

## **DISTORTIONARY IMPACTS OF THE 1982 AND 1986 U.S. TAX CODES ON CAPITAL INVESTMENTS: A CASE STUDY OF INVESTMENT IN ORANGE GROVES**

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### **Abstract**

The 1980s have been a period of dramatic change for the income tax code in the United States. Although numerous modifications were considered in policy deliberations, two key goals, the reduction of the importance of tax considerations in investment decisions and tax simplification, emerged from the discussion and guided drafting of the 1986 Tax Reform Act. This study examines the importance of tax considerations in investment decisions under the provisions of the Tax Reform Act of 1986 and its predecessor, the Tax Equity and Fiscal Responsibility Act of 1982. The study then compares the tax liability under these tax codes with a nondistortionary tax scheme. Results indicate that the Tax Reform Act of 1986 reduced the distortionary effects of the tax code on capital investment decisions. However, a large portion of the reduction can be attributed to the change in the average tax rate.

*Key words:* investment, Tax Reform Act, Tax Equity and Fiscal Responsibility Act, tax distortions.

**D**ramatic changes in the income tax code have occurred in the United States in the 1980s. In 1981, the Kemp-Roth Economic Recovery Tax Act (ERTA) (U.S. Congress, 1981) reduced the number of tax brackets, accelerated depreciation allowances, changed investment tax credit, changed capital expensing, and made numerous other adjustments to the U.S. income tax code. By 1982 the mood had shifted to fiscal responsibility as federal

budget deficits mounted. With the Tax Equity and Fiscal Responsibility Act of 1982 (TEFRA) (U.S. Congress, 1982), some tax breaks granted in ERTA were modified. Most notably, the amount of investment tax credit (ITC) allowed, the basis for depreciable assets, and the amount of capital expenditures that could be expensed were all reduced.<sup>1</sup> However, the essence of the ERTA was preserved. The tax code continued to favor capital formation via the Accelerated Cost Recovery System, ITC, and a 60-percent reduction in the marginal tax rate for capital gains. The tax system recently underwent another, possibly more dramatic, revision with the Tax Reform Act of 1986 (TRA) (Commerce Clearing House, Inc.). This act, among other things, drastically reduced the number of tax brackets and the allowable deductions from adjusted gross income. It also eliminated preferential capital gains treatment and ITC.

These changes in the federal income tax code could have major impacts on the profitability and structure of agriculture in the United States. Since the end of the second world war, U.S. agriculture has become increasingly mechanized. This increased mechanization typically gives rise to assets with finite useful lives which are expensed (i.e., depreciated) against income over the productive life of the asset. Hence, the increased mechanization in agriculture has left the sector more sensitive to changes in cost recovery or depreciation allowances and ITC in the federal income tax code. Also, since a large portion of the assets in the sector are real estate, recent changes in the tax rate for capital gains could also signifi-

<sup>1</sup>Technically, TEFRA allowed the taxpayer to either reduce the depreciable basis of the asset by half of the ITC claimed, or to reduce the amount of ITC to 8 percent. For the purpose of this study, we assume that the decision maker chooses to reduce the amount of ITC claimed.

cantly affect U.S. agriculture. Specifically, a higher marginal tax rate on capital gains could reduce agricultural real estate prices which would decrease net worth in the farm sector.

The agricultural economics profession has been quick to embrace research on the effects of tax policy on individual producers, as well as on the sector as a whole (Durst). Most of the work has centered around implications of tax considerations, such as capital gains exclusions, investment tax credit, capital expensing (Sec. 179 or additional first-year depreciation), and depreciation (Chisholm, Kay and Rister, Lynne). These studies suggest that tax considerations are a significant factor in agricultural financial decisions. In fact some have labeled the tax code the "silent farm bill," (Senator Thomas Daschle in the *Washington Post*, 14 October 1985) whose benefits appear to be skewed to the high-income investor (Hanson and Bertelson, Hanson and Eidman).<sup>2</sup>

This study examines how income tax code provisions for depreciation, investment tax credit, and capital gains exclusions distort investment decisions. The relative distortionary effects on the investment decisions are then compared for the 1982 and 1986 legislation using an empirical version of Samuelson's model of a nondistortionary tax policy.

In the following section, Samuelson's derivation of a nondistortionary tax policy is examined, and a procedure for measuring the distortion of tax policies on the profitability of an investment is developed. This technique is then applied to a hypothetical investment in a Florida orange grove to determine the relative importance of income tax considerations under the 1982 and 1986 tax codes as amended by the Technical and Miscellaneous Revenue Act of 1988. Results are then presented that indicate that the distortionary effects, though still significant, declined under the TRA of 1986.

## A NON-DISTORTIONARY TAX POLICY

This study is primarily interested in determining the effect of the change in depreciation allowance, investment tax credit, capital gains exclusions, and tax rates on investment in orange groves. These tax provisions were designed to stimulate business investment.

<sup>2</sup>The argument that the current tax policies may encourage firm growth is not unanimous. Davenport et al. cite a study that provides evidence that the current tax structure may retard firm growth.

<sup>3</sup>Some arguments for the use of marginal tax rates over the average tax rates exist. For example, the effect of firm wealth on the marginal unit of investment is the marginal tax rate. However, the average tax rate embraces more aspects of the change in tax legislation. In addition, it is probably more consistent with the spirit of Samuelson's model.

Unfortunately, the resulting distortion of investment decisions may lead to misallocation of resources between industries, misallocation of labor versus capital, and undesirable distributional effects across income brackets. Hence, this section develops a theoretical model to measure the effect of these distortions on the economic value of capital investments.

The theoretical model is a discrete form of Samuelson's continuous model. Samuelson developed a depreciation stream for a capital investment such that the economic value of the investment is independent of the tax rate. He proposes the following general theorem:

Fundamental theorem of tax-rate invariance—  
If, and only if, the true loss of economic value is permitted as a tax-deductible depreciation expense will the present value of a cash-receipt stream be independent of the rate of tax (Samuelson, p. 604).

A discrete version of Samuelson's theorem is developed along with a method for empirically deriving the distortion in investment profitability arising from the tax code.

The present value of an investment in period  $t$ , given average tax rate<sup>3</sup>  $T$ , is:

$$(1) V_t(T) = \sum_{j=t}^n \frac{[N_j - T \cdot (N_j - D_j)]}{\prod_{k=t}^j (1 + i_k \cdot (1 - T))}$$

where  $N_t$  is the cash flow from the investment in period  $t$ ,  $i_k$  is the discount rate in period  $t$ , and  $D_t$  is depreciation allowance in period  $t$ . The numerator of equation (1) is after-tax cash flow in each period  $t$ . Appendix 1 shows that equation (1) can be rewritten expressing  $V_t(T)$  as a function of its own first difference:

$$(2) V_t(T) = \frac{1}{i_{t-1}(1-T)} [1 + i_{t-1} \cdot (1-T)] \cdot \Delta V_{t-1}(T) + N_{t-1} - T \cdot (N_{t-1} - D_{t-1}).$$

The present value of an asset in period  $t$  is thus a function of the change in the value of the asset in the preceding period  $\Delta V_{t-1}(T)$  and the cash flow generated in the preceding period  $(N_{t-1} - T \cdot (N_{t-1} - D_{t-1}))$ .

Using the relationship in equation (2), Appendix 2 shows that the tax invariant sequence of depreciation charges becomes:

$$(3) D_t = - [i_t \cdot V_{t+1} - N_t] / (1+i_t),$$

or

$$(4) D_t = - \Delta V_t(T) = - \Delta V_t(0).$$

To obtain this result, Samuelson notes that if the economic value of the investment is invariant to tax considerations, then  $V_t(T)$  for a general average tax rate,  $T$ , must equal  $V_t(0)$  or the case of a zero average tax rate by construction.<sup>4</sup> This result also implies that  $\Delta V_t(T)$  must equal  $\Delta V_t(0)$  or the theorem would not hold at every point in time. Thus, the present value of an investment in a nondistortionary tax policy would, by definition, equal the present value of the same investment under any tax policy with an average tax rate of zero. Hence, the distortionary effect of a given tax regime on a particular investment, given an average tax rate, is simply the difference between the present value of the investment at a zero tax rate and the present value of the investment considering taxes.

A depreciation allowance structured as in equation (4) would produce identical investment decisions for producers regardless of their average income tax rate. Thus, this tax code would remove the impetus to "farm the tax code." Investments would be made on the basis of their economic value implied by the market. Such a tax policy would be scale neutral. Further, it would remove tendencies to overinvest in intermediate assets because of tax considerations. Therefore, such a code could reduce the possibility of the financial stress in agriculture observed in the early and mid 1980s.

### DATA

The tax results for this study are based on a case study of Hamlin oranges in southern Florida. The results represent the cost and returns from the establishment and operation of a grove of Hamlin oranges over a 25-year period. The Hamlin grove investment includes the cost of the land, land preparation, and microsprinkler irrigation. The cumulative costs for the first five years represent the capitalized costs for the grove investment (i.e., cost of land preparation, cost of planting and caring for the citrus trees through the fourth year after planting). Year one is assumed to be devoted to land preparation and government

permitting for water and land use. Trees are assumed to be planted in year two of the investment. Total trees planted per net grove acre is 140 with an annual tree attrition rate of three trees per acre. Oranges are assumed to be marketed on tree for \$5/box in each year.<sup>5</sup> For further detail on the physical layout of the orange grove or the fixed and variable costs see Muraro and Fairchild.

### EFFECT OF TAX LEGISLATION ON INVESTMENT IN ORANGE GROVES

From an orange producer's perspective, the 1986 TRA had four major consequences. First, the TRA made major changes in the tax rate schedules. Second, capital gains deductions were eliminated. Third, the TRA removed the investment tax credit. These three changes have broad implications for agriculture as a whole, and they are important considerations for most Florida producers. The final effect which is more specific to Florida orange groves reclassifies orange groves from five-year ACRS property to 10-year straightline property for depreciation purposes (U.S. Congress, 1988).

The effect of these changes in the tax code on orange grove investments can be computed by measuring the difference in the economic profitability of the investment with and without taxes, divided by the present value of the investment without taxes. Thus, it is possible to determine empirically whether the change in tax policy actually reduced the importance of tax considerations in the investment decision. In addition, results calculated using equation (4) will allow comparison of the actual taxes paid under each tax policy with the taxes paid under the Samuelson tax invariant policy.

### DISTORTIONARY EFFECTS OF TAX CODE CHANGES ON INVESTMENT DECISIONS

The TRA changed both the tax rate, depreciation, and investment tax credit, and the capital gains provisions. While it is useful to know the net effect of the tax code changes on the distortion of investment decisions under the TRA, it is also useful to know the contribution of each component. In other words, what is the effect of the change in the average tax rate relative to the change in depreciation, investment tax credit, and capital exclusion on the investment decision?

<sup>4</sup>Note that the tax flow in the numerator and the after-tax discount rate both change as the average tax rate changes.

<sup>5</sup>On-tree price equals farm-gate price net of picking and hauling costs of harvested fruit.

The problem then is to determine what portion of the change in investment decision is due to changes in the average tax rate versus changes in tax depreciation allowances, investment tax credit, and capital gains exclusion. In order to do this, one of the factors has to be held constant while the other varies. This study will value the change in depreciation allowances, investment tax credit, and capital gains exclusion while maintaining the TRA average tax rates. The effect of the change in tax rates will be calculated holding the TEFRA tax preferences constant. Hence, the results may tend to overestimate the effect of the changes in tax rates and underestimate the effect of the change in depreciation, investment tax credit, and capital gains exclusion.<sup>6</sup>

### **Effect of the Change in Average Tax Rates**

The results in Table 1 indicate that the distortionary effects of the tax code on the investment decision increase as tax rates increase. The percentage increase in net present value for the orange grove attributable to tax considerations reaches a high of 202 percent of the net present value without taxes for trees, irrigation, and land preparation under TEFRA. It is apparent by moving down the tax scale that a large portion of this distortion is attributable to the tax rate. For example, the gain due to tax considerations under TEFRA falls to 168 percent if family income declines to \$100,000. Applying the average tax rate under TRA to the tax allowances under TEFRA confirms this suspicion. A family with annual income of \$200,000 experienced a 46 (i.e., 202 - 156) percent reduction in the net present value resulting from the decline in tax rates under the TRA.

The portion of the tax distortion attributable to the change in average tax rate is an increasing function of taxable income because the TRA legislated larger declines in average tax rates for higher taxable incomes. For example, a family with a combined income of \$20,000 received only a 0.6 percent decline in their average tax rate under TRA, while a family with combined income of \$200,000 received an 11.4-percent decline in their average tax rate (Hanson and Bertelson).<sup>7</sup> The tax

distortion for a family with combined income of \$20,000 under the TEFRA code at the TEFRA rate was 90.0 percent and under the TRA rate 87.7 percent. Thus, the reduction in tax distortion resulting from the decline in average tax rates was only 2.3 percent at a \$20,000 income compared to 46.4 percent at a \$200,000 income.

The higher the average tax rate, the greater the distortionary effect of any tax policy which deviates from the Samuelson model. Thus, a large part of Congress' goal to reduce the distortionary effect of taxes on investment decisions can be accomplished simply by decreasing the average tax rate. If the current tax depreciation schedules and other preferences in the code deviate from the nondistortionary depreciation schedules, the only way to eliminate the tax distortion is to set the average tax rate to zero. Thus, to examine whether or not the tax code has become less distortionary without confusing the effect of changes in the tax rates, the average tax rate is held constant and the distortionary effects due to changes in depreciation, investment tax credit, and capital gains exclusions are determined.

### **Effect of Changes in Depreciation, Investment Tax Credit, and Capital Gains Exclusion**

The change in distortion of the investment decision attributable to changes in depreciation, investment tax credit, and capital gains provisions indicates that the TRA is less distortionary than TEFRA. However, the distortionary effects of the TRA are still significant. For example, Table 1 indicates that for a family with income of \$200,000, the TRA tax provisions result in a 13-percent increase in the net present value of an investment in trees, irrigation, and land preparation.

The distortionary effects of the changes in depreciation, investment tax credit, and capital gains provisions are also an increasing function of income. However, the distortionary effect of these changes is less sensitive to income levels than is the effect of changes in average tax rates. For a family with total income of \$20,000, the distortionary effect on the net present value of an investment in an orange grove fell from 88 percent under the TEFRA preferences to 74 percent under the

<sup>6</sup>Assuming that the TEFRA tax codes were more distortionary than the TRA tax codes, any change in average tax rate would result in a larger change in distortion *ceteris paribus*. This phenomenon is similar to the use of a Laspeyres index to measure inflation.

<sup>7</sup>The average tax rates in Hanson and Bertelson include adjustments for the change in zero bracket amounts, the effect of tax surcharges, and changes in self-employment taxes.

TRA preferences. For a family income of \$200,000, the distortion fell from 156 percent to 131 percent.

**Total Effect of the Change in Tax Code**

It is technically incorrect to compare the relative change in investment decisions between the change in average tax rate and the change in depreciation allowances, investment tax credit, and capital gains exemption. Under Samuelson's tax invariance principle, any tax rate could be nondistortionary depending on the depreciation schedule. However, given that the tax considerations distort the investment decision, a higher average tax rate leads to greater distortion. Thus, the average tax rate is a multiplier for any distortion in investment considerations arising from tax considerations.

At lower levels of income, changes in depreciation allowances, investment tax credit, and capital gains exclusions are the most significant component of distortion. If family income is \$10,000, the change in distortion due to the change in tax preferences is 12 percent versus the change in distortion due to the change in average tax rate of 1 percent. At the \$50,000 level, the change in distortion due to the average tax rate is 5 percent versus a change due to tax preferences of 19 percent. Hence, the change in distortion for lower income levels is primarily attributable to changes in depreciation allowances, investment

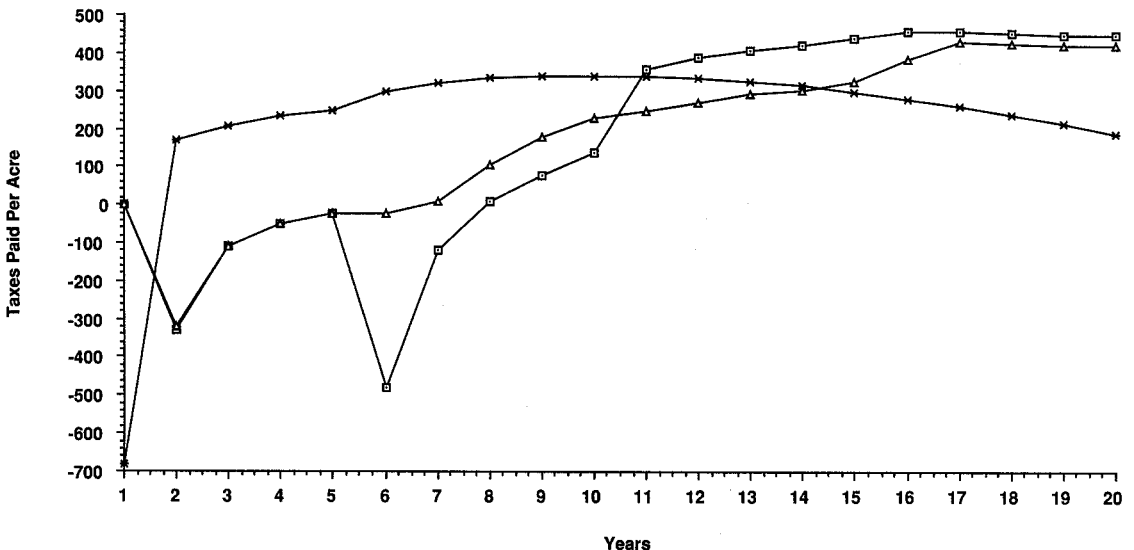
tax credit, and capital gains exclusion. However, for families with incomes of \$100,000 or greater, the effect of the change in average tax rate is greater than the effect of the change in tax preferences.

These results have implications for the distributional effects of the TRA. If the change in distortion arises primarily from the change in the average tax rate for higher income levels, then the primary effect of TRA may be to decrease taxes paid by higher income taxpayers. Thus, the differential reductions in average tax rates under TRA may be interpreted as favoring higher income producers. However, the distortion due to depreciation, investment tax credit, and capital gains exclusion also declined, holding the average tax rate constant. Therefore, the TRA also took a step toward a nondistortionary tax policy.

**COMPARISON OF TAXES PAID THROUGH TIME**

In the preceding section, the distortionary effects of the tax code were evaluated in terms of the net present values of an investment in an orange grove. However, additional information may be obtained by looking at the distortionary effect of the tax codes over time. This section presents taxes paid in each year on an acre of oranges under two income levels, and for TEFRA, TRA, and nondistortionary tax scenarios.

Figure 1 shows the taxes paid per acre under each policy scenario assuming a family in-



Tax Assumptions	□ TEFRA Tax Rate and Depreciation
	△ TRA Tax Rate and Depreciation
	* TRA Tax Rate, Samuelson Depreciation

Figure 1. Taxes Paid Per Acre With Family Income of \$50,000.

TABLE 1: PERCENTAGE CHANGE IN NET PRESENT VALUE OF INVESTMENT IN AN ORANGE GROVE ATTRIBUTABLE TO TAX CONSIDERATIONS

Income	Average Tax Rate <sup>a</sup>		TEFRA Tax Provisions <sup>b</sup>		TRA Tax Provisions and Rates
	TEFRA	TRA	TEFRA Tax Rates	TRA Tax Rates	
\$	----- percent -----				
	<b>Trees Only<sup>c</sup></b>				
10,000	7.7	7.5	39	39	32
20,000	11.9	11.3	47	46	39
50,000	21.5	20.3	67	64	54
100,000	31.4	25.5	88	75	63
200,000	40.0	28.6	106	82	68
	<b>Trees, Irrigation, and Land Preparation<sup>d</sup></b>				
10,000	7.7	7.5	74	73	61
20,000	11.9	11.3	90	88	74
50,000	21.5	20.3	128	123	104
100,000	31.4	25.5	168	144	121
200,000	40.0	28.6	202	156	131
	<b>Trees, Irrigation, Land Preparation, and Land<sup>e</sup></b>				
10,000	7.7	7.5	271	269	225
20,000	11.9	11.3	330	321	271
50,000	21.5	20.3	467	450	379
100,000	31.4	25.5	614	526	442
200,000	40.0	28.6	742	572	478

<sup>a</sup> Source: Hanson and Bertelsen.

<sup>b</sup> Tax Provisions are defined for the purpose of this table as depreciation, investment tax credit, and capital gains exclusion.

<sup>c</sup> The present value of the cash flows arising from the trees alone.

<sup>d</sup> The present value of the cash flows arising from the trees, the irrigation system, and land preparation.

<sup>e</sup> The present value of the cash flows arising from the trees, the irrigation system, land preparation, and changes in land values.

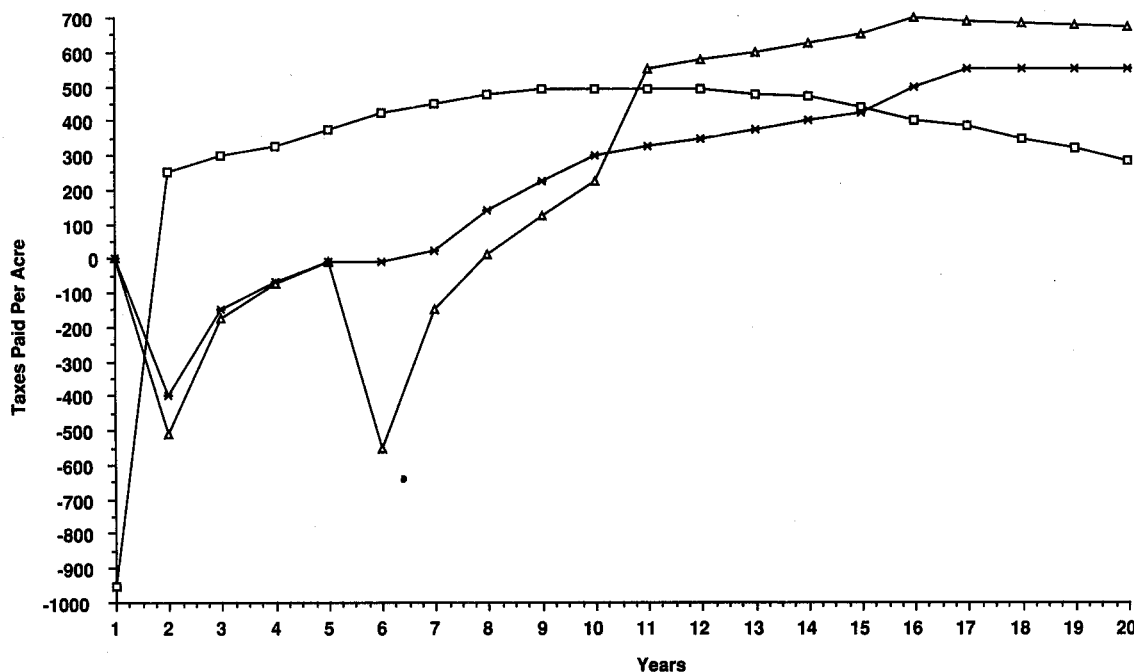
come from all sources, both farm and off farm, of \$50,000. A negative taxes paid number indicates that the investment is generating tax savings in either tax credits or deductions that can be used to offset other income. The figure indicates that planting an orange grove yields tax savings through year seven under TEFRA and year six under TRA. The large tax savings in year six under TEFRA is due primarily to investment tax credit, since recognition of investment tax credit is delayed until the investment yields positive cash flow. Further, while both TEFRA and TRA yield tax savings in early years, TEFRA yields the largest tax savings.

During the productive phase, after the trees begin to yield sufficient oranges to meet annual cost, taxes paid under TRA exceed taxes paid under TEFRA for four years. After the fourth year, however, taxes paid per acre under TEFRA exceed taxes paid under TRA. This divergence is due to two changes, the change in average tax rate and the change in depreciation schedules. From year 17 to the end of the investment, income is taxed at a higher rate under TEFRA, and the depreciation expense with 10-year straightline ends in year 16. Thus, over this time period, taxes paid under TRA are proportionately lower. From

year six to year 17, however, the difference is the result of the change in tax rate and the change from five-year ACRS to 10-year straightline depreciation.

Taxes paid under TEFRA and TRA have similar patterns through time: tax advantages in early years with tax disadvantages in latter years. Both tax codes are markedly different from the tax liability under the Samuelson or tax-invariant depreciation scheme. Under the Samuelson depreciation, the initial period offers a large tax advantage. However, the taxes paid per acre become positive in year two and remain positive throughout the life of the asset. The increase in taxes paid per acre until year 10 is due to the fact that an orange grove increases in value until the average age of a tree in the grove reaches 11 years. Note that the major deviation of TRA and TEFRA from the tax invariant depreciation occurs early in the grove's life. Therefore, the higher the discount rate, the greater the distortion.

Figure 2 presents the taxes paid per acre for a family with annual income of \$200,000. The overall patterns for the higher income family are consistent with the taxes paid per acre with a family income of \$50,000. However, since the change in average tax rate was



Tax Assumptions	□ TEFRA Tax Rate and Depreciation
	△ TRA Tax Rate and Depreciation
	* TRA Tax Rate, Samuelson Depreciation

Figure 2. Taxes Paid Per Acre With Family Income of \$200,000.

relatively higher for family incomes of \$200,000, the separation between the taxes paid under TRA and TEFRA is wider for years one to five and after year 17. Further, taxes paid under TRA now cut the Samuelson tax liability in year 15 instead of year 14. Hence, the distortionary effect is probably more sensitive to changes in the discount rate as income increases.

## CONCLUSIONS

This study investigated the change in distortionary effects on capital investments between the 1982 TEFRA and the 1986 TRA. The analysis focused on the investment decision for an acre of oranges in southern Florida. The study found that the distortion-

ary effect of tax policies on the investment decision under both the TEFRA and TRA were relatively large. Further, consistent with intuition, the stimulus to investment increased as taxable income increased.

In aggregate, the change in tax law in 1986 reduced the significance of tax considerations in the investment decision. Furthermore, the reduction in distortion increased as income increased. Unfortunately, the majority of the decrease in distortion for high-income households resulted from the reduction in average tax rates. However, the reduction of tax distortion due to changes in depreciation and other tax preferences was also significant for all income levels.

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## APPENDIX 1

Equation (1) in the text can be reexpressed as a difference equation to facilitate the remainder of the proof. Using Goldberg's definition, the first difference of equation (1) from the text is

$$(A.1) \Delta V_t(T) = V_{t+1}(T) - V_t(T)$$

$$\begin{aligned} \Delta V_t(T) &= \sum_{j=t+1}^n \frac{N_j - T(N_j - D_j)}{\prod_{k=t+1}^j (1 + i_k(1-T))} - \sum_{j=t}^n \frac{N_j - T(N_j - D_j)}{\prod_{k=t}^j (1 + i_k(1-T))} \\ &= V_{t+1}(T) - \frac{N_t - T(N_t - D_t)}{(1 + i_t(1-T))} - \frac{1}{(1 + i_t(1-T))} \\ &\quad \sum_{j=t+1}^n \frac{N_j - T(N_j - D_j)}{\prod_{k=t+1}^j (1 + i_k(1-T))} \\ &= V_{t+1}(T) - \frac{N_t - T(N_t - D_t)}{(1 + i_t(1-T))} - \frac{V_{t+1}(T)}{(1 + i_t(1-T))} \end{aligned}$$

$$(A.2) V_{t+1}(T) = \frac{1}{i_t(1-T)} (1 + i_t(1-T))\Delta V_t(T) + N_t - T(N_t - D_t).$$

## APPENDIX 2

Using a form of A.2, the difference of the present value of the investment becomes

$$(A.3) \Delta V_t(T) = \frac{i_t(1-T)V_{t+1}(T) - N_t + T(N_t - D_t)}{(1 + i_t(1-T))}.$$

If the marginal tax rate is zero, this becomes

$$(A.4) \Delta V_t(0) = \frac{i_t V_{t+1}(0) - N_t}{(1 + i_t)}.$$

Recognizing that if the sequence of depreciation is chosen so that the economic value of the investment is invariant to the marginal tax rate, then  $\Delta V_{t+1}(T) = \Delta V_{t+1}(0)$  and  $V_t(T) = V_t(0) = V_t$ . To derive the sequence of depreciation, solve for  $\{D_t\}_{t=1}^n$  such that

$$(A.5) \quad \Delta V_{t+1}(T) = \Delta V_{t+1}(0)$$

$$\frac{(i_t(1-T)V_{t+1} - N_t + T(N_t - D_t))}{(1 + i_t(1-T))} = \frac{(i_t V_{t+1} - N_t)}{(1 + i_t)}$$

$$N_t - D_t = \frac{[i_t N_t + i_t V_{t+1}]}{(1 + i_t)}$$

$$D_t = \frac{[i_t V_{t+1} + i_t N_t]}{(1 + i_t)},$$

or just as Samuelson's theorem states

$$(A.6) D_T = -\Delta V_t(0).$$

