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A NOTE ON THE EQUIVALENCE OF  
PRODUCER SURPLUS AND FACTOR RENTS

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# A NOTE ON THE EQUIVALENCE OF PRODUCER SURPLUS AND FACTOR RENTS

## Introduction

It is now twenty years since the publication of the article titled "What is Producer Surplus" by Mishan (1968). Most articles of similar vintage have long since been forgotten, but it is still common practice to cite this article in reading lists for courses on applied welfare economics, as well as in research articles.<sup>1</sup> It will be argued below that notwithstanding the age of this article, Mishan erred in concluding that the area above the long run industry supply curve and below the market price line, commonly referred to as producer surplus, in general, "carries no economic significance" (Mishan, 1968, p.1277). While there undoubtedly are problems (such as path dependency, the index number problem, etc) in using producer surplus as a measure of the gains from trade in production, such problems are, in Mishan's words, equivalent to those involved in using consumer surplus as a measure of the gains from trade in consumption.

There were two principal arguments in Mishan's article. The first was that in any competitive industry, producer surplus is at best a misnomer because any gains from trade in production accrue to the owners of scarce factors of production, and not to the conceptually distinct producers who convert these factors into output. With this argument, there can be no disagreement, and it is now universally accepted that the appropriate measure of gains from trade in production is the aggregate of the economic rents accruing to the owners of scarce factors of production.

The second, and substantial, part of Mishan's article consists of an argument that producer surplus as conventionally defined only provides an accurate measure of the corresponding economic rent under special and, by implication, rather atypical circumstances. Specifically, Mishan concludes that the two measures of gains from trade and production are equal if and only if the supply of only one factor of production is perfectly inelastic, while the supply of all other factors are perfectly elastic. By contrast, where one or more factors are in partly elastic supply, Mishan concludes that producer surplus does not accurately measure the gains from trade accruing to factor owners in the form of higher economic rents.

The plan for the rest of this paper is first to demonstrate that Mishan's conclusion is false in selected special cases, including most notably that of production involving fixed factor proportions. A more general proof then will be provided that producer surplus in a competitive industry does accurately measure economic rent under all factor supply conditions as long as it (ie, producer surplus) is measured relative to the appropriate supply curve.

Note that the analysis in this paper is restricted to the long-run; that is, to those time periods when producers have had sufficient

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1. A recent brief and incomplete citation search using the SCISEARCH database in DIALOG uncovered 43 citations to Mishan's article since its publication date. Of this subset of all citations, 15 had a publication date no earlier than 1982. While this evidence admittedly is incomplete, it is not inconsistent with the hypothesis that the citation rate for Mishan's article is not declining, and may even be increasing with time.

time to fully adjust output to a change in product price. As Just, Hueth and Schmitz (1982) note, producer surplus measured with respect to the long-run supply curve overstates quasi-rents earned by the producer during those time periods when production has not fully adjusted to the change in product price. By the same token though, during such short-run periods when it is only possible to adjust use of those factors in perfectly elastic supply, there is no change in factor rents received by resource suppliers, and producer surplus measured with respect to the short-run supply curves does accurately measure the quasi-rents accruing to producers in the short run.

Selected Counterfactual Examples to the  
Postulated Non-Equivalence of Producer Surplus and Economic Rent

Consider the hypothetical case illustrated in Figure 1 where two factors, K and L, are in partly inelastic and perfectly elastic supply respectively. Also assume that these two factors must be combined in equal proportions to yield a level of output,  $Y, = \min (K,L)$ . For such a highly simplified case, factor supply as well as product supply curves can be graphed on the same diagram with common horizontal and vertical axes and common scales. Furthermore, the product supply curve is obtained by simple vertical summation of the two factor supply curves. It is then a matter of elementary geometry to conclude that the product supply curve is parallel to the supply curve for factor K, and hence that producer surplus is equal to the rent accruing to factor K.

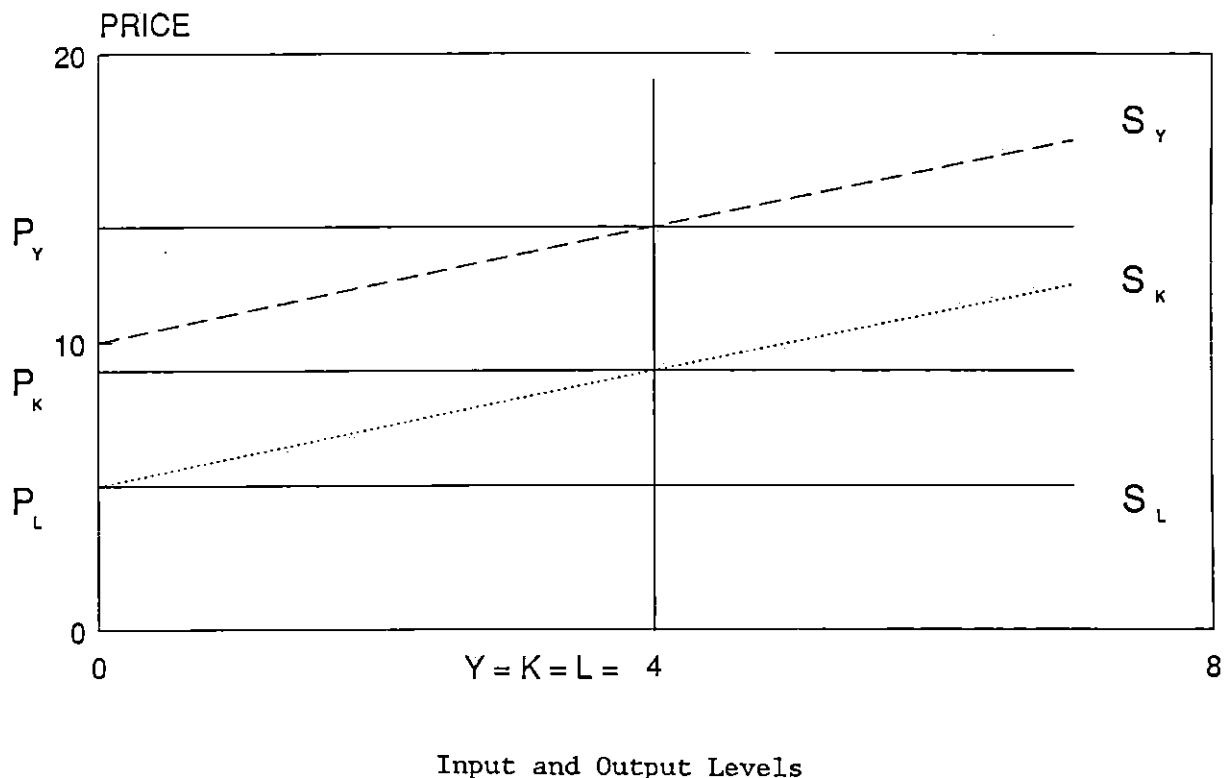


Figure 1: Producer Surplus and Factor Rent with Fixed Factor Proportions

A somewhat less simple counter-example is provided in Table 1, based again on the assumption that production involves fixed factor proportions. Specifically, it is assumed that  $L = K/4$ , and that

$$Y = \min (K/8, L/2)$$

and that supply of both factors is partly inelastic as follows:

$$r = 1 + 3K$$

and

$$w = 3 + L.$$

In keeping with the assumption that the industry is perfectly competitive, product price is set equal to average cost of production. It can be seen that the sum of the rents accruing to the two factors K and L equals the measure of producer surplus. More complicated numerical examples are available on request from the author, but are not reproduced here. Instead, a more general proof of the equivalence of producer surplus to aggregate economic rent is set out below.

Table 1: Numerical Estimates of Producer Supply and Factor Rents for Production with Fixed Factor Proportions

K	r (=1+3K)	L (=K/4)	w (=3+L)	Y (=K/8)	P (=A)	RENT TO K	RENT TO L	PRODUCER SURPLUS
0	1	0	3	0.0	14			
4	13	1	4	0.5	112	24	0.5	24.5
8	25	2	5	1.0	210	96	2.0	98.0
12	37	3	6	1.5	308	216	4.5	220.5
16	49	4	7	2.0	406	384	8.0	392.0
20	61	5	8	2.5	504	600	12.5	612.5
24	73	6	9	3.0	602	864	18.0	882.0
28	85	7	10	3.5	700	1176	24.5	1200.5
32	97	8	11	4.0	798	1536	32.0	1568.0
36	109	9	12	4.5	896	1944	40.5	1984.5
40	121	10	13	5.0	994	2400	50.0	2450.0

A More General Proof that Producer Surplus  
Equals Aggregate Economic Rent

First, define an industry production function by aggregating the individual production functions of actual and potential firms in the industry in the most efficient manner possible, as follows:

$$(1) \quad Y = f(X_1, X_2, \dots, X_n)$$

where:

$X_i$  = aggregate industry level of use of factor  $i$  ( $i=1, \dots, n$ ).

$Y$  = industry output

and for each factor  $X_i$ , assume a factor supply equation which is inelastic to the industry<sup>2</sup>:

$$(2) \quad r_i = g(X_1, X_2, \dots, X_n)$$

where:

$r_i$  = price of the  $i^{\text{th}}$  factor

and where:

$$\partial r_i / \partial X_i > 0 \quad \text{for all } i$$

and:

$$\partial r_i / \partial X_j = 0 \quad \text{for } i \neq j.$$

Note that if this industry production function exhibits constant returns to scale, then:

$$(3) \quad dY = \sum_{i=1}^n f_i dX_i ;$$

and by multiplying each side of equation (3) by product price,  $P$ ,

$$(4) \quad P \cdot dY = \sum_{i=1}^n P f_i dX_i ;$$

which must equal  $\sum_{i=1}^n r_i dX_i$  if all firms follow the profit maximising condition that  $VMP = Pf_i = \text{factor price} = r_i$ .

Now define producer surplus, PS, as:

$$(5) \quad PS = PY^* - \int_0^{Y^*} P(Y) \cdot dY$$

where  $Y^*$  is the level of output supplied by a perfectly competitive industry at price  $P$ .

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2. Note that although factor supply to the industry is less than perfectly elastic, in a competitive industry with a large number of firms each individual firm will be a price taker in the product market and in all factor markets, and therefore will equate value marginal product to factor price when maximising profits.

Note that only normal profits will be earned in such an industry, so:

$$(6) \quad PY^* = \sum_{i=1}^n r_i^* X_i^*$$

where  $X_i^*$  is the aggregate level of factor  $i$  employed in the industry to efficiently produce  $Y^*$ , and  $r_i^*$  is the price necessary to induce supply of  $X_i^*$ .

By substituting equations (4) and (6) into equation (5), we obtain:

$$(7) \quad PS = \sum_{i=1}^n r_i^* X_i^* - \sum_{i=1}^n \int_0^{X_i^*} r_i \, dX_i$$

which can be rewritten as:

$$(8) \quad PS = \sum_{i=1}^n \left[ r_i^* X_i^* - \int_0^{X_i^*} r_i \, dX_i \right]$$

In other words, given the relatively innocuous assumptions made above, producer surplus when measured as the area above a long run industry supply curve defined so as to hold constant factor supply conditions must equal the sum of rents accruing in aggregate to all scarce factors used to produce industry output.

### Conclusions

Contrary to the assertions by Mishan (1968), it is concluded that there is an exact equivalence between producer surplus and aggregate factor rents as long as the supply curve is defined appropriately. If the aim is to measure long run factor rents, then the appropriate supply curve is the long run industry supply curve defined holding factor supply conditions (but not factor prices) constant.

As a corollary, note that econometrically estimated elasticities of product supply derived holding factor prices constant do not provide a valid basis for estimating producer surplus. The challenge then is to estimate product supply elasticities holding factor supply conditions constant. This challenge is left for others to tackle.

### References

- Just, Richard E., Hueth, Darrell L. and Schmitz, Andrew (1982) Applied Welfare Economics and Public Policy. Prentice Hall, Englewood Cliffs, New Jersey.
- Mishan, E.J. (1968) "What is Producer's Surplus?," American Economic Review 58:1269-82.