AN ANALYSIS OF DELIVERY-PERIOD BASIS DETERMINATION FOR LIVE CATTLE*

William Vollink and Ronald Raikes

Because level and variability of the basis at the time a hedge is lifted affects level and variability of returns from hedging [1, 4], an understanding of the determination of basis values is important both to hedgers and to those with regulatory responsibilities. This analysis focuses on the determination of par-delivery-point basis values during the delivery period of a live-cattle futures contract; i.e., on determination of the difference between futures price and cash price for a lot of live cattle that meets all futures contract specifications (including place and time of delivery). Results of the analysis suggest that, during the delivery period, par-delivery-point basis values for live cattle frequently differ from zero by more than the transaction costs associated with arbitrage, and that fluctuations in basis values may be partly explained by trader expectations and by risk associated with returns to arbitrage.

An initial hypothesis about the determination of par-delivery-point basis values during the delivery period is that arbitrage will force these values to differ from zero by not more than the transaction costs associated with arbitrage [2, 7]. This hypothesis may be derived from the following reasoning: If the basis is positive and greater than transaction costs, then an arbitrageur can profit by simultaneously buying the cash commodity, selling a futures contract and delivering on the contract. If, on the other hand, the basis is negative and its absolute value exceeds transaction costs, an arbitrageur can buy a futures contract, accept delivery, and sell on the cash market. Thus, if the absolute value of the basis exceeds transaction costs, profit opportunities are provided for arbitragers, and arbitrage will force the basis toward an absolute value equal to or less than transaction costs.

The hypothesis that the absolute value of the par-delivery-point basis does not exceed transaction costs during delivery periods for live-cattle contracts is examined in the following section. Then, a general model of par-delivery-point cash price and basis determination in the delivery period is developed and analyzed. Next, an estimable reduced-form equation is derived for the equilibrium par-delivery-point basis in the delivery period. Results obtained by estimating this equation are presented and discussed. Finally, some conclusions and policy recommendations are offered.

DATA AND INITIAL RESULTS

Data and summary statistics in Table 1 are used to examine the hypothesis that the absolute value of the par-delivery-point basis for live cattle is less than the transaction cost throughout the delivery period. The table presents: (a) basis values for each business day of the delivery period for each live-cattle contract maturing from February 1974 through February 1976, (b) the mean basis value for each of these contracts, (c) the standard deviation of the mean value and (d) the t-ratio computed by dividing the absolute value of the mean basis, less transaction costs, by the standard deviation of the mean basis. The basis values were calculated by subtracting the midpoint of the closing price range for choice, 1000-1200 lb. steers at the Omaha terminal market.

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Table 1. Delivery-Period Basis Values and Summary Statistics for Live-Cattle Contracts Maturing From February 1974 to February 1976

<table>
<thead>
<tr>
<th>Contract</th>
<th>Beginning of delivery period</th>
<th>End of delivery period Mean</th>
<th>$\sigma$</th>
<th>t-value $^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>February</td>
<td>-0.03</td>
<td>-0.15</td>
<td>-0.78</td>
<td>-0.17</td>
</tr>
<tr>
<td>April</td>
<td>0.92</td>
<td>1.10</td>
<td>0.98</td>
<td>1.38</td>
</tr>
<tr>
<td>June</td>
<td>-1.41</td>
<td>-1.35</td>
<td>-1.88</td>
<td>-0.63</td>
</tr>
<tr>
<td>August</td>
<td>0.65</td>
<td>0.55</td>
<td>1.55</td>
<td>-0.25</td>
</tr>
<tr>
<td>October</td>
<td>1.60</td>
<td>0.53</td>
<td>0.85</td>
<td>0.55</td>
</tr>
<tr>
<td>December</td>
<td>1.67</td>
<td>1.03</td>
<td>0.55</td>
<td>0.85</td>
</tr>
<tr>
<td>1975</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>February</td>
<td>-0.78</td>
<td>-0.28</td>
<td>0.00</td>
<td>-0.10</td>
</tr>
<tr>
<td>April</td>
<td>1.45</td>
<td>1.40</td>
<td>0.63</td>
<td>0.40</td>
</tr>
<tr>
<td>June</td>
<td>-0.40</td>
<td>-0.30</td>
<td>2.55</td>
<td>0.72</td>
</tr>
<tr>
<td>August</td>
<td>-2.80</td>
<td>-1.41</td>
<td>-2.60</td>
<td>-0.72</td>
</tr>
<tr>
<td>October</td>
<td>-0.93</td>
<td>-0.93</td>
<td>-1.61</td>
<td>-2.33</td>
</tr>
<tr>
<td>December</td>
<td>1.53</td>
<td>1.32</td>
<td>1.15</td>
<td>0.55</td>
</tr>
<tr>
<td>1976</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>February</td>
<td>-0.75</td>
<td>-1.10</td>
<td>-0.73</td>
<td>-0.15</td>
</tr>
</tbody>
</table>

$^a$ During February 1974 to February 1976, the delivery period for each contract began on the first business day after the 5th of the delivery month and ended on the last business day before the 21st of the month.

$^b$t-ratio is computed by dividing the absolute value of the mean basis less $0.10/cwt. transaction cost by the standard deviation of the mean.

$^*_P < 0.1.$

from the closing price of the maturing Chicago Mercantile Exchange live-cattle futures contract. Transaction costs, which are approximately equal to brokerage fees, were estimated to be $0.10/cwt.¹

Four characteristics of the data and summary statistics in Table 1 deserve emphasis. First, daily-basis values frequently differ from zero by more than the estimated transaction costs. Second, even mean values of the basis in the delivery periods differ from zero by substantially more than transaction costs. Mean values for three of the contracts differ from zero by significantly more than transaction costs, and absolute values of the mean basis less transaction costs exceed $1.00/cwt. for four contracts. Third, the mean basis values are not all of the same sign; eight are positive and five negative. Fourth, the daily-basis values are quite variable. They range from $-2.80/cwt. to $2.55/cwt., and day-to-day changes of $1.00/cwt. or more are not infrequent.²

Data in Table 1 offer evidence that the par-delivery-point basis for live cattle fluctuates considerably during the delivery period and often differs from zero by more than transaction costs associated with arbitrage. Because basis variability reduces the attractiveness of hedging, efforts to reduce this variability may be warranted. Such efforts are more likely to be successful if based on an accurate description of basis determination and a knowledge of factors affecting the level and variability of basis values. In the following section, a model describing cash-price and basis determination is developed, exogenous factors affecting basis determination are identified and the qualitative impacts of changes in these factors on basis values are determined.

A Model of Delivery-Period Cash-Price and Basis Determination

The model of cash-price and basis determination presented here involves two markets, the cash-commodity market and the futures market. First, supply and demand functions and equilibrium con-
ditions for the cash and futures markets are presented and discussed. Then the two-market equilibrium condition and static and comparative-static results are derived. The model presented here builds on Stein's [6] analysis of spot and futures price determination but differs from his analysis in that it explicitly focuses on the delivery period of a futures contract, on daily basis values and on a continuously-produced commodity that is costly to store.

Throughout development of the model, it is assumed that market participants are expected utility maximizers, have utility functions that display positive absolute risk aversion, and have decreasing absolute risk aversion over wealth. Sandmo [5] has shown that, if an individual has these characteristics, and if \( x^*_i \) is the utility maximizing level of activity \( i \) when the expected return from a unit of activity \( i \) is \( R_i \), then, for a given variance of the return, \( \delta x^*_i / \delta R_i > 0 \). That is, for a given variance of the return, the utility maximizing level of an activity (e.g., arbitrage) is positively related to its expected return.

The total daily supply of cash commodity (\( Q_{cs} \)) is given by:

\[
Q_{cs} = Q_{secu}(P) + Q_{su}(P, B, T) + Q_{shd}(B)
\]

where \( \partial Q_{secu} / \partial P > 0, \partial Q_{su} / \partial P > 0, \partial Q_{su} / \partial B > 0, \partial Q_{shd} / \partial B > 0, \) and \( \partial Q_{shd} / \partial B < 0 \).

Total daily supply of the cash commodity is composed of four parts: quantity not hedged before this day and sold in the cash market on this day (\( Q_{secu} \)); quantity not hedged but arbitrated [i.e., futures contracts are sold and immediate delivery is made] (\( Q_{su} \)); quantity hedged and delivered (\( Q_{shd} \)); and quantity hedged and offset [i.e., futures contracts are purchased and the commodity is sold in the cash market](\( Q_{sho} \)). \( Q_{secu} \) and \( Q_{su} \) are positively related to current cash price, \( P \). \( Q_{su} \) and \( Q_{shd} \) are positively related to current basis, \( B \). \( Q_{sho} \) is negatively related to \( B \), \( B = F - P \) where \( F \) is the current futures price. \( Q_{su} \) is negatively related to \( T \), the transaction cost.

The total daily demand for the cash commodity (\( Q_{cd} \)) is given by:

\[
Q_{cd} = Q_{dcu}(P) + Q_{du}(P, B, T) + Q_{dha}(B)
\]

where \( \partial Q_{dcu} / \partial P < 0, \partial Q_{du} / \partial P < 0, \partial Q_{du} / \partial B < 0, \partial Q_{dha} / \partial B < 0, \) and \( \partial Q_{dha} / \partial B > 0 \).

Futures contracts are supplied by those who perform arbitrage by selling futures contracts and delivering the cash commodity and by long hedges who lift their hedges by offsetting. Thus, total daily supply of futures contracts is positively related to the current basis and current cash price and negatively related to the transaction cost.

The total daily demand for futures contracts (\( Q_d \)) is given by:

\[
Q_d = Q_{sho}(B) + Q_{du}(P, B, T)
\]

where \( \partial Q_{sho} / \partial F_e > 0, \partial Q_{du} / \partial P < 0, \partial Q_{sho} / \partial B < 0, \) and \( \partial Q_{sho} / \partial T > 0 \).

Equilibrium in the futures market on a given day during the delivery period occurs when total supply of futures contracts equals quantity demanded:

\[
Q_{cs} = Q_{cd}
\]

Equilibrium in the cash-commodity market on a given day during the delivery period occurs when quantity supplied equals quantity demanded:

\[
Q_{cs} = Q_{cd}
\]

The total daily supply of futures contracts (\( Q_s \)) is

\[
Q_s = Q_{su}(P, B, T) + Q_{dha}(B)
\]

where \( \partial Q_{su} / \partial P > 0, \partial Q_{su} / \partial B > 0, \partial Q_{su} / \partial T > 0, \partial Q_{dha} / \partial B > 0, \) and \( \partial Q_{dha} / \partial B < 0 \).

Net demand by speculators, \( Q_{fsp} \), is positive if speculators desire a net long position and negative if they desire a net short position. Net demand is positively related to the expected futures price, \( F_e \), and negatively related to the current futures price, \( P + B \). The sign of \( \partial Q_{fsp} / \partial T \) depends on the sign of \( Q_{fsp} \); if \( Q_{fsp} \) is positive, \( \partial Q_{fsp} / \partial T < 0 \), but if \( Q_{fsp} \) is negative, \( \partial Q_{fsp} / \partial T > 0 \).

Equilibrium in the futures market on a given day during the delivery period occurs when total quantity of futures contracts supplied equals the
total quantity demanded.

\[ Q_d = Q_s \quad (6) \]

Simultaneous equilibrium in the two markets occurs when both equilibrium conditions, equations (3) and (6), are satisfied. These form a system of two equations in the two endogenous variables, B and P, and the two exogenous variables, \( F_e \) and \( T \).

To derive static and comparative-static results for the model, two additional assumptions must be introduced: They are: \( \partial Q_{sho}/\partial B - \partial Q_{sho}/\partial B - \partial Q_{sho}/\partial B > 0 \), and \( \partial Q_{do}/\partial B - \partial Q_{dha}/\partial B - \partial Q_{do}/\partial B > 0 \). Briefly, they presume a change in the current basis will have a greater impact on quantities supplied and demanded by hedgers who offset than on hedgers and arbitragers who make or accept delivery on futures contracts. Given these assumptions and the assumption that equations (3) and (6) have continuous partial derivatives, the endogenous variable Jacobian is negative, and the unique equilibrium values for B and P exist and may be expressed as

\[ \tilde{B} = \tilde{B}(F_e, T), \text{ and} \quad (7) \]
\[ \tilde{P} = \tilde{P}(F_e, T). \quad (8) \]

Comparative static results show the qualitative impacts of changes in the exogenous variables on the equilibrium values of the endogenous variables. The comparative static derivatives are obtained by totally differentiating the equilibrium conditions with respect to B, P, \( F_e \), and T. First, setting \( dF_e = 0 \) and, then, \(dT = 0\), we find that

\[ \frac{dB}{dT} > 0, \quad \frac{dP}{dT} > 0. \]

Thus, ceteris paribus, an increase in the transaction cost may cause either an increase or a decrease in the equilibrium basis and either an increase or a decrease in the equilibrium cash price on a given day during the delivery period. An increase in the expected futures price will increase the equilibrium basis and cash price.

Although the importance of risk and participants' attitudes toward risk was recognized in the model's development, parameters representing the amount of risk associated with returns for various activities (e.g., arbitrage) were not explicitly incorporated in the model and therefore do not appear in the solution equations (7) and (8). However, the degree of risk and participants' attitudes toward it likely affect relationships between endogenous and exogenous variables in equations (7) and (8) and, ceter. par., a change in the degree or risk or attitudes toward risk likely will affect the equilibrium values \( \tilde{B} \) and \( \tilde{P} \).

**EMPIRICAL ANALYSIS**

Results of an empirical analysis of par-delivery-point basis values for live cattle provide two types of information. First, they provide a test of the hypothesized sign of one of the comparative static derivatives, \( dB/dF_e \). And second, they provide information about the proportion of variation in the par-delivery-point live-cattle basis that is explained by futures-price expectations of speculators and the proportion that remains to be explained by other factors, including risk.

The first step in the empirical analysis was to derive an estimable equation relating the equilibrium basis to the exogenous variables. Several assumptions were introduced, the first being that the reduced-form equation is linear

\[ \tilde{B}_t = a_0 + a_1 T_t + a_2 F_{et} \quad (9) \]

where \( t \) denotes the day of the delivery period. Second, it was assumed that \( T \) is constant; thus, equation (9) becomes

\[ \tilde{B}_t = \alpha_0 + a_2 F_{et} \quad (10) \]

where \( \alpha_0 = (a_0 + a_1 T) \). Third, it was assumed that \( F_{et} \) is generated by the adaptive expectations generating mechanism

\[ F_{et} = F_{et-1} + \gamma(F_{t-1} - F_{et-1}) \quad (11) \]

where \( 0 < \gamma < 2 \). Upon substitution, equation (11) becomes

\[ \tilde{B}_t = [\alpha_0 - (1-\gamma)a_0] + [1-\gamma] \tilde{B}_{t-1} + [a_2\gamma] F_{t-1}. \quad (12) \]

Finally, an error term was added

\[ \tilde{B}_t = b_0 + b_1 \tilde{B}_{t-1} + b_2 F_{t-1} + u_t \quad (13) \]

where \( b_0 = \alpha_0 - (1-\gamma)a_0, b_1 = 1-\gamma, \) and \( b_2 = a_2\gamma \).

Equation (13) is the estimable one. From the comparative-static derivative, the sign of \( b_2 \) is hypothesized to be positive. Because this equation is a first-order linear-difference equation, properties of the time path of the equilibrium basis during the delivery period can be obtained from parameter estimates.
RESULTS

Results of the empirical analysis are presented in Table 2. F-tests indicated that all data sets except the one for October contracts could be combined. Three regression equations were estimated. Results in the first row of Table 2 are for an equation with both independent variables that was estimated by using the five combined data sets (the October data set was excluded). The second row presents results for an equation estimated by using the same combined data set, but with the lagged futures price deleted. Results for an equation estimated by using data for the October contract are presented in the third row.

Results suggest that price expectations of speculators do have an impact on delivery-period basis determination for live cattle. Price expectations explain nearly 40 percent of the basis variance in the pooled data set and nearly 90 percent of the basis variance in the October data set. The coefficient of the lagged futures price has the hypothesized sign in the equation for the pooled data set, but is not significant.

Properties of the time path of the par-delivery-point live-cattle basis during the delivery period can be obtained from the solution equation for the first-order linear-difference equation. The solution equation for the equation in the second row of Table 2 (i.e., the reduced-pooled equation) is

\[ B_t = (0.6154)^t(C_0 - 0.3614) + 0.3614 \]  

where \( C_0 \) is the basis on the first day of the delivery period. This expression shows that the time path of the basis in the delivery period is stable and non-oscillatory, and that the basis converges toward a value of $0.36/cwt. which is substantially larger than the estimated transaction cost of $0.10/cwt.

SUMMARY AND CONCLUSIONS

An examination of daily par-delivery-point basis values for live cattle during the delivery periods of 13 contracts revealed that the basis values vary considerably and quite often differ from zero by more than estimated transaction costs associated with arbitrage. To identify factors affecting delivery-period basis values, a two-market model of cash-price and basis determination was developed and analyzed. Results of an empirical analysis provided some support for the model and suggested that price expectations of speculators explain about 40 percent of par-delivery-point basis variation for live cattle during the delivery period.

Several factors may account for the remaining part of the basis variation, but risk associated with delivery and arbitrage may be an especially important factor. There are some generally recognized uncertainties associated with deliveries. For those making deliveries, there is risk that the cattle delivered may not be judged to meet contract specifications. For those accepting deliveries, there are uncertainties in exact time of delivery, exact quality of live cattle delivered, and elapsed time between delivery and resale of live cattle [3, p. 242]. Further research is needed to determine whether risk associated with delivery is an important source of delivery-period basis variability and to determine whether and why basis variability differs between commodities and over time.

Because basis variability reduces attractiveness of hedging to potential hedgers, consideration of efforts to reduce basis variability, or hedgers’ exposure to basis variability, may be warranted. Among efforts that might be considered are: (a) development of more objective criteria and procedures for determining whether delivered live cattle meet contract specifications, (b) establishment of a basis reporting service and (c) introduction of basis futures contracts which would provide short and long hedgers the opportunity to fix in advance the basis in a specified cash market during a specified period.

TABLE 2. ESTIMATED LEAST-SQUARES COEFFICIENTS FOR DELIVERY-PERIOD BASIS EQUATIONS

<table>
<thead>
<tr>
<th>Equation</th>
<th>Intercept</th>
<th>Lagged Basis, ( B_{t-1} )</th>
<th>Lagged Futures Price, ( F_{t-1} )</th>
<th>( r^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pooled, full</td>
<td>0.0832</td>
<td>0.4145</td>
<td>0.001</td>
<td>0.393</td>
</tr>
<tr>
<td>Pooled, reduced</td>
<td>0.1390</td>
<td>0.4154</td>
<td>(0.0827)</td>
<td>0.395</td>
</tr>
<tr>
<td>October, full</td>
<td>0.1462</td>
<td>0.0743</td>
<td>-0.3756</td>
<td>0.861</td>
</tr>
<tr>
<td>October, reduced</td>
<td>0.1465*</td>
<td>0.3868</td>
<td>(-5.1495)*</td>
<td>0.880</td>
</tr>
</tbody>
</table>

*P < 0.01.

REFERENCES


