Economics of Beef Cow Culling and Replacement Decisions Under Genetic Progress

Bryan E. Melton

Beef cow managers annually face the question of which animals to cull from the herd and replace. The results of this decision affect not only current revenues, but, by altering the genetic composition of the herd, also affect the future profitability of the herd. These genetic changes of the herd may, therefore, be represented as a form of endogenous technological progress to the cow calf producer. This article derives general asset replacement criteria for assets undergoing either exogenous or endogenous progress and illustrates their application with a Florida cow herd example.

Probably no single aspect of modern beef herd management is as complicated, or has as potentially great an economic impact, as the cow culling and replacement decision. Cattlemen must annually face the problem of deciding at what age a cow should be culled from the herd and replaced. Not only are substantial revenues involved in the decision, but if the genetic ability of replacements exceeds that of the animals culled then genetic technological progress occurs in the breeding herd. Thus, unlike most microeconomic analyses in which technology is assumed to be exogenous, the level of genetic technology and its rate of progress are directly affected by culling and replacement strategies and must, therefore, be considered endogenous to the cattlemen's management decisions [Ladd and Gibson].

The general principles of asset replacement have been well developed in the literature. Chisholm [1966], Dillon [Ch. 3], Faris, Perrin, Preinrick and Winder and Trant, among others, have each made significant contributions to the development of these principles. In this paper, the work of Perrin is expanded by the inclusion of animal breeding principles to analyze the cow culling and replacement decision for a producer raising his own replacements and thereby achieving genetic progress[1]. The effects of genetic progress on optimal culling and replacement strategies are then illustrated for a herd of Brahman-cross cows in South Florida.

A Cow Culling and Replacement Model

Although alternative objectives certainly exist [Anderson, et al; Chisholm, 1966], for the purpose of this study it is reasonable to assume that the cattleman's objective is one of maximizing the present value of the entire stream of residual earnings attributable to any given cow in his herd. Under conditions of certainty, and disregarding the effects of

Bryan E. Melton is an assistant professor in the Food and Resource Economics Department and an adjunct assistant professor in the Animal Science Department, University of Florida, Gainesville.

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[1]A large herd, supplying its own replacements, facilitates a genetic analysis that disregards such problems as migration and inbreeding in the herd [Lush]. These aspects may be considered for other herd situations by respecification of the genetic response function.

taxation [Kay and Rister; Chisholm, 1974], the producer's basic replacement problem may be stated as one of determining the age at which the marginal benefits of keeping a cow for an additional period are just equal to the marginal benefits of immediate replacement.

Asset Replacement Principles

Although cow culling and replacement is typically an annual decision (implying a discrete-time analysis), the algebraic and graphical presentation of the culling decision is simplified by a continuous-time treatment. Thus, following Perrin's notation [pp. 60-61], the following variables are defined for use in a continuous-time culling and replacement model:

$\rho = \ln(1+r)$ = the interest rate which, when compounded continuously, yields an annual growth rate of r; i.e. $e^{\rho t} = (1+r)^t$

t = an integer number of years;

M_a = the expected market value of a cow of age a;

R_a = the flow of expected residual earnings (current expected revenues less current expected costs) for a cow of age a; and

$C(b,s,m)$ = the present value of the stream of expected residual earnings of a Challenger acquired at age b and replaced at age s by a series of m Challengers

where "Challenger" refers to a potential replacement animal.

In the case where no genetic progress is occurring, i.e. an animal is replaced with another animal of the same genetic make-up and production ability, the replacement problem may be viewed as one of self-replacement as developed by Perrin. The present value of residual earnings from an infinite stream of identical Challengers — defining the objective to be maximized in this situation — is

$$(1) \qquad C(b,s,\infty) = \frac{1}{1-e^{-\rho(s-b)}}\, C(b,s,l)$$

where

$$C(b,s,l) = \int_b^s R_t e^{-\rho(t-b)}dt + M_s e^{-\rho(s-b)} - M_b$$

The function (l) is maximized by the value of s satisfying the first-order condition

$$(2)\; R_s + M_s' = \rho\left[M_s + \frac{1}{1-e^{-\rho(s-b)}}C(b,s,l)\right]$$

where

$$M_s' = \frac{\partial M_s}{\partial s}$$

Thus, in the absence of genetic technological progress optimal self-replacement occurs at the age that equates marginal revenues to the marginal opportunity costs (foregone earnings) of replacement. Perrin then goes on to apply this basic model to the case of a one time technological increase. As with self-replacement, however, a one-time technological increase replacement model also tends to over-simplify the reality of genetic progress. In practice, most breeding programs are on-going processes, resulting in continual genetic technological changes.

Animal Breeding Principles

The cow-calf producer, acting as an animal breeder, is concerned with altering the level of the trait in the herd (as judged by changes in the mean, μ) through genetic mechanisms such that profits are improved. Selection is the only means by which these directional genetic changes can be achieved. Put most simply, genetic selection is the process of allowing certain individuals to reproduce at greater lifetime rates than others. For the

cow-calf producer, maintaining a constant herd size (as is required by animal breeding theories), two different, but related, actions are therefore required in genetic selection programs: 1) the identification of those animals in the cow herd that are to be culled and 2) the identification of those animals to be used as replacements from the group of all potential replacement animals. Assuming that these identifications can be made and that the replacements selected are superior to the cows culled, the change in the mean of the herd due to selection, or genetic response (GR), for each generation of selection may be obtained as [Falconer]

$$(3) \quad GR = (1/2)\, h^2\sigma_x i_m + (1/2)\, h^2\sigma_x i_f$$

where

h^2 = the heritability of the trait — or the proportion of a trait's observed value that is genetic in nature and may, therefore, be inherited by succeeding generations;

σ_x = the observed, or phenotypic, standard deviation of the trait (X); and

i = the intensity of selection of either males (m) or females (f), which under the assumption of normality, may be computed as

$$i = \frac{z}{\alpha}$$

where α is the proportion of the relevant replacement population (either m or f) that is selected as replacements — i.e. the proportion of the population with values of X falling beyond a point of truncation and z is the height of the ordinate of the normal distribution at the point of truncation.

To obtain expected annual response, (3) is divided by the generation interval (GI) — defined as the average age of the parent animals — or

$$GI = 1/2\,(a_m + a_f)$$

where a_m and a_f are the average ages of bulls and cows in the herd, respectively. Thus, the expected annual response to selection for X in the herd may be expressed as

$$(4) \quad \frac{GR}{GI} = \frac{h^2\,\sigma_x}{a_m + a_f}\left[\frac{z}{\alpha}_m + \frac{z}{\alpha}_f\right]$$

Genetic Progress in the Cow Culling Decision

To consider genetic progress as an integral part of the cow culling and replacement decision, it must first be expressed in economic terms. For this study the economic value of a trait is, therefore, defined as the effect on the total flow of an animal's residual earnings due to increases in the level of the trait [Hazel; Melton, et al], or

$$w = \frac{dC(b,s,l)}{dX}$$

The annualized economic value of genetic progress is then obtained by multiplying w times the expected annual genetic response specified by (4). The economic returns from genetic selection may then be assumed to be growing at an annual rate defined by

$$\gamma = \ln\,[1+g]$$

and where

$$g = \frac{w\frac{GR}{GI}}{\int_b {}^sR_t e^{-\rho(t-b)}dt}$$

In this form it is clear that the annual rate of genetic progress realized, and hence the annual rate of economic growth, is dependent upon the age at which cows are culled and replaced. Furthermore, the dependency of γ on s exists even if all genetic progress is due to the selection of sires, with replacement cows representing a random selection of the heifers produced. For instance, not only does the age at culling (s) affect the economic value of the trait (w), but for a cow

of average ability

$$a_f = \tfrac{1}{2}(s+b)$$

which, in turn, affects the generation interval and thus the annual genetic response.

When cow selection is also practiced, the maintenance of a fixed herd size requires that the number of replacements brought into the herd each year is

$$\frac{1}{s-b} + (1-\Theta) \text{ times the herd size}$$

where $(1-\Theta)$ is the average proportion of the cows naturally culled due to death loss and the calves born each year are equally (½) distributed between males and females. Thus, the proportion of the potential replacements that must be selected as replacements (α_f) is also dependent on culling age:

$$\alpha_f = \frac{2[1+(s-b)-\Theta(s-b)]}{(s-b)\beta}$$

where β is the average calving rate.

Because genetic progress is measured in terms of changes in the mean of the herd, it is appropriate to initially consider the optimal culling and replacement decision for a cow just equal to the mean. For simplicity C(b,s,l) is therefore defined as the present value of the average cow. (The culling and replacement of cows more or less productive than the mean may then be related to the culling of the average cow, as will be shown later.) The present value of residual earnings from an infinite stream of Challengers (continuously improving at the rate γ) may then be expressed as

$$(5)\quad C(b,s,\infty) = \frac{1}{1-e^{(-\rho+\gamma)(s-b)}}\, C(b,s,1)$$

When the noted relationships between the annual rate of genetic progress (γ) and culling age (s) are recognized, the derivative of (5) with respect to s is

$$(6)\quad \frac{dC(b,s,\infty)}{ds} = \frac{-\left[\rho-\gamma_s-(s-b)\gamma_s'\right]e^{(-\rho+\gamma_s)(s-b)}}{\left[1-e^{(-\rho+\gamma_s)(s-b)}\right]^2}\, C(b,s,1) + \frac{e^{-\rho(s-b)}\left[R_s+M_s'-\rho M_s\right]}{1-e^{(-\rho+\gamma_s)(s-b)}}$$

where γ_s is the partial derivative of γ with respect to s and γ_s is the value of γ (each evaluated at age s). Setting (6) equal to zero and rearranging terms produces the first-order replacement condition under genetic progress:

$$(7)\quad R_s+M_s' = \rho\left[M_s + \frac{\left[1-\frac{\gamma_s}{\rho}-\frac{(s-b)}{\rho}\gamma_s'\right]e^{\gamma_s(s-b)}}{1-e^{(-\rho+\gamma_s)(s-b)}}\, C(b,s,1)\right]$$

An examination of this condition indicates that it may actually be regarded as the general replacement criterion for any asset undergoing technological progress at an annual rate of γ — whether that progress is endogenous, as in the case of livestock, or exogenous, as in the case of equipment. For instance, if $\gamma_s = 0$ for all values of s (i.e. γ is a constant), then (7) reduces to the general replacement criterion for continuous exogenous technological progress:

$$(8)\quad R_s+M_s' = \rho\left[M_s + \frac{\left(1-\frac{\gamma}{\rho}\right)e^{\gamma(s-b)}}{1-e^{(-\rho+\gamma)(s-b)}}\right]$$

If $\gamma = 0$, then (8) obviously further reduces to the optimal asset replacement criterion developed by Perrin for self replacement, as shown in (2).

For non-negative asset values (C(b,s,l) ≥ 0),[2] a comparison of these alternative

[2]An existing asset with a negative net present value would typically be sold when its own value was at a maximum often as soon as possible. For beef cows this leads to a reduction in the size of the cow herd when

scenarios of technological progress indicates that the optimal replacement age of an asset undergoing exogenous technological progress at an annual rate of γ ($\gamma>0$ and $\gamma' = 0$) will be earlier than the same asset in the absence of technological progress ($\gamma = 0$).[3] Furthermore, the optimal replacement age under endogenous technological progress will be earlier than the case of constant exogenous progress whenever γ_s is negative and later whenever γ_s is positive, as shown in Figure. 1.

Culling Genetically Superior Cows

The derivation of this general first order replacement criterion (9) and its subsequent solution for the optimal replacement age (s) should not, however, be taken to imply that every cow should be replaced at age s. This replacement criterion is appropriate only for the average cow in the herd. In any herd, some cows will be better while others are worse than the average.

The culling and replacement of these genetically superior (and inferior) cows may be viewed as analogous to the case of asset replacement under a one-time technological increase, as derived by Perrin. In this situation, the superior (or inferior) cow is to be replaced by an infinite series of genetically improving Challengers. The superior cow would logically have a flow of residual earnings (R_c) greater than the average (R_s) in each year, such that

$$R_c > R_s$$

at every age ($c=s$). The left-hand-side of (7) would thus be greater for the superior cow than for the average cow, leading to a later optimal replacement age for superior cows than is observed for the average cow in the herd. The opposite would obviously hold for an inferior cow. How much later (or earlier) would depend upon the relative superiority (or inferiority) of the cow under consideration.

Multiple Trait Genetic Progress

It should also be recognized that the cow culling and replacement principles presented thus far have dealt with but a single trait. Because traits are correlated, however, selection for one trait (X_i) often affects other traits (X_j). In such cases the correlated annual response in X_j due to selection for X_i (CGR_{ij}) would be

$$\frac{CGR_{ij}}{GI} = \frac{h_i h_j \sigma_{xi}}{a_m + a_f} \frac{\sigma_{gij}}{\sigma_{gi}\sigma_{gi}} \left(\frac{z}{\alpha_m} + \frac{z}{\alpha_f} \right)$$

where σ_{gij} is the genetic covariance between the *ith* and *jth* traits and σ_{gi} and σ_{gi} are the respective genetic standard deviations [Falconer].

Hazel seized upon this fact to introduce selection based upon a multiple-trait index in animal breeding. By this procedure the effects of correlated responses are already reflected in the value of the index, thereby effectively reducing the problem to one of single trait selection where the magnitude of the index itself is the single trait of interest.

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cattle prices are low because the number of potential replacements with positive net present values is small relative to the number of cows culled with negative net present values (i.e. the cattle cycle phenomena). Because animal breeding theories can not adequately deal with a variable herd size, this analysis is restricted to the realm of non-negative net present values ($C(b,s,1) \geq 0$).

[3]This result is obtained by comparing the multipliers of ($C(b,s,1)$ in (2) and (8). This comparison necesssitates evaluating the sign of

$$e^{\gamma(s-b)}\left[1 - \frac{\gamma}{\rho}(1 - e^{-\rho(s-b)})\right]$$

which is obviously positive whenever $\frac{\gamma}{\rho} \leq 1$.

It is logical for γ to have an upper limit of ρ, since if $\gamma>\rho$ returns are compounded at a rate of $\gamma-\rho$. In the long-run, market forces would tend to eliminate such a situation — implying the existence of economic profits and a negative time preference — by bidding up current prices (either nominal rents, sales or ρ) until γ were again less than ρ [Perrin, p. 64].

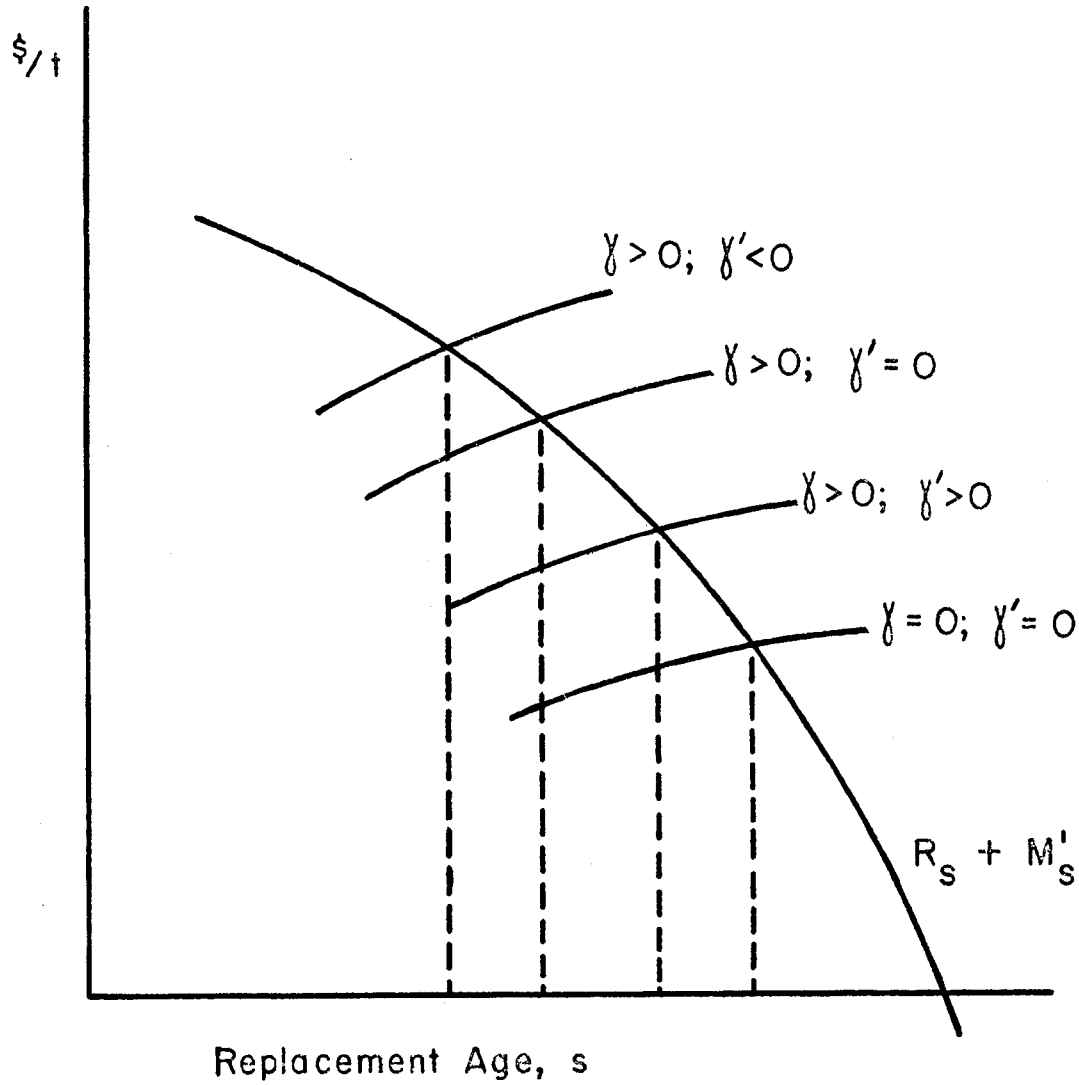

FIGURE 1. Effect of Genetic Technological Progress (γ) on Optimal Replacement Age.

Thus, the preceding discussion of replacement under genetic progress may be easily extended to multiple-trait genetic progress through the derivation of an appropriate selection index and selection on that basis.

The Discrete-Time Analog

As noted previously, the cow culling decision is typically made on an annual basis, implying a discrete-time analysis. The present value of future earnings specified by (5) would then become

$$(9)\quad C(b,s,\infty) = \frac{1}{1-(1+r)^{-(s-b)}(1+g)^{(s-b)}}$$

$$C(b,s,1)$$

where

$$C(b,s,1) = \sum_{t=b+1}^{s} R_t(1+r)^{-(t-b)}$$

$$+ M_s(1+r)^{-(s-b)} + M_b$$

The discrete-time analog of the general, continuous-time replacement condition (7) is, therefore,

$$(10)\quad R_{s+1} + \Delta M_{s+1} = r\left[M_s + \frac{\left(1-\frac{g_s}{r}-\frac{(s-b)}{r}\Delta g_s\right)(1+g_s)^{(s-b)}}{1-(1+r)^{-(s-b)}(1+g_s)^{(s-b)}}C(b,s,1)\right]$$

where as Perrin notes [p. 64], it is more logical to compare returns in the forthcoming year $R_{s+1}+\Delta M_{s+1}$ to returns from current replacement because it is the forthcoming year's returns that will be foregone by a current year replacement policy.

The obvious problem with this discrete-time replacement criterion is that it is only by accident that the equality specified by (1) would be satisfied at the time the culling decision is made. There is, therefore, the potential of a one year error being made in optimal culling age with the discrete-time replacement criterion specified by (1). Burt developed marginal replacement criteria that avoid this error, which in terms of the variable used here produces

$$(10.1)\quad R_{s+1} + \Delta M_{s+1} \leq r\left[M_s + \frac{\left(1-\frac{g_s}{r}-\frac{(s-b)}{r}\Delta g_s\right)(1+g_s)^{(s-b)}}{1-(1+r)^{-(s-b)}(1+g_s)^{(s-b)}}C(b,s,1)\right] \leq R_s + (1+r)\Delta M_s$$

Empirical Example

An example may better illustrate the cow culling and replacement decision under genetic progress. For this purpose data from an experimental herd of Brahman-cross (⅜ to ⅝ Brahman) cows located at the Range Cattle Station, Ona, Florida were used to estimate the necessary physical production relationships relating to cow growth, mortality, reproduction, calf weaning weight, etc. (More detailed discussions of this South Florida herd are presented by Koger, et al, Mbah and Peacock, et al.) For simplicity it was assumed that all calf sales for this herd occurred at the time calves were weaned (in October) — following an average 205-day lactation period. At the same time all culled cows were sold and replaced with 2-year old heifers just completing their first lactation. Thus, all revenues were assumed to be received at weaning, and all costs, although possibly incurred throughout the year were assumed to be paid at the same time. Both revenues and costs were estimated using average Florida prices for the period 1967-76[4]. (A more detailed discussion of the method used to calculate annual costs by age of cow is provided by Melton.)

[4]In reality there are obvious differences in animal prices due to both producer expectations and market prices of individual animals. A ten-year average is used as representative of producer expectations that minimizes the effects of the "cattle-cycle" phenomena. Price differences between animals that may be due to grade differences (and are generally independent of age beyond 4 years of age) are avoided by the use of average Florida slaughter cow prices.

For purposes of illustration, two alternative scenarios of genetic selection for increased calf (weight) production per cow were examined: 1) $g = 0$ and 2) $g = f(s)$ such that $\Delta g \neq 0$. Where $g = f(s)$ it was assumed that sire selection is also practiced independently of the cow culling decision, with the top 2% of male calves produced annually kept as replacement sires and an average sire age of six years; thus, $i_m = 2.42$ and $a_m = 6$. It was also assumed that the additional costs incurred in capitalizing genetic progress in calf production would arise from added feed requirements. For this example these costs were estimated to be $.25 per additional pound of calf produced, meaning that the annual value of an additional pound of calf production is equal to its market value less $.25 per pound, or $.067. Hence, when the expression of genetic superiority by an animal is unaffected by its age,

$$w = .067 \int_{2}^{s} e^{-\rho(t-2)}dt$$

The remaining genetic parameters for calf production used are $h^2 = .40$ [Dickerson, et al] and $\sigma_x = 48.3$ pounds [Peacock, et al].

Results

Table 1 presents the relevant parameters of the culling and replacement decision for the average cow in this herd when the producer expects the average of 1967-76 cattle prices and a discount rate of 5% (i.e. $r = .05$)[5]. Columns (1) and (2) present the estimated market value and residual earnings of this cow by year of age. Columns (4) and (5) specify the annualized returns from replacement under the appropriate conditions of genetic progress for each of the selection scenarios considered. The annualized returns from replacement must then be compared to the returns from keeping the cow — shown in columns (3) and (6) — using the optimal replacement condition specified by (10.1).

When no genetic progress is occurring ($g = 0$) the optimal culling age is 11 years of age, as shown in column (4). At this age the annualized returns from replacement of $14.27 most nearly equal, although exceed, the returns from keeping the cow for an additional year of $12.91, but are less than the cow's current earning value of $15.25. At any earlier age the returns from keeping the cow for an additional year exceed her annualized returns from replacement, indicating that an economic benefit can be gained by delaying the cow's culling. At later ages the opposite holds.

When it is recognized that the rate of genetic progress is influenced by the culling age ($g = f(s)$), there is a marked reduction in the optimal culling age of the average cow — as shown in column (5). Under these conditions the optimal replacement criteria specified by (10) and (10.1) are most nearly satisfied by culling the average cow at 8 years of age when the annualized returns from replacement are $19.57[6].

The decrease in the optimal culling age when $g = f(s)$ can primarily be attributed to the decline in the annual rate of genetic progress that accompanies increased average culling age. For instance, at 8 years of age $g = .033$, while at 9 years of age $g = .029$ and at 11 years of age $g = .026$. Thus, the annual rate of genetic progress as defined in this study decreases at a decreasing rate with increases in culling age ($\gamma_s' < 0$). An earlier culling age is, therefore, warranted to capture the benefits of genetic progress. The

[5]The selection of a 5% discount rate is admitted arbitrary, but should represent a reasonable average required rate of return over the 1967-76 period. Perrin [p. 65] discusses alternative interpretations and basis for the selection of a discount rate; each of which may be equally appropriate in the cow culling decision model.

[6]An interesting paradox is presented by this example: the optimal replacement criterion specified by (10.1) is also satisfied by culling the average cow at 9 years of age, when the annualied returns from replacement are $18.31. Calculating the net present value of the infinite series of genetically improving cows directly yields $C(2,8,\infty) = \$175.08$ and $C(2,9,\infty) = \$150.66$, indicating a maximum at 8 years of age.

TABLE 1. Estimated Parameters of the Cow Culling Decision Under Average 1967-1976 Cattle Prices for Brahman-cross Cows in South Florida.

Cow age (t)	Market Value (M_t)	Residual Earnings (R_t)	Annualized Returns from Replacement[a]			
			$R_t+\Delta M_t$	g = 0	g = f(s)	$R_t+(1+r)\Delta M_t$
	(1)	(2)	(3)	(4)	(5)	(6)
3	204.10	− 6.20	− 2.10	b	b	− 1.90
4	211.98	3.92	11.80	b	b	12.19
5	215.40	11.33	14.75	b	b	14.92
6	216.62	16.28	17.50	10.56	10.83	17.56
7	216.73	19.08	19.21	12.13	19.56	19.22
8	216.29	20.09	19.65	13.21	19.57	19.63
9	215.59	19.70	19.00	13.89	18.31	18.97
10	214.76	18.27	17.44	14.22	17.44	17.40
11	213.86	16.19	15.29	14.27	16.62	15.25
12	212.93	13.84	12.91	14.12	15.81	12.86
13	211.99	11.57	10.63	13.82	15.04	10.58
14	211.05	9.77	8.82	13.46	14.40	8.78
15	210.10	8.78	7.83	13.10	13.95	7.78

[a]Annualized returns from replacement are computed as the right hand side of the general replacement criterion specified by (10).

[b]Culling at this age will not allow for the maintenance of a constant herd size under the given reproductive level.

amount of these benefits can be calculated as the difference between the net present values of the infinite series of cows when g = 0 and g = f(s). At the optimal culling ages $C(2,11,\infty)$ = \$71.57 when g = 0 and $C(2,8,\infty)$ = \$175.08 when g = f(s). The discounted value of genetic progress through infinity is, therefore, \$103.51 per cow. This amount represents the maximum investment that can economically be made in a genetic selection program by this producer.

At the optimal culling age under genetic progress of 8 years approximately 55 percent of the heifer calves produced would be required as replacements for the herd. The intensity of selection produced by this replacement rate is approximately .71, leading to an increase in weight of calf produced of approximately five pounds per year. Over 77 percent of this incrase is due to the selection of sires, while 23 percent is due to the selection of dams. Thus, the discounted value of sire selection through infinity is approximately \$79.50 per cow while the discounted value of cow selection is approximately \$23.81 per cow. Because of the greater value due to sire selection, and the fact that many fewer bulls are required than cows, it is clear that the selection of replacement bulls warrants the special attention it frequently receives. The full benefits of even the best sire can not, however, be realized without an optimal cow selection and culling program.

Effects of Genetic Superiority and Inferiority

As discussed previously, the culling of genetically superior and inferior cows may be related to the culling of the average cow through differences in their annual residual earnings, R_t. To illustrate this, the culling of cows one phenotypic standard deviation better and worse than the mean for calf production is examined. The superior cow, therefore, generates residual earnings of \$3.24 per year more than the average cow ($\sigma_x(P_c - .25)$ = \$3.24) while the inferior cow's residual earnings are \$3.24 per year less than the average. These differences in the value of R_t, assuming M_t and M_t' are unchanged by superiority or inferiority for calf production, have the effect of increasing the optimal

culling age for superior cows and reducing it for inferior cows.

Specifically, the superior cow is culled at 12 years of age when $g = 0$ and at 11 years of age when $g = f(s)$. The inferior cow, on the other hand, is culled at 10 years of age when $g = 0$ and at 5 years of age when $g = f(s)$, if the proportion of the herd represented by these inferior cows is sufficiently small that the early culling will still allow for a constant herd size. Hence, when $g = 0$ a one year change in optimal culling age accompanies a one standard deviation change in the productivity of the cow. When $g = f(s)$, however, the same change in cow productivity leads to a three year change in the optimal culling age. The effects on optimal culling age of genetic superiority and inferiority are, therefore, clearly more significant when genetic progress is occurring than when the herd mean is constant. Such a result is to be expected, and is entirely consistent with animal breeding practices and theories [Lush].

Implications

There are obvious limitations to the results presented in this paper. Most notable of these regards the producer's assumed certainty with respect to prices and interest rates. Obviously, these are neither known with certainty nor fixed through infinity. Additional research is therefore needed into the manner in which these expectations are formed by beef producers and their effect on optimal cow replacement.

The example used in this study is also limited in that it deals only with one trait, calf production. A more general replacement problem would deal with a selection index that recognizes the fact that as weaning weights increase there are correlated increases in mature cow weights and, thus, both greater feed-carrying costs and market value (M_s).

Despite these limitations and the fact that the primary focus of this study has been the analysis of optimal culling and replacement of beef cows under genetic technological progress, it is not limited to that decision. Within the study's relatively narrow focus a more general asset replacement criterion has been developed. By this criterion the replacement age of any asset undergoing technological progress may be determined, such that the net present value of the asset and its infinite series of replacements is maximized. The criterion developed allows for the rate of technological progress to be either endogenous to the producer's replacement decision, or the exogenous parameter it has typically been assumed to be. Thus, greater insight into any producer's asset replacement decision and his rate of technological progress (or adoption) is possible. Such results should prove valuable not only to producers, but also to researchers and scientists engaged in the development of new technology, and to extension personnel and educators charged with disseminating the new technology (or knowledge).

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