Verification in Contracts with Random Changes in Quality

by

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Working Paper No. 00-17

Date of Submission: October 2000

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Abstract: Many goods and services are subject to random changes in quality during the time between sale and delivery, resulting in markets characterized by lemons-market equilibria. We examine two forms of costly verification, ex post inspection and ex ante certification. In equilibrium, ex post verification is used to verify buyers’ reports of low quality, while ex ante certification is used to verify sellers’ shipments of high quality. Ex post inspection allows buyers to earn rents; in some circumstances, ex ante certification allows sellers to earn rents. One would expect buyers to prefer ex post inspection while sellers prefer ex ante certification.

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**Abstract:** Many goods and services are subject to random changes in quality during the time between sale and delivery, resulting in markets characterized by lemons-market equilibria. We examine two forms of costly verification, ex post inspection and ex ante certification. In equilibrium, ex post verification is used to verify buyers’ reports of low quality, while ex ante certification is used to verify sellers’ shipments of high quality. Ex post inspection allows buyers to earn rents; in some circumstances, ex ante certification allows sellers to earn rents. One would expect buyers to prefer ex post inspection while sellers prefer ex ante certification.
Verification in Contracts with Random Changes in Quality

Many goods and services are subject to random changes in quality during the time between sale and delivery. Agricultural commodities are a prime example. Despite efforts to control quality during production (for example, by controlling the varieties grown, in-season production decisions like pesticide application, and timing of harvest), problems in ensuring quality may persist because most agricultural products are perishable, making it possible for on-farm quality to differ from delivered quality. For example, fruit may contain mold spores that are unobservable at the time of shipping, but blossom into mold during shipment. Grains may absorb moisture and insect eggs may hatch during shipment, reducing quality prior to delivery.

When quality deteriorates stochastically in this manner, there will likely be disputes between sellers and buyers over quality that cannot be resolved without independent verification. In other words, it will not generally be possible to write self-enforcing contracts because even when quality is completely observable by either party, each agent’s observation of quality is insufficient to determine whether its partner in the contract has complied with agreed-on contract terms. When uncontrollable stochastic factors account for a significant share of quality deterioration, such problems will persist even in repeated transactions. Without third-party verification, markets in such cases will be characterized by lemons-market equilibria where trade in low quality commodities prevails.

Third party verification may make contract enforcement feasible in such cases. The bulk of the literature has considered ex ante verification (that is, certification by sellers) in cases where quality does not deteriorate. Costly, perfect certification of quality
can suffice to replicate first best market equilibria (Viscusi). Costly, imperfect certification, however, only partially overcomes adverse selection problems (De and Nabar); it does so to a greater extent when the cost of verification is higher (Mason and Sterbenz). Third party ex post verification of quality has not been explicitly addressed in the literature. Related literature, however, considers third party verification in a variety of settings. Auditing, one form of third party verification, may induce conformity with regulations, for example, taxpayer compliance (Reinganum and Wilde) or truthful cost reporting by a monopoly (Baron and Besanko). Costly, perfect verification can explain why insurance contracts might not completely share risk (Townsend). If there is no commitment to costly verification, contracts designed by a better informed agent have a lower verification cost than do contracts designed by an agent with less information (Choe).

This paper considers the role of third-party verification in competitive markets when quality changes stochastically in this manner. We consider allocations of a commodity with exogenously determined quality between a high value market subject to exogenous deterioration and an alternative market in which quality does not change. Without independent verification, lemons market equilibria will predominate. We compare two forms of verification: ex ante certification and ex post verification. We show that ex post verification rectifies the lemons market problem to some extent, but allows high value market buyers to earn rent. Ex ante certification also rectifies the lemons market problem but allows some high value market sellers to earn rent. Thus, buyers prefer ex post verification while sellers prefer ex ante certification.
I. Structure of Consignment and FOB Contracts

Consider a set of one-time transactions between perfectly competitive, risk-neutral sellers and buyers. Each seller possesses one unit of a commodity of quality $q$, which is perfectly observable. In the absence of (costly) independent verification, $q$ is private information. The distribution of quality across sellers $G(q)$ and its associated density $g(q)$ are common knowledge. The minimum quality is $q$ and maximum quality is $\overline{q}$. Each seller has a reservation price $l(q)$, which equals the value of a commodity of quality $q$ in an alternative market. This reservation price is increasing in quality, $l'(q) > 0$. The quality of the commodity in this alternative use is not subject to random deterioration. It is common knowledge and exogenously given.

The value of the commodity in the high value market, $p(r)$, depends on its quality at the time of receipt, $r$. The relationship between received quality and value to the buyer, $p(r)$, is common knowledge. Received quality $r$ is perfectly observable to the buyer. In the absence of (costly) verification it is the buyer's private information. The value of the commodity is increasing in quality, $p'(r) > 0$. During delivery from the seller to the buyer, the commodity undergoes exogenous stochastic deterioration (and so moral hazard is not a factor). The probability that the quality of the commodity at the time of delivery is no greater than $r$, conditional on quality at the time of shipment $q$, $H(r;q)$, and its associated density $h(r;q)$ are common knowledge. We assume that quality cannot deteriorate to less than the minimum $q$ and that quality does not appreciate during shipment. The upper support of $H(r;q)$ is shipped quality $q$. We assume that increases in $q$ induce the equivalent of first order stochastic dominant shifts in the distribution function $H(r;q)$. In what follows, we concentrate on the case of economic interest, that in
which the expected value of the commodity in the high value market, $E_p(q) = \int_q^\infty p(z) h(z; q) dz$, exceeds the reservation price $l(q)$ for at least some $q \in [q, \overline{q}]$. To simplify the analysis of market equilibria, we make the further assumption that

$$\frac{\partial E_{p}(q)}{\partial q} > l'(q) \forall q \in [q, \overline{q}],$$

i.e., an increase in quality increases the expected high value market price more than the alternative market price.

We consider two forms of contract: a consignment contract in which the price paid to the seller by high value market buyers depends on received quality $r$ and an FOB contract in which the price paid to the seller by high value market buyers depends on shipped quality $q$. In the consignment contract, the high value market buyer reports delivered quality ($r$ is delivered quality, and $\hat{r}$ is the report of delivered quality) to the seller, who then decides whether to accept the buyer’s report or whether to obtain a neutral third-party verification of quality. In the FOB contract, the seller decides whether to certify quality $q$ prior to shipping the product to the high value market buyer.

II. First-Best and No-Verification Equilibria in a Competitive Industry

The basic structure of the transaction is as follows. First, the seller observes the quality of the commodity, $q$, and decides whether to ship the commodity to the high value market or to sell it in the alternative market. If the seller ships the commodity to the high value market, the commodity undergoes stochastic deterioration in quality. The high value market buyer receives the commodity and observes received quality $r$ and thus knows its value $p(r)$. The price the buyer pays to the seller may be based on either the seller’s reported shipped quality $q$ (the FOB contract) or on the buyer’s reported received
quality r (the consignment contract). The price paid to sellers under FOB contracts is \( w(q) \), and the price paid to sellers under consignment contracts is \( w(r) \). Perfect competition among buyers leads to equilibrium prices paid to sellers equal to the value of the commodity, so that \( w(q) = E_p(q) \) under the FOB contract and \( w(r) = p(r) \) under the consignment contract.

Our analysis focuses on two different cases. In the first case, it is optimal for the high quality commodities to be sold in the high value market, i.e., \( E_p(q) > l(q) \) for some \( q > q^* \), and the low quality commodities to be sold in the alternative market. In the second case, it is optimal for all commodities, regardless of quality, to be sold in the high value market. The assumptions needed for the first case, expressed as Assumption 1a below, are that the expected high value market price for a commodity of minimum quality is less than the price for minimum quality in the alternative market while the expected high value market price for a commodity of maximal quality is greater than the price for minimum quality in the alternative market. Coupled with the assumption that \( \frac{\partial E_p(q)}{\partial q} > l'(q) \), this assumption ensures that there exists a \( q^* \) such that \( E_p(q) > (\leq) (\geq) l(q) \) for \( q > (\leq) (\geq) q^* \).

**Assumption 1a:** \( p(q) < l(q) \) and \( E_p(q) > l(q) \).

For the second case, we assume that expected prices for each possible level of shipped quality are greater than prices in the alternative market.

**Assumption 1b:** \( E_p(q) > l(q), \forall q \in [q, \bar{q}] \).

The first best allocation maximizes social welfare, which equals the total value of the commodity. Let \( \delta(q) \) denote the share of the commodity of quality \( q \) sold to a buyer
in the high-value market, while the remainder, \( 1 - \delta(q) \), is sold on the alternative market. The first best allocation is found by choosing \( \delta(q) \) to

\[
max \int \left\{ \delta(s) \int p(z; s) dz + (1 - \delta(s))l(s) \right\} g(s) ds
\]

subject to \( 0 \leq \delta(q) \leq 1 \).

Under Assumption 1a, high quality commodities will be sold in the high value market (see Figure 1) and low quality commodities will be sold in the alternative market. The cut-off quality level, \( q^* \) in Figure 1a, is determined by the contract’s expected prices relative to prices in the alternative market. In other words, it is optimal to allocate high quality commodities to the high value market and low quality commodities to the low value market. Under Assumption 1b, all commodities will be sold in the high value market and none will be sold in the alternative market. In other words, it is optimal to allocate all units to the high-value market and none to the low-value market (Figure 2).

In the absence of independent verification, there will be a lemons market equilibrium in which all commodities shipped receive the same price regardless of shipped quality. An FOB contract will be subject to the Lake Wobegon effect: All sellers will report that their commodity is the highest possible quality \( \bar{q} \), so that their reports reveal no information to the buyer. In response, buyers will offer a single price regardless of reported quality. It will be useful to define

\[
E_p^u(q) = \int_{\bar{q}}^{q} \int_{\bar{q}}^{s} p(z; s) g(s) dz ds
\]
the expected high value market price of all commodities no greater than \( q \). Competition among buyers should generate an offered price equal to \( \text{Ep}^u(\tilde{q}) \), where \( \tilde{q} \) is the maximum quality sold in the high value market.

There are three possible equilibria. If \( \text{Ep}^u(\tilde{q}) > l(\tilde{q}) \), then all commodities will be sold in the high value market at a price \( \text{Ep}^u(\tilde{q}) \). If \( \text{Ep}^u(\tilde{q}) < l(\tilde{q}) \), Assumptions 1a and 1b generate different equilibria. Under Assumption 1a, \( l(q) > \text{Ep}^u(q) \) \( \forall q \), so that all commodities will be sold in the alternative market (see Figure 1). Under Assumption 1b, there will exist a \( \tilde{q} \) such that all commodities of quality \( q \leq \tilde{q} < \tilde{q} \) will be sold in the high value market while the remainder \( \tilde{q} \leq q \leq \tilde{q} \) will be sold in the alternative market (see Figure 2). Note that this last equilibrium features a perverse allocation of quality: The highest quality commodities are sold in the low value alternative market while the high value market receives only low quality items, if anything.

Under a consignment contract, buyers have no incentive to report received quality truthfully. This is the case because actual delivered quality is not observable to seller and because buyers pay a lower price for lower quality. Thus, under a consignment contract every high value market buyer will offer to pay every seller the value of a minimum quality commodity, \( p(q) \), regardless of shipped quality. Under Assumption 1a, nothing will be sold in the high-value market, since \( l(q) > p(q) \) (see Figure 1). Under Assumption 1b, such an offer will elicit the sale of commodities with quality less than \( \tilde{q} \) while commodities with quality greater than \( \tilde{q} \) will be sent to the alternative market (see Figure 2). Note that high value market buyers will earn positive rent in this case because they pay less than the full value of commodities shipped. For this reason, there is an
incentive for buyers to deviate from this offer. One buyer could increase sales volume and thus total profit by offering a single price slightly higher than \( p(q) \). Competition of this sort among buyers would lead to an equilibrium identical to one of the FOB equilibria discussed previously.

III. Equilibrium Under Consignment Contracts with Verification of Delivered Quality

Next, consider the case where the commodity is sold under consignment in the high-value market and costly verification is available. The sequence of events in this case is as follows. First, the seller decides whether to ship the commodity to the high value market or sell for the reservation price in the alternative market. If the seller chooses to ship the commodity to the high value market, she receives a report of delivered quality from the buyer. The seller either accepts the report and receives the associated high value market price or orders a costly inspection that reveals true received quality.

We derive a perfect Bayesian equilibrium of the contract with inspection, which consists of a set of prices and allocation of quality between the high value and the alternative markets.

III.A. The Seller’s Inspection and Shipping Decisions

Consider first the seller’s inspection decision on receipt of reported received quality. The seller makes this decision by comparing expected profit with inspection and profit from accepting the buyer’s report of delivered quality, \( r \). The fact that received quality is observed without error implies that verification is perfect, that is, that
inspection reveals the true quality. Let \( m(r; \hat{r}, q) \) be the seller’s posterior distribution over received quality given the buyer’s report \( \hat{r} \) and shipped quality \( q \). Let \( R \) be the set of possible levels of received quality \( r \) that correspond to each report of quality \( \hat{r} \). Then

\[
m(r; \hat{r}, q) = \frac{h(r; q)}{\int_{\hat{r}} h(z; q) dz}.
\]

Let \( c \) be the cost of an inspection. We assume that the cost of inspection is not prohibitive, specifically, that \( c < \text{Ep}(\overline{q}) - l(\overline{q}) \).

If the seller ships quality \( q \), the expected net return with inspection equals

\[
\int_{\hat{r}} p(z)m(z; \hat{r}, q) dz - c.
\]

The return without inspection is \( p(\hat{r}) \). The seller maximizes expected profit by choosing the share to be inspected \( \psi(q) \) to

\[
\max \psi(q) \left( \int_{\hat{r}} p(z)m(z; \hat{r}) dz - c \right) + (1 - \psi(q)) p(\hat{r})
\]

It will be optimal to order an inspection when the expected return from inspection exceeds the price associated with the reported received quality,

\[
\int_{\hat{r}} p(z)m(z; \hat{r}, q) dz - c > p(\hat{r}).
\]

Next consider the seller’s decision as to whether to sell in the high value market or the low value alternative market. Because deterioration is exogenously stochastic, the seller does not know received quality with certainty. However, the seller does know the buyer’s decision rule for reporting quality and can infer from it the probability that the buyer will report received quality \( \hat{r} \) conditional on shipped quality \( q \). Denote this probability \( v(\hat{r}; q) \). Then the seller’s decision problem involves choosing the share of
the commodity $\zeta(q)$ to ship to the high value market in order to maximize expected profit given the anticipated optimizing choice of whether to order an inspection $\psi^*(q)$:

$$
\zeta(q) \int_0^q \psi^*(q) \int_\hat{r} \left( p(z)m(z;\hat{r}) - c \right) dz + \left[ 1 - \psi^*(q) \right] p(\hat{r}) \right \} v(\hat{r};q)d\hat{r} + (1 - \zeta(q))l(q).
$$

The seller will choose to ship to the high value market if the expected return from high value market sales given anticipated inspection decisions,

$$
Ew^*(q) = \int_0^q \psi^*(q) \int_\hat{r} \left( p(z)m(z;\hat{r}) - c \right) dz + \left[ 1 - \psi^*(q) \right] p(\hat{r}) \right \} v(\hat{r};q)d\hat{r}.
$$

exceeds the alternative market price l(q).

III.B. The High Value Market Buyer’s Report of Quality

Next consider the high value market buyer’s decision as to the report of received quality. The buyer observes received quality $r$ with certainty but does not know shipped quality. She chooses the report of quality to maximize her profit. In doing so, she faces a tradeoff. Reporting received quality below the actual level allows her to earn rent by paying less than the full value of the commodity. Reductions in reported received quality increase that rent. At the same time, reducing reported received quality increases the likelihood that the seller will order an inspection, in which case the buyer earns no rent. The buyer’s optimal report maximizes the expected rent.

Formally, the buyer can form a posterior distribution of shipped quality conditional on observed received quality $j(q;r)$ using Bayes’ Rule:

$$
j(q;r) = \frac{h(r;q)g(q)}{\int_r h(r;s)g(s)ds}.
$$
Let $U(\hat{r})$ be the set of shipped qualities $q$ such that a given report of received quality $\hat{r}$ does not trigger an inspection, i.e., $U(\hat{r}) = \left\{ q : \int_{\hat{r}} p(z) m(z; \hat{r}, q) dz - c \leq p(\hat{r}) \right\}$. Then the buyer’s posterior probability that a given report of received quality will not induce the seller to order an inspection, conditional on observed received quality, is

$$K(\hat{r}; r) = \int_{U(\hat{r})} \frac{h(r; q) g(q)}{\int_{\hat{r}} h(r; s) g(s) ds} dq$$

and the buyer’s expected rent is

$$p(r) - p(\hat{r}) K(\hat{r}; r) - p(r) [(1 - K(\hat{r}; r))]$$

The first order condition for a maximum can be written

$$[p(r) - p(\hat{r})] \frac{k(\hat{r}; r)}{K(\hat{r}; r)} - p'(\hat{r}) = 0.$$ 

Since $p'(r) > 0$ and $k(\hat{r}; r)/ K(\hat{r}; r) > 0$, it follows that $p(r) - p(\hat{r}) > 0$ and thus that $r > \hat{r}$.

**Proposition 1:** With ex post verification the high value market buyer always underreports quality and, on the average, earns positive rent.

One implication of Proposition 1 is that the seller’s ex ante expected return from a high value market sale, $E_{w^v}(q)$, will be less than the first best expected return $E_{p}(q)$ (for qualities greater than the minimum $q$) because the first best does not involve an inspection cost and because, under the consignment contract, the buyer always underreports quality. As a result, the equilibrium allocation of commodities under the consignment contract will generally differ from the first best.
Proposition 1 also allows us to define R, the set of received qualities consistent with the buyer’s report of \( \hat{r} \), more precisely in terms of the optimal report of received quality for a high value market buyer, i.e.,

\[
R = \left\{ r : \hat{r} \in \arg \max \{ p(r) - p(\hat{r}) K(\hat{r}; r) - p(r)[(1 - K(\hat{r}; r)) \} \} \right\}.
\]

III.C. Separating Equilibrium

The nature of the equilibrium allocation of commodities depends on the information conveyed by the report of received quality \( \hat{r} \). Suppose first that the solution to the buyer’s optimization problem is unique, i.e., that there is a unique optimal report of received quality \( \hat{r} \) for each given received quality \( r \). In this case, the seller will be able to deduce received quality \( r \) exactly from reported quality \( \hat{r} \). The seller’s posterior distribution of received quality \( r \) will be degenerate: \( m(z; \hat{r}, q) = 1 \) for \( z \) equal to true received quality \( r \) and 0 otherwise. The seller will thus order an inspection whenever reported received quality \( \hat{r} \) falls below a critical level \( \hat{r}^* \) defined by

\[
p(r) - c - p(\hat{r}^*) = 0.
\]

The buyer’s subjective assessment of the probability that the seller will order an inspection is similarly degenerate: \( K(\hat{r}; r) = 1 \) whenever \( \hat{r} \) is below \( \hat{r}^* \) and 0 otherwise. The buyer will therefore report \( \hat{r}^* \), avoid an inspection, and earn a rent equal to the cost of inspection \( c \). As a result, the expected return to selling in the high value market will equal the true expected high value market price given shipped quality \( q \)

\[
\int_{q}^{q^*} p(z) h(z; q) dz,
\]

less the cost of inspection \( c \). There will be a critical level of shipped quality \( q^* \) defined
by \( \int_2^{q^*} p(z)h(z; q^*)dz - c - l(q^*) = 0 \). All commodities of quality exceeding \( q^* \) will be sold in the high value market while the remainder is sold in the alternative market.

Under Assumption 1a (see Figure 3), this critical quality \( q^* \) will generally exist and exceed \( q^* \), the critical quality level defining the first-best allocation of quality. In this case ex post verification will replicate the first-best allocation of quality for the highest and lowest qualities but not for some intermediate qualities \( q^* < q < q^* \). Under Assumption 1b (see Figure 4), two possibilities arise. If the cost of inspection \( c \) is sufficiently small (that is, less than \( p(q) - l(q) \)), then the equilibrium allocation under the consignment contract will be the same as the first best allocation. If the cost of inspection is sufficiently large, specifically, \( c > p(q) - l(q) \), however, there will exist a critical quality \( q^* \) as in the case of Assumption 1a.

These results can be summarized as follows:

**Proposition 2:** With ex post verification, if the buyer’s optimal report of quality and received quality is one-to-one function of received quality, then the buyer will report quality \( \hat{r}^*(r) \) defined by \( p(r) - c - p(\hat{r}^*) = 0 \). Sellers will never inspect. The equilibrium allocation of the highest quality commodities will equal that under the first best. Under Assumption 1a, the equilibrium allocation of the lowest quality will also equal the first best, while some intermediate quality commodities will remain misallocated. The degree of misallocation will depend on both the signal of reported quality and the cost of verification. Under Assumption 1b, the lowest quality commodities will be misallocated if the cost of inspection \( c > p(q) - l(q) \) or \( E_p(q) - E_w(p^v(q)) > p(q) - l(q) \); otherwise, the equilibrium allocation will equal the first best for all commodities.

Note that in this case the possibility of verification suffices to make contracts enforceable and thus to rectify misallocation of quality, at least in part. Even though no inspections will occur in equilibrium, the threat of inspection induces buyers to report the equivalent
of true received quality. Interestingly, commercial provision of inspections will not be economically viable in such circumstances and government subsidies may be needed to make them available. In other words, contract enforcement measures for addressing lemons market problems will be feasible in such cases only through government intervention.

**III.D. Pooling Equilibrium**

Next, suppose that the solution to the buyer’s optimization problem is not unique, i.e., that there are multiple optimal reports of received quality \( \hat{r} \) for each given received quality \( r \). In this case, the seller will be not able to deduce received quality \( r \) exactly from reported quality \( \hat{r} \). As before, the seller’s ex ante expected return from a high value market sale, \( E_w^\hat{y}(q) \), will be less than the first best expected return \( E_p(q) \) because the first best does not involve an inspection cost and because, under the consignment contract, the buyer always underreports quality. Under Assumption 1a (see Figure 3), there will generally exist a critical quality \( q^y > q^* \). All commodities of quality \( q \geq q^y \) will be sold under the consignment contract in the high value market, while all commodities of quality \( q < q^y \) will be sold in the alternative market. Under Assumption 1b (see Figure 4), two possibilities arise. If the difference between \( E_p(q) \) and \( E_w^\hat{y}(q) \) is sufficiently small (that is, less than \( p(q) - l(q) \)), then the equilibrium allocation under the consignment contract will be the same as the first best allocation. If the difference \( E_p(q) - E_w^\hat{y}(q) > p(q) - l(q) \), however, there will exist a critical quality \( q^y \) as in the case of Assumption 1a.

These results can be summarized as follows:
**Proposition 3:** With ex post verification, the equilibrium allocation of the highest quality commodities will equal that under the first best. Under Assumption 1a, the equilibrium allocation of the lowest quality will also equal the first best, while some intermediate quality commodities will remain misallocated. Under Assumption 1b, the lowest quality commodities will be misallocated if $E_p(q) - E_w(q) > p(q) - l(q)$; otherwise, the equilibrium allocation will equal the first best for all commodities.

In contrast to the separating equilibrium case, inspections will occur in equilibrium. Thus, commercial inspection services may be economically viable in pooling equilibria even though they are not in separating equilibria.

**IV. Equilibrium Under FOB Contracts with Certification of Shipped Quality**

An alternative to verifying received quality ex post is for sellers to have shipped quality certified, at a cost, by an independent inspector. Certification makes enforceable FOB contracts in which the price the seller receives from high value market buyers is based on shipped quality. Contracts of this type have been investigated by Viscusi, De and Nabar, Mason and Sterbenz, and others.

The high value market is perfectly competitive and high value market prices are common knowledge. As a result, buyers will offer to purchase commodities at prices equal to their expected value on receipt. The equilibrium FOB contract price of certified commodities will thus equal the expected high value market price conditional on shipped quality $q$, i.e.,

$$w^c(q) = E_p(q) = \int_q^\infty p(z) h(z; q) \, dz.$$  

Uncertified commodities will receive the average high value market price of all uncertified qualities shipped to the high value market.
\[ w^u(q) = \int_{q^c}^{q^u} \int_{q^c}^{s} p(z)h(z;s)g(s)dzds, \]

where \( q^c \) and \( q^u \) are, respectively, the lowest and highest qualities shipped uncertified to the high value market.

Assume that the cost of certification is not prohibitive, specifically, that

\[ a < \int_{q^c}^{q^u} p(z)h(z;q)dz - \int_{q^c}^{q^u} \int_{q^c}^{s} p(z)h(z;s)g(s)dzds \quad \text{and} \]

\[ a < \int_{q^c}^{q^u} p(z)h(z;q)dz - l(\bar{q}). \]

These assumptions imply that sellers with the highest possible quality commodities will find selling in the high value market with certification more profitable than either selling without certification in a lemons market equilibrium or selling in the alternative market.

It will be useful to define two critical quality levels. First, since the expected price with certification has a lower intercept and higher slope than the uncertified price curve

\[ E^u_p(q) = \int_{q^c}^{q^u} \int_{q^c}^{s} p(z)h(z;s)g(s)dzds \]

(the latter because \( p(q)h(q;q) > \int_{q^c}^{q^u} p(z)h(z;q)g(q)dz \) for all \( q \)), there exists a \( q^{***} \) such that

\[ \int_{q^c}^{q^{***}} p(z)h(z;q)dz - a = \int_{q^c}^{q^{***}} \int_{q^c}^{s} p(z)h(z;s)g(s)dzds \quad (\text{alternatively, } E^u_p(q^{***}) - a = E^u_p(q^{***})). \]

Second, there exists a \( q^{**} \) such that \( E^u_p(q^{**}) - a = l(q^{**}). \) The fact that \( E_p(q) \) is increasing in \( q \) implies that \( q^{**} > q^*, \) where \( q^* \) is the critical quality level that prevails in the first best.

Several types of equilibria are possible.
Under Assumption 1a, commodities of the lowest quality will be sold in the alternative market since, by assumption, \( l(q) > Ep^u(q) = Ep(q) > Ep(q) \). If \( q^{**} < q^{***} \), then \( l(q) > (\leq) Ep(q) \) for all \( q < (\geq) q^{**} \) and \( l(q) > Ep^u(q) \) for all \( q \) (Figure 5a).

Thus, all commodities of quality \( q \geq q^{**} \) will be sold certified in the high value market while all commodities of quality \( q < q^{**} \) will be sold in the alternative market. Sellers will find it optimal to certify all commodities shipped to the high value market.

Commodities of intermediate qualities (\( q^* < q < q^{**} \)) will remain misallocated; the cost of certification prevents achievement of a first best allocation of quality.

If \( q^{***} < q^{**} \), \( l(q) > Ep^u(q) > Ep(q) \) for all \( q \in [q, q^{***}) \), so that all commodities of qualities \( q \leq q < q^{***} \) will be sold in the alternative market (Figure 5b). At \( q^{***} \),

\[
\int_{q}^{q^{***}} p(z)h(z; s)g(s)dzds > Ep(q^{***}), \quad \text{so that } q^n = q^{***} \quad \text{and } q^c \text{ is defined by }
\]

\[
\int_{q}^{q^c} p(z)h(z; q)dz = \int_{q^{***}}^{q^{***}} p(z)h(z; s)g(s)dzds.
\]

If \( q^c < \bar{q} \), commodities of intermediate quality (\( q^{***} \leq q < q^c \)) will be sold uncertified in the high value market while commodities of the highest quality will be sold certified in the high value market. If \( q^c > \bar{q} \), all commodities sold in the high value market will be uncertified.

Under Assumption 1b, commodities of the lowest quality will be sold uncertified in the high value market since, by assumption, \( l(q) < Ep^u(q) = Ep(q) > Ep(q) \). If \( q^{**} < q^{***} \) (or if \( l(q) < Ep(q) \) for all \( q \)), then \( Ep^u(q) > l(q) \) and \( Ep^u(q) > Ep(q) \) for \( q \in [q, q^{***}) \), so that all commodities of qualities \( q \leq q < q^{***} \) while \( Ep(q) > l(q) \) and \( Ep(q) > Ep^u(q) \) for all \( q > q^{***} \) (Figure 6a). In this case, all commodities will be sold in the
high value market. Those of qualities lower than $q^{**}$ will be sold uncertified while those of qualities $q^{**}$ and higher will be sold certified.

If $q^{**} < q^*, \ Ep^u(q) > l(q) > Ep(q)$ for all $q < q^n$ such that $Ep^u(q^n) = l(q^n)$. Thus, all commodities of qualities $q \leq q < q^n$ will be sold uncertified in the high value market (Figure 6b). Further, $l(q) > Ep^u(q) > Ep(q)$ for all $q \in [q^n,q^{**})$, so that all commodities of qualities $q^n \leq q < q^{**}$ will be sold in the alternative market. Finally, $Ep(q) > l(q) > \int q q s q z d z d s$ for all $q > q^{**}$, so that commodities of the highest qualities will be sold certified in the high value market.

Note that in any equilibrium in which some commodities are sold uncertified in the high value market, some sellers will earn rent. Specifically, sellers with the lowest qualities of commodity sold uncertified in the high value market will receive a price greater than the social value of their commodities. Thus, in contrast to equilibria with ex post verification, in which high value market buyers earned rent, equilibria with ex ante certification may permit some sellers to earn rent.

These results can be summarized as follows:

**Proposition 4:** If the cost of certification is not prohibitive, ex ante certification will replicate the first best allocation of commodities of the lowest and highest qualities. Commodities of intermediate quality may remain misallocated.

**Proposition 5:** Ex ante certification may allow some sellers to earn rent.

**V. Discussion**

This paper has investigated the ways in which costly independent third party verification can be used to address misallocation of product quality due to adverse selection problems when quality deteriorates in an exogenous stochastic manner between
shipment and receipt of a commodity. Agricultural commodities provide the classic examples. Fruits and vegetables that appear in perfect condition at the time of shipment may rot or become moldy while in transit to distant markets. Shipments of grain may become contaminated with insects that hatch from eggs that were not apparent at shipping time. Contract enforcement problems of this kind appear to have been a major impetus for the establishment of both ex post receiving market and ex ante shipping point inspection services under the auspices of the U.S. Department of Agriculture.

Discussions of fruit marketing from the early years of the 20th century indicate that widespread disputes over quality between growers and wholesalers in urban markets motivated growers to exert political pressure for the creation of an independent inspection agency (Dimitri). The agency created in response to this pressure, operated by the Department of Agriculture but financed primarily by growers, has provided both kinds of inspections since the World War I years.

Similar problems can occur with nonagricultural goods as well. For example, the percentages of valid addresses or phone numbers on specialized mailing or telephone lists may be less than advertised because of changes in residence unmonitored by the list seller. As a result, the user of the list may experience a larger than anticipated incidence of wasted mailings or calls. Those employing the services of lobbyists offering contacts with key government officials may similarly find that, due to unmonitored turnover in employment, the personal contacts of the lobbyist hired will be fewer and less valuable than previously supposed. Similarly, goods may suffer random breakage or other forms of damage during shipment, for example, during marine transit in unexpectedly heavy seas.
We examine two forms of verification, ex post inspection verifying received quality and ex ante certification of shipped quality, in competitive markets in which sellers and buyers are risk neutral and in which sellers bear the cost of verification. We show that both can rectify the misallocation of quality occurring in lemons market equilibria, at least in part. Because verification is costly, some misallocation of quality may persist, however. Furthermore, because verification is costly, some agents may be able to appropriate rent. We also show that the two differ in interesting ways. First, we show that in equilibrium the two are used on different qualities of commodities. Ex post verification is used to verify buyers’ reports of low quality, while ex ante certification is used to verify sellers’ shipments of high quality. Thus, for example, producers of agricultural goods will find it optimal to verify reports that low quality was received but will certify shipments of high quality. Similarly, lobbyists will find it optimal to verify reports that low level staffers have changed jobs but will certify the current positions of highly influential political figures.

Second, we show that the distribution of rent differs significantly between the two methods of verification. Ex post inspection allows buyers to pay less than the social value of commodities they receive, on average. In some circumstances, ex ante certification allows sellers of uncertified commodities to receive a price exceeding the social value of the commodities they ship. As a result, one would expect buyers to prefer ex post inspection while sellers prefer ex ante certification.
Figure 1. First-Best and No-Verification Equilibria Under Assumption 1a

Figure 2. First-Best and No-Verification Equilibria Under Assumption 1b
Figure 3: Comparison of First Best, Separating, and Pooling Equilibrium Consignment Contracts under Assumption 1a.

Figure 4: Comparison of First Best, Separating, and Pooling Equilibrium Consignment Contracts under Assumption 1b
Figure 5a. Comparison of First Best and FOB with Certification Equilibria Under Assumption 1a

Figure 5b. Comparison of First Best and FOB with Certification Equilibria Under Assumption 1a
Figure 6a. Comparison of First Best and FOB Contract with Certification Equilibrium Under Assumption 1b.

Figure 6b. Comparison of First Best and FOB Contract with Certification Equilibrium Under Assumption 1b.
References


