. This summation occurs in the maximand of the model discussed here.

	1 2 Site 1 (existing)		3 4 Site 2 (proposed)		5 6 Site 3 (proposed)			7	8	9	10 11 Use by Zone of Origin		Type of Con- straint	RHS
	Z ₁	Z ₂	Z ₁	Z ₂	Z ₁	Z ₂	T ₃₂	TC	MC	OC	Z ₁	Z ₂		
1 Travel Cost (TC)	T ₁₁	T ₁₂	T ₂₁	T ₂₂	T ₃₁	T ₃₂	-1						=	0
2 Management Cost (MC)	M ₁₁	M ₁₂	M ₂₁	M ₂₂	M ₃₁	M ₃₂			-1				=	0
3 Opportunity Cost (OC)	O ₁₁	O ₁₂	O ₂₁	O ₂₂	O ₃₁	O ₃₂				-1			=	0
4 Combine RVD's by	1	1	1	1	1	1					-1	-1	=	0
5 Zone of Origin													=	0
6 Capacity Constraints	1	1											=	A ₁
7			1	1									=	A ₂
8					1	1							=	A ₃
9 Objective Function							-1	-1	-1	-1	B ₁	B ₂	=	Max

Figure 1. A Simple RECAM Structure with Three Sites and Two Origins (Z₁ and Z₂).

the quantity demanded variable in the piecewise approximated benefit functions—one for each origin. Row 9 represents the objective function. The B's in row 9 represent the nonlinear benefit functions. Rows 6 through 8 constrain the RVD's to the maximum capacity of each site.

A "carrying capacity" (for example, maximum allowable RVD's per acre) might be used to convert optimal RVD's at any site to optimal site size or level of development. In some cases, it may be more appropriate to use a mixed integer formulation and treat the first six columns in Figure 1 as discrete choice variables indicating development or nondevelopment.

Obviously, any number of other embellishments on this basic structure are possible. In particular, the RECAM structure in Figure 1 involves optimization over a given single time period (for example one year). In this sense, it is a type of "steady state" model that does not consider scheduling options in the optimization. A dynamic version of RECAM would be an interesting extension of the model structure discussed in this paper. This basic structure should suffice, however, to demonstrate the use of an operations research approach to efficiently planning for recreation at the regional level, employing the travel cost information that travel cost-based demand analysis should provide.

It should be clear at this point that RECAM can be viewed in either of two ways: as a travel cost-based evaluation model or as a regional recreation planning (optimization) model. In the context of a regional planning model, it is likely that a given planning situation will involve several different types of recreation or several different market areas. The planner could build one RECAM model for each of these recreation types or market areas if net benefit estimation were all that is desired. If RECAM is to be used as a

planning optimization tool, however, this may not be desirable. If a global or overall agency budget constraint or output "target" applies to all recreation types or market areas, then one RECAM should be built for all of them. So long as the demand functions for these different types or market areas are independent, this is a straightforward extension of the simple example in Figure 1. This is the type of model demonstrated in the next section.

An Example Application of RECAM

In order to briefly demonstrate the workings of RECAM, a model was built for part of the Colorado National Wilderness Preservation System. Three existing wilderness areas (Eagles Nest, Rawahs, and Weminuche) were taken as the sites for which six proposed sites might substitute. Eagles Nest is closest to Denver (70 miles west of Denver), Rawahs is located 120 miles northwest of Denver, and Weminuche is in the southwest portion of Colorado. The proposed sites are the "Further Planning Areas" designated by RARE II (Roadless Area Evaluation Study II). It was assumed that (1) the St Louis Peak and Williams Fork Further Planning Areas will perfectly substitute for the Eagles Nest Wilderness Area; (2) the Lost Creek, Service Creek, and Davis Peak Further Planning Areas will perfectly substitute for the Rawahs Wilderness area; and (3) the Cannibal Plateau Further Planning Area will perfectly substitute for the Weminuche Wilderness Area. This application of the model could be viewed as demonstrating either a case where three different types of wilderness are being provided, or where three groups of wilderness sites are spatially removed such that they serve different markets.

Naturally, some of the proposed sites have some current use, even though they are not currently designated wilderness. For the purposes of this demonstrative example, this will be ignored. RECAM obviously applies best to situations where

proposed sites have no current use—for example, a case where without wilderness designation, no access into the proposed sites is available.

For this simple example, only relatively close origins (within roughly 300 miles) are included in the linear program: 5 origins for the Eagles Nest group, 6 origins for the Rawahs group, and 10 origins for the Weminuche group. Obviously, this example application should not be given policy interpretation with regard to the suitability of specific wilderness areas. It is presented here to demonstrate the applicability of the RECAM modeling approach and the types of (commonly extant) data needed.

The travel cost demand curves were estimated from all relevant origin-destination data collected over several summers by the U.S. Forest Service.⁴ The aggregate (zonal) travel cost method was employed instead of the individual observation approach because the data did not track a particular individual's visitations per year. The general form of this simple travel cost demand function was visits per 1,000 population as a function of the round-trip travel costs and value of travel time. In the initial regressions, origin per capita income was included as an independent variable. Because of its consistent insignificance, the income variable was dropped. As Cicchetti *et al.* point out, the finding of an insignificant "t" value on this variable does not imply that income is necessarily unimportant, only that our measure—county per capita income—was not a significant determinant of visitation rates. The total travel cost of a visit was made up of the variable transportation cost per mile and the value of travel time. The

⁴ Zip code/use data came from surveys directed by B. L. Driver and Perry J. Brown, funded by the USDA Forest Service, Rocky Mountain Forest and Range Experiment Station, and supported by Colorado State University McIntire-Stennis funds. The surveys were based on representative samples of summer-season users of the wilderness areas.

value of travel time was assumed to be one-third the average wage rate in Colorado along the lines suggested by Cesario. The variable transportation costs were 12.6 cents per mile (U.S. Department of Transportation). This was divided by the average number of persons per vehicle to get the transportation cost per visitor.

It has been shown that the aggregate (zonal) travel cost model can often be expected to be heteroskedastic (Bowes and Loomis; Christensen and Price; Vaughan *et al.*). For this simple example, the approach taken is the one suggested by Bowes and Loomis—a simple weighted regression where the weights are square roots of the zonal populations. This approach is equivalent to generalized least squares estimation if unequal zonal populations are the *only* source of heteroskedasticity. To the extent that heteroskedasticity and misspecification problems probably remain in the demand functions, these should be interpreted only as examples, not as reliable benefit estimators.

A semilog functional form was selected somewhat arbitrarily over quadratic and linear forms on the basis of the R². A log transformation on the dependent variable was not tested, because the presence of a travel cost axis intercept makes the linear program objective function (piecewise) easier to derive (without an intercept, an arbitrary cut-off point must be assumed). Also, the combination of the square-root transformation and a linear dependent variable ensures exact prediction of total use at current travel costs (Bowes and Loomis). This is judged to also be a desirable characteristic in RECAM. If a log-linear or log-log transformation is judged to be a superior functional form for the demand functions, these forms can be utilized in a RECAM model structure, but with a bit more difficulty.

The resulting per capita (actually per 1,000 persons) demand equations for Weminuche, Eagles Nest, and the Rawahs, respectively, are:

$$V/pop = .0424 - .0394 \ln TC$$

(3.084) $\bar{R}^2 = .335$

$$V/pop = .0345 - .0335 \ln TC$$

(3.699) $\bar{R}^2 = .564$

$$V/pop = .3000 - .0296 \ln TC$$

(2.177) $\bar{R}^2 = .290,$

where

- V/pop = visits per 1000 population
- TC = total travel cost in hundreds of dollars.

The numbers in parentheses are the “t” values. The t values on Weminuche and Eagles Nest are significant at the 99-percent level. The t value for the Rawahs is significant at the 95-percent level. For use in this study, these equations were scaled to account for the sampling density of overall annual use. Visits were converted to RVD’s at the rate of three RVD’s per visit. Data for this conversion were provided by the USDA Forest Service, Rocky Mountain Region (unpublished data).

As noted earlier, cross prices for existing sites other than Weminuche, Eagles Nest, and Rawahs are held constant. Thus, if they were included in the demand functions, they would enter the objective function in the same manner as the intercepts. It should be pointed out that if the existing sites included in the model (Weminuche, Eagles Nest, and Rawahs) are interrelated (for example, the Rawahs and Eagles Nest prices appear in each other’s demand functions), then the problem becomes more complex. The cross prices would not be constants, so the objective function would be a line integral of the system of equations. Duloy and Norton present a means of incorporating such an objective function in a linear program for the case of symmetrical cross-price partial derivatives. With ordinary Marshallian demand functions, the cross-price partial deriva-

TABLE 3. Run With \$250,000 Budget Constraint on Management Costs.

Total Benefits		
Travel Costs	\$14,043,000	
Management	2,616,100	
Costs	250,000	
Opportunity Costs	29,750	
Total Costs	2,895,850	
Net Benefits	\$11,147,150	
	RVD's	% of Capacity
Eagles Nest	7,749	6%
St. Louis Peak	0	0%
Williams Fork	10,783	14%
Lost Creek	23,000	100%
Rawahs	6,555	24%
Service Creek	0	0%
Davis Peak	93	1%
Weminuche	62,688	16%
Cannibal Plateau	31,990	100%
Total RVD's	142,858	

al net benefits they create.⁵ Thus, the current net benefits of existing sites should be deducted from the net benefits in Table 2, if valuation of the new sites (in solution) is desired. Knetsch demonstrated that if the value of a site is the additional consumer surplus it creates, then the value foregone because of reduced use at other sites is already taken into account.

Table 3 presents a solution where a budget constraint (\$250,000) is imposed on site management. Obviously, the budget constraint results in reduced expansion of the wilderness system. This run demonstrates the potential usefulness of a model such as RECAM in budget analysis. In this example, the budget reduction from the optimal \$425,026 to \$250,000 to meet the budget constraint is indicated to result in a loss of net benefits on the order of \$1.3 million. This type of information

⁵ "Net Benefits" are referred to here instead of "consumer surplus" so that changes in costs at existing sites between the current situation and the RECAM solution will also be accounted for. It should be noted that the demand curves in RECAM are not kinked (Knetsch, p. 126), but are the entire demand curves "including" current consumer surplus.

would be very useful in evaluation of different budget levels.

Table 4 presents two solutions where "target levels" were imposed on the total number of RVD's produced by the system of sites. This has become common practice in public land management analysis. Table 4 demonstrates that in this example, the imposition of targets has the potential for significantly affecting the optimization solution. With a target of 500,000 RVD's (as compared to an optimal 242,872 RVD's), management costs are about double those in the efficient solution, and the objective function is reduced by about \$9,000. With a target of 750,000 RVD's, management costs are tripled, and a loss of about \$1 million in net benefits is incurred. In contrast, if a target is set at or below the optimal number of RVD's, then the target would have no effect on the solution. If costs were minimized subject to a target rather than net benefits being maximized, then the target would always affect the solution. If the benefit measures are "believed," then imposition of target levels is somewhat illogical and can impact optimal solutions significantly. In the common situation where targets are imposed because of a lack of confidence in benefit measures, it must be recognized that benefit measurement has not been avoided—it has been done implicitly by setting the target levels. Setting targets in a tenable manner may actually be more difficult than measuring benefits in a tenable manner.

Conclusion

This paper started with a conceptual view of travel cost models such that a new site's net benefits are created by the implicit recreation price decreases associated with that new site's development. In that context, if a number of proposed sites are included in a regional planning problem, then optimizing across these proposed sites as they substitute (perfectly) for an exist-

TABLE 4. Analysis of Target Levels.

	Run With Target = 500,000 RVD's	Run With Target = 750,000 RVD's
Total Benefits	\$18,380,000	\$19,599,000
Travel Costs	4,966,200	6,692,400
Management Costs	875,000	1,312,500
Opportunity Costs	29,750	56,100
Total Costs	5,870,950	8,061,000
Net Benefits	\$12,509,050	\$11,538,000
	RVD's (% capacity)	RVD's (% capacity)
Eagles Nest	37,844 (29%)	130,915 (100%)
St. Louis Peak	0 (0%)	12,800 (100%)
Williams Fork	21,565 (29%)	74,770 (100%)
Lost Creek	23,000 (100%)	23,000 (100%)
Rawahs	27,464 (100%)	27,464 (100%)
Service Creek	0 (0%)	39,860 (100%)
Davis Peak	140 (1%)	7,801 (68%)
Weminuche	357,997 (89%)	401,400 (100%)
Cannibal Plateau	31,990 (100%)	31,990 (100%)
Total RVD's	500,000	750,000

ing site is a very natural extension of the traditional travel cost model. This is a basic optimization procedure that solves for the mix of inputs (travel and recreation sites) and the level of final output (recreation RVD's) that maximizes net benefits. Because of the origin-destination structure of the problem with downward sloping demand functions, its solution is rather similar to that of the traditional "transportation problem" and to those of "spatial equilibrium" models.

The example application demonstrated that the approach is feasible with commonly available data. And it showed how the model can be used in budget analysis and in planning for administrative targets. The sensitivity of solutions to these considerations was also discussed. An actual application of this model structure would generally require a larger model (in particular, a model with more recreator origins included). The foreseeable size would by no means be prohibitive, however, given current linear programming solution procedures.

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