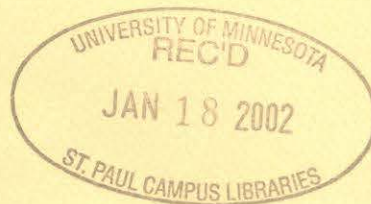# The University of Western Australia

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MIGRATION AND THE DEGREE OF COMMON
PROPERTY FOR A NATURAL RESOURCE

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MIGRATION AND THE DEGREE OF COMMON
PROPERTY FOR A NATURAL RESOURCE

Abstract

Migration, along with growth and harvest, is one of three fundamental ways a natural resource stock can change. Migration across property lines makes the resource common property. This study models migration and the degree of common property along the continuum from the exclusive access of private property through limited access to open access. It was found that wildlife travelling under their own power are more likely to be common property than resources powered by exogenous forces such as the wind. Pests have a higher degree of common property than valuable resources, faster growing resources a lower degree, and inexpensively managed resources a higher degree. Both exclusive access and open access are difficult to reach and most migratory resources will have limited access between the two extremes.

By definition common property has more owners than one. Multiple owners may over-exploit the common property and dissipate its scarcity rent. An open-access fishery is the textbook example. Other examples are open-access forest and grazing lands. Many natural resources, however, are common property but with limited access. Rivers, aquifers, oilpools, pollution, pests, diseases and wildlife can migrate across fence lines but are accessible only to local landowners. These landowners may capture some of the rent, especially if migration is slow and landowners are few.

Almost all studies of common property simplify by assuming a highly mobile resource and a large number of landowners. An open-access equilibrium is compared to the private property equilibrium as if the rent were completely dissipated. A study of limited access with partially dissipated rent must model migration of the resource and imperfect competition among landowners. Migration is one of three fundamental ways a resource stock can change and has been extensively studied by hydrologists and mathematical ecologists but not by economists. An exception is pollution transport models [1]. Imperfect competition for a natural resource has been studied by Cornes and Sandler [3] in a static model without migration.

The purpose of this article is to model the degree of common property on the continuum from the exclusive access of private property to open access. It emphasizes migration of a resource and briefly discusses imperfect competition among a few landowners.

First a dynamic bioeconomic model is specified to include both migratory pests and valuable resources. Next, functional forms for migration from the mathematical ecology literature are generalized and adapted to the study of common property. The roles of migration and imperfect competition in determining the degree of common property are defined. Finally, the model is illustrated for different types of natural resources.

I. A Dynamic Model

Wildlife are natural resources which are harvested directly and also affect the production of other commodities. Animals can be hunted for food or recreation. They may be pests and damage crop and domestic livestock production, or they may be desirable species and produce amenities. Hereafter the resource will be called wildlife, and the local landowners who exploit the wildlife will be called ranchers. Other types of resources are special cases and can be described with an appropriate alteration of terms.

A rancher manages a proportion of the total wildlife in his neighborhood. How much control he can exert depends on the mobility of the wildlife and the size of his ranch. He seeks to maximize the value of his initial stock of wildlife which equals the net present value of direct benefits from harvesting plus indirect benefits of producing livestock and amenities, minus the costs of ranching.

$$(1) \qquad j\left(s_0\right) = \underset{h}{\text{Max}} \int_0^\infty e^{-\delta t} \left[p_h h_t + p_y y\left(s_t, k\right) - p_c c\left(h_t, s_t, k\right) \right] dt \ ;$$

where j is the net present value of the ranch; h is harvest of wildlife; y is yield of livestock and amenities; c is the quantity of inputs; s is the stock of wildlife on the ranch; k is the carrying capacity of the ranch; p_h, p_y and p_c are prices; and δ is the interest rate. Each of the neighboring ranchers has a similar decision problem.

The stock of wildlife can change over time. The rate at which wildlife reproduce and grow on a ranch with a limited carrying capacity depends on intrinsic growth and population density. The rate at which they migrate depends on population densities, their mobility and forces such as prevailing winds. And the rate at which they are harvested is controlled, of course, by the rancher.

$$(2) \qquad \dot{s}_t = g\left(s_t, k, r\right) + m\left(s_t, S_t, k, K, v\right) - h_t \ ;$$

where $\dot{s}$ is the change over time in the stock of wildlife on the ranch; g is biological growth; m is the net migration onto the ranch; r is the intrinsic growth rate for a ranch with unlimited carrying capacity; S is the aggregate stock on neighboring ranches; K is the aggregate carrying capacity of neighboring ranches; and v is the velocity at which wildlife travel under their own power or windpower. Migration on to the ranch is influenced by stocks on neighboring ranches. Thus migration links decisions of the rancher to the decisions of his neighbors.

Rivers, aquifers and oil pools are special cases of the model with y set to zero; pollution, pests and diseases are special cases with p_h set to zero; and fisheries and forests are special cases with y set to zero but migration

defined so that the location of the resource stocks is relative to the location of fishing vessels or logging operations and not in relation to fixed fence lines.

The righthand side of equation (2) is the net quantity of wildlife used in time t. Multiplying the net quantity used by an imputed price gives the total rent paid for the wildlife. This rent is total user-cost. Subtracting total user-cost from static profits, which equal total revenue of harvest plus total revenue of livestock and amenities minus total costs of inputs, gives a dynamic measure of profit at time t.

$$(3) \quad \pi\Big[h_t,s_t,\lambda_t\Big] = p_h h_t + p_y y\Big[s_t,k\Big] - p_c c\Big[h_t,s_t,k\Big]$$
$$+ \lambda_t\Big[g\Big[s_t,k,r\Big] + m\Big[s_t,S_t,k,K,v\Big] - h_t\Big] \; ;$$

where π is dynamic profit at time t and λ is the imputed price or marginal user-cost of wildlife. π is the current-value Hamiltonian and λ is the current-value costate. Neither is discounted; both are denominated in dollars at time t. Because the costate captures the effect of current decisions on the future, maximizing the Hamiltonian in each time period is equivalent to maximizing the net present value of the ranch in equation (1) subject to the change in the wildlife population in equation (2).

If functions y, g and m are concave and c is convex, the Hamiltonian is concave and the optimum is characterized by first-order conditions with respect to harvest, wildlife and marginal user-cost plus an initial condition on wildlife and a terminal condition on marginal user-cost.

(4a) $\quad \partial\pi/\partial h_t = 0 = p_h - p_c \partial c/\partial h_t - \lambda_t \; ; \qquad 0 \le t$

(4b) $\quad -\partial\pi/\partial s_t = \dot{\lambda}_t - \delta\lambda_t = -p_y \partial y/\partial s_t + p_c \partial c/\partial s_t - \lambda_t\Big[\partial g/\partial s_t + \partial m/\partial s_t\Big]; \; 0 \le t$

(4c) $\quad \partial\pi/\partial\lambda_t = \dot{s}_t = g + m - h_t \; ; \qquad 0 \le t$

(4d) $\quad s_0$ is given ;

(4e) $\quad \lim_{t\to\infty} e^{-\delta t}\lambda_t = 0 \; .$

Condition (4a) equates marginal revenue to marginal costs from harvesting plus marginal user-cost. If there were open access, the rent due the scarce wildlife would be dissipated and the marginal user-cost would be zero. The harvesting decision would maximize current profits with no regard for the future. If there were partially limited access, the marginal user-cost would be non-zero but less than the full rent due the wildlife.

Marginal user-cost is defined by condition (4b) which can be rearranged into a form somewhat like that of condition (4a).

(4b') $0 = \left[p_y \partial y/\partial s_t - p_c \partial c/\partial s_t\right] / \left[\delta - \dot{\lambda}_t/\lambda_t - \partial g/\partial s_t - \partial m/\partial s_t\right] - \lambda_t$.

The marginal revenue minus the marginal costs with respect to wildlife are capitalized by an appropriate discount rate because a change in the stock of wildlife affects all future stocks. The marginal user-cost is the present value of all these effects. The appropriate discount rate is the interest rate less the rate of capital gains, the marginal growth rate and the marginal migration rate. Migration can dissipate marginal user-cost in two ways. The first is directly through marginal migration, causing the same effect as discounting the future too heavily. The second is indirectly through changing stocks and altering marginal revenue, marginal costs and marginal growth. First-order conditions for each of the neighboring ranches are linked through migration and must be solved simultaneously to determine stocks on the ranch and stocks on neighboring ranches.

II. Migration

Growth and harvest have been studied by Clark [1] and many others but migration has not been included in bioeconomic models. In the mathematical ecology literature, Skellam [8] was among the first to study migration in heterogeneous environments. His work has been supplemented by many authors with surveys by Levin [4] and McMurtrie [6] and a text by Nisbet and Gurney [7]. The foundation of this literature is a partial differential equation describing migration at a single point in an infinitely large environment. Wildlife may move about the environment but movement will not lead to migration if it is aimless and undirected. Nor is directed movement, called flux, sufficient for migration. Migration is the change in directed movement.

This can be explained beginning with wildlife movement per unit of time defined as:

$$vn \; ;$$

where v is wildlife velocity and n is wildlife numbers. Wildlife will move if velocity is positive. Movement may not be directed, however, unless it changes systematically.

$$F/dt = \left[v\partial n/\partial x + n\partial v/\partial x\right] \; ;$$

where F is directed movement called flux, dt is a unit of time and x is a position in the environment. Wildlife will move aimlessly if they are uniformly distributed and their velocity is constant. Conversely, wildlife movement is directed with non-zero flux if (a) there is a population gradient, or (b) velocity changes. But flux is not migration. Migration is a change in the flux over the environment.

$$\partial n/\partial t = \left[\partial F/\partial x\right]/dt = \left[v\partial^2 n/\partial x^2 + 2\left(\partial v/\partial x\right)\left(\partial n/\partial x\right) + n\partial^2 v/\partial x^2\right] \; .$$

Movement can be directed and flux non-zero without causing migration. Linearly distributed wildlife travelling at a constant velocity will move

down the population gradient but each animal arriving at position x is matched by an animal departing with no net migration. Uniformly distributed wildlife travelling at a linearly changing velocity will move down the velocity gradient but, again, with no net migration. Wildlife migrate if (a) wildlife are nonlinearly distributed, (b) there are both population and velocity gradients, or (c) velocity is nonlinear. With suitable functional forms, these three conditions can describe a heterogeneous environment of changing population densities, prevailing winds, territorial behavior, hunting pressure by man, water holes, and refuges from predators.

Migration at a point in an infinitely large environment does not fit the bioeconomic model of the previous section, however. A point corresponds to one unit of carrying capacity whereas a ranch may comprise many units. The ranch is surrounded by a neighborhood suitable for livestock and the neighborhood is part of the total environment occupied by wildlife. If the environment were infinite, the degree of common property would be indistinguishable between small and large ranches. Measured against infinity, a ranch of one point would appear the same as a ranch encompassing the entire neighborhood.

A large ranch has not been modeled in the literature but a finite environment has been modeled by two approaches (McMurtrie, p.18 [6]). Either wildlife are assumed to travel uniformly around a habitat but die immediately if they go beyond fixed boundaries or wildlife slow as they travel up a gradual incline from favorable to less favorable habitat. The model to be derived combines elements of both these approaches. Boundaries are fixed to allow comparison with the size of the ranch but the boundaries, themselves, slow the velocity of approaching wildlife. The finite environment is heterogeneous, causing slower migration onto larger ranches.

The model of migration can be summarized by a theorem and its corollary.

THEOREM 1: For a large ranch in a finite environment, wildlife will migrate under their own power or under wind power according to:

$$m_t = 2Z\left(\rho_k\right)\left[\sigma^2\left(\eta_3 + \eta_4\right) + w\eta_1\right]k .$$

The term in square brackets is average migration at a point in an infinite environment. Variance of wildlife movement, σ^2, multiplies terms for the second derivative of wildlife numbers, $\eta_3 + \eta_4$. The distance wildlife travel on a prevailing wind, w, multiplies a term for the first derivative of wildlife numbers, η_1. Average migration at a point is scaled up by k, the size of the ranch, and modified by $2Z(\rho_k)$, the probability that mobile wildlife at the edge of the ranch will not reach a boundary of the environment minus the probability they will hit a boundary. In an infinite environment, wildlife would never hit a boundary and $2Z$ would converge to unity. In a finite environment, $2Z$ falls toward zero as the size of the ranch

increases. Wildlife migrate onto the ranch if the size of the ranch is less than the size of the environment and (a) wildlife are nonlinearly distributed, or (b) a wind combines with a population gradient parallel to the wind. A population gradient perpendicular to the wind does not affect migration. Territorial behavior, hunting pressure by man, water holes and refuges from predators would make the variance of wildlife movement a function instead of a constant and add a third reason for migration.

COROLLARY 1: <u>The three parameters for the distribution of wildlife numbers can be observed by three censuses. Migration becomes:</u>

$$m\left[s, S_r, S_\ell, k, K, \sigma^2, w\right] = 2Z\left[\rho_k\right]\left\{\sigma^2\left[\left[S_r + S_\ell\right]/K - s_t/k\right]4\pi/\left[K + k\right]\right.$$

$$\left. + w\left[S_r - S_\ell\right]3\pi^{1.5}/4\left[\left[K + k\right]^{1.5} - k^{1.5}\right]\right\}k \; .$$

The three censuses are for total stock on the ranch, s, total stock upwind to right of the ranch, S_r, and total stock downwind to the left, S_ℓ. Wildlife will migrate onto the ranch if it is smaller than the environment and (a) wildlife are more densely populated on neighboring ranches or (b) wildlife are concentrated upwind. This form for migration will be incorporated into the bioeconomic model.

PROOF: A sketch of the proof follows. A detailed proof is in the Appendix.

The ranch and neighborhood are shown in Figure 1. Each point on the x-y plane is one unit of carrying capacity. The small circle centered at the origin of the x-y plane represents the ranch of radius ρ_k and area k; the larger concentric circle represents the neighborhood of radius ρ_K and area k + K. Not shown is the size of the environment which is larger yet of radius ρ_κ. The number of wildlife at each point, n(x,y,t) is the vertical distance to the sloping and convex surface above. Total wildlife on the ranch and over the neighborhood are s and s + S. The x-axis is parallel to the prevailing wind and θ is the angle from the wind of a line through the origin. A point on the line can be expressed in either Cartesian coordinates (x,y), or polar coordinates (ρ, θ) where x = $\rho\cos\theta$, y = $\rho\sin\theta$ and ρ is a radius. Often in the literature only flux in the x-direction is considered. Nisbet and Gurney (1982, p.132) consider both the x- and y-directions. In Figure 1, the direction can be at any angle but flux must be either toward or away from the origin.

Flux of wildlife from right to left past point (ρ, θ) along a line through the origin equals the numbers of wildlife on the right multiplied by the probabilities they will move to the left minus the numbers on the left multiplied by the probabilities they will move to the right.
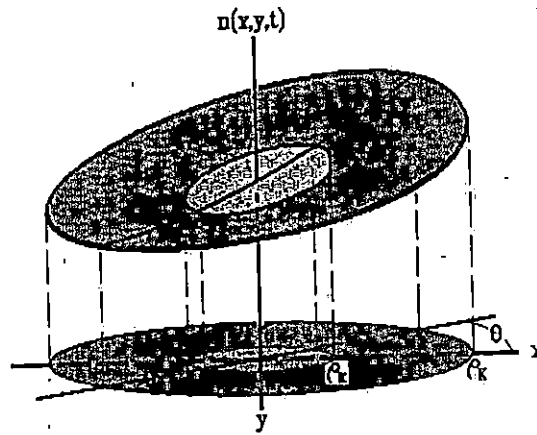
Figure 1. Distribution of Wildlife Numbers

$$F\left(\rho,\theta\right) = \int_{-\rho_\kappa}^{\rho} \int_{\rho}^{\rho_\kappa} \left[n\left(\rho_r,\theta,t\right) \phi\left(d\left(\rho_r - \rho_\ell,w,\theta\right)\right) \right.$$

$$\left. - n\left(\rho_\ell,\theta,t\right) \phi\left(d\left(\rho_r - \rho_\ell,w,\pi - \theta\right)\right) \right] d\rho_r d\rho_\ell \; ;$$

where ρ_r and ρ_ℓ are radii to the right and left of ρ; ϕ is the probability wildlife will travel either from right to left or from left to right; d is the distance wildlife must travel on their own legs; $\rho_r - \rho_\ell$ is the total distance travelled; and w is the distance travelled on the wind. The double integral evaluates wildlife movement from all points on the right to each point on the left.

Figure 2 graphs probabilities of moving distances to the right or left. Distances are standardized by deviation σ. The distribution is symmetric and applies to wildlife travelling under their own power or on the wind in a finite environment of diameter $2\rho_\kappa$. Territorial behavior, hunting pressure, water holes or wildlife refuges will alter wildlife velocity by shifting deviation σ. A larger deviation increases the chances wildlife will travel longer distances. Except for hunting pressure, these shifts to σ have been included in models of infinite environments (McMurtrie [6]). Shifts to σ in a finite environment are more complex and not fundamental to the study of common property. Therefore, σ will be assumed constant.

Flux at a point will be approximated to first-order by Taylor expansions. Wildlife numbers are expanded around point (ρ,θ).
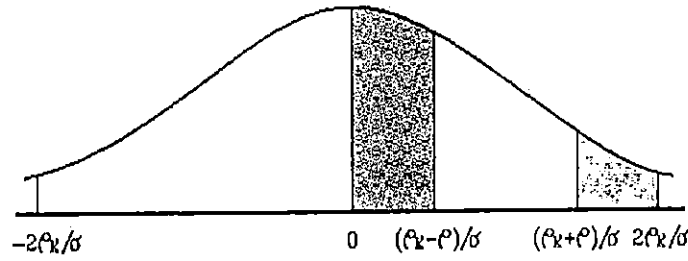
Figure 2. Probabilities of Travelling Distances Left and Right from a Point

$$n\left[\rho_r,\theta,t\right] = n\left[\rho,\theta,t\right] + \left[\rho_r - \rho\right]\partial n/\partial\rho + O\left(\left[\rho_r - \rho\right]^2\right) \ ;$$

$$n\left[\rho_\ell,\theta,t\right] = n\left[\rho,\theta,t\right] - \left[\rho - \rho_\ell\right]\partial n/\partial\rho + O\left(\left[\rho - \rho_\ell\right]^2\right) \ ;$$

where $O(\cdot)$ are second-order and higher terms. Probabilities are expanded around distances with no wind, $d(\rho_r - \ell_\ell,0,\theta)$.

$$\phi\left[d\left[\rho_r - \rho_\ell,w,\theta\right]\right] = \phi\left[d\left[\rho_r - \rho_\ell,0,\theta\right]\right] + w\left[\partial\phi/\partial d\right]\left[\partial d/\partial w\right] + O\left[w^2\right];$$

$$\phi\left[d\left[\rho_r - \rho_\ell,w,\pi-\theta\right]\right] = \phi\left[d\left[\rho_r - \rho_\ell,0,\pi-\theta\right]\right] + w\left[\partial\phi/\partial d\right]\left[\partial d/\partial w\right] + O\left[w^2\right] \ .$$

The expansions for distances can be simplified. Figure 3 shows distances on the x-y plane. Wildlife must travel as if their destination was distance w upwind from their actual destination. By the law of cosines, the distances wildlife must travel on their legs are:

$$d\left[\rho_r - \rho_\ell,w,\theta\right] = \left[\left[\rho_r - \rho_\ell\right]^2 + w^2 - 2\left[\rho_r - \rho_\ell\right]w\ \cos\theta\right]^{0.5};$$

$$d\left[\rho_r - \rho_\ell,w,\pi-\theta\right] = \left[\left[\rho_r - \rho_\ell\right]^2 + w^2 - 2\left[\rho_r - \rho_\ell\right]w\ \cos\left[\pi-\theta\right]\right]^{0.5}.$$

Both distances equal $\rho_r - \rho_\ell$ when evaluated with w set to zero. Derivatives evaluated with w set to zero are:

$$\partial d\left[\rho_r - \rho_\ell,0,\theta\right]/\partial w = - \cos\ \theta\ ;$$

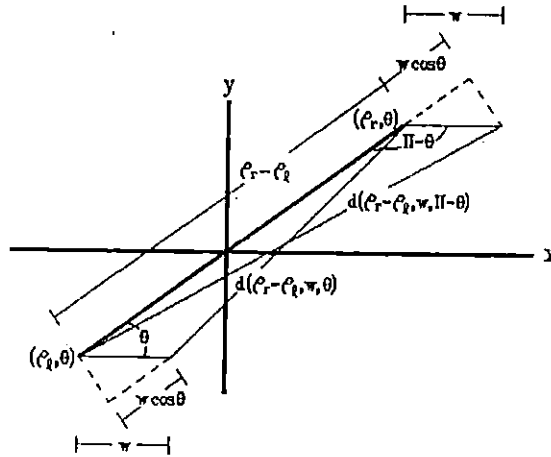$$\partial d\left[\rho_r - \rho_\ell,0,\pi-\theta\right]/\partial w = \cos\ \theta\ .$$

Figure 3.   Distances Travelled in a Prevailing Wind

If aided by a wind, wildlife must travel under their own power approximately w $\cos\theta$ less than total distance, $\rho_r - \rho_\ell$. If hindered by a wind, wildlife must travel approximately w $\cos\theta$ farther.

Substitute the expansions into flux and simplify.

$$F\left[\rho,\theta\right] = \partial n/\partial\rho \int_{-\rho_\kappa}^{\rho} \int_{\rho}^{\rho_\kappa} \left[\rho_r - \rho_\ell\right]\phi\left[\rho_r - \rho_\ell\right]d\rho_r d\rho_\ell$$

$$- n \int_{-\rho_\kappa}^{\rho} \int_{\rho}^{\rho_\kappa} 2w\cos\theta \ \partial\phi/\partial\left[\rho_r - \rho_\ell\right]d\rho_r d\rho_\ell + O\left[\cdot\right]^2 + O\left[\cdot\right]^3 \quad .$$

The first term corresponds to $v\partial n/\partial x$ dt in the introductory explanation of flux and the second term to $n\partial v/\partial x$ dt. Thus the first double integral defines wildlife velocity unaffected by wind. The second double integral defines the change in velocity due to wind. Velocity will be seen to equal a variance weighted by a cumulative probability. In an infinite environment, the cumulative probability converges to 0.5 but in a finite environment an explicit probability must be assumed. Distances are sampled over a large environment and will tend toward a normal distribution.

$$\phi\left[\rho_r - \rho_\ell\right] = \left[e^{-(\rho_r - \rho_\ell)^2/2\sigma^2}/\sigma(2\pi)^{0.5}\right] dt \ ;$$

where the mean distance is zero and the variance is σ^2.

Substitute probabilities into the first double integral for velocity. Evaluate the inner integral and change the remaining variable of integration to standard normal variable z.

$$vdt = Z\sigma^2 dt;$$

where

$$(5) \quad Z\left[\rho\right] = \int_0^{(\rho_\kappa - \rho)/\sigma} e^{-z^2/2} (2\pi)^{-0.5} dz - \int_{(\rho_\kappa + \rho)/\sigma}^{2\rho_\kappa/\sigma} e^{-z^2/2} (2\pi)^{-0.5} dz .$$

Thus velocity is the variance weighted by cumulative probability Z which equals the difference between the two shaded areas under the standard normal curve of Figure 2. Doubling the shaded areas for the symmetric distribution gives the total probabilities of travelling either short or long distances. The area between is the total probability of travelling intermediate distances. Wildlife travelling short, long or intermediate distances will hit a boundary never, always or half the time. Consequently, the first shaded area plus one-half the middle area represents the probability of not reaching a boundary. One-half the middle area plus the second shaded area represents the probability of hitting a boundary. The difference between the probabilities of not reaching and hitting a boundary is 2Z. It is greatest at the center of the environment and zero at a boundary. It is less for a large variance making it impossible for wildlife in a small environment to travel at more than a moderate velocity.

Differentiate probability and substitute into the second double integral for the change due to wind. Evaluate.

$$\partial v/\partial w \, dt = -2Zw \cos\theta \, dt .$$

Because wind both assists travel from the right and hinders travel from the left, its effect on flux from right to left is doubled. The effect of wind is greatest at the center and least at a boundary of the environment.

Flux becomes, approximately,

$$(6) \quad F\left[\rho,\theta\right] = 2Z\left[\left(\sigma^2/2\right)\left(\partial n/\partial\rho\right) + nw \cos\theta\right]dt .$$

Flux along an infinite line, in brackets, is weighted by the cumulative probability to give flux along a finite line.

Migration is the change in flux per unit of time. To show this, note that flux from right to left past point (ρ,θ) also equals the increase over time in the numbers of wildlife at all points to the left.

$$F\left[\rho,\theta\right] = dt \int_{-\rho_\kappa}^{\rho} \left[\partial n\left[\rho_\ell,\theta,t\right]/\partial t\right]d\rho_\ell$$

If ρ was larger, length would be added on the left and flux would change as wildlife on the added length changed.

$$\left[\partial F/\partial\rho\right]/dt = \partial n/\partial t ;$$

Differentiating (6), the change in numbers over time is migration at a point, not considering wildlife growth and harvest.

(7) $\quad \partial n/\partial t = 2\left[\partial Z/\partial \rho\right]\left[\left(\sigma^2/2\right)\left(\partial n/\partial \rho\right) + nw\,\cos\theta\right]$

$\qquad\qquad + 2Z\left[\left(\sigma^2/2\right)\left(\partial^2 n/\partial \rho^2\right) + w\,\cos\theta\left(\partial n/\partial \rho\right)\right]$.

To apply equation (7) the distribution of wildlife must be estimated. A quadrative functional form will estimate up to second derivatives.

(8) $\quad n\left(\rho,\theta,t\right) = \eta_0(t) + \left[\eta_1(t)\cos\theta + \eta_2(t)\sin\theta\right]\rho + \left[\eta_3(t)\cos^2\theta + \eta_4(t)\sin^2\theta\right]\rho^2$.

Only the intercept need be positive but if all coefficients are positive, the distribution of wildlife slopes down and to the left and is convex as in Figure 1. The coefficients are functions of time and will change with migration.

The ranch is not just a point on a line, however. Wildlife can migrate onto the ranch past every point on its circumference. Migration toward the center of the ranch past one point on the circumference will increase wildlife numbers along the radius of the ranch. Total migration onto the ranch equals the sum of migration onto all of its radii.

$$m_t = \int_0^{2\pi} \rho_k \int_0^{\rho_k} \partial n/\partial t \; d\rho d\theta \; .$$

Substitute in equations (7) and (8) and integrate to obtain THEOREM 1.

Integrate over the distribution of wildlife numbers to find total stocks.

$$s_t = \int_0^{\rho_k} \int_0^{2\pi} \rho n \; d\theta d\rho = k\eta_0 + k^2\left[\eta_3 + \eta_4\right]/4\pi \; ;$$

$$S_r = \int_{\rho_k}^{\rho_K} \int_{-\pi/2}^{\pi/2} \rho n \; d\theta d\rho = K\eta_0/2 + \left[\left(K + k\right)^{1.5} - k^{1.5}\right]\eta_1 2/3\pi^{1.5}$$

$$\qquad\qquad + K\left(K + 2k\right)\left[\eta_3 + \eta_4\right]/8\pi \; ;$$

$$S_\ell = \int_{\rho_k}^{\rho_K} \int_{\pi/2}^{3\pi/2} \rho n \; d\theta d\rho = K\eta_0/2 - \left[\left(K + k\right)^{1.5} - k^{1.5}\right]\eta_1 2/3\pi^{1.5}$$

$$\qquad\qquad + K\left(K + 2k\right)\left[\eta_3 + \eta_4\right]/8\pi \; .$$

Solve for $\eta_3 + \eta_4$ by adding S_r and S_ℓ to get total stock on neighboring ranches, dividing by K to get average density, also dividing s_t by k and subtracting to get the difference in average densities, and rearranging.

$$\eta_3 + \eta_4 = \left[\left(S_r + S_\ell\right)/K - s_t/k\right] 4\pi/\left[K + k\right] .$$

Solve for η_1 by subtracting S_ℓ from S_r and rearranging.

$$\eta_1 = \left[S_r - S_\ell\right] 3\pi^{1.5}/4\left[\left[K + k\right]^{1.5} - k^{1.5}\right] .$$

Finally, substitute into migration to obtain COROLLARY 1.

III. The Degree of Common Property

One rancher could have exclusive access to wildlife. Many ranchers could have open access. But more likely, a few ranchers will have limited access to wildlife and the degree of common property will be between the two extremes.

Wildlife with legs can be common property whether or not they actually migrate. It is enough they potentially could migrate because an increase in stock would reduce migration.

$$\partial m/\partial s_t = -2Z\sigma^2 4\pi/\left[K + k\right] .$$

Negative marginal migration causes marginal user-cost in equation (4b') to be too small. The effect is less on a larger ranch because the magnitude of marginal migration declines.

$$\partial^2 m/\partial s_t \partial \rho_k = -2\left[\partial Z/\partial \rho_k\right]\sigma^2 4\pi/\left[K + k\right] ;$$

where the size of the neighborhood, $K + k$, is constant. Except in an infinite environment, the change in cumulative probability is negative and the change in marginal migration positive. Therefore, marginal migration increases toward zero and decreases in magnitude.

To reach exclusive access, the ranch must become large enough to encompass the entire environment traversed by wildlife. If the neighborhood suitable for ranching is smaller than the environment, wildlife can travel beyond the boundaries of the neighborhood. Even the largest possible ranch could not gain exclusive access. At the other extreme, open access may seldom occur. A large variance is not enough because the chance of hitting a boundary is also large and cumulative probability small. The environment must be large as well. Nor is this result specific to the normal distribution in Figure 2. For open access, wildlife must travel unpredictably in a very large environment.

Wildlife travelling on the wind are common property only if they migrate. Marginal migration does not depend on the wind and marginal user-cost is affected only indirectly by changes in the stock on the ranch.

Non-cooperative, Nash behavior was assumed in deriving marginal migration, but other types of behavior are possible [3]. A rancher must conjecture about the variations to wildlife stock by his neighbors in reaction to his own variations.

$$\partial m/\partial s_t \;=\; 2Z\Big\{\sigma^2\Big[\big[dS_r/ds + dS_\ell/ds\big]/K - 1/k\Big]4\pi/\Big[K + K\Big]$$

$$+\; w\Big[dS_r/ds - dS_\ell/ds\Big]3\pi^{1.5}/4\Big[\big[K + k\big]^{1.5} - k^{1.5}\Big]\Big\}k \;;$$

where dS_r/ds and dS_ℓ/ds are conjectural variations. A rancher who does not cooperate with his neighbors would have zero conjectural variations. But suppose wildlife were uniformly distributed over the neighborhood and each rancher were convinced his neighbors would react to maintain a uniform distribution. Cooperative conjectural variations would be

$$(dS_r + dS_\ell)/K = ds/k \;;\quad dS_r = dS_\ell \;.$$

Both migration and marginal migration would be zero and the common property problem would be eliminated.

This cooperative approach is sometimes advocated as a "land stewardship" policy. Cooperative equilibrium would be unstable unless each rancher could monitor the actions of his neighbors and credibly threaten to retaliate against exploitive actions. Cooperative management of wildlife migrating on their own legs might be possible, but not of wildlife migrating on the wind. Downwind ranchers have no credible threat against upwind ranchers.

In the illustrations to follow, noncooperative behavior is assumed. The degree of common property among ranchers is measured by the proportion of rent dissipated. The proportion dissipated equals one minus the proportion captured and the proportion captured is the actual marginal user-cost divided by the hypothetical marginal user-cost of privately-owned wildlife.

IV. Illustrations

Migration in the equation of COROLLARY 1 causes economies of scale. Illustrations are clearest if yield of livestock, quantities of inputs and growth of wildlife are scale-neutral functions of population densities.

(9a) $\qquad y = \alpha\Big[1 - \beta s_t/k\Big]k;$

(9b) $\qquad c = \Big[s_t/k\Big]^{-\gamma}h_t;$

(9c) $\qquad g = r\Big[1 - s_t/k\Big]s_t;$

Yield of livestock in equation (9a) has a maximum yield per unit of carrying capacity equal to α. Yield declines as the degree of competition between wildlife and livestock goes from no competition, with β equal to zero, to complete competition, with β equal to one. Inputs in equation (9b) decrease with population density, where elasticity γ is greater than or equal to one. Growth of wildlife in equation (9c) is described by a logistic function.

No migration

Nonmigratory wildlife with exclusive access are the benchmark in determining the degree of common property for migratory wildlife with limited

access. Table 1 contains model parameters for a hypothetical species, crocodiles, which have some value for harvesting but are predominantly predators. In a steady-state, harvest equals growth. Multiplying by prices gives steady-state revenue from harvest of crocodiles, revenues from yield of livestock and costs of inputs. These are graphed versus crocodile population in Figure 4. The two ranch sizes demonstrate the scale neutrality of the growth, yield and cost functions.

Table 1. Model Parameters for Crocodiles

Parameter[†]	Value
p_h	15
p_y	5
p_c	50
δ	0.05
α	20
β	0.95
γ	1.25
r	0.75
σ^2	0
w	0
K + k	9000
κ	10000

[†]p_h: harvest price; p_y: livestock yield price; p_c: input price; δ: interest rate; α: maximum livestock yield; β: percent competition with livestock/100; γ: input elasticity; r: intrinsic growth rate; σ^2: variance of travel; w: wind distance; K + k: carrying capacity of neighborhood; κ: carrying capacity of environment.

Solving optimality conditions (4a), (4b) and (4c) with $\dot{s}$ and $\dot{\lambda}$ set to zero[1] gives a steady-state of 0.30 crocodiles per unit of carrying capacity for all ranch sizes. Crocodiles are managed at a low density because they are pests with a negative marginal user-cost of -$209.40/unit of stock. An open access pest would be managed at the other extreme with a density of 1, a

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1. The first-order conditions are solved by General INterative Optimizer (GINO) (Liebman et al, 1986).

marginal user-cost of zero, and total revenue from livestock yield minus total cost of inputs as small as possible in Figure 4.

Another nonmigratory species, koalas, are predominantly valuable and compete little against livestock with price p_h of 150 and parameter β of 0.05. The exclusive access steady-state is 0.67 koalas per unit of carrying capacity. Koalas are managed at a high density because they are valuable with a positive marginal user-cost of $67.47/unit of stock. A valuable open-access species would be managed at a density of only 0.42 with a marginal user-cost of zero and total revenue of harvest equal to the total cost of inputs.[2]
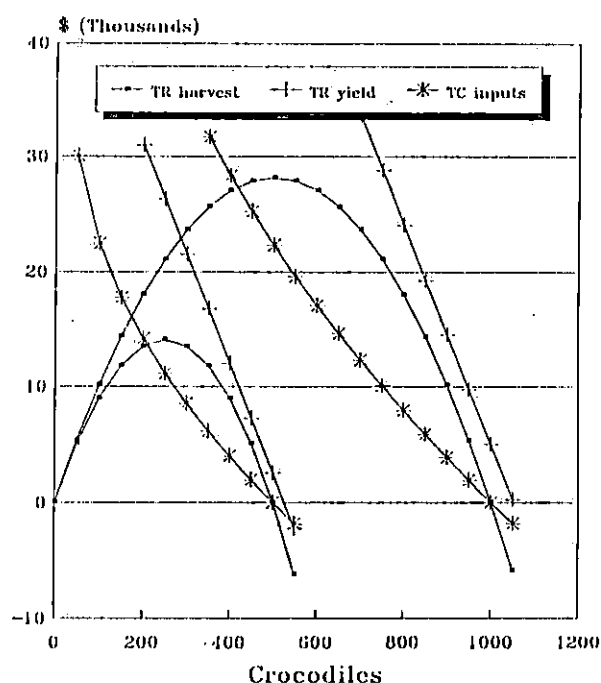

Figure 4. Total Revenues and Costs for Carrying Capacities of 500 and 1000

Density-dependent migration

Even if wildlife do not actually migrate, potential migration causes economies of scale. To demonstrate, consider two mobile species, dingoes and

2. Total revenue equal to total cost is sometimes quoted as the rule for open-access. It is not a general rule, however, but follows from assuming marginal cost of inputs is constant with respect to harvest. If marginal cost of inputs is constant, inputs receive no rent and total revenue equals total cost once user-cost is dissipated. But if marginal cost increases with harvest, inputs receive rent and total revenue exceeds total costs, even in open access. Zero marginal user-cost with no rent to the resource is the general rule for open access.

kangaroos. Assume both are .uniformly distributed over the neighborhood so there is no actual migration. Dingoes are pests like crocodiles, kangaroos are valuable like koalas but both have a standard deviation of travel equal to the radius of the environment, 56.42. Figures 5 and 6 compare the population densities and marginal user-costs among species. Immobile crocodiles and koalas are managed the same on all sized ranches. Dingoes are under-controlled yet even the smallest ranch captures some of the marginal user-cost. No ranch captures all the marginal user-cost. Similarly, kangaroos are over-exploited, particularly on small ranches. The marginal user-cost is never completely dissipated nor captured.
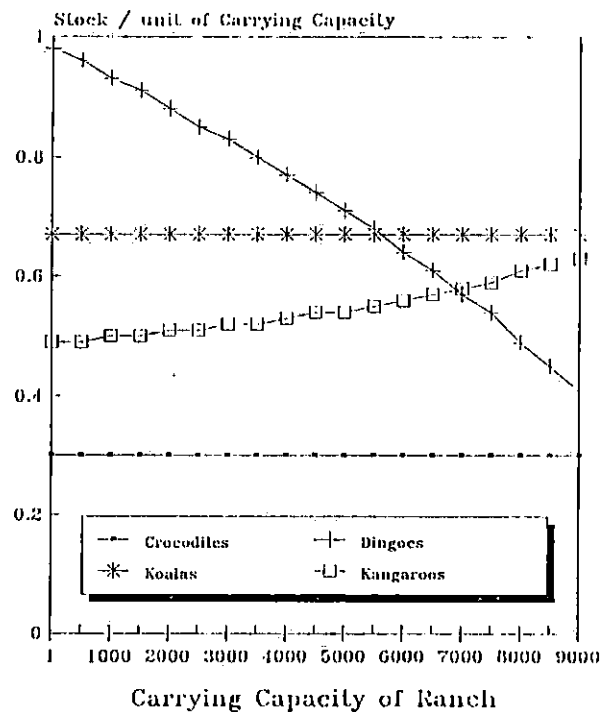

Figure 5. Population Densities

In Figure 7, the proportion of marginal user-cost dissipated measures the degree of common property. For density-dependent migration the degree of common property is greatest on small ranches but never reaches one. Both the environment and the variance would have to increase by a factor of 10 until the smallest ranch was 1/100,000 of the environment before 98 percent of the marginal user-cost of dingoes would be dissipated. Neither does the degree of common property go to zero. The largest ranch encompassing the entire neighborhood but only 9/10 of the environment still dissipates 8 percent of the marginal user-cost of kangaroos.
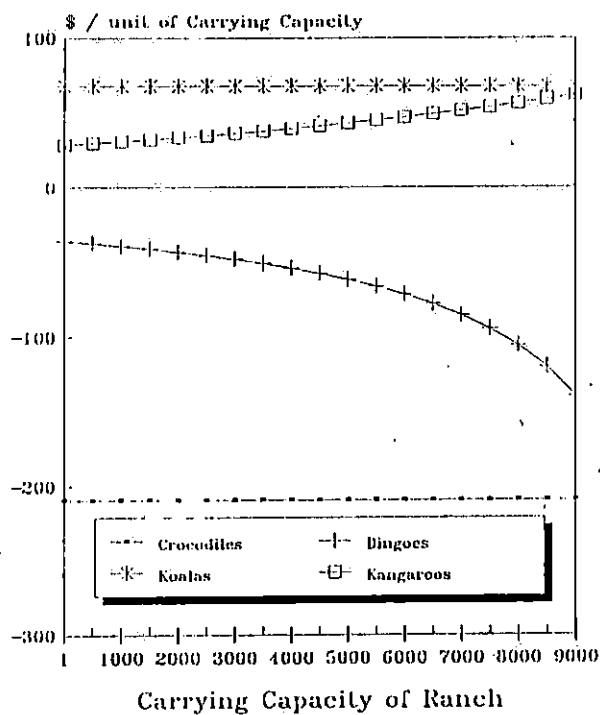
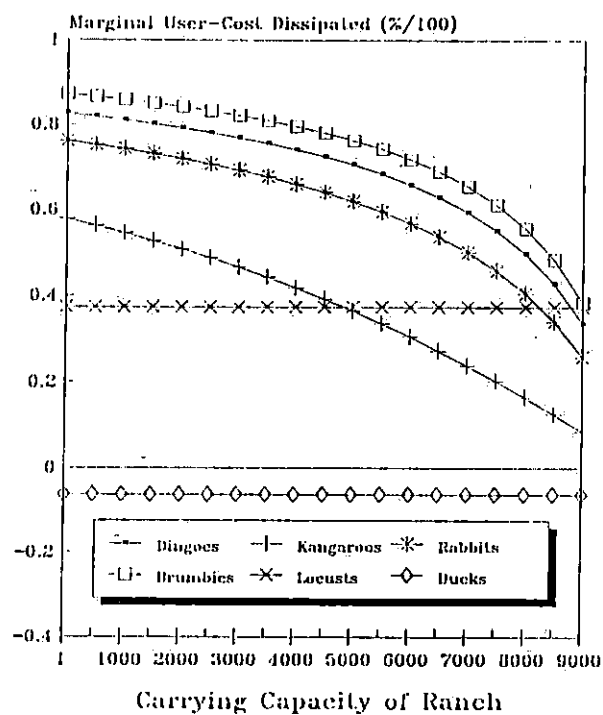**Figure 6. Marginal User-costs**

Figure 7. Degree of Common Property

Dingoes have a higher degree of common property than kangaroos because pests damage livestock yields and amplify the effect that marginal migration has on marginal user-cost in equation (4b'). Figure 7 compares two other pests, rabbits and brumbies, to dingoes. Rabbits are faster growing at rate r of 1.00 and brumbies are less expensively controlled with price p_c of 10. Curiously, rapid growth decreases the degree of common property. Rabbit populations are indeed greater but marginal migration becomes relatively less important than marginal growth. Curiously, again, inexpensive control increases the degree of common property. Brumbie populations are smaller but marginal migration frustrates more extensive control efforts.

Wind-driven migration

Figure 7 also shows the degree of common property for two wind-driven species, locusts and ducks. Locusts are pests. Ducks are valuable. Both are carried on a wind which travels the radius of the environment, 56.42, per unit of time. The population gradient in the direction of the wind is assumed constant at 0.001 because both locusts and ducks migrate seasonally. Migration is exogenous to the ranch and the degree of common property is independent of ranch size. Locusts are costly with a significant degree of common property, but ducks are beneficial with a negative degree. The rancher actually captures more rent from migrating ducks than he would otherwise.

Endogenous migration with a changing population gradient over time will reach a steady-state of uniformly distributed wildlife with no gradient, no wind-driven migration and a zero degree of common property. A larger ranch will adjust more quickly to the steady-state but, in the long-run, the size of the ranch is unimportant for wind-driven migration.

V. Conclusions

Migration, along with growth and harvest, is one of three fundamental ways in which a wildlife population can change. Migration across fence lines attenuates the right of a rancher to exclude others from the wildlife and is a root cause of common property. Yet migration has seldom been incorporated into bioeconomic models or studies of common property.

Migration at a point in an infinite environment has been modeled extensively in the mathematical ecology literature. But a bioeconomic model must superimpose a pattern of land ownership. In this study, migration was derived for various sized ranches in a finite neighborhood under the two basic modes of power: wildlife travelling on their own legs and wildlife powered by an exogenous force such as the wind.

It was found that wildlife travelling under their own power are common property, even if they do not actually migrate. Their potential to enter or leave a ranch restricts a rancher's ability to control them as pests or conserve them as valuable wildlife. As a general rule, pests have a higher degree of common property than valuable wildlife; faster growing wildlife

have a lower degree of common property; and inexpensively controlled wildlife have a higher degree.

Wildlife travelling under wind power are common property only if they actually migrate. For seasonal migration across a large environment, the degree of common property is independent of ranch size. Pests have a higher degree of common property, as before, but valuable wildlife have a negative degree. A rancher receives "manna from heaven" and captures more rent than if wildlife did not migrate. For non-seasonal migration under wind power, a uniformly distributed population with no migration and a zero degree of common property will eventually be reached. Larger ranches may adjust more quickly but in the long run the size of the ranch is unimportant.

Finally, migration can convey economies of scale to larger ranches with lower degrees of common property on the continuum from open through limited to exclusive access. Open-access can occur only for wildlife travelling unpredictably in a very large environment. Even the textbook example of a fishery may not meet these conditions. A fisherman using a vessel equipped with sonar eliminates unpredictability and should capture rent from the fishery. Certainly a rancher will capture a significant portion of rent from wildlife. Nor can exclusive access be achieved if the environment is bigger than the neighborhood. The owner of all fishing vessels on the ocean or all land in the vicinity of a national park could not capture all the rent.

In conclusion, common property is a key feature of natural resource management. Migration is the root cause. Bioeconomic studies have avoided modeling migration by comparing open with exclusive access. Government policies are based on this comparison. But the degree of common property almost always lies between the two extremes and bioeconomic models and policies should begin to include migration.

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Appendix
Detailed Derivation of Migration

For explanation of the mathematical symbols and the interpretation of equations, please refer to section II "Migration". Equations with a † symbol are from the text. Begin with the definition of flux.

$$\dagger \quad F\left(\rho,\theta\right) = \int_{-\rho_\kappa}^{\rho} \int_{\rho}^{\rho_\kappa} \left[n\left(\rho_r,\theta,t\right) \phi\left(d\left(\rho_r - \rho_\ell, w, \theta\right)\right)\right.$$

$$\left. - n\left(\rho_\ell,\theta,t\right) \phi\left(d\left(\rho_r - \rho_\ell, w, \pi - \theta\right)\right)\right] d\rho_r d\rho_\ell \ .$$

Taylor expand numbers and probability.

$$\dagger \quad n\left(\rho_r,\theta,t\right) = n\left(\rho,\theta,t\right) + \left[\rho_r - \rho\right]\partial n/\partial\rho + O\left(\left[\rho_r - \rho\right]^2\right) \ ;$$

$$\dagger \quad n\left(\rho_\ell,\theta,t\right) = n\left(\rho,\theta,t\right) - \left[\rho - \rho_\ell\right]\partial n/\partial\rho + O\left(\left[\rho - \rho_\ell\right]^2\right) \ ;$$

$$\dagger \quad \phi\left(d\left(\rho_r - \rho_\ell, w, \theta\right)\right) = \phi\left(d\left(\rho_r - \rho_\ell, 0, \theta\right)\right) + w\left(\partial\phi/\partial d\right)\left(\partial d/\partial w\right) + O\left(w^2\right) ;$$

$$\phi\left(d\left(\rho_r - \rho_\ell, w, \pi-\theta\right)\right) = \phi\left(d\left(\rho_r - \rho_\ell, 0, \pi-\theta\right)\right) + w\left(\partial\phi/\partial d\right)\left(\partial d/\partial w\right) + O\left(w^2\right) \ .$$

Define distance.

$$\dagger \quad d\left(\rho_r - \rho_\ell, w, \theta\right) = \left[\left(\rho_r - \rho_\ell\right)^2 + w^2 - 2\left(\rho_r - \rho_\ell\right)w \cos\theta\right]^{0.5};$$

$$\dagger \quad d\left(\rho_r - \rho_\ell, w, \pi-\theta\right) = \left[\left(\rho_r - \rho_\ell\right)^2 + w^2 - 2\left(\rho_r - \rho_\ell\right)w \cos\left[\pi-\theta\right]\right]^{0.5}.$$

Evaluate distance and its derivative at zero.

$$d\left(\rho_r - \rho_\ell, 0, \theta\right) = \rho_r - \rho_\ell;$$

$$d\left(\rho_r - \rho_\ell, 0, \pi-\theta\right) = \rho_r - \rho_\ell.$$

$$\left[\partial/\partial w\right]d\left(\rho_r - \rho_\ell, w, \theta\right)\bigg|_{w=0}$$

$$= 0.5\left[\left(\rho_r-\rho_\ell\right)^2 + w^2 - 2\left(\rho_r-\rho_\ell\right)w \cos\theta\right]^{-0.5}\left[2w - 2\left(\rho_r-\rho_\ell\right)\cos\theta\right]_{w=0}$$

$$= -\left(\rho_r-\rho_\ell\right)^{-1}\left(\rho_r-\rho_\ell\right) \cos\theta \ .$$

$$\dagger \quad \partial d\left(\rho_r - \rho_\ell, 0, \theta\right)/\partial w = -\cos\theta \ .$$

$$\left[\partial/\partial w\right]d\left(\rho_r - \rho_\ell, w, \pi-\theta\right)\Big|_{w=0}$$

$$= 0.5\left[\left(\rho_r-\rho_\ell\right)^2 + w^2-2\left(\rho_r-\rho_\ell\right)w\,\cos\left(\pi-\theta\right)\right]^{-0.5}\left[2w-2\left(\rho_r-\rho_\ell\right)w\,\cos\left(\pi-\theta\right)\right]_{w=0}$$

$$= -\left(\rho_r-\rho_\ell\right)^{-1}\left[\left(\rho_r-\rho_\ell\right)\cos\left(\pi-\theta\right)\right]$$

$$= -\cos\pi\,\cos\theta + \sin\pi\,\sin\theta \ .$$

† $\partial d\left(\rho_r - \rho_\ell, 0, \pi-\theta\right)/\partial w = \cos\,\theta.$

Substitute into the expansion of probability.

$$\phi\left[d\left(\rho_r-\rho_\ell\right), w, \theta\right] = \phi\left(\rho_r-\rho_\ell\right) - w\,\cos\theta\,\,\partial\phi/\partial\left(\rho_r-\rho_\ell\right) + 0\left(w^2\right) \quad ;$$

$$\phi\left[d\left(\rho_r-\rho_\ell\right), w, \pi-\theta\right] = \phi\left(\rho_r-\rho_\ell\right) + w\,\cos\theta\,\,\partial\phi/\partial\left(\rho_r-\rho_\ell\right) + 0\left(w^2\right) \quad .$$

Substitute expansions into flux.

$$F\left(\rho,\theta\right) = \int\int\left\{\left[n\left(\rho,\theta,t\right)+\left[\rho_r-\rho\right]\partial n/\partial\rho+0\left(\cdot\right)^2\right]\left[\phi\left(\rho_r-\rho_\ell\right)-w\,\cos\theta\,\,\partial\phi/\partial\left(\rho_r-\rho_\ell\right)+0\left(\cdot\right)^2\right]\right.$$

$$- \left[n\left(\rho,\theta,t\right)-\left[\rho-\rho_\ell\right]\partial n/\partial\rho + 0\left(\cdot\right)^2\right]\left.\left[\phi\left(\rho_r-\rho_\ell\right)+w\,\cos\theta\,\,\partial\phi/\partial\left(\rho_r-\rho_\ell\right)+0\left(\cdot\right)^2\right]\right\}d\rho_r d\rho_\ell$$

$$= \int\int\left\{n\left[\phi\left(\rho_r-\rho_\ell\right) - \phi\left(\rho_r-\rho_\ell\right) - 2w\,\cos\theta\,\,\partial\phi/\partial\left(\rho_r-\rho_\ell\right)\right]\right.$$

$$+ \partial n/\partial\rho\left[\rho_r-\rho + \rho-\rho_\ell\right]\phi\left(\rho_r-\rho_\ell\right)$$

$$\left.+ n\left[0\left(\cdot\right)^2\right] + \partial n/\partial\rho\left[0\left(\cdot\right)^2 + 0\left(\cdot\right)^3\right]\right\}d\rho_r d\rho_\ell$$

† $$F\left(\rho,\theta\right) = \partial n/\partial\rho \int_{-\rho_K}^{\rho}\int_{\rho}^{\rho_K}\left[\rho_r - \rho_\ell\right]\phi\left(\rho_r - \rho_\ell\right)d\rho_r d\rho_\ell$$

$$- n\int_{-\rho_K}^{\rho}\int_{\rho}^{\rho_K}2w\,\cos\theta\,\,\partial\phi/\partial\left(\rho_r - \rho_\ell\right)d\rho_r d\rho_\ell + 0\left(\cdot\right)^2 + 0\left(\cdot\right)^3 \ .$$

Define probability as normal.

$$\dagger \quad \phi\Big(\rho_r - \rho_\ell\Big) = \Big[e^{-(\rho_r-\rho_\ell)^2/2\sigma^2}/\sigma(2\pi)^{0.5}\Big]\,dt\ .$$

Evaluate the first double integral from flux.

$$\int_{-\rho_\kappa}^{\rho}\int_{\rho}^{\rho_\kappa}\Big[\rho_r - \rho_\ell\Big]\phi\Big(\rho_r - \rho_\ell\Big)d\rho_r d\rho_\ell$$

$$= dt\int\int\Big[\rho_r - \rho_\ell\Big]e^{-(\rho_r-\rho_\ell)^2/2\sigma^2}/\sigma(2\pi)^{0.5}\,d\rho_r d\rho_\ell$$

$$= dt\int -\sigma\ e^{-(\rho_r-\rho_\ell)^2/2\sigma^2}(2\pi)^{-0.5}\Big|_{\rho_r=\rho}^{\rho_r=\rho_\kappa}\,d\rho_\ell$$

$$= dt\int_{-\rho_\kappa}^{\rho} -\sigma\Big[e^{-(\rho_\kappa-\rho_\ell)^2/2\sigma^2} - e^{-(\rho-\rho_\ell)^2/2\sigma^2}\Big]\Big(2\pi\Big)^{-0.5}\,d\rho_\ell\ .$$

$$z = \Big(\rho_\kappa - \rho_\ell\Big)/\sigma \qquad z = \Big(\rho - \rho_\ell\Big)/\sigma$$

$$dz = -\,d\rho_\ell/\sigma \qquad dz = -\,d\rho_\ell/\sigma$$

$$= dt\ \sigma^2\Big[\int_{2\rho_\kappa/\sigma}^{(\rho_\kappa-\rho)/\sigma} e^{-z^2/2}(2\pi)^{-0.5}dz - \int_{(\rho_\kappa+\rho)/\sigma}^{0} e^{-z^2/2}(2\pi)^{-0.5}\,dz\Big]$$

$$= dt\ \sigma^2\Big[\int_{0}^{(\rho_\kappa-\rho)/\sigma} e^{-z^2/2}(2\pi)^{-0.5}dz - \int_{(\rho_\kappa+\rho)/\sigma}^{2\rho_\kappa/\sigma} e^{-z^2/2}(2\pi)^{-0.5}\,dz\Big]\ .$$

$$\dagger \quad vdt = Z\sigma^2 dt.$$

Differentiate probability.

$$\partial\phi/\partial\Big[\rho_r-\rho_\ell\Big] = -\Big[\Big(\rho_r-\rho_\ell\Big)e^{-(\rho_r-\rho_\ell)^2/2\sigma^2}/\sigma^3(2\pi)^{0.5}\Big]\,dt\ .$$

Evaluate the second double integral from flux.

$$\int_{-\rho_\kappa}^{\rho} \int_{\rho}^{\rho_\kappa} 2w \, \cos\theta \, \partial\phi/\partial\left[\rho_r - \rho_\ell\right] d\rho_r d\rho_\ell$$

$$= -2w \, \cos\theta\left[dt/\sigma^2\right] \int \int \left[\rho_r - \rho_\ell\right] e^{-(\rho_r - \rho_\ell)^2/2\sigma^2} /\sigma(2\pi)^{0.5} d\rho_r d\rho_\ell \quad .$$

† $\partial v/\partial w \, dt = -2Zw \, \cos\theta \, dt$.

Simplify flux.

† $F\left[\rho,\theta\right] = 2Z\left[\left[\sigma^2/2\right]\left[\partial n/\partial\rho\right] + nw \, \cos\theta\right]dt$.

Differentiate to get migration at a point.

† $\partial n/\partial t = 2\left[\partial Z/\partial\rho\right]\left[\left[\sigma^2/2\right]\left[\partial n/\partial\rho\right] + nw \, \cos\theta\right]$

$$+ 2Z\left[\left[\sigma^2/2\right]\left[\partial^2 n/\partial\rho^2\right] + w \, \cos\theta\left[\partial n/\partial\rho\right]\right] \quad ..$$

Define the wildlife distribution.

† $n\left[\rho,\theta,t\right] = \eta_0(t) + \left[\eta_1(t)\cos\theta + \eta_2(t)\sin\theta\right]\rho + \left[\eta_3(t)\cos^2\theta + \eta_4(t)\sin^2\theta\right]\rho^2$.

Define migration onto the ranch.

† $m_t = \int_0^{2\pi} \rho_k \int_0^{\rho_k} \partial n/\partial t \, d\rho d\theta$.

$$= \int_0^{2\pi} \rho_k\left[F\left[\rho_k,\theta\right] - F\left[0,\theta\right]\right] d\theta/dt \quad .$$

Differentiate the wildlife distribution.

$$\partial n/\partial \rho = \eta_1\cos\theta + \eta_2\sin\theta + 2\left[\eta_3\cos^2\theta + \eta_4\sin^2\theta\right]\rho .$$

Substitute into migration within fluxes, $F(\rho_k,\theta)$ and $F(\rho,\theta)$, and integrate. Odd powers of $\cos\theta$ and $\sin\theta$ are zero.

$$m_t = \int_0^{2\pi} \rho_k\ z\left[\rho_k\right]\left\{\sigma^2\left[\eta_1\cos\theta + \eta_2\sin\theta + 2\left[\eta_3\cos^2\theta + \eta_4\sin^2\theta\right]\rho_k\right]\right.$$

$$+ 2w\left[\eta_0\cos\theta + \left[\eta_1\cos^2\theta + \eta_2\sin\theta\ \cos\theta\right]\rho_k + \left[\eta_3\cos^3\theta + \eta_4\sin^2\theta\cos\theta\right]\rho_k^2\right]\right\}\ d\theta$$

$$- \int_0^{2\pi} \rho_k\ z\left[0\right]\left\{\sigma^2\left[\eta_1\cos\theta + \eta_2\sin\theta\right] + 2w\eta_0\ \cos\theta\right\}\ d\theta$$

$$= \rho_k\ z\left[\rho_k\right]\left\{\sigma^2\ 2\left[\eta_3 + \eta_4\right]\ \pi\rho_k + 2w\eta_1\ \pi\rho_k\right\} .$$

$$\dagger \quad m_t = 2Z\left[\rho_k\right]\left[\sigma^2\left[\eta_3 + \eta_4\right] + w\eta_1\right]k .$$

Integrate stocks on the ranch.

$$\dagger \quad s_t = \int_0^{\rho_k}\int_0^{2\pi} \rho n\ d\theta d\rho$$

$$= \int_0^{\rho_k}\int_0^{2\pi} \rho\left[\eta_0 + \left[\eta_1\cos\theta + \eta_2\sin\theta\right]\rho + \left[\eta_3\cos^2\theta + \eta_4\sin^2\theta\right]\rho^2\right]d\theta d\rho$$

$$= \int_0^{\rho_k} \rho\eta_0 2\pi + \rho^3\left[\eta_3 + \eta_4\right]\pi\ d\rho$$

$$= \rho_k^2\ \eta_0\pi + \tfrac{1}{4}\rho_k^4\left[\eta_3 + \eta_4\right]\pi$$

$$\dagger \quad s_t = k\eta_0 + k^2\left[\eta_3 + \eta_4\right]/4\pi .$$

Integrate the stocks to the right of the ranch.

$$S_r = \int_{\rho_k}^{\rho_K} \int_{-\pi/2}^{\pi/2} \rho\left[\eta_0 + \left(\eta_1\cos\theta + \eta_2\sin\theta\right)\rho + \left(\eta_3\cos^2\theta + \eta_4\sin^2\theta\right)\rho^2\right]d\theta\,d\rho$$

$$= \int_{\rho_k}^{\rho_K} \rho\eta_0\left[\pi/2 + \pi/2\right] + \rho^2\eta_1\left[1 + 1\right] + \rho^3\left[\eta_3 + \eta_4\right]\left[\pi/4 + \pi/4\right]\,d\rho$$

$$= 1/2\left[\rho_K^2 - \rho_k^2\right]\eta_0\pi + 2/3\left[\rho_K^3 - \rho_k^3\right]\eta_1 + 1/8\left[\rho_K^4 - \rho_k^4\right]\left[\eta_3 + \eta_3\right]\pi$$

$$= 1/2\left[K+k-k\right]\eta_0 + 2/3\left[\left(K+k\right)^{1.5} - k^{1.5}\right]\eta_1/\pi^{1.5} + 1/8\left[\left(K+k\right)^2 - k^2\right]\left[\eta_3 + \eta_4\right]/\pi$$

† $\quad S_r = K\eta_0/2 + \left[\left(K + k\right)^{1.5} - k^{1.5}\right]\eta_1 2/3\pi^{1.5} + K\left(K + 2k\right)\left(\eta_3 + \eta_4\right)/8\pi$.

Integrate stocks to the left of the ranch.

$$S_\ell = \int_{\rho_k}^{\rho_K} \int_{\pi/2}^{3\pi/2} \rho\left[\eta_0 + \left(\eta_1\cos\theta + \eta_2\sin\theta\right)\rho + \left(\eta_3\cos^2\theta + \eta_4\sin^2\theta\right)\right]d\theta\,d\rho$$

$$= \int_{\rho_k}^{\rho_K} \rho\eta_0\left[3\pi/2 - \pi/2\right] + \rho^2\eta_1\left[-1 - 1\right] + \rho^3\left[\eta_3 + \eta_4\right]\left[3\pi/4 - \pi/4\right]d\rho \quad .$$

† $\quad S_\ell = K\eta_0/2 - \left[\left(K + k\right)^{1.5} - k^{1.5}\right]\eta_1 2/3\pi^{1.5} + K\left(K + 2k\right)\left[\eta_3 + \eta_4\right]/8\pi$.

Solve for $\eta_3 + \eta_4$.

$$S_r + S_\ell = K\eta_0 + K\left[K + 2k\right]\left[\eta_3 + \eta_4\right]/4\pi \quad .$$

$$\left(S_r + S_\ell\right)/K - s_t/k = \eta_0 - \eta_0 + \left[K+2k-k\right]\left[\eta_3 + \eta_4\right]/4\pi = \left[K+k\right]\left[\eta_3 + \eta_4\right]/4\pi$$

or

† $\quad \eta_3 + \eta_4 = \left[\left(S_r + S_\ell\right)/K - s_t/k\right] 4\pi/\left[K + k\right]$.

Solve for η_1.

$$S_r - S_\ell = \left[\left(K + k\right)^{1.5} - k^{1.5}\right]\eta_1 \; 4/3\pi^{1.5}$$

or

$$\dagger \quad \eta_1 = \left[S_r - S_\ell\right] 3\pi^{1.5}/4\left[\left(K + k\right)^{1.5} - k^{1.5}\right] .$$

Finally, substitute $\eta_3 + \eta_4$ and η_1 into migration.

$$\dagger \quad m\left(s, S_r, S_\ell, k, K, \sigma^2, w\right) = 2Z\left(\rho_k\right)\left\{\sigma^2\left[\left(S_r + S_\ell\right)/K - s_t/k\right]4\pi/\left[K + k\right]\right.$$

$$\left. + w\left[S_r - S_\ell\right]3\pi^{1.5}/4\left[\left(K + k\right)^{1.5} - k^{1.5}\right]\right\}k .$$