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Efficiency and equity in a society-economy integrated model*

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A model that only focuses on economic relations, and in which efficiency and equity are defined in terms of resource allocation may miss an important part of the picture. We propose an integrated model that embeds economic activities in a larger game of social interactions, and define comprehensive notions of efficiency and equity. Social interactions provide additional sources of inefficiencies and inequalities, including through the mere lack of coordination between the regulation mechanisms of the economic sphere and the social sphere. Remedies may require more than market solutions and redistribution of resources. In particular, social norms are likely to play an important role.

1 Introduction

This paper aims at making three contributions. First, it presents a stylized general model of society that embeds the economic market equilibrium in other non-market social interactions, and encapsulates key tenets of the main models of social interactions. Second, it examines the conditions for achieving efficiency and equity in such a broad "socio-economy model" and compares these with the conditions for efficiency and equity in the standard general equilibrium model, focused on the economy alone. Third, the paper explores implications for some of the most significant ideas to have emerged from the discipline of economics, such as the workings of the Invisible Hand (as elucidated by the two fundamental theorems of welfare economics), the role of markets in coordinating economic activities, the relation between efficiency and equity, the welfare effects of profit-maximization and self-interested utility maximization, the welfare effects of redistribution, the sources of market discrimination, and the exploitation of potential gains from trade. These implications constitute an important motivation for our socio-economic model.

Our model relates straightforwardly to the existing economic thinking on economic and social interactions. The core models of economic theory focus on markets, contracts and trade and tend to ignore

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other forms of social interactions. Since Adam Smith's *Theory of Moral Sentiments*, Veblen's *Theory of the Leisure Class* (Veblen 1934) and Becker's "Theory of social interactions" (Becker 1974), however, many economists have been aware that there is an important social side to people's lives that cannot be ignored because it influences the economic world and is affected by it in return. A very large economic literature on social interactions has blossomed in the last decades.

This literature can be divided into three broad categories. One branch studies social influences with game-theoretic and related tools and focuses on the specific patterns that emerge in social interactions, as distinct from market transactions. It highlights specific phenomena that naturally appear in such interactions, such as conformism and social multiplier effects, as well as multiple equilibria, and it generally focuses on one variable of choice, such as a particular cultural, moral or consumption behavior (e.g., Bernheim 1994, Akerlof 1997, Brennan and Pettit 2004, Brock and Durlauf 2001, Durlauf 2001, Durlauf and Ioannides 2010, Manski 2000). In some approaches there is a direct desire to conform to the others' behavior, whereas in others there is external policing through approval and blame. Diffusion through networks has recently been extensively studied (Jackson 2008, Bramoullé et al. 2016).

The other important branch of the literature studies the interdependence between social mechanisms and economic activity. It has adopted the sociological notion of social capital (Bourdieu 1979, Coleman 1994, Putnam 2002) and examined how social capital, competition for social status, social identities and social norms or even mere narratives influence economic decisions, in particular human capital investment, savings and consumption behavior, as well as how, conversely, economic decisions and economic transformations shape social structures such as class hierarchies and segregation (Akerlof and Kranton 2000, Cole, Mailath and Postlewaite 1992, 1998, Becker and Murphy 2000, Benabou 1993, 1996, Frank 1985, Kolm 2005, Loury 1977, Coate and Loury 1993, Mailath and Postlewaite 2003, 2006, Snower and Bosworth 2016, Shiller 2019).

A third branch of the literature explores the microfoundations of social influences. Taking inspiration from psychology (e.g., Fiske 2005, Kahneman et al. 1999) and direct experimental evidence, it reveals how sensitive people are to social comparisons and fairness evaluations, how influenced they are by the social context, and how much they care about others (Rabin 1993, Fehr and Schmidt 1999, Fehr et al. 2002, Cherchia et al. 2017, Layard 2005). Brennan and Pettit (2004) propose to consider the desire to be esteemed as a key social enforcement lever beside the fear of punishment and economic incentives, and this can be enlarged to other desires such as genuine caring, a desire for belonging, and so on.

As this broad literature provides many illuminating analyses of the way in which real-life economies, embedded in social settings, differ considerably from the textbook economic model, the time may be ripe for rethinking the basic general equilibrium model and revisit the standard economic concepts of efficiency and equity that have been developed for this model. This is the purpose of our socio-economic model.

The paper starts with a basic model, which provides key insights into the channels of interdependence between economic and social interactions. This model is a standard economic Arrow-Debreu model embedded in a social game, resembling the integrated models of the economy and the environment (e.g., Nordhaus and Boyer 2000). The social game is a simple interactive game in which every individual directs some action to every individual (including oneself), and social success depends on the level of

support or reinforcement received from others. The Walras-Nash equilibrium of this model contains the competitive equilibrium in the economy in the Nash equilibrium of the larger social game.

This model is general enough to encompass the conformism mechanism of social interaction models, the enforcement of norms by social pressure, patterns of grouping and segregation, power relations as well as competition for status and power. The model is static and has a fixed population, and therefore leaves out a key set of dynamic social phenomena, in particular intergenerational transmission. But, as in economic theory, static analysis is able to uncover key structural insights of steady-state dynamic equilibria.

This model distinguishes two channels of interdependence between the economic module and the larger social game. One channel operates through people's preferences, or more generally character formation, which may involve influence of the social context on economic preferences. The other channel lies in the determination of social success, i.e., in the rules of the social game, in which economic activities may influence social outcomes, and therefore may be determined by social strategic considerations. These are the two senses in which the economic model of Walrasian equilibrium is embedded in the social model of Nash equilibrium. Each channel brings in externality effects and thereby important sources of inefficiency.

As far as equity is concerned, this model makes it possible to examine the relationship between economic inequality and social inequality, and raises the question of defining equity in a way that encompasses the economic and the social dimensions of people's lives simultaneously. As can be expected, economic inequality is perfectly correlated with social inequality only when the social game heavily depends on economic assets. As far general socioeconomic inequality is concerned, it can be measured by extending money-metric utilities, already familiar in the literature on well-being (Fleurbaey and Blanchet 2013), to include social dimensions. This provides a convenient generalization of the measurement of economic inequality and offers the possibility to disentangle the contribution of economic inequality from other social factors in the measure of general socioeconomic inequality.

Three limitations of the basic model are that it assumes perfect competition and the absence of market failures on the economic side, that it ignores the fact that some social interactions occur through market transactions, which is likely to generate market power as well, and that it excludes social impacts on the available production technology. In later sections of the paper (6, 7), extensions of the basic model remedy these limitations and enable us to accommodate further interactions between economic activities and social interactions, such as social remedies to economic ills, and social interactions flourishing through economic interactions.

The paper is structured as follows. Section 2 presents an overview of the underlying ideas and the main insights. Section 3 introduces the basic socio-economy model and three special cases of this model that disentangle the various types of interactions between the economic sphere and the broader social sphere. Sections 4 and 5 then examine what happens to the study of efficiency and equity, respectively, in this socio-economy model. Section 6 generalizes the model and examines how social interactions and social norms can help alleviate problems of inefficiency and inequality in the economy. Section 7 proposes another more general model in which the economic and the social spheres are more closely enmeshed than in the basic model, because every economic transaction in this model can be the site

of social phenomena as well. Section 8 concludes.

2 Insights from this analysis

Our analysis offers several important insights and suggests open questions that we would like to highlight here, before diving into the details of the model. The ideas emerging from this paper are not all new, and many have been suggested by various existing models of social interactions. Our socio-economy model however provides a general framework for understanding and exploring these various dimensions, which together have far-reaching implications for our understanding of how the market economy interacts with society and how economic policy affects this interaction. Numbers in parentheses refer to the subsections of the paper where these points appear.

A first set of insights has to do with the fact that the economy is embeddged in society, and that ignoring this interdependence is profoundly misleading:

- The fundamental theorems of welfare economics are undermined: In the presence of the social sphere, the two fundamental theorems of welfare economics no longer hold. In particular, the perfectly competitive general equilibrium no longer leads to a Pareto efficient allocation of resources, even under complete markets, symmetric information and no economic externalities (4.1, 4.4). Since competitive markets do not lead to efficiency, the conventional theoretical basis for the Invisible Hand is undermined. Lump-sum transfers combined with free markets would not suffice to simultaneously achieve efficiency and equity (5.1). More precisely, Pareto efficiency requires (i) efficient economic markets, (ii) efficient social interactions, and (iii) efficient coordination between the economic and social spheres. In other words, Pareto efficiency in the economic sphere and in the social sphere is not sufficient to ensure the Pareto-efficiency of the socio-economy (whereby it is impossible to make one individual better off, across both spheres, without making another individual worse off). The reason is that socioeconomic Pareto efficiency additionally rests on the condition that coordination between the economic and social spheres be efficient (4.1, 4.4). This condition is both important and difficult to fulfill, since the commodification of many social relations is impossible, while the typical coordination mechanisms of the social sphere (social norms, tacit reciprocity, or centralized coordination) are usually not designed for such a task. However, social interactions generated through market transactions may in some cases have their value incorporated into market prices, making it possible to coordinate some forms of social bonding through prices without actually commodifying the relations themselves (7).

- Conventional economic models make unrealistic implicit assumptions about society: Conventional economic analysis may be understood as a special component of a more general socioeconomic theory. In particular, conventional general equilibrium theory, as in the Arrow-Debreu model, is a depiction of a well-functioning economy, for which it has been implicitly assumed that society (unmodelled) is well-functioning, too and there is perfect coordination between the social and economic spheres. It is on account of this implicit assumption that conventional analysis does not take account of social dysfunctions as a source of economic dysfunctions. Economic theories of market imperfections (such as theories of asymmetric information and market power) and most environmental economic theories recognize the existence of externalities and other market failures, but still assume

a well-functioning society and ignore the issue of the coordination between the spheres. This implicit assumption has important implications for the predictions of conventional economic analysis. A less charitable interpretation of the conventional economic models is that they assume away social interactions altogether, even if a world in which people interact economically but not socially is totally foreign to any reasonable conception of humanity.

A second set of insights concerns the relative strengths and weaknesses of the economic and social spheres in meeting human needs. Although these insights are not new, they appear vividly when the economic market and the social sphere are put together in the same model.

- **The strength of the social sphere relative to market coordination**: The relative strength of the social sphere lies in its ability to mobilize other sources of motivation than self-interest, such as altruism or moral and social norms, as well as its ability to involve other forms of coordination and communication. This may actually serve to address problems originating in the economy, such as market failures and inequities. These market failures and inequities may be mitigated through social mechanisms, in particular through appropriate social norms and values (6). Needless to say, socially-driven remediation does not happen automatically. For example, Elinor Ostrom's Core Design Principles aim to summarize conditions under which market failures may be overcome in the social sphere, supported by appropriate governance mechanisms.

- **The strength of the economic sphere relative to the social sphere**: The relative strength of the economic sphere lies in its ability to coordinate decisions across markets. This coordination, involving prices as information carriers, is generally not possible in the social sphere. For example, if one invites a friend for dinner at home, that friend may reciprocate with a dinner in her home; but it is not acceptable for the friend to instead pay for the dinner and even less acceptable for someone else to pay on the friend's behalf. The social sphere provides more limited opportunities for coordination than does the economic sphere, since the social sphere operates through interpersonal relationships, involving channels such as tacit reciprocity, care, culturally scripted norms, or centralized coordination that is cognitively demanding. These channels are characteristically individual-specific (I help you if you help me) and/or action-specific (a dinner invitation in return for a dinner invitation). The flexibility of coordination that is characteristic of the economic sphere is missing, since social relations tend to be less interchangeable than economic relations (3.1, 3.2, 4). An additional advantage of the market comes from its ability to reduce social tensions triggered by rivalry in access to resources. This, substantially, is the idea underlying "doux commerce," the notion that commerce tends to civilize people, making them less prone to direct pressure or violence (6). This advantage must be set against the concomitant problem that market activity can generate harmful social externalities due to materialistic status-seeking (5.1).

A third set of insights is about inequalities, which now have an economic and a social dimension.

- **Socio-economic inequality**: This paper develops a way of conceptualizing and measuring socio-economic inequality. While economic inequalities can generate social inequalities, there is not necessarily a perfect correlation between economic inequality and social inequality (3.1, 5.1, 7). Because economic and social standing are linked but distinct, measuring well-being in a comprehensive way covering economic and social dimensions is important and offers ways to analyze the contribution of each

sphere to well-being inequalities (5.2). It also sheds light on the limited benefits for social optimality of economic transactions that transfer resources from rich to poor and reduce inequalities in consumption but widen the gap in their social standing, e.g., when they enter master-servant, or employer-employee relations. Actually, we show that for a social welfare criterion respecting individual preferences, it is logically impossible, when individuals have heterogeneous preferences over the relative importance of economic and social dimensions, to always treat reducing inequalities in resources as socially desirable, even among individuals with identical social standing.

- Complementarity between efficiency and equity: A huge literature has devoted attention to the correlation between economic inequalities and economic productivity and efficiency. Here we find that the interdependence between social relations and economic inequalities provides a natural channel by which reducing economic inequality can enhance general socio-economic efficiency (as distinct from economic efficiency). In particular, when economic inequality is sufficiently high and sufficient status-seeking activities sufficiently intense, economic redistribution can create more commonality among people and thereby enables a wider range of mutually beneficial social relations. The resulting social redistribution leads to further benefits by reducing the pursuit of status-seeking activities (which involve effort but provide no overall gain). Under suitable circumstances, redistribution is good for everyone (4.2, 5.1).

The socio-economic model suggests new research questions about the interactions between the two spheres and the relative responsibilities of various actors.

- Positive and negative interactions between the social and economic spheres: Activities in the social and economic spheres affect one another in many ways. Economic resources are key inputs to social interaction (3.1), as highlighted in the previous point. Conversely, social relations may shape economics relations by influencing individuals' motivations and imposing non-monetary constraints on their economic decision.

"Positive" feedback between the economic and social spheres obtains when economic cooperation rests on social cooperation and vice versa, such as when:

- responsible economic behavior prevails more readily among people who are socially bonded (6),
- mutual care within a social group leads to informal mechanisms for cushioning group members from economic hardship (6), or conversely,
- social bonding depends negatively on the extent of economic inequalities (5.1).

"Negative" feedback may occur when, from society to the economy,

- market discrimination is used for social pressure, meaning that the incentive for economic discrimination may lie in the social sphere (3.1),
- status seeking leads individuals to value relative income rather than absolute income (5.1), or, in the other direction, when

- technological choices that are profit-maximizing may be welfare-destroying, in the sense that the economic gains may be dominated by the associated social losses (7).

Furthermore, when globalization and automation increase aggregate output while breaking social connections (such as communities that collapse under the pressures of outsourcing and robotics), these phenomena may turn out to be socially undesirable, since the measured economic gains may fall short of the generally unmeasured social losses (4.2, 4.3, 5.1). Therefore, a natural question emerges about assessing the relative importance and strength of these positive and negative feedbacks, and finding ways to enhance the positive ones and loosen the interconnection between the spheres in the negative feedback channels.

- The appropriate division of economic responsibilities: In the conventional division of economic responsibilities, firms maximize profits (or shareholder value), households maximize their self-interested utility from consumption, and the government shapes market conditions (through taxes, subsidies, regulations, finance or provision of public goods, and management of the commons) so that market failures and inequities are mitigated. In the presence of the social sphere, this conventional division of economic responsibilities appears no longer appropriate.

- Since profit-maximization and the maximization of self-interested utility from consumption may hurt social relations and the resulting social costs may exceed the economic gains, firms and households must assume both economic and social responsibilities (6).
- Furthermore, the government's responsibility is to use its instruments of economic and social policy with a view to promoting efficiency and equity across the economic and social spheres, taking into account the strengths and weaknesses of the two spheres (4.2-4.4, 5.1).

This suggests that

- business performance needs to be measured by more than profit (6, 7),
- economic policy needs to be measured by more than economic gains (5.2), and
- civil society has responsibilities to fulfill in the application of norms and values (6). Our model is not designed to depict government intervention and studying this issue would require adding structural elements to it.

The insights above call into question the conventional view of Adam Smith's Invisible Hand and bring us closer to Arrow's (1973) views about the complementarity between economic interests and moral norms. What makes economic markets function as well as they do—despite widespread market failures and imperfect coordination between the economic and social spheres due to the impossibility of commodification of social relations—is a combination of economic and social enablers: Market economies function well not only on account of voluntary exchange in the economic sphere (allowing coordination across economic markets), but also on account of cooperative social relations that enable individuals to internalize externalities, address other market failures and mitigate inequities. These two

drivers of the Invisible Hand—a well-functioning economy that permits coordination across markets and a well-functioning society that permits internalization of externalities and other market failures—correspond to Adam Smith's two contributions in this regard, the former in *The Wealth of Nations* and the latter in *The Theory of Moral Sentiments*. Conventional general equilibrium theory focuses exclusively on the former, leading to a consequential neglect of the latter, with dangerous policy implications.

In practice, needless to say, economies and societies often do not function well. When economies function badly and societies function well, we find that human needs are not adequately met, despite "caring societies," in some less-developed economies (e.g., Bellah et al. 1992, Glenn 2000). When economies function well and societies function badly, we observe the discontented consumerist societies of developed economies (e.g., Scitovsky 1976, Hilton 2008). In the so-called "failed states," both economies and societies function badly.

The history of capitalism may be understood in terms of the gradual extension of economic markets into domains that used to be governed by social relations. In his magnum opus, *The Great Transformation*, Karl Polanyi describes how economies that were based on reciprocity and communal relationships became transformed into competitive markets, accompanied by industrialization and rising influence of the state. It was this transformation that enabled us to conceive of human behavior in terms of the satisfaction of consumption wants through voluntary exchange. In the process, economics lost sight of Polanyi's insight that "man's economy, as a rule, is submerged in his social relationships." (p. 48) This paper is a tentative attempt to right the balance.

The insights above are also a first step towards a balanced appreciation of two major themes, emerging from two different disciplines: The first is "the magic of markets" theme of economics, explaining why people trading freely with one another under competitive conditions can make each other better off. The second is "the magic of societies" theme of anthropology, explaining how much of the evolutionary success of humanity can be explained by our capacities for social cooperation (see, for example, Christakis (2019), Henrich (2016), Novak (2012) and Turchin (2016)). The first may be understood as the economist's Invisible Hand; the second as the anthropologist's Invisible Hand. We need an appreciation of both themes within a single conceptual framework in order to gain a more balanced understanding of how economies, in interaction with the societies in which they are embedded, help people coordinate their actions in order to address the challenges they face. This paper is meant as a step towards such a more balanced understanding.

3 The basic model

The basic model starts with a standard Arrow-Debreu general equilibrium model and embeds it in a social game, in similar fashion as integrated climate-economy models feature a standard Solow growth model connected to a climate module, but with the key difference that the social sphere is still the locus of human strategic action. Even though this is only a first step toward the integration of the economic and the non-economic, it already contains several channels of interactions, and therefore we

will introduce specific subcases of this basic model in which the specific channels are isolated. This section introduces the general basic model and its relevant three special cases.

3.1 General framework

To keep things simple, and take the most favorable outlook for the economic sphere of the model as a starting point, the economy part of the model is an Arrow-Debreu economy with constant returns to scale in production, perfect competition and no externalities. Market imperfections will be introduced in sections 6-7.

3.1.1 Individual behavior

There is a finite number of individuals $i \in N = \{1, ..., n\}$. Each individual's situation is described by a pair (x_i, y_i), where $x_i \in X_i \subset \mathbb{R}^\ell_+$ is a vector of $\ell > 1$ commodities consumed by i, and y_i a vector of personal and collective outcomes in the social sphere that are relevant for individual i. Total production takes the form of a transformation of commodities $q \in Q \subset \{0\} \cup \left(\mathbb{R}^\ell \setminus \mathbb{R}^\ell_+\right)$, where a positive component of q is an output and a negative component an input, and Q is a cone. We assume constant returns to scale (i.e., Q is a cone) in order to avoid having the track the distribution of profit among individuals. When $Q = \{0\}$, there is no production and the model describes a pure exchange economy.

Individual preferences are represented by a utility function $u_i(x_i, y_i)$. This utility function can actually capture much more than standard preferences. Since social influences really shape individuals' mindset and determine their personal development, this function can capture deep and formative impacts of social interactions on the individual. In that sense, this model allows for dramatic departures from the standard economic model in which preferences over x_i are stable. What we keep from the standard approach in the main part of this paper, though, is that individual behavior is assumed to rationally strive for personal well-being according to the true function u_i. This means that in our analysis, individuals choose social relations that make them grow and systematically shun social interactions that would have a nefarious influence on their personal development. It is obvious that this assumption is not realistic. But we retain it here because our focus is on the fact that, in spite of such a demanding rationality assumption, there are serious obstacles to achieving a social optimum in this model. It is obvious that things are much harder if individuals adopt self-destructive social strategies.

Each individual faces two contraints, the economic and the social one. The economic constraint is a typical budget, and it is assumed in sections 1-6 that individuals are price-takers:

$$x \in X_i \text{ and } px_i \leq p\omega_i,$$

where ω_i is i's endowment. Commodities can include labor services.

The other constraint brings the social game into the picture. It says that y_i is obtained through a game form of the type

$$y_i = F_i(p, x, s),$$

where $x = (x_1, ..., x_n)$ is the economic allocation and $s = (s_1, ..., s_n)$ is the profile of social strategies in the population. The function F_i encapsulates how i's social outcome depends on social strategies but also on the economic allocation. The interdependence between commodities and social interactions can actually go both ways. Some social patterns may require certain economic distributions, but conversely, it may be impossible for an individual to adopt an economic lifestyle without the realization of certain social strategies. To keep the analysis simple, we will assume that the set of strategies available to i is a fixed S_i, and that all interdependent feasibility constraints between x and s, or between s_i and s_{-i}, are embodied in the function F_i.

Many examples of social games can be drawn from the literature on social interactions cited in the introduction, and many examples are provided in this paper. For concreteness, think of s_i as a strategy which can consist in a combination of the following actions:

- extend, accept or decline invitations to and from other people (leading to family formation, friendships, joint activities including economic activities);
- directly engage in interactions and collective activities (e.g., cheering, booing, fighting, joining demonstrations);
- pass or withhold (mis)information, advice and admonitions, expressive symbolic messages that may influence or affect the others;
- behave in a way that enhances or undermines the relevant others' esteem for the individual (e.g., being truthful, faithful, courageous, empathetic, or the opposite);
- seek, keep or change one's or someone else's position in a network or a hierarchy of power or influence.

The game produces social outcomes, and the relevant outcomes for i are contained in the vector y_i. This vector can for instance contain a list of pairs (j, r) meaning that i is in relation r with individual j (e.g., i has authority over j regarding road use), or it can describe how i is viewed by other people in terms of identity, esteem, prestige, what type of social role, power and status i enjoys or endures, and so on. It can includes features about the whole social situation and need not be limited to strictly personal outcomes, since the individual may care about who is related to whom, who has such or such position or social status and so on.

Moreover, since $y_i = F_i(p, x, s)$, the vector of social outcomes y_i can include many things about the social situation that the individual may care about: the distribution of resources x, the strategies played by everyone s. This means that our model is able to capture preferences about procedures and not just about final outcomes. For instance, individuals may resent being forced to do certain things (e.g., practice social distancing), even though these things are not by themselves particularly unpleasant. This can be depicted in this model by listing two components in y_i, one representing the actual outcome, and another recording how it came about through specific strategies. There are many aspects of social relations, especially, in which spontaneity is viewed as important and therefore individuals may accept certain interventions that foster social relations by facilitating meetings or

creating favorable occasions, but would be repelled by incentives and injunctions that would kill the spontaneous element that gives value to the relations.

3.1.2 Fleshing out the social game

As there is a strikingly similar structure in most social interactions, it is possible and useful to provide a more precise description of the social game, at a similar level of abstraction as the Arrow-Debreu economic model. The common structure in social games comes from their distributed reinforcing nature. That is, every individual directs a more or less supportive action at every individual (including oneself), and the outcome for an individual is an increasing function of the level of support received by this individual. The support itself may be aimed at two different types of outcomes: a collective achievement (joint activity, common beliefs), or a personal outcome (status, power). Hybrid outcomes are also commonplace, when group gatherings and actions play a role in social competition for status or power.

Formally, let $s_i = (s_{ij})_{j=1,...,n}$, where each $s_{ij} \in \mathbb{R}^m$ is a level of support in dimensions $d = 1, ..., m$. The individual outcome $y_i \in \mathbb{R}^{m+\ell n+mn^2}$ includes the same dimensions $d = 1, ..., m$ as well as elements of x and s. The function $F_{id}(p, x, s)$ in dimension d is assumed to be an increasing function of the vector $(s_{jid})_{j=1,...,n}$. The number of dimensions m can be large because it can include the existence of a link between any pair of individuals in a network, and there may be several different networks for different joint activities.

Different types of functions $F_{id}(p, x, s)$ embody different social norms for various outcomes. Here are salient examples, with illustrations for collective achievements and personal outcomes, when relevant:

- the veto function $y_{id} = \min_{j \in N_{id}} s_{ji}$ enables every j in a certain subset N_{id} to limit the outcome y_{id} :
  - collective: whether i befriends l can be vetoed by i, l and perhaps a few others among their relatives;
  - personal: i's credibility may be determined by the individual who trusts him the least, if such opinions are common knowledge;
- the claim function $y_{id} = \max_{j \in N_{id}} s_{ji}$ enables every j in a certain subset to up the outcome y_{id} :
  - collective: how often the Smiths and the Joneses have dinner together may depend on the one who extends the most invitations;
  - personal: in order to get a position, one acceptance may suffice;
- the additive function $y_{id} = \sum_{j \in N_{id}} s_{ji}$ enables every j in a certain subset to add up to the outcome :
  - collective: community life depends on multiple contributions (as for a public good);
  - personal: people heap praise or blame on i;

- the rank function $y_{id} = r$ if $\sum_j s_{ji}$ has rank r in the distribution of $\sum_j s_{jl}, l = 1, ..., n$:
  - personal: status or power depends on the relative support received, e.g. getting a plurality of votes in an election;
- the affordability function $y_{id} = \min\{x_{ik}, s_{ijd}\}$:
  - personal or social: a certain social strategy (e.g., initiating a relation) is possible only with the required wherewithal in resources;
- the gatekeeper function $y_{id} = x_{ik}\text{sgn}\left(s_{ijd} - s^*_{ijd}\right)$;
  - personal: the individual suffers a social penalty for consuming x_{ik} unless she "conforms" by having $s_{ijd} > s^*_{ijd}$; this can serve to police social behavior through market access discrimination;
- the contest function $y_{id} = f(x_{jk}, j = 1, ..., n)$ where f is increasing in x_{ik} and decreasing in x_{jk} for $j \neq i$, and k is a commodity (or a subset of commodities) representing expenses in the contest:
  - personal: status may be obtained by ostentatious consumption.
- the solo function $y_{id} = s_{iid}$ marks a purely individual decision:
  - personal: i joins an institution (e.g., a religious denomination), makes a public statement (e.g., an outing), or adopt any observable behavior that may not directly affect others' personal outcomes but will potentially shift their strategies in equilibrium;

The next to last example only involves economic actions, and one can interpret buying a quantity of commodity k as supporting one's own status, while the others do the same on their own count. No support for others is considered in this example, but one could introduce it by allowing for gifts. More examples will be given in the paper, but it should be clear that this model encompasses most of the social interaction games of the literature, with the exception of dynamic and intergenerational models.

3.1.3 Walras-Nash equilibrium

In summary, individual i selects his economic and social behavior by solving the following program:

$$\max_{x_i \in X_i, s_i \in S_i} u_i(x_i, y_i)$$

$$\text{such that } \begin{cases} px_i \leq p\omega_i, \\ y_i = F_i(p, (x_i, x_{-i}), (s_i, s_{-i})). \end{cases}$$

This program assumes that the individual takes prices and other individuals' strategies as given, corresponding to competitive economic behavior and Nash-type strategic behavior.

A Walras-Nash equilibrium of this model is a pair (x, y) such that, for a price vector p and a strategy profile s:

WN-i) every i solves the above program;

WN-ii) the markets clear: $\sum_i x_i = \sum_i \omega_i + q$;

WN-iii) q maximizes pq for $q \in Q$.

Observe that the market clearance condition is separate from individual maximization, whereas the feasibility constraints on social strategies and outcomes are included in the functions F_i.

A Walras-Nash equilibrium includes two component subequilibria in the two spheres. The Walras subequilibrium is defined for a given strategy profile s as follows. The economic allocation is a Walras subequilibrium if there is a price vector p such that:

W-i) every i solves the following program:

$$\max_{x_i \in X_i} u_i(x_i, y_i) \text{ such that } \begin{cases} px_i \leq p\omega_i, \\ y_i = F_i(p, (x_i, x_{-i}), s). \end{cases}$$

W-ii) the markets clear (same as WN-ii);

W-iii) production choice (same as WN-iii).

In the Walras subequilibrium, the individual takes account of the influence that the choice of x_i has on the social outcome y_i.

The Nash subequilibrium is defined as follows, for a given economic allocation x. The social outcome y is a Nash subequilibrium if there is a strategy profile s such that:

N-i) every i solves the following program:

$$\max_{s_i \in S_i} u_i(x_i, y_i) \text{ such that } y_i = F_i(p, x, (s_i, s_{-i})).$$

It is worth noting an apparent asymmetry between the Walras and the Nash subequilibria. In the former, the individual takes account of the social consequences y_i of economic decisions x_i, via the function $F_i(p, (x_i, x_{-i}), s)$, whereas in the latter, social strategies appear not to have any economic consequences. This may seem odd, since the function $F_i(p, (x_i, x_{-i}), s)$ does encapsulate the possibility that certain economic actions are possible only in conjunction with certain social strategies (e.g., access to a particular market may be barred unless a social relation is established with a gatekeeper). But social outcomes and economic outcomes are inherently different. Once economic decisions are fixed, there is nothing that can be done about it through social actions, because physical transactions are given: in the economic realm, strategies and outcomes are the same thing. In contrast, an economic decision may have the power to alter social outcomes, which have symbolic dimensions, even if social strategies s are also physically given and unalterable.

Proposition 1 *A pair (x, y) is a Walras-Nash equilibrium, associated to a price vector p and a strategy profile s, only if x is a Walras subequilibrium for the given s (and associated to the same vector p) and*

y is a Nash subequilibrium for the given x (and associated to the same s). The converse ("if") is true if for every i, there is an increasing transform φ such that the function $h_i(p,x,s) = \varphi \circ u_i(x_i, F_i(p,x,s))$ is concave and C^1 in (x_i, s_i).

Proof. The "only if" part follows directly from the definitions and the fact that the maximum of a multivariate function must be a maximum in every subset of dimensions.

The "if" part comes from the fact that condition WN-i is satisfied when x is a Walras subequilibrium for a given s and y is a Nash subequilibrium for the same x and associated to the same s. Indeed, individual utility is equal to

$$u_i(x_i, y_i) = u_i(x_i, F_i(p, (x_i, x_{-i}), (s_i, s_{-i}))).$$

If the right-hand side is concave and C^1 in (x_i, s_i) and is maximized separately in x_i and in s_i, then it is maximized in the pair (x_i, s_i). But one does not need concavity of this function. It is sufficient that it is the monotonic transform of a concave function.[1]

■

Conditions for the existence of a Walras-Nash equilibrium can be derived from adapting Ghosal and Polemarchakis (1997, Prop. 1).

Proposition 2 *A Walras-Nash equilibrium exists under the following assumptions:*

- *The function $U_i(p,x,s) := u_i(x_i, F_i(p,x,s))$ is continuous in (p,x,s) and non-satiable[2] in x_i;*
- *The set X_i is closed and convex;*
- *The set S_i is compact and convex;*
- *The individual endowment $\omega_i \gg 0$;*
- *The cone Q is closed;*
- *For every p and (x_{-i}, s_{-i}), the set of (x_i, s_i) maximizing $U_i(p,x,s)$ such that $px_i \leq p\omega_i$ is convex.*

Proof. See the appendix. ■

[1] It is necessary to introduce differentiability, otherwise the result would not hold. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by:

$$f(x,y) = \begin{cases} -x + 2y & \text{if } x \geq y \\ 2x - y & \text{if} x \leq y. \end{cases}$$

This function is concave, and at any point where $x = y$ it is maximal with respect to x and y separately, but this is not a maximum.

[2] Non-satiation means that for every (p,x,s), every neighborhood $N \subset X_i$ of x_i, there is $x_i' \in N$ such that $U_i(p, x_i', x_{-i}, s) > U_i(p,x,s)$.

3.2 Three restricted models

The reason why the model is not simply written in terms of two variables (x_i, s_i) but also includes y_i is that it enables us to distinguish interactions between the social and the economic that take place in the individuals' preferences and interactions that come through feasibility constraints. This distinction makes it possible to identify different mechanisms. For this purpose, it is useful to disentangle the various interactions by looking at specific restricted variants of the model.

3.2.1 The park model

The first variant is named after the park because when people's social interactions occur in the park, this is a break from the economic part of life and everyone comes in casual outfit, so that it is hard to notice economic inequalities.

Reflecting this intuition, the park model involves the following specification. First, individuals have separable preferences over commodities and social interactions:

$$u_i(x_i, y_i) = v_i(f_i(x_i), g_i(y_i)),$$

where f_i is the subutility on commodities and g_i the subutility on social outcomes. In other words, what people want to do in the park (social interactions) is independent of what they do at home (consumption). The second restriction is that social interactions are not constrained by the economic allocation:

$$y_i = F_i(s).$$

This model completely separates the economic subequilibrium from the social subequilibrium. The economic subequilibrium in the park model is an allocation x such that, for a price vector p :

i) every i chooses $x_i \in \mathbb{R}^\ell_+$ so as to maximize $f_i(x_i)$ such that $px_i \leq p\omega_i$;

ii) the markets clear: $\sum_i x_i = \sum_i \omega_i + q$;

iii) q maximizes pq for $q \in Q$.

The social subequilibrium is a social situation y such that, for a strategy profile s:

iv) every i chooses $s_i \in S_i$ so as to maximize $g_i(y_i)$ such that $y_i = F_i(s)$.

As one can see, (i)-(iii) form a standard Walrasian equilibrium, while (iv) forms a standard Nash equilibrium.

3.2.2 The backyard model

This second variant keeps preferences separable but reintroduces economic affairs into social interactions:

$$u_i(x_i, y_i) = v_i(f_i(x_i), g_i(y_i)) \text{ and } y_i = F_i(p, x, s).$$

The name of this variant is inspired by the idea that in one's backyard, social interactions happen only when one can afford to invite people and offer them drinks and food, but on the other hand there is little interaction between economic and social aspects in preferences.

Can the separability of individual preferences separate and simplify the subequilibria? The Nash subequilibrium indeed is now based on the individual program:

$$\max_{s_i \in S_i} g_i(y_i) \text{ such that } y_i = F_i(p, x, (s_i, s_{-i})),$$

where the influence of x is confined to the feasibility of social outcomes, and no longer bears on preferences.

This is why in the Walras subequilibrium, the individual still has to take account of the influence of x_i over y_i:

$$\max_{x_i \in \mathbb{R}^{\ell}_{+}} v_i(f_i(x_i), g_i(y_i)) \text{ such that } \begin{cases} px_i \leq p\omega_i, \\ y_i = F_i(p, (x_i, x_{-i}), s). \end{cases}$$

For instance, being better dressed may make the individual able to invite a friend to a restaurant, and have a better chance that the friend will be interested in spending time with him.

3.2.3 The club model

This third variant does the opposite of the previous one. It drops the separability in preferences but shuts down any interaction between economic and social affairs on the feasibility side: $y_i = F_i(s)$.

In this case, it is now the Walras subequilibrium that is simplified, since the individual program boils down to:

$$\max_{x_i} u_i(x_i, y_i) \text{ such that } px_i \leq p\omega_i.$$

This does not mean, however, that when solving the full program, the individual neglects the social side when making the economic decision about x_i. This choice will alter the optimal strategy s_i through the preference interaction. This can occur, for instance, when certain social interactions make one seek certain commodities (e.g., gifts of a specific kind), which, conversely, makes these social relations more attractive when one has access to these commodities. It also happens when certain commodities can be substitutes for certain social relations (e.g., buying a TV set may reduce the need or time for chats with neighbors, having access to private insurance may reduce the need for solidarity arrangements with relatives and friends).

In summary, we have the four models depicted in the following table:

	$y_i = F_i(s)$	$y_i = F_i(p, x, s)$
$v_i(f_i(x_i), g_i(y_i))$	Park	Backyard
$u_i(x_i, y_i)$	Club	Full model

By symmetry, one could be interested in a separability condition applied to the function F_i :

$$F_i(x, s) = G_i(h_i(x), k_i(s)).$$

This has the interesting effect of making strategy choice independent of x, but it does not preserve x from being influenced by s even when preferences are separable in x_i and y_i. For instance, assume

$$v_i(f,g) = fg$$

$$G_i(h,k) = \sqrt{h+k},$$

with $g_i(y_i) = y_i$. Then the marginal rate of substitution (MRS) between goods 1 and 2 is equal to

$$\frac{\frac{\partial f_i}{\partial x_{i1}}(h_i+k_i) + 0.5 f_i \frac{\partial h_i}{\partial x_{i1}}}{\frac{\partial f_i}{\partial x_{i2}}(h_i+k_i) + 0.5 f_i \frac{\partial h_i}{\partial x_{i2}}},$$

which depends on k_i in general, in a systematic way: The greater k_i (in positive values), the more the MRS depends on consumption preferences (f_i) rather than social considerations (h_i).

4 Efficiency

Efficiency can be defined for the whole situation, or separately for the economic and the social situtions.

- An allocation (x,y) is efficient if there is no other feasible allocation (x',y') such that $u_i(x_i,y_i) \leq u_i(x'_i,y'_i)$ for all i and $u_i(x_i,y_i) < u_i(x'_i,y'_i)$ for some i.
- The economic allocation x is efficient given the strategy profile s if there is no other feasible allocation x' such that, letting $y'_i = F_i(p,x',s)$, one has $u_i(x_i,y_i) \leq u_i(x'_i,y'_i)$ for all i and $u_i(x_i,y_i) < u_i(x'_i,y'_i)$ for some i.
- The social situation y is efficient given the economic allocation x if there is no other feasible strategy profile s' such that, letting $y'_i = F_i(p,x,s')$, one has $u_i(x_i,y_i) \leq u_i(x'_i,y'_i)$ for all i and $u_i(x_i,y_i) < u_i(x'_i,y'_i)$ for some i.

General conditions for the efficiency of the economic allocation are well-known, and in particular involve the absence of externalities from economic decisions. Obviously, if x_{-i} influences i's utility via $F_i(p,x,s)$, externalities are likely to arise, and therefore the park model and the club model, where the technology $F_i(s)$ prevails, are better suited for economic efficiency.

Efficiency in the social game can be analyzed by distinguishing three cases, which may jointly appear in various dimensions of the outcome vectors. These cases correspond to different technologies introduced in section 2:

1. The joint activity case where the veto or the claim technology prevails is likely to produce efficient equilibria, because at least one individual obtains her preferred option.
2. The case of a public good that is collectively produced, where private optimization tends to lead to underproduction, and therefore is generally plagued with inefficiency.

3. The case of competition for status or power, via social and/or economic strategies, also generally leads to inefficiency because of excessive exertion in the competition.

Clearly, the introduction of a social game next to the economic equilibrium introduces several potential causes of inefficiency: externalities via social outcomes influenced by economic actions, public good effects and expensive competition for status or power. In this section, we further study the channels by which inefficiency may appear. The three restricted variants of the model, introduced in section 2.2, are useful for this analysis.

4.1 The park model

The park model gives the simplest relation between general efficiency and efficiency in the economic sphere and the social sphere. First, it is possible to define efficiency separately for each sphere: An economic allocation x is efficient if there is no other feasible allocation x' such that $f_i(x_i) \leq f_i(x'_i)$ for all i and $f_i(x_i) < f_i(x'_i)$ for some i; a social allocation y is efficient if there is no other feasible allocation y' such that $g_i(y_i) \leq g_i(y'_i)$ for all i and $g_i(y_i) < g_i(y'_i)$ for some i.

Since the Arrow-Debreu equilibrium is separable from the rest of the model, the First Welfare Theorem applies and under the usual assumptions about preferences, the economic equilibrium is efficient. But general efficiency may be hard to obtain.

Proposition 3 *An allocation (x, y) is efficient only if x and y are each separately efficient, but this is not sufficient in general.*

Proof. If either x or y is inefficient, it is possible to find a Pareto-dominating allocation, which will increase u_i for some individual i because v_i is increasing in each of its arguments $f_i(x_i), g_i(y_i)$, and no other individual will be harmed. Therefore joint efficiency of x and y is necessary for general efficiency of (x, y).

The counterexample proving that this is not sufficient is provided below (Example 1). ■

Here is an example illustrating how an allocation (x, y) which is separately efficient in x and in y can nevertheless be grossly inefficient.

Example 1. Consider a society with two individuals. On the economic side, there is only one good, so that there is no possibility of economic trade and every individual consumes her endowment: $x_i = \omega_i$. This is trivially efficient. On the social side, there is only one dimension (y_i is a real number, and s_i, too) and the game is defined by $y_i = \min_{j=1,2} s_j$. One recognizes the veto function introduction in section 2.[3] For instance, in the park the length of the conversation may be determined by the individual who stops first. Or the warmth of the relationship may be determined by the colder individual.

In this game, every individual has a preferred y_i^* which maximizes $g_i(y_i)$, and in the Nash equilibrium can play $s_i = y_i^*$, generating the social outcome $y_1 = y_2 = \min_{i=1,2} y_i^*$. Assuming there is a unique y_i^*

[3] In the formalism introduced in section 2, one would interpret s_i as being actually s_{ij}, i.e., the proposal for joint activity made by i to j, and s_{ii} would be irrelevant.

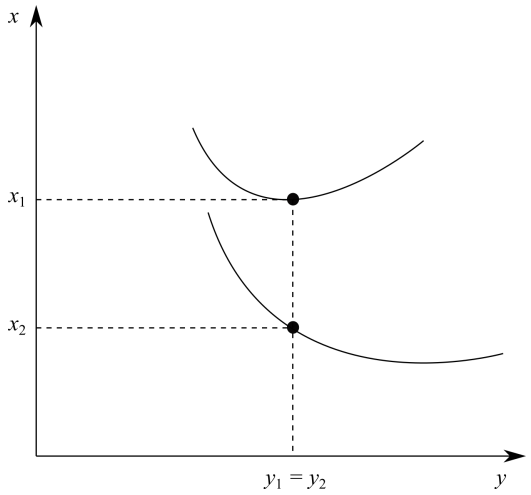

Figure 1: Equilibrium in Example 1

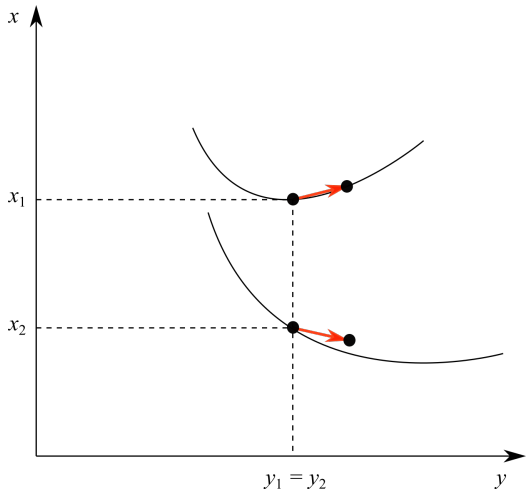

Figure 2: How to improve on the equilibrium in Example 1

for each i, this is efficient because one individual has his best possible outcome, and there is no way to improve the allocation for the other one without harming the former.

Figure 1 illustrates the allocation, with the economic consumption on the vertical axis and the social outcome on the horizontal axis, and the situation of the two individuals is depicted in the figure. Individual 1 is richer than individual 2 and has a lower y_i^*, therefore determines the outcome of the social game. There is no way to improve the situation of both individuals by altering the economic allocation only, or the social strategies only.

Figure 2 shows that, in spite of being separately efficient in x and in y, this equilibrium is not efficient. Combining transfers of consumption from individual 2 to individual 1 with an increase in the "conversation" can make both of them better off.

This improvement looks very much like the introduction of a market for conversation, in which indi-

vidual 2 pays individual 1 for a longer chat. What this example is meant to suggest, however, is that commodifying the social interaction may not be feasible. A market for the timing of conversations may be self-defeating, when conversation is supposed to be a self-motivated activity which cannot be incentivized by external means.[4]

To illustrate this point, imagine the introduction of a market for social intercourse. Individuals may make the difference between genuine interaction y_i and paid interaction z_i, which is a new commodity created by this market. Individual 2 represented in Fig. 2 may have the following preferences (taking x_2, y_2 as the quantities in the initial equilibrium of Fig. 1, and introducing z_2 as the quantity of paid interaction into the vector (x_2, y_2, z_2)), for relevant values of δ, ε:

$$u_2(x_2 - \delta, y_2 + \varepsilon, 0) > u_2(x_2, y_2, 0) > u_2(x_2 - \delta, 0, y_2 + \varepsilon).$$

In other words, this individual would rather stay at the initial equilibrium than give money for a "paid interaction", even if this individual would actually like this move, with the same quantities of money lost and interaction gained, if the interaction was genuine.

Similarly, a direct Coasean bargain between the individuals might not work if it is in terms of material payment, or any form of exchange requiring direct and immediate reciprocation, but it might work if it involves other, more subtle ways. For instance, rather than proposing to pay for more conversation, individual 2 in Fig. 2 might make a gift and this might induce individual 1 to "spontaneously" reciprocate by trying to be nicer and stay longer. To describe these considerations explicitly in the model, one would have to add to the social game the possibility to make transfers of resources and introduce these transfers into the budget constraint on the economic side. This avenue is not explored in this paper, but social solidarity through other means is examined in section 6.

One way to restore efficiency without involving transfers of resources consists in changing the technology of production of social outcomes. In the conversation example, norms of politeness play a key role in determining the actual length of conversation. Example 1 features a norm that gives the power to every individual to stop the interaction at any time ("sorry, I have to go"). The opposite norm would impose to stay whenever the other person still wants to chat. This would correspond to the technology $y_i = \max_{j=1,2} s_j$, also introduced in section 2. Similar inefficiency problems would arise with this technology, now with bored individuals willing to "pay" to stop the conversation earlier. Casual observation suggests that, for conversation, many people have developed a more subtle norm of politeness that involves body language. A sudden change of position or tone is meant to gently suggest that one would like to move on, without abruptly signaling an injunction to stop. Then the conversation slowly winds down, depending on the interest of the participants in the substance. Under this subtle politeness norm, the actual outcome lies between $\min_{j=1,2} s_j$ and $\max_{j=1,2} s_j$. Hopefully, this norm can approximate an efficient outcome, illustrated in Figure 3.[5]

Example 1 is about a joint activity, and similar results can be obtained when the social game is a competition for status or power, even considering special cases in which the competition is not

[4] For general analyses of the limits of markets, see Kanbur (2004), Satz (2012), Sandel (2012).

[5] A similar outcome may also be obtained not through external social norms, but through internalized values, when individuals care about their behavior being conducive to optimal social outcomes. However, this requires discussing whether such internalizing process implies a revision to the measurement of individual well-being.

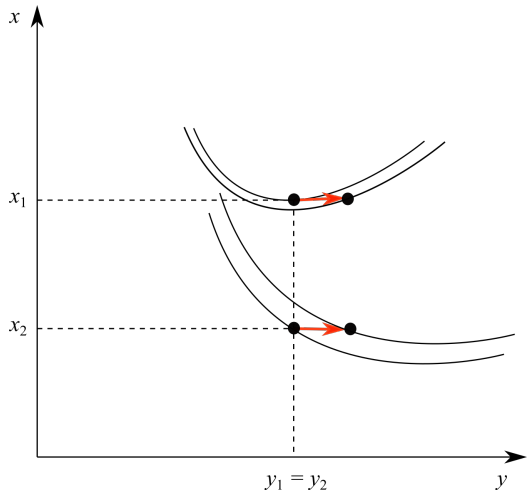

Figure 3: Efficiency through altered social norm

wasteful.

This subsection provides three key lessons. First, for a given technology of the social game, the only way to improve efficiency may involve a combined alteration of the economic distribution and the social strategies. Second, extending the scope of market transactions may not be a universally effective way to address inefficiency problems. Third, social norms may alter the technology of the social game so as to reduce inefficiency.

4.2 The backyard model

In the backyard model, feasibility constraints include interactions between the economy and society. Now it is much harder for the economic equilibrium to be efficient.

Define a new function, which embodies the indirect utility individual i derives from choosing x_i once the social strategy is defined:

$$h_i\left(x_i, x_{-i}, s, p\right) = v_i\left(f_i\left(x_i\right), g_i\left(F_i\left(p, \left(x_i, x_{-i}\right), s\right)\right)\right).$$

As explained in subsection 2.2.2, the individual's behavior in the economic subequilibrium maximizes this indirect utility function by choosing x_i. But the First Welfare Theorem no longer applies, due to the externalities in consumption. When choosing x_i, individual i does not take into account that this will influence the utility $h_j\left(x_j, x_i, x_{-ij}, s, p\right)$ of the other individuals j. These externalities are entirely due to the influence of x on the social game. This feature of the model can describe phenomena like the rat race, when people seek social status through economic prowess.

Another interesting feature of this model is that, even in absence of such externalities, the efficiency of the whole allocation may depend on the distribution of initial resources. For instance, inequalities in the economy may hinder social relations in a way that is harmful to everyone. Redistribution can

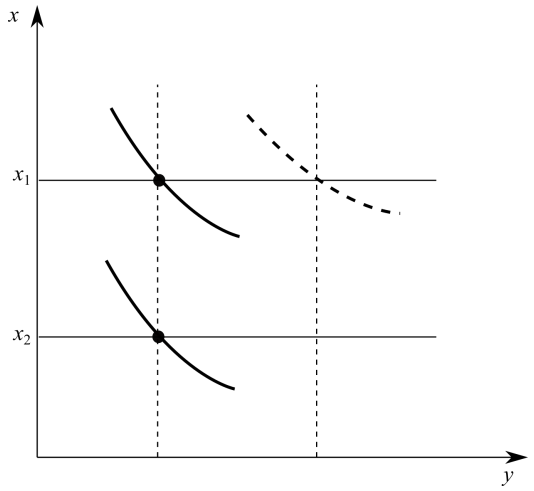

Figure 4: Equilibrium in Example 2

promote a more cohesive community in which the benefits of social bonding compensate the rich for their loss of economic privilege. This can be illustrated as follows.

Example 2. Consider a society with two individuals $i = 1, 2$. As in Example 1, there is only one commodity in the economy and the equilibrium is trivial, with $x_i = \omega_i$ for every i. In this example, the Walras subequilibrium is therefore efficient, even if this is not the case in general in the backyard model.

The individuals like to invite each other for a backyard party. But the host has to pay for the catering, and there is therefore a limited amount of invitations that each of them can extend. Moreover, out of reciprocity they maintain an equal number of parties in either backyard. The economic constraint on hosting parties is represented by the function

$$F_i(x, s) = \min\{x_1, x_2, s_1, s_2\},$$

meaning that individuals can invite less than they can afford if they wish ($s_i < x_i$) but cannot effectively invite more than they can afford.

Figure 4 illustrates the equilibrium in this example. Individual 1 is richer and would like to host more parties (dashed indifference curve), but is limited by individual 2's smaller wealth and the reciprocity norm. The vertical axis represents personal consumption, the horizontal axis the partying, and preferences over personal consumption and parties take the conventional form $u_i(x_i, y_i)$, where u_i is a quasi-concave function. This is separable in x_i and in y_i when x and y are one-dimensional.

Figure 5 shows how a Pareto-improvement is possible with redistribution of resources from individual 1 to individual 2. By giving some resources to individual 2, more parties are possible, and this compensates the loss of resources for individual 1 if the transfer is not excessive.

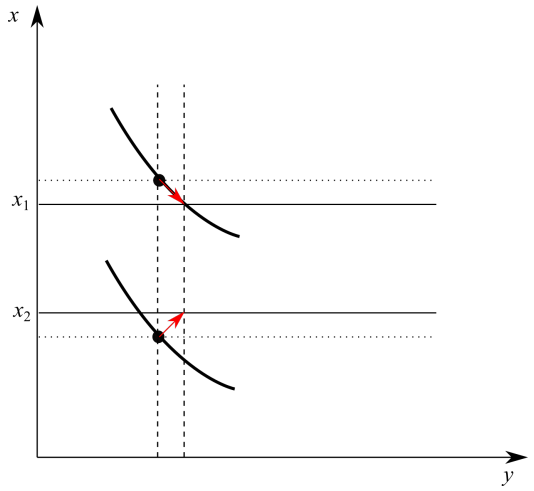

Figure 5: How to improve on the equilibrium in Example 2

4.3 The club model

In the club model, private consumption is relevant to social interactions but does not entail externalities on other people directly. In the absence of externalities originating from economic activities, the economic equilibrium is generally efficient for a given value of the social strategies. But the two spheres, the economic and the social, are interdependent, since s influences the economic equilibrium and x influences the social game, both through the non-separability of preferences over x_i and y_i. This introduces a new source of inefficiency in the general allocation, which can be illustrated as follows.

Example 3. There are two individuals and, this time, two commodities, video streaming services and biking equipment. The social game is similar to the park model, with the same min technology. But a strategy consists in offering time biking together, which directly influences the economic preferences for biking equipment. Preferences are such that there is no interest for buying equipment beyond what is needed for the time spent biking, and conversely, there is no interest in biking more than made possible with the available equipment. And an individual will invite the other to bike according to her own optimal bundle of streaming and biking equipment in her budget. These preferences are represented in Fig. 6 (good 1 is streaming, good 2 is biking equipment) for the case in which individual 2 has a smaller budget and limits the biking activity of both individuals. The preferences represented for individual 1 are the preferences this individual would have if the other's invitation to biking was not constraining. But given the actual constraint, individual 1 stops at a lower level of consumption, equal to individual 2's level because their biking time is identical.

In this example, it is assumed that total consumption can vary according to a CRS technology transforming streaming services into biking equipment and conversely (at the rate represented by the individuals' budgets on the figure). As in the previous examples, there are many equilibria, since any allocation with lower biking activity (and consumption) than at the allocation illustrated in Fig. 6 is an equilibrium. What is different from the previous cases, though, is that the social subgame can be trapped in an inefficient allocation if the economic equilibrium has a low consumption of biking

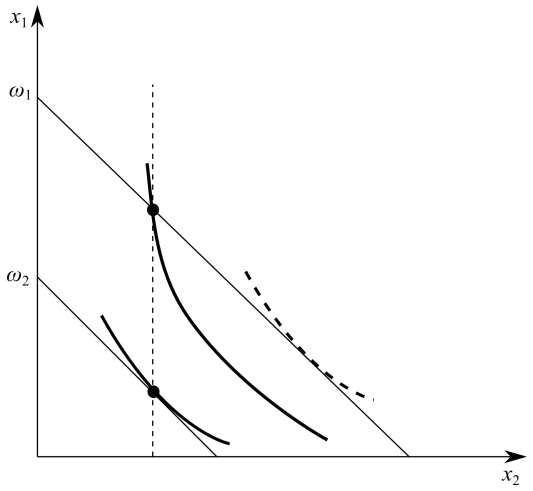

Figure 6: Equilibrium with endogenous economic preferences

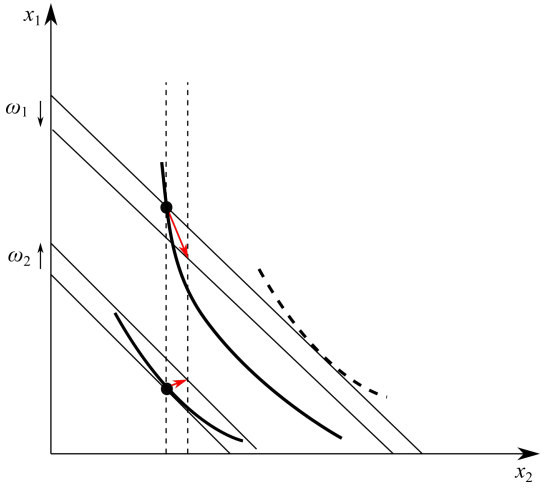

Figure 7: How to improve on the equilibrium in Example 3

equipment, thereby restricting the preferences for biking time. In order to avoid being trapped in such an inferior equilibrium, a joint change in the economy and in the social game is needed. Such a coordination may be hard to achieve with the available institutions, i.e., a market and an open club. It may seem simple, here, for the individuals to talk and coordinate in practicing more biking and buying more biking equipment, but this pattern can emerge in situations in which many different agents are involved and a critical mass is needed to make the move attractive.

As illustrated here, contrary to Example 1 and more like Example 2, it may happen in the club model that redistribution of resources can enhance efficiency. Unlike Example 2, this comes from preference non-separability rather than feasibility constraints, but this is a somewhat minor distinction, since the difference between taste and possibility is sometimes blurry in practice. Figure 7 illustrates how a Pareto improvement can be implemented through redistribution.

4.4 The full model

A variant of Prop. 1 can be formulated for the general model.

Proposition 4 *For the allocation (x, y) to be efficient, it is necessary but not sufficient that the social subequilibrium be efficient (for the given x) and that the economic subequilibrium be efficient (for the given s).*

Proof. Suppose that, for the given x, the social subequilibrium strategy profile s is not efficient. Then it is possible to find another profile s' such that

$$u_i\left(x_i, F_i\left(x, \left(s_i, s_{-i}\right)\right)\right) \leq u_i\left(x_i, F_i\left(x, \left(s'_i, s'_{-i}\right)\right)\right)$$

for all i, with a strict inequality for some i. Let $y'_i = F_i\left(x, \left(s'_i, s'_{-i}\right)\right)$. The allocation (x, y') is thus such that it Pareto-dominates the allocation (x, y).

Suppose that, for the given s, the economic subequilibrium is not efficient. Then it is possible to find another allocation x' such that

$$u_i\left(x_i, F_i\left(x, s\right)\right) \leq u_i\left(x'_i, F_i\left(x', s\right)\right)$$

for all i, with a strict inequality for some i. The allocation (x', y'), where $y'_i = F_i\left(x', s\right)$, is thus such that it Pareto-dominates the allocation (x, y).

The fact that sufficiency does not hold is proven by the examples provided in the previous sections, since these are special cases of this model. ■

4.5 A further variant

Although the Nash equilibrium approach is commonly considered adequate to model prisoner's dilemma situations which can easily occur in social interactions, in some settings social interactions involve greater cooperation and commitment possibilities. Let us go to the opposite extreme and assume that the social game relies on full coordination, so that in effect a social objective is maximized simultaneously by all individuals. Formally, assume that the strategy profile is selected to maximize a social objective $W\left(u_1\left(x_1, y_1\right), ..., u_n\left(x_n, y_n\right)\right)$, taking x as given. This guarantees in particular that the social sphere is efficient, provided the W function is increasing.

Even in this variant, the whole allocation (x, y) can be inefficient for a similar reason as in the previous sections, i.e., due to a lack of management of the trade-offs between x and y. In order for full efficiency to be guaranteed, one would need the social coordination to include the economic sphere, or at least to take account of the economic consequences of social strategies. For a given strategy profile s, one can define the corresponding Walrasian equilibrium as a function $x\left(s\right)$ if there is a unique equilibrium. Then full social coordination would maximize $W\left(u_1\left(x_1(s), F_1\left(x(s), s\right)\right), ..., u_n\left(x_n\left(s\right), F_n\left(x(s), s\right)\right)\right)$ by choosing s.

4.6 Summary

Let us recap the main facts about efficiency in this section, in light of the lessons of the various models:

1. The efficiency of the two spheres (the economic and the social) does not guarantee full efficiency, due to interactions between the spheres even when preferences are separable and social interactions have no economic constraints. The interactions come from the fact that people may be willing to make trade-offs between economic and social outcomes.

2. Even if each sphere is efficient, it may be difficult to achieve full efficiency when the changes in the allocation and the strategies must go together and cannot be implemented jointly with existing institutions. In other words, the equilibrium may not provide them with tools to express their preferences over the trade-offs between economic and social outcomes.

3. In general, when economic activities affect social outcomes, the economic equilibrium is plagued with consumption externalities, and makes the Walras subequilibrium inefficient.

4. The distribution of resources may have efficiency impacts, through the ability of individuals to obtain social outcomes as a function of their economic resources. In particular, economic equality may foster beneficial social cohesion.

5. The interaction may generate multiple equilibria, which are potentially ranked in terms of Pareto-domination. This creates a coordination problem which involves the two spheres simultaneously.

6. Social norms that govern the social game may themselves be an impediment to achieving efficiency. In our examples, reciprocity constraints put limits on the quantity of enjoyable interactions, and this is only one instance of how social conventions can reduce opportunities for beneficial social interactions. Altering the social norms may be a remedy.

Arguably, the most interesting source of inefficiency problems in this model is the lack of coordination across the two spheres. It is worth exploring the reasons why such a lack of coordination may occur. One particular angle on this question is to wonder why commodification of the social interactions, in other words, their incorporation into the economic sphere, would not provide a ready-made solution. One can list at least four reasons why commodification is unlikely to work out. First, the size of many markets for social interactions would be vanishingly small, so that specific prices could hardly form in a competitive way, thereby undermining any hope of efficiency gains in this direction. Second, commodification would undermine the essential nature of many social relations that are based on authentic feelings and deferred reciprocity, if not outright disinterested motivations. Economic theory commonly praises market trades for making every party better off, but trade is a venial type of social intercourse, where every party expects immediate reciprocation and pursues its own interest selfishly. For psychologically normal human beings, there are higher forms of social relations and they involve selfless motivations, or at least deferred, not automatic, reciprocity. In the park example, a friendly chat could not be bought with money and still be a friendly chat with the same enjoyment. Third, another key feature of many social relations is that they involve beliefs and feelings (as when having esteem for someone means holding certain beliefs and feelings about this person) which are inherently

non-contractible and therefore cannot be subject to transactions. There is no way to pay people to believe that one is worthy of high trust, friendship or love. Truly, many economic activities are entangled with the creation of reputation and tacit reciprocity leading to more or less intimate social relations (Zelizer 2005), but the social relations themselves have no explicit price. Fourth, people often have mental accounting habits which prevent them from doing the trade-offs that would be required for efficiency. Some of this mental accounting may come from conventions requiring to keep different social interactions separate lest some of them would be spoiled (e.g., by venial motivations), but there may be mental accounting above and beyond such considerations. For instance, people may keep track of reciprocity in a particular sphere (e.g., small non-monetary favors among neighbors) and ignore possible compensation through other spheres (e.g., contributing to the budget of a local association).

More broadly (i.e., looking beyond commodification), what obstacles prevent a more general integration of the two spheres? The fact that the economic sphere is governed by market rules makes it quite difficult to devise an integration that neither destroys the market (the planning route) nor expands it to cover social interactions (the commodification route). The celebrated effectiveness of the economic sphere, a signature achievement of the modern era (although it was prefigured in earlier periods of history), stems precisely from its relative separation from the rest of social interactions. The only way in which a successful integration that preserves the autonomy of the economic sphere could lead to full efficiency would, in all likelihood, go through something similar to the maximization of the global social objective alluded to at the end of section 3.5. This would require the social sphere to achieve full coordination of strategies among the individuals, a feat that is very far from reality, and a perfect anticipation of the general equilibrium economic consequences of social strategies, another feat which appears at least as farfetched. In conclusion, societies are probably condemned to suffer from inefficiencies which are neither due to market failures nor inefficient social interactions, but come from the lack of coordination across the various spheres of interaction, as illustrated here with the economic and the social sphere. However, tacit coordination may occur through altered social norms that enable individuals to express the intensity of their preferences as it would be measured by their "willingness to pay" (i.e., the trade-offs in their preferences, not their willingness to engage in commodified relations) for the quality of social interactions.

5 Equity

This model enables us to analyze equity in more dimensions than resource equality. Indeed, inequalities in social relations, in terms of status or power, can also be explicitly examined here. In this section, we first examine how economic equity matters, depending on the degree of interaction between the two spheres, and then study how to define equity in a comprehensive way.

5.1 Economic equity

When the economic and the social spheres interact, two things may happen. Economic inequalities may become more important because they may reinforce or foster social inequalities. In addition, they may be harder to curb because they are entrenched in the social structure.

When interactions between the two spheres are low, in contrast, the social inequalities which are not entangled with economic issues will be hard to address via economic policy. In the park model, in particular, interactions between the two spheres are minimal and social inequalities can remain high in spite of perfect economic equality. For instance, if skin color is associated with deep inequalities of status in the park, independently of economic affairs, even full economic equality will fail to tackle these social inequalities.

In examples 1-3, there are no apparent social inequalities because of the type of game defined in terms of joint activity. However, there might be underlying inequities hidden in the situation. For instance, in example 1, individual 1 might be reluctant to chat with individual 2 due to racial animus. The frustration of individual 2 is then a form of social inequality, and will be analyzed in terms of overall socio-economic inequality in the next subsection.

Let us now introduce new examples which jointly include economic and (apparent) social inequalities.

Example 4 (park model). Assume that in the park the main activity now is jogging instead of chatting, and that there is a competition for social status that is ongoing through the speed s_i at which the two individuals are able to jog. Each individual has a range of possible speeds, $S_i = [0, \bar{s}_i]$, and social status is equal to $y_i = s_i + (s_i - s_j)$, where $i, j = 1, 2, i \neq j$.

In the equilibrium, every individual runs at maximum speed if social status is seen as a good thing in their own preferences, and this generates a social inequality. (There may even be some inefficiency in the social game itself, if there is a disutility associated with speed.) In this example, reducing economic inequality between the individuals cannot do much to address the social problem, due to the separation between the economic and the social spheres.

Example 5 (backyard model). Another simple modification of example 1 can introduce social inequalities. Assume that, as in a backyard type of model, it is now possible to associate social outcomes with the economic standing of the partners in social interaction. Suppose that the social outcome has two components: $y_i = (y_{i1}, y_{i2}) = (\min\{s_1, s_2\}, x_j)$, interpreted as "spending $y_{i1} = \min\{s_1, s_2\}$ time of conversation with a person of economic standing $y_{i2} = x_j$." The social outcomes are then unequal insofar as the economic standings of the two individuals are unequal. Obviously, reducing economic inequality then has a direct effect on social inequalities, but whether it matters depends on individual preferences about the second component of y_i. In particular, one can imagine that individual 1's initial lukewarm taste for interactions with individual 2 (for whatever reason) can be assuaged by improving the latter's economic situation. This introduces an interesting channel by which economic equity can alleviate social inequalities. The extent to which this works out depends on contingent facts about preferences. A hard-core racist person may be impervious to the economic standing of people from the other race.

Example 6 (club model). Modify example 3 to introduce a large population of many individuals, with unequal levels of resources but identical preferences. Individuals will spontaneously assemble by wealth group in order to find companions with the same optimal quantity of biking, and thereby obtain their preferred combination of biking and video streaming. In this example, reducing economic inequalities would contribute to homogenizing people's preferences for biking and reduce inequalities not only in economic resources but also in biking enjoyment. For a more realistic version of this

example, replace biking with the degree of exotism, distance, or luxury in vacation destinations, and consider stratification in social groups going to different places and socializing there.

In summary, we have identified in examples 5-6 two channels by which economic equity can have positive spillovers on the social sphere: through better access to social relations (in example 5, this is mediated by people's dislike for interacting with poor people, but it can also more directly come from providing economic resources for social interactions, as in example 2), and through homogenizing preferences. In the full model, these two channels can reinforce each other. Greater access to social relations may homogenize economic preferences and further enhance tighter social interactions.

For instance, reducing economic inequality may foster social relations and reduce inequalities in social status in local communities, and such social relations may induce people to spend less on private consumption and more on local public goods, providing further ways in which more social relations become possible and attractive, thereby increasing social equality. This is illustrated in the following examples.

Example 7 (full model). Individuals have an initial endowment ω_i, which they can spend on video streaming v_i or garden-sharing g_i, under the constraint $v_i + g_i = \omega_i$. Individual labels are ordered by decreasing wealth level: $\omega_1 > ... > \omega_n$. Their social strategy consists in inviting a subset of people to garden-sharing with them: $S_i = 2^{\{1,...,n\}}$. An invitation is accepted when it is reciprocated, i.e., i and j end up gardening together if they invite each other.

Thus, y_i is the subset of people who garden-share with i, associated with the wealth of these individuals. I.e., y_i is a list of pairs (j, ω_j), where each j is garden-sharing with i.[6] Assume that preferences have the following structure:

$$u_i(x_i, y_i) = v_i + n(y_i)\sqrt{g_i},$$

where

$$n(y_i) = \#\{j|(j,\omega_j) \in y_i, \omega_j \geq \omega_i/2\} - \#\{j|(j,\omega_j) \in y_i, \omega_j < \omega_i/2\}.$$

This term embodies a dislike for garden-sharing with people who are less than half as rich as oneself. For any given v_i, g_i, individual i will invite all other individuals j such that $\omega_j \geq \omega_i/2$, and the invitation will be accepted by all those such that $\omega_i \geq \omega_j/2$. At the end of the day, i ends up garden-sharing with those whose wealth is in the range $[\omega_i/2, 2\omega_i]$. Then, the optimal amount of spending on garden-sharing is $n(y_i)^2/4$, for $n(y_i) = \#\{j|\omega_i/2 \leq \omega_j \leq 2\omega_i\}$. The indirect utility is equal to

$$\omega_i - n(y_i)^2/4 + n(y_i)^2/2 = \omega_i + n(y_i)^2/4$$

Under economic equality, one has $n(y_i) = n$ for all i, and this induces maximal spending on g. On the contrary, economic inequality may seriously reduce social interactions, and thus spending on g as well. Suppose for instance that there are two homogeneous groups of equal size $n/2$, such that the richer group is more than twice as rich as the other group. Then, $n(y_i) = n/2$ for all i, and spending on g is only $1/4$ of what it is under full equality.

[6] This model is a special case of the model introduced in section 2.1, but here we use different notations that make for a simpler presentation.

In this two-tier society, can the rich benefit from an equalizing redistribution? This can happen if their wealth ω and the lower group's wealth ω' satisfy

$$\frac{\omega + \omega'}{2} + n^2/4 > \omega + n^2/16,$$

i.e., $\omega < \omega' + 3n^2/8$. Clearly, a large society could make this happen.

The possibility for the rich to benefit from redistribution policies that enhance opportunities for social interactions is a very important phenomenon, if it holds true. But opposite examples are worth considering, in which the rich benefit from enhanced status and invest in preserving this status, whereas redistribution diminishes their status and their investment jointly. This is illustrated in the following example.

Example 8 (full model). Individuals have an initial endowment ω_i, which they can spend on necessities v_i or status goods g_i, under the constraint $v_i + g_i = \omega_i$. Social status is obtained by the expenditure on status goods relative to the average expenditure in such goods in society:

$$y_i = g_i/\bar{g},$$

where $\bar{g}$ is the average over all individuals ($\bar{g}$ is taken as given by each individual). This example does not have social strategies different from expenditures. Utility is defined as

$$u_i(x_i, y_i) = \sqrt{v_i} + \sqrt{y_i g_i},$$

embodying a reinforcement effect whereby higher status induces a greater taste for status goods. Individuals then spend $g_i = \omega_i - \bar{g}/4$, which is increasing in the endowment, giving greater status to the richer individuals. Redistribution then directly redistributes resources and social status at the same time, and reduces inequalities in luxury spending, too. In this example, redistribution does not curb the social competition for status, because total spending in status goods is always 80 percent of the total wealth of the economy (the efficient level of spending in status goods would be 50 percent in an economy with equal endowments).

After these examples, it is worth seeking general conditions under which, in the general (i.e., full) model, inequalities in resources translate into lower social inclusion or social equality. Consider the case in which x_i and y_i are each associated with partial orders, both denoted $\succeq$ since no confusion is possible, which are identical across individuals and serve to compare individuals and track the morally relevant inequalities. For instance, x may be ordered by market value (for a set of possible market prices), and y may be ordered in terms of number of contacts (for social inclusion) and/or status (for social inequality). The orderings are partial and each can be thought of as the intersection of special orderings for particular dimensions in the space of resources and in the space of social outcomes, respectively.

In the next proposition, two conditions jointly induce a correlation between economic inequalities and social inequalities. The first condition stipulates that economic inequalities always offer an advantaged individual the opportunity to outperform a disadvantaged individual in social outcomes. The second

condition requires social outcomes to depend more on the individual's own strategy than on others', so that social inequalities can be reversed by an individual only when this individual unambiguously harms her own social outcome. To simplify notations, the price vector p is omitted, the economic allocation being fixed.

Proposition 5 *Let the partial orders $\succeq$ be given and assume that for all i and all x_i, $y_i \succ y_i'$ implies $u_i(x_i, y_i) > u_i(x_i, y_i')$. Consider i, j such that:*
(i) whenever $x_i \succ x_j$, for all $s_{-i} \in S_{-i}$ there is $s_i \in S_i$ for which $F_i(x,s) \succ F_j(x,s)$;
(ii) whenever $F_i(x,s) \succ F_j(x,s)$, if s_i' is such that $F_i(x, s_i', s_{-i}) \not\prec F_i(x,s)$, then $F_i(x, s_i', s_{-i}) \succ F_j(x, s_i', s_{-i})$.
Then, $x_i \succ x_j$ implies $F_i(x,s) \succ F_j(x,s)$ at any equilibrium. Moreover, each condition is necessary in the sense that the result no longer holds if condition is dropped from the statement.

Proof. Let i, j be such that $x_i \succ x_j$.

Consider equilibrium strategies s^*. By condition (i), there is s_i such that $F_i\left(x, s_i, s_{-i}^*\right) \succ F_j\left(x, s_i, s_{-i}^*\right)$.

In the Nash subequilibrium, $F_i\left(x, s_i^*, s_{-i}^*\right) \prec F_i\left(x, s_i, s_{-i}^*\right)$ is impossible because of the monotonicity assumption about preferences.

Therefore, condition (ii) applies and entails that $F_i\left(x, s_i^*, s_{-i}^*\right) \succ F_j\left(x, s_i^*, s_{-i}^*\right)$.

Necessity of (i): Consider a social game such that y is always equal over all agents. It satisfies (ii) trivially but not the conclusion.

Necessity of (ii): Consider a social game in which x and s_j have no influence on the social outcomes, and every $i \neq j$ has a strategy s_0 (e.g., slander) which destroys y_j (so that condition (i) is satisfied), but such that the only way for every i to maximize her own status is to choose strategy s_1 which supports putting j at the pinnacle (in violation of condition (ii)). The F_j function gives j the top status if everyone else chooses s_1. One of the equilibria then has j with the greatest status (independently of x). This game satisfies (i) but not the conclusion. ■

What is noteworthy about this result is how demanding the conditions are, suggesting that reversals between economic and social outcomes are not difficult to obtain. Condition (i) is missing when, for instance, a highly praiseworthy act by an economically disadvantaged person can lift her social status tremendously, so that there is little more advantaged people can do to outperform her in the social subgame. Condition (ii) is missing when, for instance, an equilibrium partnership between an investor and a genial inventor ends up putting the former on a pedestal, even though, separately, the inventor would do better than the business person in the social subgame.

This result should be qualified in two important ways. First, even if perfect correlation between economic and social rankings is rather hard to obtain, for practical purposes a large correlation, as generally observed, is what really matters, and this proposition says nothing about how large the correlation is likely to be when either of the conditions fails. Second, the conditions (i) and (ii) are necessary only in the sense that dropping either of them nullifies the result, not in the sense they must hold when perfect correlation is achieved.

5.2 Socio-economic equity

The main message of this paper is that focusing on the economic sphere is insufficient. This holds not only for efficiency but for equity as well, and here is an additional argument against a narrow focus on economic equity. To simplify, consider the case in which x_i is one-dimensional, and is interpreted as income (or wealth). It might appear reasonable to recommend reducing economic inequalities among individuals sharing the same social outcome, i.e., to endorse the following transfer principle:

Economic equity For all allocations $(x,y),(x',y')$ and all i,j, such that $y_i = y_j = y'_i = y'_j$ and $x_i > x_j$, if $(x,y),(x',y')$ differ only by a regressive transfer $x'_i = x_i + \delta$, $x'_j = x_j - \delta$, with $\delta > 0$, then (x,y) is better than (x',y').

But as is well known in the theory of fair social orderings (Fleurbaey and Maniquet 2011), this kind of principle runs afoul of the Pareto principle when individuals may have different preferences about trading off x_i against y_i. This is because situations with equally low $y_i = y_j$ and $x_i > x_j$ may be Pareto indifferent to situations with equally high $y_i = y_j$ and $x_i < x_j$. According to the Economic equity principle, the former situation could be improved by a transfer from i to j,whereas the latter could be improved by a transfer from j to i. Since individuals are Pareto indifferent, respect for the Pareto principle should treat these two situations as equivalent, hence a clash.

The current model is similar to contexts in which individual preferences bear on market commodities and non-market aspects of quality of life. For this type of context, one can follow Fleurbaey and Blanchet (2013) and restrict the application of the Economic equity principle to situations in which the non-market aspect of life is at its best for every individual. This restriction eliminates the tension with the Pareto principle. Let us say that y_i is ideal for i given x_i when y_i maximizes $u_i(x_i, y_i)$ among the possible values of y_i.

Economic equity under ideal social outcomes For all allocations $(x,y),(x',y')$ and all i,j, such that y_i, y_j, y'_i, y'_j are ideal for i,j given x_i, x_j, x'_i, x'_j respectively,

and $x_i > x_j$, if $(x,y),(x',y')$ differ only by a regressive transfer $x'_i = x_i + \delta$, $x'_j = x_j - \delta$, with $\delta > 0$, then (x,y) is better than (x',y').

Although the tension with Pareto is alleviated, combining this equity principle with the Pareto principle seriously narrows down the set of acceptable approaches. Let us first state the Pareto principle and introduce the notion of equivalent income.

Strong Pareto For all allocations $(x,y),(x',y')$ such that $u_i(x_i,y_i) \geq u_i(x'_i,y'_i)$ for all i, (x,y) is at least as good as (x',y'); and if the inequality is strict for at least one i, then (x,y) is better than (x',y').

The equivalent income is a utility representation defined as the minimal x_i that is needed to bring i to the current utility level, when full adjustment of social outcomes is possible:

$$\min\left\{z | \max_w u_i(z,w) \geq u_i(x_i,y_i)\right\}.$$

It is illustrated on Figure 5. The equivalent income obtains at a situation in which y_i is ideal given this level of income.

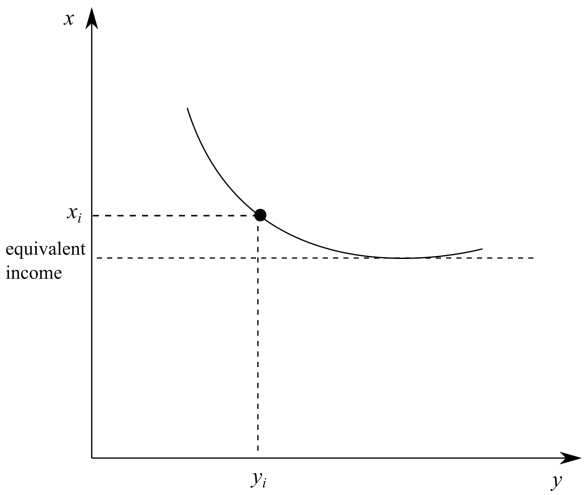

Figure 8: Equivalent income

Finally, let us say that an ordering over n-vectors of real numbers is monotonic increasing if an increase in a component moves the vector up the ordering, and inequality averse if a regressive transfer between two components moves the vector down. One then obtains a proposition similar to Willig's (1981) approach.

Proposition 6 *If an ordering of allocations (x, y) satisfies Economic equity under ideal social outcomes and Strong Pareto, the ordering is entirely defined by a monotonic increasing and inequality averse ordering on the distribution of equivalent incomes, for the allocations for which equivalent incomes are well defined.*

Proof. Consider an allocation for which the individual equivalent incomes are well defined. By the Pareto principle, one can move every individual to the equivalent income level and the associated ideal social outcome, and this yields an allocation that is as good as the initial allocation (Note: such a move may not be feasible, but the ordering of allocations is not limited to feasible allocations).

The ordering of allocations therefore has to coincide with the ordering of these "equivalent" allocations. I.e., (x, y) is at least as good as (x', y') if and only if for the equivalent allocations, (x^*, y^*) is at least as good as (x'^*, y'^*), where x_i^* is the equivalent income of (x_i, y_i) for i, associated with the ideal social outcome y_i^*, and likewise for the primed allocation.

Now, the Pareto condition requires this ordering to be monotonic increasing and the equity condition requires it to be inequality averse. ■

The above proposition is silent for allocations for which the equivalent income is not defined for some individuals. This is likely to be rare in practice, as income is a necessary good, implying that in Figure 5, indifference curves near the horizontal axis are likely to be close to horizontal, meaning that economic subsistence becomes a priority over social outcomes. But this may be debated, as poor people do complain that the worst of their condition is not so much deprivation as the lack of respect

and dignity in their social interactions with the rest of society. This might mean that certain social deprivations may be worse than falling below the subsistence level on the economic front.

The distribution of equivalent incomes can be used for valuable analysis of inequalities. In particular, a decomposition of the respective contributions of economic and of social inequalities to the overall socio-economic inequality can be performed, and can provide an estimate of how much can be gained by policies which tackle economic inequalities only.

Let us provide a few illustrations of this approach. In example 1, the economic inequality between individuals 1 and 2 is accompanied with an additional inequality due to the frustration of individual 2 not having his full lot of chat. This can be assessed by looking at the inequality in equivalent incomes, which is, as can be seen from Fig. 1, greater than inequality in resources, because individual 2's equivalent income is below her income, whereas individual 1's equivalent income is equal to her income.

In example 1, efficiency is achieved when the willingness to accept of the less chatty individual equals the willingness to accept of the chattier individual. If, as in example 1, the former is richer, this individual is likely to be choosier, so that the efficient allocation will have a lower gap between the actual and the preferred quantity of interaction for this individual (this is illustrated in Fig. 3). In a nutshell, efficiency would justify that the rich could be less polite than the others. It was suggested in section 3 that social norms of politeness tend to reduce inefficiency by letting individuals subtly express their wishes. But norms of politeness do not refer to willingness to pay and are therefore likely to produce more egalitarian results. More precisely, they are likely to reduce the inequality in equivalent incomes, compared to what the efficient allocation would be in absence of transfers of resources.

In example 2, both individuals suffer from the impossibility to have as many parties as they would wish, so that their equivalent incomes are lower than their ordinary incomes. Moreover, if partying is a normal good, the gap between income and equivalent income is larger for the richer person (who is further constrained by the lack of resources of her neighbor).

The case of Faustian socio-economic bargains provides another illustration of the approach. The threat of economic duress, which affects a large share of the population since most people cannot survive without selling something, can lead the most disadvantaged among them to accept sacrifices on their social status or their autonomy in order to get by. Economists have long been interested in the analysis of what exactly is exchanged in the labor market. Adam Smith, in the *Theory of Moral Sentiments*, offered a (not-so-well-known) invisible hand perspective on trades that transfer money from rich employers to poor employees: "They are led by an invisible hand to make nearly the same distribution of the necessaries of life, which would have been made, had the earth been divided into equal portions among all its inhabitants; and thus, without intending it, without knowing it, advance the interest of the society, and afford means to the multiplication of the species." Karl Marx believed he uncovered the secret of profit in the idea that employers only paid the value of the labor force but could then extract the full value of labor. Neoclassical economists emphasized the fact that everyone benefits from the trade, compared with their initial endowment. Labor economists noted that leisure has a value for people and offered various ways to account for the disutility of lost leisure, which include the equivalent income proposed here and other variants of the money-metric approach (Preston and

Walker 1999).

What is missing from all of this is an explicit account of social status and autonomy. Yet this was a rather prominent concern for the Founding Fathers of the US Republic. "Although most Americans in 1776 believed that not everyone in a republic had to have the same amount of property ... all took for granted, that a society could not long remain republican if a tiny minority controlled most of the wealth and the bulk of the population remained dependent servants or poor landless laborers." (Gordon S. Wood, *Empire of Liberty: A History of the Early Republic, 1789–1815*, cited in Blasi et al. 2013, p. 7). More recent surveys of job satisfaction point to the importance of autonomy for many employees (Freeman and Rogers 2006), although they generally ignore comparisons of status with independent workers and employers, since employee status has become the norm rather than the exception. Incorporating the loss of independence and autonomy into the computation of the equivalent income of employees should capture these aspects of their situation, at least to the extent that their preferences have not come to accept their inferior position as a matter of indifference.

This last remark raises the important issue of whether questionable social conventions may make a preference-based measure like equivalent income problematic for analyzing inequalities. It is of course possible to "correct" preferences to eliminate biases (as is commonly done to treat biases, such as present bias, in behavioral economics), before they are applied to the measurement of equivalent income. If some individuals come to like their servitude, social analysis can still measure how their situation fares according to more acceptable preferences.

6 Society as a remedy to market failures and economic inequalities

The thrust of this paper, so far, has been to show that a narrow focus on the economic sphere misses important dimensions of efficiency and equity. To this effect, the Arrow-Debreu model was a good starting point, as it embodies the most effective type of economic coordination. It is, however, far from realistic and obviously, the prospects for achieving efficiency and equity are dimmer than in the previous sections when market failures plague the economic sphere. The point of this section is to highlight that, as announced in the introduction, there is another way in which social facts are crucial to understand the larger picture about the economy. Namely, the social game may alleviate some of the problems generated from within the economy.

This idea is far from new, of course, and many authors have emphasized how economic outcomes crucially depended on social mechanisms. Weber (2003) described how various branches of Protestantism steered the faithful toward economic activities to unequal degrees. Coase (1937) and Williamson (1985) saw market transaction costs, due to informational problems and returns to scale, as a main reason for social associations (business firms) taking over certain production activities. Arrow (1973, 1974) has been a prolific author on the interplay between ethics, trust and economic efficiency (Cato and Lutz 2018). The huge literature on economic and institutional development and on social capital (see, e.g., Keefer and Knack 2005 for a review) has developed such insights in multiple directions and provided

crucial empirical evidence on variations across cultures, countries and social groups. The purpose of this section is not to rehash well-known ideas, but to show that the present framework can be easily expanded to incorporate these notions.

Let us first introduce a more general model that allows for market failures in the economic sphere. To keep it simple, we retain the assumption of perfect competition here (it is dropped in the next section), and only introduce externalities, which can also capture public good effects. It would be great to introduce non-contractible actions as well (representing dedication, due diligence, quality service...) but these require tracking bilateral trades in which such actions are impactful, and this extension of the model is left for the next section.[7]

There are n individuals and m firms. An individual i has an endowment ω_i of commodities and time, and can receive dividends from firms, and this income can be used to purchase and consume bundle x_i (including leisure). The individual's utility depends on the others individuals' and the firms' behavior through external effects. A firm j operates a production unit, picking a production plan q_j to maximize profit. But in its choice among the production plans on the frontier, the firm may be influenced by social reputation effects, or a sense of social responsibility.

Formally, individual i selects his economic and social behavior by solving the following program:

$$\max_{x_i \in X_i, s_i \in S_i} u_i\left(x_i, z_i, y_i\right)$$

$$\text{such that} \begin{cases} px_i \leq p\omega_i + \sum_j \theta_{ij} pq_j, \\ z_i = G_i\left(x, q\right) \\ y_i = F_i\left(p, \left(x_i, x_{-i}\right), \left(s_i, s_{-i}\right)\right). \end{cases}$$

In these notations, q is the vector of all firms' production plans q_j, pq_j measures firm j's profit and θ_{ij} is the share of j's profit received by individual i (one has $\sum_i \theta_{ij} = 1$ for all j). This formalism distinguishes two channels of consumption externalities. Economic externalities triggered by x may come from physical effects (the beauty of the neighbor's repainted house, the noise of his music, the smoke of a factory), and this appears through the presence of $z_i = G_i\left(x, q\right)$ as an argument of utility, as well as social effects (the envy of the neighbor's standing, the sympathy for a struggling subgroup), which operate through y_i.

Firm j selects its production plan from a set $Q_j\left(x, q_{-j}, s\right)$ which also, potentially, depends on external effects from individual consumption and other firms' production, as well as social attitudes, which may influence productivity. Moreover, the firm may include a social responsibility term in its optimization, which represents how its production plan affects its reputation or its sense of serving society responsibly:

$$\max_{q_j \in Q_j(x, q_{-j}, s), pq_j \geq 0} pq_j + y_j\left(q_j, s\right).$$

[7] There is a sense in which, due to the absence of markets covering them, non-contractible features of transactions can also be described as producing externalities. Even if they are often very local and targeted, since they are associated with particular transactions, their pervasiveness can sometimes resemble a general externality problem (e.g., in the case of mediocre quality dominating production, or the general features of "phishing equilibria" described by Akerlof and Shiller 2015).

Note that the firm needs to be viable and therefore respect the constraint $pq_j \geq 0$, but it may not fully maximize its profit if this would clash with social responsibility. The pressure of social responsibility may depend on social strategies.

A Walras-Nash equilibrium of this model is a pair (x, y) such that, for a price vector p and a strategy profile s:

WN-i) every individual i and every firm j solve the above programs;

WN-ii) the markets clear: $\sum_i x_i = \sum_i \omega_i + \sum_j q_j$.

To recap the multiple channels of externalities in this model, one can list the following:

- By choosing x_i, individual i may affect:
  - other individuals:
    - through externalities or public good effects in their utilities;
    - through social effects (e.g., social competition);
  - productive firms through technological effects (e.g., congestion).
- By choosing s_i, individual i may affect:
  - other individuals through social effects (possibly linked to the economic allocation);
  - productive firms:
    - through technological effects (e.g., work ethic);
    - through social norms on corporate responsibility.
- By choosing q_j, firm j may affect:
  - individuals:
    - through dividend distribution;
    - through externalities and public good effects in their utilities;
  - other firms through technological effects.

This long list shows that both economic and social strategies can impact other agents. There is still one subtle asymmetry in the fact that while economic choices can have social effects in multiple ways (the neighbor's choice of a car can alter one's social prestige), social strategies cannot directly alter the economic allocation for individuals (no social gesture by a neighbor can alter the vehicle in one's own garage once it is acquired) and can only do so through social effects. However, they can alter productivity in firms, reflecting the fact that production is largely a social activity. And it is also interesting to depict social effects impacting corporate behavior through norms of responsibility.

Equipped with this general framework, in this section we make three points which, we believe, deserve special emphasis. The first point is that the inefficiency generated by market failures may be alleviated by social norms and moral values. In the following example, social norms (or moral values) may help alleviate an externality problem coming from the consumption of polluting goods.

Example 9. Assume that the consumption of a good creates a negative externality on everyone. As in Ex. 8, individuals have an initial endowment ω_i, which they can spend on two categories of goods, namely, clean goods v_i or polluting goods g_i, under the constraint $v_i + g_i = \omega_i$. Individual utility is

$$u_i(x_i, y_i) = \sqrt{v_i g_i} - \sum_i g_i + y_i,$$

where the effect of pollution depends on total pollution $\sum_i g_i$, and the social outcome is a standing variable $y_i = -\alpha_i g_i$ that depends on the social shame, or the guilt internalized by i, for polluting. When y_i measures (the opposite of) social shame, there is a part of the social game that involves other people chastising i for polluting, and we leave this part of the game implicit here, which can be justified if the blaming behavior by others is automatic (and has no noticeable well-being effect on them) and is correctly expected by i (for whom it has a tangible well-being impact). As is clear, it is actually enough that i fears the others' blame, there is no need for this blame to be explicitly conveyed to i by the other people. This is why internalized social norms and endorsed moral values play a similar role in guiding behavior, even if the underlying mental mechanisms are somewhat different.

Individual i's demand for g is then equal to

$$g_i = \frac{\omega_i}{2}\left(1 - \sqrt{1 - \frac{1}{2 + \alpha_i(2 + \alpha_i)}}\right),$$

which is decreasing in α_i. Obviously, it is possible for the social mechanism working through α_i to be insufficient or excessive. It is not clear how a socially optimal norm could occur in this setting. But at least, starting from the laissez-faire in which $\alpha_i = 0$, the social norms operate in the same way as a Pigou tax in individual optimization and thus reduce pollution.

In similar fashion as for a Pigou tax, one should not over-interpret the meaning of the negative sign of the term $-\alpha_i g_i$. One could indeed worry that social norms reduce the pollution but also generate negative mood (guilt or shame) which could nullify the positive effect on well-being of reduced pollution. There are two ways in which this negative effect of "moralizing" behavior can be avoided. First, the final level of well-being can actually benefit from virtuous behavior, and moralizing can occur through positive reinforcement. If $g_i^* = \omega_i/2$ is taken as the bad reference set by the selfish (and naive) behavior of the individual who does not care at all about the pollution impact (including on oneself), one could have a moral term equal to $+\alpha_i(g_i^* - g_i)$, generating the same behavior as above but with a positive net effect of the moral term. In practice, it is often the case that following the crowd makes one's behavior "normal," and therefore does not lead to particular praise or blame and does not trigger any strong feeling. Shame is for those who deviate in the bad direction, and high praise is reserved for supererogatory virtue.

Another interesting issue raised by this example is that internalizing may be unequal among individuals, either because they are not equally virtuous, or because they genuinely disagree about the optimal allocation (e.g., transfers may not be possible and, in this light, some may want to exempt poor people from the abatement effort, while others may disagree). Social norms, therefore, generate issues not just of level but also of coordination about the underlying social objective. This topic is studied in a

companion paper.

A complementary example deals with firms, where the influence of social attitudes can affect not only responsible behavior but also productivity.

Example 10. There are a large number of identical firms. They produce food f either from a green good g or a brown good b. The latter generates pollution when it is used. The technology is linear: $q_{jf} + \gamma (q_{jg} + q_{jb}) = 0$, where $q_{jf} > 0 \geq q_{jg}, q_{jb}$. There are social norms in society, and individuals play a conformism game, in which they are led to share the prevailing ethos. The equilibrium of the conformism game is essentially indeterminate, since any uniformly adopted norm is a Nash equilibrium of the social subgame (no one dares to challenge the prevailing norms). Norms bear on two aspects. First, they determine the work ethic of people (employees and managers), which influences the productivity at work γ. Second, they determine the repulsion for the brown good and the induced pollution. With a very high repulsion, the firm bears a social responsibility virtual cost equal to q_{jb}. With no repulsion, there is no such cost.

Ignoring the utility coming from adopting the prevailing norms in the social game, individual utility is $3\left(\sqrt[3]{x_{if}x_{ig}x_{ib}} + \theta q_b\right)$, where q_b is the total pollution. Individuals have endowments ω_i in goods g, b, and their budget constraint is $x_{if} + p_g x_{ig} + p_b x_{ib} = p_g \omega_{ig} + p_b \omega_{ib}$. There is no pollution from consumption use of good b (e.g., use the resource ground for recreational activities instead of extraction).

In this example, several equilibria can occur, and here a brief description. (Variables without an agent's label depict total quantities.)

- no moral aversion to pollution: $p_g = p_b = \gamma$, total food production is $q_f = \frac{\gamma}{3}(\omega_g + \omega_b)$, and total pollution is $-q_b = \frac{1}{3}\omega_g + \frac{2}{3}\omega_b$. Individual i's utility is then $\gamma^{1/3}(\omega_{ig} + \omega_{ib}) - \theta(\omega_g + 2\omega_b)$.
- high aversion to pollution (i.e., socially responsible objective for firms is $q_{jf} + p_g q_{jg} + p_b q_{jb} + q_{jb}$, where the last term captures the virtual social cost):
  - low work ethic ($\gamma < 1$): firms do not use good b (if they used it, its price would be $\gamma - 1$), there is no pollution. Prices are $p_g = \gamma$, $p_b = \frac{\gamma}{2}\frac{\omega_g}{\omega_b}$. Total production is $q_f = \frac{\gamma}{2}\omega_g$. Individual i's utility is then $\left(2\gamma\frac{\omega_b}{\omega_g}\right)^{1/3}\left(\omega_{ig} + \frac{\omega_g}{2\omega_b}\omega_{ib}\right)$.
  - high work ethic ($\gamma > 1$), high brown endowment ($\omega_b > \frac{\gamma}{2(\gamma-1)}\omega_g$): $p_g = \gamma, p_b = \gamma - 1$, implying total production equal to $q_f = \frac{\gamma}{3}(\omega_g + \omega_b) - \frac{\omega_b}{3}$, and total pollution is $-q_b = \frac{2}{3}\omega_b - \frac{\gamma}{3(\gamma-1)}\omega_g$. Individual i's utility is then $\frac{1}{(\gamma(\gamma-1))^{1/3}}(\gamma\omega_{ig} + (\gamma-1)\omega_{ib}) - \theta\left(2\omega_b - \frac{\gamma}{(\gamma-1)}\omega_g\right)$.
  - high work ethic ($\gamma > 1$), low brown endowment ($\omega_b \leq \frac{\gamma}{2(\gamma-1)}\omega_g$): same equilibrium as for low work ethics, but with greater clean production.

This example illustrates that social norms that enhance productivity may be a double-edged sword, if greater productivity encourages polluting activities. For the fixed aversion to pollution in the example, when $\omega_b > \frac{\omega_g}{2}$, there is always a level of work ethic that is high enough to induce pollution, and can possibly decrease the utility of many individuals. Thus, in this example, promoting high work ethic should generally go hand in hand with promoting high concern for pollution.

The second point we want to make in this section is that social solidarity may serve to reduce economic inequalities. This may come through sheer concern for economic inequality, or through a broader concern for the distribution of well-being, or a less altruistic desire for social bonding when inequalities erect barriers against social relations.

Recall Ex. 2, in which inequality and norms of reciprocity may prevent social interactions from happening as often as the individuals would wish. One obvious solution operating through the social game is to alter the social norms and diminish the norms of reciprocity. This is often observed when someone having a more spacious place offers to host more events, taking the excuse of practical convenience to relieve others of their guilt for failing to reciprocate. Here is a variant of Ex. 2 illustrating the idea that social contributions to alleviating inequalities can come through other channels than direct resource redistribution and with the help of social feelings of solidarity.

Example 11. There is a private good and a public good in the economy. Individuals start with an endowment in the private good and can contribute some of it to the public good. In addition, they can devote social attention to targeted other people, in the form of inclusive gestures (direct transfers of resources are excluded for simplicity). Individual utility is

$$u_i = x_i + \ln \sum_j g_j + \sum_{j \neq i} \left(s_{ji} - \frac{1}{2} s_{ij}^2 \right) + \theta \min_j u_j,$$

where x_i is private consumption, g_j the contribution by j to the public good (which everyone enjoys equally), and s_{ji} the inclusive gesture from j to i (which benefit the receiver and cost the donor). The last term is a special concern for the worst-off in society. Observe that, as quasi-linear utility is a money-metric measure, this utility function is consistent with the equivalent income approach proposed in the previous section (up to an additive constant).

When $\theta = 0$ (no concern for the worst-off), the equilibrium has a public good equal to 1 (with multiple equilibria, the distribution of contributions being indeterminate), and there are no social gestures toward others.

When $\theta > 0$, in contrast, there is a group of m worst-off people, and the production of public good is then increased to $1 + \theta$, while all worst-off people receive $s_{ij} = \theta/m$ from each of the others.

The main point about this example is that social support can come in multiple forms, without direct resource transfers and through public goods as well as greater social inclusion.

The third and final point of this section is that social remedies may have limitations due to the scope of social interactions. Consider Ex. 9, and imagine that most of the pollution is affecting far away populations to which the social group under consideration feels no particular affinity. In that case, it is possible that social norms would not evolve to discipline polluting behavior, because the victims would have no way to influence how this behavior is regarded. An example of this type of behavior, although it involves formal policy, is provided by norms on chemicals that are laxer for exports than for domestic use in many rich countries. Likewise, consider Ex. 11 and imagine that this inequality and lack of interaction is so entrenched that the two parties live separate lives and do not feel a strong urge to alter this situation. The rich population may have a low coefficient θ, because it does not care

much about the disadvantaged population's welfare, but also because it does not put high value on interactions with that population (and might have a low valuation for s_{ji} received from someone from that group).

As a matter of fact, markets and social interactions sometimes compete to address certain economic issues. The earlier reference to Adam Smith's positive view of the labor market (section 5.2) is relevant here. The following example illustrates how the market can not only eliminate inefficiencies but also reduce poverty, surpassing social mechanisms.

Example 12. There are two equally sized groups of individuals with unequal wealth. They all have the same endowment in time (good 1), but the rich also have an endowment in good 2 (a generic staple), whereas the poor have no such endowment. As a slight alteration of the model of this section, direct transfers are allowed. Two cases are distinguished. In the first case, there is no market and the poor can only survive by begging the rich, or threatening them. Individual constraints on choice are: a resource constraint $x_{i2} = \omega_i + r_i - d_i$, where r_i is the amount received and d_i the amount donated; and a time constraint $x_{i1} = 1 - s_i$, where s is the time spent begging for good 2.

In the second case, there is a market where service time can buy good 2, so that the two constraints become

$$x_{i1} + px_{i2} = 1 - s_i + p(\omega_i + r_i - d_i)$$

$$x_{i1} = 1 - s_i - l_i$$

where p is the relative price of good 2, and l is the net amount of time sold (it is negative is i is a net buyer of service time).

Individual utilities are equal to

$$\ln x_{i2} - \frac{1}{2}(2 - x_{i1})^2 - 2w_i^2 x_{i2}^2,$$

where x_{i2} the consumption of good 2, and w_i the pressure of beggars (i.e., the sum of the s_j targeted at i). This functional form includes a benefit from services bought (if any), a cost of lost leisure (also capturing the benefit of hiring labor services), and the high inconvenience of being subject to pressure, which is reduced as donations are made and exhaust the endowment. Altruistic solidarity is not introduced here for simplicity. Even if the rich were moderately altruists, begging would ultimately push them to the point where donating has a net disutility and is done only for the sake of alleviating the pressure of beggars.

To simplify, we assume that there is an equal number of identical rich individuals and of identical poor individuals, and they are paired. Thus, begging time exerted by a poor person is received as pressure by one rich person. When a market is opened, individuals are assumed to behave as price takers. Moreover, let us assume that $\omega_i = 1$ for every rich individual.

First case: no labor market. Consider individual i who is rich and relents under pressure. This individual's utility is then

$$\ln(\omega_i - d_i) - 2w_i^2(\omega_i - d_i)^2,$$

leading to $d_i = \omega_i - \frac{1}{2w_i}$.

Now consider a poor individual i who needs to beg to earn a living by begging from the paired rich individual j. This individual's utility is

$$\ln\left(\omega_j - \frac{1}{2s_i}\right) - \frac{1}{2}\left(s_i + 1\right)^2$$

leading to $s_i \simeq 0.83$; thus, $r_i \simeq 0.40$ and utility equals -2.60 for the poor, whereas it equals -1.01 for the rich. Note that the rich has no incentive to beg because the pressure from beggars forces x_{i2} to fall back to the same level. Clearly, this allocation is inefficient because with the same distribution of consumption and no begging, everyone would be better off.

Second case: a labor market is opened and begging is outlawed (or intolerable to employers). Labor supply and demand are determined by maximizing

$$\ln\left(\frac{l}{p} + \omega_i\right) - \frac{1}{2}\left(1 + l\right)^2,$$

implying $l = \frac{-1+\sqrt{5}}{2} \simeq 0.62$ for a poor and $l = \frac{-(1+p)+\sqrt{(1+p)^2-4(p-1)}}{2}$ for a rich individual, so that a market equilibrium is obtained if $p = 2\frac{\sqrt{5}-1}{3-\sqrt{5}} \simeq 3.23$. Utilities are then -2.96 for the poor and -0.29 for the rich.

In this example, opening a labor market is good for the rich, who escape the pressure of beggars and can hire new hands, whereas it is not positive for the poor because the returns to begging were higher in the previous allocation than the wage rate in the labor market. But it would suffice to add a small additional cost to begging, for instance in the form of social shame or uncertainty, to make the new situation advantageous for everyone.

To sum up, the scale of social remedies may not always be at the level of the market failures and economic inequities that need to be addressed. As described in Polanyi (1944), social solidarity and norms of civility were operative at the community level before industrialization and the emergence of the market economy has freed economic activities from traditional shackles but also deprived them of the social safeguards that came with community bonds. The emergence of government regulation and the welfare state is partly a rationalization and universalization of old-style safeguards, but also a response to the inadequacy of old social recipes to the scope of the issues raised by the expanded market economy.

However, given how pervasive market failures are, in particular through local externalities and non-contractible aspects of transactions, public agencies seldom have the information, or the proper incentives, needed to address these market failures at the fine-grained level at which they arise. This means that a natural division of labor emerges, and indeed this is more or less what is observed. Norms of good behavior are primarily aimed at local impacts, where caring attention to fellow citizens, co-workers or partners in transactions helps smooth economic interactions. They reduce the transaction costs that would be staggering if pure ruthless and selfish advantage-seeking was the norm. Government regulation, meanwhile, provides general guidelines and is especially useful for problems with diffuse impacts and larger societal or geographical scope. In other words, a division of labor between private social initiative and public policy is needed to address the ills of the market economy.

This is relevant not just for inefficiency due to market failures, but also for inequalities. Putnam and Garrett (2020) describe the social consequences of parallel trends in the economy, politics, social relations and culture throughout the 20th Century. The core element of such trends is the battle between individualism and solidarity. When a solidarity mindset prevails, private behavior and public policy help reduce inequalities, whereas the opposite occurs when individualism is on the rise. This analysis suggests that, far from stepping in to palliate the limitations of social patches to societal ills, public policy operates at its level and is subject to the same general trends affecting the general norms and collective mindset of the population. The division of labor between bottom up and top down initiatives is operative, but with a coordination between private and public action that is reflected in the general mindset and produces dramatic swings in the economy and the social fabric.

7 Social relations in the economy

One aspect that is missing in the model presented so far is the possibility for social relations to take place through economic transactions. We propose a minimal alteration to the model that can accommodate this phenomenon.

Let z_{ijk} be the net purchase of good k that individual i makes from individual j, at prices p_{ijk}. We introduce the possibility that social relations between i and j depend on such trades. If z denotes the whole matrix $(z_{ijk})_{i,j\in N,k\in K}$, let the outcome y_i now be determined as:

$$y_i = F_i(p, x, z, s).$$

Both x and z appear in this function because they may play different roles. The vector x_i represents the economic standing of the individual whereas z_i represents economic interactions through trades.

Notice the similar form of the vectors z and s, both based on bilateral components. Examples of technologies may include, for component d of social outcomes representing the social relation between i and j:

- $y_{id} = (p_{ij}z_{ij})^{\alpha} \min\{s_{ij}, s_{ji}\}$: the social bonding between i and j depends, in a multiplicative way, on the value of the trade and on their social overtures;
- $y_{id} = (\sum_l \min\{p_{il}z_{il}, p_{jl}z_{jl}\})^{\alpha} \min\{s_{ij}, s_{ji}\}$: the social bonding between i and j depends on their being both in relations with the same set of l agents (employers, providers) and on making the usual social overtures;
- $y_{id} = |z_{ijk}| + \min\{s_{ij}, s_{ji}\}$: the exchange of commodity k (e.g., a haircut) by itself requires social interaction, to which can be added other social overtures.

Individual preferences can still bear on the pair (x_i, y_i), where $x_i = \omega_i + \sum_{j\neq i} z_{ij}$ and z_{ij} denotes the vector $(z_{ijk})_{k\in K}$. This means that preferences over bilateral trades do not come from genuine economic preferences over consumption, and this can be justified by the fact that a commodity is a commodity, independently of its origin. The taste of an orange juice is not modified by the smile of the shopkeeper.

But the consumer may care about the social import of the origin of her consumption, and enjoy the chat with a particular shopkeeper while buying oranges.

This model is, however, amenable to capturing non-contractible economic actions (diligence, quality service...) which enhance or dampen the value of a particular trade. We do not explore this avenue in this section, but one could use the strategies s to represent such non-contractible actions as well.

The economic equilibrium is now defined in terms of bilateral trading. The equilibrium involves individual beliefs about what could be obtained in bilateral trading with other agents.

Let B_i denote the set of pairs $(z_i, p_i) = (z_{ij}, p_{ij})_{j \neq i}$, such that agent i believes that the trade z_{ij} with agent j is possible at prices p_{ij}. Each i solves the following program:

$$\max_{(z_i, p_i, s_i) \in B_i \times S_i} u_i(x_i, y_i)$$

$$\text{such that } \begin{cases} x_i = \omega_i + \sum_j z_{ij} \\ \sum_{j \neq i} p_{ij} z_{ij} = 0 \\ y_i = F_i(p, x, z, s) \end{cases}$$

An equilibrium is defined as a vector of belief sets B_i, trades z_{ij}, prices p_{ij} and strategies s_i such that every i solves the above program, and individual beliefs are mutually fulfilled at every realized bilateral trade, i.e., the realized $(z_{ij}, p_{ij}) \in B_i$ corresponds to the realized $(z_{ji}, p_{ji}) \in B_j$, with same prices and opposite quantities traded.

Whether beliefs should also be common across individuals about non-realized transactions can be considered, and introduces restrictions on the possible equilibria (see Zame 2007 for a discussion of this issue in a different model).

Production can no longer be treated as a single unit operation in this model, since a single price vector no longer applies in general. A simple option is to posit that each individual has access to a set Ω_i from which ω_i is drawn, with the convention that negative quantities are admissible and correspond to inputs to be bought. This modeling approach does not imply that production is an individual activity. An entrepreneur can pick a production plan involving buying inputs from, and selling outputs to, many individuals.

This model is a direct generalization of the competitive model. The competitive case is the special case when individuals believe that there is only one price vector that is admissible for all transactions and no restriction on transactions at these prices.

The general proof of existence of an equilibrium is trivial when there are no constraints on (possibly common) beliefs, since it suffices to posit that individuals believe only the current trade is possible. Existence is less obvious when the belief sets are expanded. This model is quite general and can represent many forms of competition, including fixed price rationing equilibria. Rather than exploring various specifications for the beliefs, we focus here on examples illustrating how the two spheres can interact in this model.

Example 13. There are two goods and two types of identical individuals. Individuals of type i are

the only suppliers of good $k = i$. All individuals have the same preferences $u(x_1, x_2, y)$ where $y = z_{121}$ is the sale of good 1 by an individual of type 1 to an individual of type 2. In this particular model, one actually has $z_{121} = x_{21}$, i.e., the net trade is equal to the consumption of good 1 by an individual of type 2.

If the types contain many individuals, one can reasonably consider that they behave as price-takers. At the equilibrium, letting $u_k^i = \frac{\partial u}{\partial x_{ik}}(x_{i1}, x_{i2}, x_{21})$, one has

$$\frac{u_1^1 - u_3^1}{u_2^1} = \frac{u_1^2 + u_3^2}{u_2^2},$$

inducing individuals of type 2 to consume more of good 1 than individuals of type 1, other things equal. The social benefit of trading good 1 boosts its market, compared to good 2.

The next example illustrates how the choice of techniques through profit maximization may miss externalities linked to the provision of human service, an issue highlighted in Atkinson (2015).

Example 14. Individual 1 is the only entrepreneur and has a monopoly on the production of good 1. Two technologies are available, one is labor (good 2) intensive whereas the other uses good 3 as input. Both exhibit constant returns to scale of the type $q_1 = x_k/c_k$, where q_1 is the output, x_k the input and c_k the unit cost. Individuals all have the same utility

$$av(x_1) + v(x_2) + x_3 + b.$$

There is a constant returns to scale technology transforming good 2 into good 3 one for one. Taking good 3 as numeraire, one then has $p_2 = 1$. In the utility function, parameters a and b are sensitive to the technology used for the production of good 1. If good 2 is used as input, then consumers enjoy human contact with the service providers, so that $a > 1, b > 0$, whereas if good 3 is used as input, there is no such human contact and $a = 1, b = 0$. One can view b as representing the benefit of having human contact at all, while a increases the marginal utility of good 1 and leads to an increase in the quantity consumed, other things equal.

Individual 1 is assumed to use a standard monopoly pricing policy, while all other individuals are price-takers. Also, although individual 1 has a slight personal preference, qua consumer, for the labor technology, it is assumed to be a good approximation to consider that profit maximization is the leading criterion for the choice of technology (because the population is large, and so is the profit).

Let us focus on the case in which v is a constant elasticity function $\frac{\varepsilon}{\varepsilon-1}x^{\frac{\varepsilon-1}{\varepsilon}}$, yielding demand elasticity ε. Then the monopoly's profit, per capita, is equal to

$$\left(\frac{c_k}{\varepsilon - 1}\right)^{1-\varepsilon}\left(\frac{a}{\varepsilon}\right)^{\varepsilon},$$

whereas consumer surplus from good 1, per capita, is equal to

$$\frac{\varepsilon}{\varepsilon - 1}\left(\frac{c_k}{\varepsilon - 1}\right)^{1-\varepsilon}\left(\frac{a}{\varepsilon}\right)^{\varepsilon} + b.$$

If b did not depend on the technology, consumer surplus and profit would have be aligned in the choice of technology. But the presence of b may separate what is good for profit from what is good for consumer utility.

The point of this section is primarily to convey the point that the basic model from the earlier sections can be easily modified to accommodate in-market social interaction. Such interactions further reinforce the point that social externalities are pervasive in economic activities. From the park model where such interactions are minimal to this latest version, this class of models makes it easy to disentangle the various ways in which social interdependence matters for the analysis of the socioeconomic situation at large.

8 Conclusion

This paper offers a versatile model and conceptual toolkit which can be used as a useful umbrella to encapsulate many aspects uncovered in the economic literature on social interactions. The stylized depiction of the economy and the society that it contains helps fleshing out how economic activities are part of a broader social setting and why they should not, in general, be studied separately. Ideally, a model such as this one should replace the canonical models that shape how people, and especially experts and policymakers, view the economy and its rules.

The literature on social interactions often refers to *externalities* as the key phenomenon brought in by the mutual interdependence in which people navigate their social lives. Externalities, in economic theory, are normally seen as a problem for the market, in the form of an afterthought. In the standard outlook of economic analysis, as well as in usual teaching curricula, one starts with the analysis of the perfect market and only later consider market failures. What the literature on social interactions suggest, and the model of this paper vividly affirms, is that externalities are actually the primary factor, and that market transactions can at most carve a space in social interactions but never escape from the general social interdependence between people. People's preferences and their social success are in substance made of externalities, and these externalities do not disappear when people interact in markets. In a nutshell, instead of viewing externalities as icebergs in the market sea, one should view market transactions as a small fleet in an ocean of externalities.

We have seen, however, that even though the social game seems well represented by an influence model, it does not always by itself generate inefficiency, as in prisoner's dilemma situations. In particular, veto and claim (as well as in-between) technologies and social norms for joint activities tend to produce outcomes that lie between the preferred options of the various players and are therefore efficient, though not necessarily equitable. But inefficiency can still emerge from the lack of coordination between the economic and the social spheres. People might be dissatisfied with the social outcomes and would be willing to spend resources to improve them, but resources may be the wrong currency under the spontaneity and sincerity rules that govern many social interactions. Many people would be willing to be less affluent and have a better social environment, but there may be no available technology that transforms resources into decency, friendship, esteem, genuine reciprocity and the like. Therefore, societies must find ways to tend their social garden with appropriate social norms.

Resources can, however, be devoted to education, to sharing information, and to raising awareness about the importance of social interdependence. They can also be used, sometimes, in subtle exchanges that do not undermine the value of social interactions (Zelizer 2005, 2017). Moreover, in the economic sphere, a mix of economic incentives and norms can be mobilized to enhance responsible behavior that takes account of the social consequences of economic decisions.

All of this undermines the idea that efficiency theorems of general equilibrium theory are of any relevance to the real world. But the results of this paper should not be interpreted as pessimistic about the possibility to improve the efficiency and equity of the general socioeconomic system. They call for a serious examination of the multiple instruments which can be used to this purpose. The fact that social norms are likely to play a prominent role is particularly interesting. As analyzed in IPSP (2018), very long-term cultural trends seem to spontaneously favor more tolerant and supportive attitudes. Supportive functions of various sorts have become a burgeoning industry and a flourishing part of charitable activism, while laws curbing abusive practices and protecting weak parties in long interactions such as marriage and jobs become ever sharper and expansive. These prevent negative externalities and enhance the capacity of individuals to produce positive externalities. These evolutions also contribute to reducing socioeconomic inequalities by improving the life of the former victims of exclusion and harassment. But medium term fluctuations and backlash can set societies back in a significant way. In particular, Putnam and Garrett (2020) analyze how the US society has abandoned its solidarity ideals and embraced destructive individualism in the last decades. Comparing the impact of such trends over general efficiency and socioeconomic equity to the effect of standard policy instruments would be very interesting.

Deciphering all the implications of the strong social interdependence among individuals and groups appears to us as a promising research program, to which this paper seeks to contribute.

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Appendix: Proof of Prop. 2

The assumptions are:

- The function $U_i(p,x,s) := u_i(x_i, F_i(p,x,s))$ is continuous in (p,x,s) and non-satiable in x_i;
- The set X_i is closed and convex;
- The set S_i is compact and convex;
- The individual endowment $\omega_i \gg 0$;
- The cone Q is closed;
- For every p and (x_{-i}, s_{-i}), the set of (x_i, s_i) maximizing $U_i(p,x,s)$ such that $px_i \leq p\omega_i$ is convex.

Since production is limited by the available inputs $\sum_i \omega_i$, there is a compact and convex truncation of Q, denoted Q^*, in which every feasible allocation takes its production plan. Likewise, there is a compact and convex truncation of X_i, denoted X_i^*, in which every feasible allocation takes i's consumption. The truncation must be large enough (by going beyond the set spanned by feasible allocations, in the relevant directions) so that whenever $q \in \arg\max\{pq | q \in Q^*\}$ and belongs to a feasible allocation, then $q \in \arg\max\{pq | q \in Q\}$. Likewise, whenever $x_i \in \arg\max\{U_i(p,x,s) | x_i \in X_i^*, px_i \leq p\omega_i\}$ belongs to a feasible allocation, then $x_i \in \arg\max\{U_i(p,x,s) | x_i \in X_i, px_i \leq p\omega_i\}$. Let individual 1 be declared the owner of Q^*, and receive the profit pq (in equilibrium, this profit is null, therefore this is without loss of generality).

Let $P = \left\{p \in R_+^\ell \,|\, \|p\| = 1\right\}$, where $\|\|$ is the Euclidean norm, $X = \prod_i X_i$, $S = \prod_i S_i$.

Consider the correspondence over $P \times X \times S$ defined as follows:

- $p \in \left\{ \frac{p+\sum_i (x_i-\omega_i)-q}{\|p+\sum_i (x_i-\omega_i)-q\|} | q \in \arg\max \{pq | q \in Q^*\} \right\}$
- $x_i \in \arg\max \{U_i(p,x,s) | x_i \in X_i^*, px_i \leq p\omega_i\}$
- $s_i \in \arg\max \{U_i(p,x,s) | s_i \in S_i\}$

This correspondence, in each of its components, is upper hemicontinuous. In particular, $\arg\max \{pq | q \in Q^*\}$ is also upper hemicontinuous in p, while

$$\frac{p+\sum_i (x_i-\omega_i)-q}{\|p+\sum_i (x_i-\omega_i)-q\|}$$

is continuous in (p,x,q). The correspondence

$$\arg\max \{U_i(p,x,s) | x_i \in X_i^*, px_i \leq p\omega_i\}$$

is upper hemicontinuous since $X_i^* \cap \left\{x_i \in \mathbb{R}_+^\ell | px_i \leq p\omega_i\right\}$ is compact and continuous in p (i.e., both upper and lower hemicontinuous, the latter depending on the assumption $\omega_i \gg 0$) while $U_i(p,x,s)$ is continuous in (p,x,s).

The images of the correspondence are convex for each component. For p, this comes from the fact that $\arg\max \{pq | q \in Q^*\}$ is convex, and thus

$$\left\{ \frac{p+\sum_i (x_i-\omega_i)-q}{\|p+\sum_i (x_i-\omega_i)-q\|} | q \in \arg\max \{pq | q \in Q^*\} \right\}$$

is also convex, as it is the projection of the convex set

$$\left\{ p+\sum_i (x_i-\omega_i)-q | q \in \arg\max \{pq | q \in Q^*\} \right\}$$

on the set P. For x_i and s_i this directly comes from the assumptions, and the truncation via X_i^* does not invalidate this assumption.

Therefore, Kakutani's fixed-point theorem can be applied, implying that this correspondence has a fixed point (p^*,x^*,s^*). For p^*, given that

$$p\left(\sum_i (x_i-\omega_i)-q\right)=0$$

by non-satiation, this obtains only if

$$\sum_i (x_i-\omega_i)-q=0$$

for some

$$q^* \in \arg\max \{p^*q | q \in Q^*\} \, i.e., if \sum_i (x_i^*-\omega_i)-q^*=0.$$

To see this, consider the two possible cases.

First case: $\|p^* + \sum_i (x_i^* - \omega_i) - q^*\| = 1$. In this case, one has

$$p^* = p^* + \sum_i (x_i^* - \omega_i) - q^*,$$

implying $\sum_i (x_i^* - \omega_i) - q^* = 0$.

Second case: $\|p^* + \sum_i (x_i^* - \omega_i) - q^*\| \neq 1$. In this case, one has

$$p^* = \frac{\sum_i (x_i^* - \omega_i) - q^*}{\|p^* + \sum_i (x_i^* - \omega_i) - q^*\| - 1},$$

implying

$$\|p^*\| = \frac{p^* \left(\sum_i (x_i^* - \omega_i) - q^*\right)}{\|p^* + \sum_i (x_i^* - \omega_i) - q^*\| - 1} = 0,$$

which is impossible since $\|p^*\| = 1$ by construction.

Thus, this allocation is feasible, so it also satisfies

$$q^* \in \arg\max \{p^* q | q \in Q\}$$

and

$$x_i^* \in \arg\max \{U_i (p^*, x, s^*) | x_i \in X_i, p^* x_i \leq p^* \omega_i\},$$

implying that it is an equilibrium.