



**AgEcon** SEARCH  
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search  
<http://ageconsearch.umn.edu>  
[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

## VERIFICATION OF LINEAR PROGRAMMING SOLUTIONS, WITH EMPHASIS ON SUPPLY IMPLICATIONS\*

C. Richard Shumway and Hovav Talpaz

Linear programming (LP) models have been developed for a wide range of normative purposes in agricultural production economics. Despite their widespread application, a pervading concern among users is reliability — how well does a particular model actually describe and/or predict real world phenomena when it is so designed.

Much attention has been devoted in recent years to methods for making programming models produce results more in line with those actually observed. These efforts have included development of more detail in production activities and restrictions, incorporation of flexibility constraints into recursive programming systems [12], specification of more realistic behavioral properties [1], and development of guidelines for reducing aggregation error [11].

The objective of this paper is to develop procedures for verifying static multicommodity LP models constructed with a long run adjustment horizon.<sup>1</sup> Standard procedures for adjusting or calibrating “best guess” data used in the model are briefly discussed. Major attention is then devoted to additional methods of evaluating model reliability. In particular, two procedures are developed for combining supply function parameters from the LP model with time series data in order to predict short run adjustments in commodity production levels. Predictions of 1974 production of 15 vegetables and field crops are reported and compared.

### INITIAL MODEL CALIBRATION<sup>2</sup>

The static LP model used here was based on the assumption of a long run adjustment period (i.e., no short run constraints on resources such as labor or operating capital), and was developed to evaluate the interaction in resource demand among 15 major groups of vegetables and field crops in California. The state was divided into 95 production areas based on similarities in soil, climate and water resources. Acreage suitable and potentially available for crop production in each area during the period 1961-65 was inventoried. The 15 commodity groups utilized 79 percent of actual total crop acreage during that period. Acreage in excluded high-value crops (orchard crops and minor vegetables) and in non-agricultural uses was deducted from the inventory. Maximum individual crop acreages were limited only by available land and water resources and by rotation requirements.

Since production area boundaries did not coincide with counties, yield estimates were developed by commodity specialists and county extension agents. The first step in model calibration was multiplying these yields by crop acreage in each production area; this determined implied state output in the base period 1961-65, assuming yield estimates were correct. All yield estimates for a crop were scaled up or down until implied state output equaled actual output.

C. Richard Shumway and Hovav Talpaz are Associate Professors of Agricultural Economics, Texas A & M University.

\*Technical Article 12738 of the Texas Agricultural Experiment Station. This paper is a revised and expanded version of “Combining LP Results and Time Series Data for Prediction of Supply: Two Approaches,” contributed paper presented at AAEEA annual meeting, State College, Pennsylvania, 15-18 August, 1976. The authors wish to thank Peter Barry, John Penson, C. Robert Taylor and anonymous *Journal* reviewers for constructive comments on earlier drafts. Anne Chang provided much computer assistance for which we are most appreciative.

<sup>1</sup>This paper is a continuation of research reported in Shumway and Chang [13]. The same empirical data and similar methodology are used. Alternative econometric procedures are reported for deriving supply elasticities from the LP results and for using LP estimates as prior information for econometric analysis.

<sup>2</sup>Details of the linear programming model are reported in Shumway, et al.

Beginning with budgets representing a high management level, the second step in model calibration required solution of the LP model constrained to produce actual 1961-65 state output with minimum costs. Shadow prices representing marginal crop producing costs were thus obtained. Assuming (1) producers equate marginal cost with expected price and (2) expected price equals lagged actual price, production costs were adjusted so that the profit maximizing model results approximated actual 1961-65 average output of each commodity at 1960-64 average price.

### EVALUATING MODEL RELIABILITY

Model results generated with the objective function of minimizing actual cost of producing 1961-65 output resulted in a resource demand distribution among nine state regions (i.e., aggregate of production areas) that was highly correlated with actual use in that period. The square of the correlation coefficient of actual and predicted harvested acreage across regions was .84 and of actual and predicted water usage, .82. These correlations were very high, as noted by Perrin [10], when compared with figures of .02-.50 reported by Wallace [18, p. 16] in an evaluation of major variables in nine earlier spatial studies. Although higher correlation among larger aggregates than among individual production areas would be expected, the model's ability to approximate actual spatial resource use appeared reasonable.

Remaining considerations for reliability evaluation included estimating sensitivity of implied supply to changes in expected prices. From the profit maximizing model, a total of two hundred parametric observations on production levels of all crops was obtained by sequentially changing product prices, largely in equal increments within the range of 1950-73 deflated prices.<sup>3</sup> The parametric solutions for each commodity were treated as independent observations in formulating linear multiple regression supply equations of the following form:<sup>4</sup>

$$Y_i = a_{i0} + a_{i1}P_1 + \dots + a_{in}P_n + \mu_i \quad (1)$$

<sup>3</sup>California product prices [2, 3, 4, 5] were deflated by the USDA [16, 17] index of prices paid by farmers for factors of production (1960-64 = 100).

<sup>4</sup>Use of regression analysis is reported here with two qualifications: (1) observations are not really independent since prices were not selected randomly; (2) price selection procedure implies a uniform distribution of prices between the high and low observations. Since typical distribution of actual prices is more bell-shaped, use of parametric programming observations here gives undue weight to the outliers. In retrospect, these problems could have been reduced by developing a factorial design for the LP parametrization instead of adhering to sequential, equal-interval parametrization.

<sup>5</sup>As Debertin and Freund point out, the standard stepped regression procedures, including these, indicate an upward-biased level of significance on selected variables (i.e., increases our chances of a Type 1 error). However, stepwise regression was used here only for discarding obviously unimportant independent variables and estimating parameter values on the others. These data were never used to test the hypothesis that parameters of specific variables were significantly different from zero. Bias in significance level was recognized but did not pose a serious problem. Consequently, no attempt was made to adjust for degrees of freedom associated with the original variables set [8, p. 215]. Supply elasticities estimated from simple regression equations fit to these data are reported in Shumway and Chang [13].

where

$Y_i$  = derived output vector of commodity  $i$   
 $P_1 \dots P_n$  = 15 commodity vectors of parametrized price used in the LP model, and  
 $\mu$  = error term.

The equations were first fit with untransformed (linear) data and then in logarithms to obtain estimates of the relation between prices of 15 commodity groups and the output of each. In all cases, the linear equations resulted in a better or almost equivalent fit (based on  $R^2$  value). Accordingly, estimates reported here were derived from the linear equations.

Stepped variable selection procedures were used to select and estimate price variable parameters significantly different from zero at the five percent level or less (based on final degrees of freedom).<sup>5</sup> Supply elasticities obtained from these regressions were computed at average 1961-65 output levels and lagged representative crop prices, and are reported in Table 1. Supply elasticities estimated from quadratic and simple linear regression equations fit to these data are reported in Shumway and Chang [13, p. 350]. More than half of the cross-price elasticities estimated to be non-zero using simple regression are reported as zero here using stepped variable selection on multiple regression equations. Our direct elasticity estimates are generally higher than the Shumway and Chang simple linear regression estimates, but lower than their quadratic equation estimates.

All direct elasticity estimates are positive, and most cross elasticities non-positive. Cross elasticities are positive only between grain sorghum and barley, two crops frequently double-cropped in the state. Consequently, no elasticities violate *a priori* expectations as to sign. Actual average 1961-65 crop output and 1960-64 price are reported in Shumway and Chang [13, p. 347].

### Combining LP Results with Time Series Data for Prediction

The price coefficients derived from LP are estimates of long run responsiveness. Since long run

TABLE 1. LINEAR PROGRAMMING—STEPPED REGRESSION ESTIMATES OF LONG-RUN ELASTICITIES OF SUPPLY

Commodity	No.	Elasticity with respect to the price of:														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Alfalfa hay (and seed)	1	<u>6.6</u>	-1.6	-0.1	0	0	0	0	0	0	-0.2	-0.2	-0.1	-0.1	0	-0.1
Barley (small grains)	2	-1.5	<u>5.1</u>	-0.1	0	0	1.7	-0.5	0	0	-0.1	-0.1	-0.1	-0.1	0	-0.1
Corn for grain (and silage)	3	0	0	<u>3.6</u>	0	0	0	0	0	0	0	0	0	0	0	0
Cotton	4	-0.7	0	-0.1	<u>1.5</u>	0	0	0	0	0	-0.2	-0.2	0	-0.1	0	-0.1
Dry Beans	5	0	0	-0.1	0	7.2	0	0	0	0	-0.2	-0.2	0	-0.1	0	-0.1
Grain sorghum (and sorghum silage)	6	-0.9	8.1	-0.1	0	0	<u>10.9</u>	0	0	0	-0.1	0	0	0	0	0
Rice	7	-1.0	-1.6	-0.1	0	-0.3	-0.5	<u>6.6</u>	0	-0.2	-0.1	-0.1	-0.1	-0.1	0	-0.1
Safflower	8	0	0	0	0	0	0	0	<u>9.6</u>	0	0	0	0	0	0	0
Sugar beets	9	0	0	0	0	0	0	0	0	<u>3.7</u>	-0.2	-0.2	0	-0.1	0	0
Asparagus	10	0	0	0	0	0	0	0	0	0	<u>18.0</u>	0	0	0	0	0
Broccoli (cole crops)	11	0	0	0	0	0	0	0	0	0	0	<u>14.1</u>	0	0	0	0
Cantaloups (melons)	12	0	0	0	0	0	0	0	0	0	0	0	<u>6.0</u>	0	0	0
Lettuce	13	0	0	0	0	0	0	0	0	0	0	0	0	<u>6.2</u>	0	0
Potatoes	14	0	0	0	0	0	0	0	0	0	0	0	0	0	<u>2.0</u>	0
Tomatoes, processing (and fresh)	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	<u>3.4</u>

output levels are never observed in the real world, no observations exist against which predictive accuracy of the LP-derived output projections can be measured. Therefore, one could not simply insert one year's prices into the LP-derived supply equations, predict the next year's output and meaningfully compare that prediction against actual output. Consequently, the LP-derived long run price coefficients were used as prior information in two models with annual price and output data for the period 1950-73 in order to predict 1974 output levels. Both prediction models presume a geometric lag adjustment process; the first in adjusting to desired output levels, and the second in modifying expectations of future prices.<sup>6</sup> The first model is the same as that used by Shumway and Chang [13, pp. 353-4].

**Partial adjustment premise.** Let us suppose producers revise desired output levels based on prices of relevant crops in the previous year and, in each year, move toward that level by a constant proportion of

the difference between actual and desired output. This adjustment process can be expressed in terms of a partial adjustment model consisting of two equations (see Nerlove [9, pp. 502-503]):

$$y_{it} - y_{i,t-1} = \gamma_i(y_{it}^* - y_{i,t-1}) + u_{it} \quad (2)$$

$$y_{it}^* = c_{i0} + \sum_{j=1}^n c_{ij}p_{j,t-1} \quad (3)$$

where

$y_{it}$  = actual output of commodity  $i$  in year  $t$

$y^*$  = desired output

$0 < \gamma \leq 1$  = estimated coefficient of adjustment (a constant)

$p_{jt}$  = price of commodity  $j$  in year  $t$

$j$  = commodity index including  $j = i$

$c_{ij}$  = estimated parameter of the  $j$ th commodity price in affecting desired

<sup>6</sup>A compound geometric lag model could have been developed incorporating both assumptions in a single model. The estimation equations for that model are similar to the equations for the partial adjustment model expanded such that the independent price variables,  $p_j$  are lagged 1,2, ...  $n$  periods, and the output  $y$ , is lagged 1,2, ...  $n+1$  periods. This model was not used here because of the very high degree of collinearity evident for most crops among the independent variables with different lag lengths.

supply of the  $i$ th commodity, and  
 $u$  = error term.

To preserve the relative magnitudes of the LP-derived direct and cross-price parameters, a composite variable was defined from those parameters and a time series of crop prices,

$$p'_{it} = \sum_{j=1}^n \hat{a}_{ij} p_{jt} \quad (4)$$

where

$p'_{it}$  = composite price variable for commodity  $i$  in year  $t$

$\hat{a}_{ij}$  = LP-derived estimate of the  $j$ th commodity price parameter in the supply of the  $i$ th commodity, and

$p_{jt}$  = deflated price of the  $j$ th representative commodity in year  $t$ .

The  $\hat{a}_{ij}$  values in some of the LP-derived equations estimate responsiveness in the supply of crop groups to changes in the price of individual commodities. Here, we are concerned with developing predictive equations for individual commodities only. Therefore,  $\hat{a}_{ij}$  values for crop groups were adjusted based on individual crop acreage as a percent of the group total in 1961-65.

Estimation equations derived from equations (2), (3), and (4) and then fit to annual output and composite price data for the period 1950-73. Two estimation equations per commodity were fit, regressing output on lagged output and composite price, with and without a trend variable:

$$y_{it} = b_{i0} + b_{i1} p'_{i,t-1} + b_{i2} y_{i,t-1} + u_{it} \quad (5)$$

$$y_{it} = b_{i0} + b_{i1} p'_{i,t-1} + b_{i2} y_{i,t-1} + b_{i3} t + u_{it} \quad (6)$$

where  $t$  for year 1950 equals 1.

Equations in which the lagged dependent variable appears as an independent variable frequently exhibit serially correlated error terms and also bias the test for serial correlation in OLS estimation. Consequently, the less restrictive Cochrane-Orcutt [6] model with the Cooper [7] transformation was used to obtain parameter estimates and standard statistics with asymptotically desirable properties for these equations.

**Adaptive expectation premise—maximum likelihood formulation.** An analogous model to the above is the adaptive expectations model. When there is only one price variable, it can be derived from the

same estimation equation. Assuming (1) producers instantaneously adjust output levels to expected price and (2) expected price is a geometrically lagged function of previous prices, the model consists of two equations. These are formulated below for the current case where some of the equations have more than one relevant price variable:

$$p_{ijt}^* - p_{ij,t-1}^* = \beta_{ij} (p_{j,t-1} - p_{ij,t-1}^*) \quad (7)$$

$$y_{it} = c_{i0} + \sum_{j=1}^n c_{ij} p_{ijt}^* + v_{it} \quad (8)$$

where

$p_{ij}^*$  = expected price of commodity  $j$  in supply equation  $i$

$0 < \beta \leq 1$  = coefficient of expectation (a constant) which is uniquely estimated for each price variable in each equation, and

$v$  = error term.

The parameters of this model can be derived from estimation equations (5) and (6) expanded to include independent variables  $p$  and  $y$  lagged 1, 2, ...  $n$  years. Alternatively, they can be estimated as maximum likelihood functions of  $\beta_{ij}$  from equation (8) where  $P_{ijt}^*$  is defined as in equation (7) [9, pp. 503-504]. The latter method was used because of the high degree of collinearity among the independent variables with different lag lengths.

Rearranging terms, equation (7) becomes:<sup>7</sup>

$$p_{ijt}^* = \sum_{m=1}^{\infty} \beta_{ij} (1 - \beta_{ij})^{m-1} p_{j,t-m} \quad (9)$$

Incorporating the price parameters derived from the LP model, a composite variable may be defined as follows:

$$p_{it}^{*'} = \sum_{j=1}^n \hat{a}_{ij} p_{ijt}^* \quad (10)$$

where the  $\hat{a}_{ij}$  values are the price parameters from the LP-derived supply equation for commodity  $i$ . The  $p_{ijt}^*$  variables can be obtained from equation (9) by solving for the  $\beta_{ij}$  parameters as maximum likelihood estimates that minimize the sum of squared error (equivalent to maximizing  $R^2$  with a given set of independent variables) in the following modified equation (8):

$$y_{it} = b_{i0} + b_{i1} p_{it}^{*'} + v_{it} \quad (11)$$

Because equation (9) is intrinsically nonlinear, a

<sup>7</sup>Since  $\beta_j$  must be estimated by an iterative search procedure,  $m$  (length of lag) was truncated on pragmatic grounds at 9 years.

nonlinear search procedure developed by Talpaz [15] was used to search for the values  $\beta_{ij}$  in order to approach within a specified tolerance the maximum  $R^2$  value for the equation.<sup>8</sup> The search procedure was conducted with these alternative conditions:

- (a) no further restrictions
- (b) 
$$p_{ijt}^* = \frac{\sum_{m=1}^9 \beta_{ij}(1-\beta_{ij})^{m-1} p_{j,t-m}}{\sum_{m=1}^9 \beta_{ij}(1-\beta_{ij})^{m-1}}$$
- (c)  $0 < \beta_{ij} \leq 1$ ,
- (d) conditions (b) and (c).

Condition (b) scales every  $p_{ij}^*$  so that the sum of coefficients on  $p_j$  over the relevant time horizon is the same (i.e., 1.0). This condition assures that the relative magnitudes of LP-derived direct and cross-price coefficients are preserved in each equation. Condition (a) on the other hand permits modification of the relative magnitudes of LP-derived price parameters ( $a_{ij}$ ) but only by simultaneously changing the shape of the lag distribution. An increase in  $\beta_{ij}$  implies both an increase in the relative weight on the LP-derived price parameter of commodity  $j$  and a rapidly declining lag distribution as  $m$  increases. A decrease in  $\beta_{ij}$  implies both a decrease in the relative weight and an evening out of the lag distribution toward a rectangular lag. Condition (c) forces satisfaction of the *a priori* expectation that the difference between last year's and current expected price is positively related to but no greater than the difference between last year's expected and actual price. Condition (d) simultaneously imposes both conditions (b) and (c) on the estimation.

Length of lag was truncated for all commodities on pragmatic grounds at nine years.

### Empirical Results<sup>9</sup>

Two equations based on the partial adjustment conceptual model and three based on the adaptive expectations model were fit to time series data for 1950-73 using the LP-derived price parameters as prior information. Selected statistics and parameter estimates are reported below; first for the partial adjustment equations and subsequently for the adaptive expectation equations.

**Partial adjustment model equations.** The price parameter and coefficient of adjustment for each crop estimated by the two partial adjustment equations are reported in Table 2. Since the number of independent variables in the two equations differs,  $R^2$  is reported as the relevant measure of goodness of fit rather than  $R^2$ . *A priori* expectations are: (1)  $b_{11} > 0$  since a direct response in supply to a change in the composite commodity price is expected, and (2)  $0 \leq b_{12} < 1$  since the partial adjustment hypothesis implies that producers adjust toward desired level but do not overadjust.

Nineteen of the 30 equations satisfied *a priori* expectations as to sign and magnitude of the relevant parameters. At least one equation satisfied expecta-

TABLE 2. SELECTED RESULTS OF PARTIAL ADJUSTMENT LP-TIME SERIES EQUATIONS

Commodity	Independent variables	Parameter estimates			$\bar{R}^2$
		Price, lagged		Output, lagged ( $b_{12}$ )	
		Composite ( $b_{11}$ )	Own ( $b_{11}, a_{11}$ )		
Alfalfa hay	p,y	-.002	-3,721	.921 <sup>***</sup>	.86
	p,y,t	-.002	-3,721	.360 <sup>***</sup>	.88
Barley	p,y	.031 <sup>*</sup>	6,806 <sup>*</sup>	.938 <sup>***</sup>	.50
	p,y,t	-.004	-878	.768	.50
Corn for grain	p,y	.742 <sup>***</sup>	12,393 <sup>***</sup>	.553	.91
	p,y,t	.681 <sup>***</sup>	11,373 <sup>***</sup>	.587 <sup>***</sup>	.91
Cotton	p,y	.214	3,306	.223	.38
	p,y,t	.317 <sup>*</sup>	4,897 <sup>*</sup>	.289	.41
Dry beans	p,y	.066 <sup>***</sup>	388 <sup>***</sup>	1.016 <sup>***</sup>	.77
	p,y,t	.048 <sup>*</sup>	282 <sup>*</sup>	.325 <sup>***</sup>	.83
Grain sorghum	p,y	.005	631	.853 <sup>*</sup>	.90
	p,y,t	.018	2,270	.496 <sup>*</sup>	.90
Rice	p,y	-.019	-986	.926 <sup>***</sup>	.71
	p,y,t	.034	1,765	.068	.77
Safflower	p,y	.011	322	.832 <sup>***</sup>	.77
	p,y,t	.009	259	.844 <sup>***</sup>	.76
Sugar beets	p,y	.268	509,325	.950 <sup>***</sup>	.59
	p,y,t	.714 <sup>***</sup>	1,356,932 <sup>***</sup>	.498 <sup>***</sup>	.82
Asparagus	p,y	-.009	-62	.771 <sup>***</sup>	.63
	p,y,t	-.001	-9	.774 <sup>***</sup>	.63
Broccoli	p,y	.037	276	1.111 <sup>***</sup>	.87
	p,y,t	.090 <sup>***</sup>	677 <sup>***</sup>	.596 <sup>***</sup>	.91
Cantaloups	p,y	.095 <sup>***</sup>	2,264 <sup>***</sup>	.953 <sup>***</sup>	.76
	p,y,t	.095 <sup>***</sup>	2,252 <sup>***</sup>	.948 <sup>***</sup>	.74
Lettuce	p,y	.028	2,625	1.080 <sup>***</sup>	.94
	p,y,t	.039 <sup>***</sup>	3,626 <sup>***</sup>	.862 <sup>***</sup>	.95
Potatoes	p,y	.182 <sup>***</sup>	8,701 <sup>***</sup>	.939 <sup>***</sup>	.69
	p,y,t	.181 <sup>***</sup>	8,654 <sup>***</sup>	.942 <sup>***</sup>	.67
Tomatoes, processing	p,y	-.017	-5,990	.967 <sup>***</sup>	.57
	p,y,t	.215 <sup>*</sup>	74,769 <sup>*</sup>	-.452 <sup>***</sup>	.77

\* $b_{11}$  coefficient significant at the 5 percent level.

\*\* $b_{11}$  coefficient significant at the 1 percent level.

<sup>8</sup>This nonlinear optimization procedure is an improved version of the Fletcher-Powell-Davidon method (see Talpaz [15] for details). The gradient vector and the Hessian matrix are numerically approximated avoiding need for any mathematical transformation. A fast convergence is achieved by efficient stepwise selection. Stopping criterion used here was  $R^2 - R^2_{\ell-1} \leq .001$ , where  $\ell$  is the iteration index.

<sup>9</sup>In the interest of space, only selected results from the specified equations are reported here. Full details on these equations are available from the authors.

tions for all but three crops—alfalfa, asparagus and tomatoes. Two of these are perennial crops and the third experienced a change in production practices midway through the time series from labor-intensive to a mechanized harvest. Consequently, it is not surprising that supply response was not logically explained by the simple geometric lag models.

**Adaptive expectations model equations.** Price parameter and coefficient of expectation for own price are reported for each adaptive expectation equation along with  $\bar{R}^2$  values in Table 3. Three equations were fit for each crop. Only 17 of the 45 equations satisfied *a priori* expectations as to sign and magnitude on  $b_{11}$  and most  $\beta_{ij}$  (i.e.,  $b_{11} > 0$ ,  $0 < \beta_{ij} \leq 1$ ), although two equations did meet those expectations for alfalfa.  $\bar{R}^2$  values for the adaptive expectation equations were generally lower than for the partial adjustment equations.

### Comparison of Predictive Accuracy

The percent error in predicting 1974 output is reported for each estimation equation in Table 4. Across crops, the error in prediction ranged from 0.4

TABLE 4. PERCENT ERROR IN PREDICTING 1974 OUTPUT

Commodity	LP-based predictive equations						Econometric equations with independent variables	
	Partial adjustment model with independent variables:		Adaptive expectation model with restriction:					
	P.V	P.V,t	a	b	c	d	P.V	P.V,t
Alfalfa	1.9	3.5	-2.1	-3.7	0.4		1.0	2.3
Barley	14.0	0.9	12.7	26.5	18.0		15.0	1.4
Corn	43.6	43.3	-16.6	-16.6	-16.6		43.6	43.3
Cotton	-41.6	-37.2	-49.2		-48.3	-48.2	-38.3	-21.3
Dry beans	1.5	-15.7	-36.6		-39.1	-35.5	-1.8	-19.0
Grain sorghum	38.2	66.2	50.8	49.5		52.2	40.6	70.1
Rice	-12.4	-8.3		-11.8	-10.3	-11.8	-12.3	2.1
Safflower	-9.5	-12.2	27.3	27.3	27.3		-9.5	-12.2
Sugar beets	28.1	91.9	39.8	27.1	17.8		28.7	83.0
Asparagus	0.4	-3.0	1.1	1.1	1.1		0.4	-3.0
Broccoli	-8.3	-12.3	-20.8	-20.8	-20.8		-8.3	-12.3
Cantaloups	13.3	13.5	27.8	27.8	27.8		13.3	13.5
Lettuce	3.4	3.0	-23.1	-23.1	-25.2		3.4	3.0
Potatoes	2.9	2.5	1.5	1.5	1.5		2.9	2.5
Tomatoes	-15.4	-26.7	-36.1	-36.1	42.3		-15.4	-26.7

Underscored numbers are from equations that satisfied *a priori* expectations with regard to parameter signs and magnitudes.

TABLE 3. SELECTED RESULTS OF ADAPTIVE EXPECTATION LP-TIME SERIES EQUATIONS

Commodity	Restriction set	Parameter estimates <sup>a/</sup>			$\bar{R}^2$
		Price, lagged		Coefficient of expectation, own price ( $B_{11}$ )	
		Composite ( $b_{11}$ )	Own ( $b_{11} a_{11}$ )		
Alfalfa hay	a	.326	606,509	-.029	.36
	b	.055	102,325	.832	.20
	c	5.055	9,404,605	-.005	.63
Barley	a	.532	116,793	.483	.87
	b	.095	20,856	.533	.86
	c	.537	117,890	.487	.84
Corn for grain	a	-2.609	-43,581	.113	.74
	b	-1.720	-28,731	.113	.74
	c	-2.613	-43,648	.112	.74
Cotton	a	.381	5,885	1.333	.50
	c	.466	7,198	1.000	.40
	d	.468	7,229	1.000	.40
Dry beans	a	-2.854	-16,787	-.007	.80
	b	-5.262	-30,951	0	.68
	d	-.170	-1,000	.041	.09
Grain sorghum	a	.001	126	-.675	.49
	b	-.051	-6,433	-.725	.51
	d	-.090	-11,352	0	.35
Rice	b	-.763	-39,607	.016	.52
	c	-1.180	-61,254	.095	.38
	d	-.762	-39,555	.022	.51
Safflower	a	.319	9,390	.347	.21
	b	.312	9,184	.347	.21
	c	.319	9,390	.347	.21
Sugar beets	a	.067	127,331	-.038	.63
	b	-1.752	-3,329,615	.137	.36
	c	-2.434	-4,625,732	.032	.19
Asparagus	a	-.055	-361	.354	.70
	b	-.054	-354	.351	.70
	c	-.055	-361	.351	.70
Broccoli	a	-1.888	-14,200	.063	.75
	b	-.842	-6,333	.063	.75
	c	-1.893	-14,237	.063	.75
Cantaloups	a	.960	22,779	.113	.64
	b	.634	15,044	.113	.64
	c	.955	22,660	.114	.64
Lettuce	a	0	0	-8.484	.35
	b	-.157	-14,583	-8.484	.35
	c	.385	35,761	.303	.04
Potatoes	a	.853	40,720	.297	.70
	b	.817	39,001	.297	.70
	c	.852	40,672	.297	.70
Tomatoes	a	0	0	4.689	.09
	b	-.265	-92,186	4.689	.09
	c	-.008	-2,783	1.000	0

<sup>a/</sup> Statistics for maximum likelihood estimates were not calculated.

to 91.9 percent for the partial adjustment equations and from 0.4 to 52.4 for the adaptive expectation equations. Compared without regard to satisfaction of *a priori* expectations, the best prediction for 11 of the 15 commodities was provided by partial adjustment equations.

Percent error is also reported in Table 4 for two straight econometric models, i.e., equations (5) and (6) with  $p_1^* = p_1$ , using only time series data and no prior information from the LP results. These equations are equivalent to LP-based partial adjustment equations in which the only prior information used from LP was the own-price parameters. Predictions from the LP-based equations compared favorably to these single commodity time series equations for 1974. The best prediction was provided by time series equations for two crops, by LP-based equations for seven, and was tied for six. On average, accuracy of the best LP-based predictive equations was comparable to accuracy of the best of the straight econometric models. Shumway and Chang [13, p. 355] also report comparable predictive accuracy between straight econometric models and predictive equations based on simple linear regressions of LP results and the partial adjustment premise.

### Implications of Prediction Equation Parameters

With the preceding evidence that LP-based supply equations depicted the real world as precisely as did single commodity econometric equations, attention is focused finally on implications of the LP-based parameter estimates. Although the LP results were used as prior information, additional parameters were

estimated when time series data were added. Some of these parameters provided evidence that the LP model either over or underestimated supply responsiveness. For example, equations (5) and (6) imply that real world direct and cross price parameters in the supply of commodity  $i$  are estimated by LP-derived parameters multiplied by  $\hat{b}_{ii}/(1-\hat{b}_{i2})$ . From equation (11),  $\hat{b}_{ii}$  is the appropriate adjuster. The closer these coefficients are to 1.0, the closer the LP price parameters to those implied by LP-based predictive equations.

Restricting our attention only to those LP-based equations satisfying *a priori* expectations, and *assuming those equations correctly specified*, implied coefficients by which LP supply parameters should be multiplied are reported in Table 5. Across all crops the average coefficient is 0.91, but individual coefficients vary widely. Five are greater than 1.0, implying that the LP model underestimated real supply responsiveness. The rest are less, implying overestimation of supply responsiveness for the majority of crops. Consequently, it appears that the LP model did not accurately estimate individual crop supply responsiveness.

The ultimate use of this information would be to recalibrate the LP model. Unfortunately, it is not clear how this could be accomplished in a systematic fashion. A first attempt might be to scale all deviations in unit production costs from actual base period crop price up or down depending on whether

the LP over or underestimated supply responsiveness. Widening production cost differences would cause the marginal cost curve to become steeper (i.e., the supply curve less elastic). However, the problem in attempted calibration is that a change in the marginal cost curve of one crop may alter supply curves of several crops because it alters comparative advantage relations. Consequently, use of this information for LP model calibration is likely to be more trial and error than a precise analytic adjustment process.

## SUMMARY

This paper focused on LP model calibration and evaluation. Two calibration procedures for scaling initial cost and yield estimates were briefly reviewed along with an evaluation procedure based on correlations in regional resource use. Major attention was devoted to two methods for combining LP-derived parameters with time series data in making short run (i.e., one year) production predictions.

In general, the predictions were not extremely accurate — of the best predictions for each commodity, six of the 15 missed the mark by more than ten percent. However, they were comparable to the predictive accuracy of single commodity straight econometric models. Furthermore, because of the great flux in the market system that year, predicting output levels in 1974 was a rigorous test. With regard to relative performance, models based on the partial adjustment premise proved to be more satisfactory for the specific cases examined here than models based on the adaptive expectations premise using a maximum likelihood estimation procedure. They satisfied *a priori* sign and magnitude expectations and provided better goodness of fit for more crops than the adaptive expectations procedure. They also predicted more accurately for most crops and were less expensive in terms of computation time.

Both methods have the capability of preserving relative magnitudes of direct and cross relationships derived from parametric iterations of the LP model. Both also permit re-estimation of the absolute magnitude of long run coefficients based on time series data. Although a systematic procedure for using this information in model calibration is not readily apparent, it does serve as an additional basis for evaluating the real world performance of the LP model. In this one regard, the LP model evaluated was found wanting due to wide variability in supply response estimates relative to the time series estimates for individual crops.

TABLE 5. COEFFICIENTS FOR CALIBRATING LP SUPPLY RESPONSIVENESS

Commodity	Coefficients from models satisfying <i>a priori</i> sign and magnitude expectations		
	Number	Range	Average
Alfalfa hay	2	0.06-5.06	2.56
Barley	4	0.07-0.49	0.35
Corn for grain	2	1.63-1.64	1.64
Cotton	4	0.27-0.47	0.40
Dry beans	1	0.07	0.07
Grain sorghum	2	0.03-0.04	0.04
Rice	1	0.04	0.04
Safflower	5	0.06-0.32	0.21
Sugar beets	2	1.41-5.26	3.34
Asparagus	0		
Broccoli	1	0.22	0.22
Cantaloups	5	0.42-2.01	1.10
Lettuce	2	0.28-0.39	0.34
Potatoes	5	0.61-3.09	1.58
Tomatoes, processing	0		



## REFERENCES

- [1] Boussard, J. M. "Time Horizon, Objective Function, and Uncertainty in a Multiperiod Model of Firm Growth," *American Journal of Agricultural Economics*, Volume 53, 1971, pp. 467-77.
- [2] California Crop and Livestock Reporting Service. *Annual Field Crop Summary*, Sacramento, 1976.
- [3] California Crop and Livestock Reporting Service. *Annual Vegetable Summary*, Sacramento, 1975, 1976.
- [4] California Crop and Livestock Reporting Service. *California Field Crop Statistics*, Sacramento, 1958, 1965, 1972, 1973, 1975.
- [5] California Crop and Livestock Reporting Service. *California Vegetable Crops*, Sacramento, 1953, 1955, 1962, 1968, 1973, 1974.
- [6] Cochrane, D. and G. H. Orcutt. "Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms," *Journal of the American Statistics Association*, Volume 44, 1949, pp. 32-61.
- [7] Cooper, J. P. "Asymptotic Covariance Matrix of Procedures of Linear Regression in the Presence of First-Order Autoregressive Disturbances," *Econometrica*, Volume 40, 1972, pp. 305-10.
- [8] Debertin, D. L. and R. J. Freund. "The Deletion of Variables from Regression Models Based on Tests of Significance: A Statistical and Moral Issue," *Southern Journal of Agricultural Economics*, Volume VII, No. 1, 1975, pp. 211-15.
- [9] Nerlove, Marc. "Estimates of the Elasticities of Supply of Selected Agricultural Commodities," *Journal of Farm Economics*, Volume 38, 1956, pp. 496-509.
- [10] Perrin, R. K. "Linear Programming in Rural Development Research," in *Quantitative Techniques with Application to Rural Development Research*, (Ed., G. L. Bradford and F. B. Saunders), Southern Farm Management Research Committee Conference, 1972.
- [11] Rausser, G. C. and Q. Paris. "Sufficient Conditions for Aggregation of Linear Programming Models," *American Journal of Agricultural Economics*, Volume 55, 1973, pp. 659-66.
- [12] Schaller, W. N. and G. W. Dean. *Predicting Regional Crop Production: An Application of Recursive Programming*, USDA ERS Technical Bulletin 1329, April 1965.
- [13] Shumway, C. R. and A. A. Chang. "Linear Programming vs. Positively Estimated Supply Functions—An Empirical and Methodological Critique," *American Journal of Agricultural Economics*, Volume 59, 1977, pp. 344-57.
- [14] Shumway, C. R., G. A. King, H. O. Carter and G. W. Dean. *Regional Resource Use for Agricultural Production in California, 1961-65 and 1980*, Gianinni Foundation Monograph No. 25, University of California, September 1970.
- [15] Talpaz, Hovav. "Nonlinear Estimation by an Efficient Numerical Search Method," *The Review of Economics and Statistics*, Volume 58, 1976, pp. 501-04.
- [16] U.S. Department of Agriculture. *Agricultural Statistics*, 1973.
- [17] U.S. Department of Agriculture. *Farm Index*, Volume 13, No. 4, 1974, p. 23.
- [18] Wallace, T. D. "The General Problem of Spatial Equilibrium: A Methodological Issue," *Interregional Competition Research Methods*, Ed. R. A. King, pp. 11-17, North Carolina State University, Agricultural Policy Institute Series 10, 1964.