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Regulation of relationships between heterogeneous farmers and an aquifer accounting for lag effects*

Cyril Bourgeois and Pierre-Alain Jayet[†]

Many environmental problems are due to damage caused by pollutants that accumulate with a time lag following their emission. In this study, we focus on nitrates used in agriculture, which can pollute groundwater many years after their initial application. A dynamic optimal control problem with heterogeneous farmers is proposed. The usual structural parameters such as the discount rate, the natural clearing rate and the lagged time interval between the occurrence of soil-level pollution and the impact on groundwater are taken into account. We also examine pollution as caused by a continuous set of farms characterised by their individual performance index and by their individual marginal contribution to the pollution. The issue is further investigated by taking account of change in the information context, successively related to perfect information and to asymmetric information. As a result, when the delay between the spreading of N-fertilizer and the impact on the aquifer increases, that is, the longer the lag, the steady-state pollution stock and the steady-state shadow price of the stock both increase. Moreover, we show that the optimal regulation may require a decreasing amount of fertilizer over time, even in the case of initial underpollution.

Key words: Time lag, delay, optimal control, nonpoint source pollution, aquifer pollution, mechanism design.

1. Introduction

Aquifers constitute about 89 per cent of the freshwater on our planet, providing most of the world's drinking water, and are vulnerable to surface pollution especially from agricultural nitrates (Koundouri 2004). When nitrates are ingested in too large quantities, they have a toxic effect on human health, causing blue-baby syndrome and stomach cancer (Addiscott 1996). Nitrates also contribute to soil eutrophication. More generally, nitrates 'have the potential to become one of the costliest and the most challenging

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environmental problems' that environmental agencies face (Stoner 2011). This problem led to World Health Organisation to recommend not to exceed 50 mg.l^{-1} (1993) in groundwater. This recommendation has been enacted by EU and US environment agencies. The U.S. EPA established a maximum contaminant level (MCL) of 10 mg.l^{-1} for nitrate in drinking water (1995). The Water Framework Directive (2000/60/EC) adopted by the European Commissions requires all groundwater bodies to achieve a 'good' status by 2015. This goal includes the nitrate limit of 50 mg.l^{-1} set by the Nitrates Directive (91/676/EEC). However, this threshold is already exceeded in many groundwater bodies in Europe (Rivett *et al.* 2008) and the U.S. (Gurdak and Qi 2012). Similar high concentrations are observed in many countries, especially in China (Ju *et al.* 2006) and in Australia, in the latter case directly threatening the Great Barrier Reef (Thorburn *et al.* 2003; Mitchell *et al.* 2009; Windle and Rolfe 2011).

Water pollution by nitrates from agriculture is a typical case of nonpoint source (NPS) pollution, because individual emissions cannot be measured precisely by the social planner. NPS pollution problems have received considerable attention in the economics literature, mainly to identify the appropriate regulatory instruments. These include various mechanisms, essentially based on the ambient concentration of pollutants (e.g. Segerson 1988; Xepapadeas 1991) and on emission proxies such as inputs (e.g. Griffin and Bromley 1982; Shortle and Dunn 1986; Shortle and Abler 1994). Managing such pollution is made difficult by the fact that the social planner is faced with a situation of moral hazard and adverse selection. Firstly, in the case of an ambient pollutant-based instrument, it could be prohibitively costly to measure with sufficient precision farmers' current efforts in pollution abatement. Indeed, the social planner can only measure ambient pollutant concentration at prespecified 'receptor points'. To eliminate this moral hazard problem, Xepapadeas (1991) proposes a system of subsidies and random penalties in cases of noncompliance with the desired ambient levels, and Bystrom and Bromley (1998) suggest the use of nonindividual contracts and collective penalties. Secondly, the social planner also faces an adverse selection problem (both in the case of ambient and input-based instruments), which may be related to soil spatial heterogeneity. This means that the same management for the same crop in different fields will not necessarily lead to similar nitrate losses (Cabe and Herriges 1992; Lacroix *et al.* 2005).

Aquifer pollution by nitrates is similar to a stock pollutant problem which requires taking into account the dynamics of pollutant accumulation in order to regulate the pollution efficiently. However, few studies have considered the dynamic characteristics of this pollutant, as noted by Shortle and Horan (2001), even if some recent papers include it in the presence of heterogeneous farmers (Xabadia *et al.* 2006, 2008). Xepapadeas (1992) shows that applying static ambient-incentive schemes in dynamic situations leads to suboptimalities (pollutant overaccumulation), particularly when polluters follow feedback strategies. He then suggests schemes that take the

form of charges per unit deviation between desired and observed pollutant accumulation paths.

However, due to the slow transfer of nitrates through the unsaturated zone of aquifers, these effects are not visible until 10–60 years after their use because the nitrate transfer velocity varies mainly between 0.60 and 2.50 m/year (Legout *et al.* 2007; Gutierrez and Baran 2009). Such a lag undermines the incentive schemes proposed by Xepapadeas (1992). Indeed, the social planner cannot impose penalties today when the pollution is due to fertilizers used several decades ago.

In this context, optimal regulation of aquifer polluted by nitrates requires taking into account the lag effect in a dynamic framework.

In this study, we examine an optimal management problem of a NPS pollution to a pollutant stock which accumulates with a lag. The delay between an agent's action and its consequences is an important feature in the literature dealing with the accumulation of capital (Rustichini 1989; Asea and Zak 1999).

Lag effects were introduced into the environmental economics literature to address the pollutant accumulation problem by Fleming *et al.* (1995). In an empirical study, these authors deal with the lag impact under public regulation but only from a steady-state perspective. They suggest basing regulation on the ambient tax principle, which applies only in the case of overpollution. However, this exogenous and nonoptimal tax does not depend, as it should do, on the delayed value of both the shadow price and the pollution stock. Moreover, this instrument is ineffective in cases of underpollution, whereas it is a key element of an environmental problem with lagged pollution. Brandt-Pollmann *et al.* (2008) and Winkler (2010) analyse a *point source* problem via a generic optimal control model of stock accumulation with a lag. They focus on the lag effect regarding steady-state existence, unicity and stability. When the production function is additively separable, they show that the optimal control of delayed stock does not pose additional analytical complexities compared to instantaneous stock accumulation. However, they study neither the lag impact on steady state nor the trajectory path or public regulation. We aim to extend this literature to the case of lagged NPS pollution with heterogeneous agents and an adverse selection problem. To our knowledge, the literature has not provided any answers to the question of how accounting for the lag effect in the pollution stock can modify the policy of the social planner.

A fortiori the problem becomes more difficult in the case of asymmetric information. To address this issue, we develop an optimal control problem with heterogeneous agents (farmers) whose emissions accumulate with a time lag. The pollution is caused by a continuous set of producers characterised by their individual index, the topsoil quality, and by their individual marginal contribution to the pollution. Following Winkler (2010), we assume a separable objective function. This allows us to solve the lag problem in the case of perfect and asymmetric information. We show that the lag acts by increasing the stock and its shadow price at the steady state. It implies that social planners have to demand more effort from farmers, at the same time as

the environmental result is weaker. This effect is augmented when the information between farmers and the social planner is asymmetric. Moreover, we show that, due to the lag, when the initial stock is underpolluted, regulation based on a decreasing value of the shadow price may be required, as in the case of overpollution. We use the theoretical model to illustrate the impacts of the time lag on the dynamics and the steady state. Doing so paves the way for estimating the impact of regulation of a river basin scale when the time lag acts significantly (Bourgeois and Jayet 2012).

The study is organised as follows. Section 2 presents the basic model. In Section 3, we set out the generic control problem with time-lagged stock accumulation when the social planner is completely informed about individual farm characteristics. In Section 4, we develop the analysis of the optimal control problem when asymmetric information drives the mechanism design needed to be implemented by the regulator. Finally, in Section 5, we look at the differences when different time lags are considered. We also compare results in the case of perfect information to those in the case of asymmetric information.

2. Basic elements of the model

Let us consider a set of farmers contributing to nitrate pollution of an aquifer. Farming activity is represented by a farmer's demand for nitrogen fertilizers, denoted by x . Activity depends on a soil quality index summarised by the one-dimensional parameter θ . The individual farm profit is represented by the function $\pi(x, \theta)$, in which the farm index θ is spread over the interval $\Theta = [\underline{\theta}, \bar{\theta}]$. The probability distribution function is denoted by $\gamma(\theta)$ and assumed to be strictly positive at any point within the interval:

$$\gamma(\theta) > 0 \quad \forall \theta. \quad (1)$$

The related cumulative function is denoted by $\Gamma(\theta)$. Agricultural nitrate losses depend on N-fertilizer demand and use at the soil-root zone. The social planner is assumed to know the aquifer characteristics and the transfer process from arable soils to aquifers at a regional scale by means of hydrogeological modelling. Asymmetric information comes with wide heterogeneity of farming conditions in a broad sense. The index, θ , could represent the heterogeneity in topsoil quality, as well as in farming practices, including the use of environmentally friendly techniques. When asymmetric information on farm characteristics comes into our analysis, an adverse selection problem arises.

The π function is assumed to be continuously differentiable twice. The assumption of decreasing returns to scale and the assumed positive marginal profit when x is close to 0 hold here:

$$\pi_{xx} < 0. \quad (2)$$

$$\pi_x(0, \theta) > 0 \quad \forall \theta. \quad (3)$$

We assume that the marginal profit increases when the topsoil quality index θ increases, as it comes with parameters of functions of the Mitscherlich-Baule class (Frank *et al.* 1990; Llewelyn and Featherstone 1997):

$$\pi_{x\theta} > 0. \quad (4)$$

Regarding the marginal farm profit and further formal analysis later in the article, let us consider the x -variable equation $\pi_x(x, \theta) = c$. Hypotheses (2) and (4) lead to the solution $x = \phi(\theta, c)$ being characterised as a decreasing function of the additional unit cost of nitrogen fertilizer, c^1 and an increasing function of the characteristics index θ :

$$\pi_x(\phi(\theta, c), \theta) = c \Rightarrow \begin{cases} (2) & \Rightarrow \phi_c < 0 \\ (2) \text{ and } (4) & \Rightarrow \phi_\theta > 0 \end{cases}. \quad (5)$$

Demand for nitrogen fertilizer increases when topsoil quality increases, and decreases when the unit cost of nitrogen fertilizer increases. Let us note that high value of c may lead some farms to stop producing (in this case x and π are equal to zero).

The farming activity is assumed to occur over time. Accordingly, the global profit, at time t , is expressed by $\int_{\Theta} \pi(x(\theta, t), \theta) \gamma(\theta) d\theta$.

Regarding the environmental impact and related damage, we can start by applying a standard framework. The state of our aquifer system is characterised by the nitrate stock per volume unit and denoted by z . Its dynamic evolution over time is the result of a twofold effect. On one hand, the clearing effect takes the form of a standard exponential decline characterised by the decline rate τ . On the other hand, the amount of N -fertilizer applied by the θ farm additively contributes to increase pollution.

The contribution to the pollution per unit of θ -input x is summed-up by a parameter, a , which accounts for the spatial heterogeneity of soil (through θ) and the aquifer's physical characteristics, which for simplicity we consider as constant over time. Aquifer θ -dependency should be seen as an extension of the model, when topsoil characteristics are correlated with subsoil ones, and more relevantly when θ farm practices interfere with soil characteristics. A difficulty arises when we introduce the lag effect of nitrogen fertilizer use on the nitrate concentration in the aquifer. The key parameter is the lag parameter, β , which represents the delay between the

¹ The really important point is based on the constant sign of the derivative of the marginal profitability with respect to the soil quality, $\Pi_{x\theta}$, which plays a role in the case of information asymmetry (see Section 4).

application of fertilizer and its environmental impact in the aquifer. Accordingly, the pollution contribution of the θ farm at time t is $a(\theta)x(\theta, t-\beta)$, and the time evolution of the environmental system is described by the equation:

$$\dot{z}(t) = -\tau z(t) + \int_{\Theta} a(\theta)x(\theta, t-\beta)\gamma(\theta)d\theta. \quad (6)$$

Expecting that the regulatory body will be asked to design the optimal individual farm demand for input $x(\theta, t)$ at time 0 for any further time t , we assume that the social planner or environmental regulator integrates knowledge related to the initial state of the aquifer and to the farming activity in the recent past. In addition, the input has to be non-negative. This is expressed by the following assumption:

$$z(0) = z_0; x(\theta, t) = \varepsilon(\theta, t) \forall \theta \in \Theta \forall t \in [-\beta, 0[; x(\theta, t) \geq 0 \forall \theta \in \Theta, \forall t \geq 0. \quad (7)$$

When the social planner is misinformed about the individual characteristics θ (i.e. the asymmetric information case), the ‘initial condition’ $x(\theta, t) = \varepsilon(\theta, t)$, $t \in [-\beta, 0[$ should be viewed from a statistical standpoint. The social planner should have statistical knowledge of the $\varepsilon(\theta, t)$ fertilizer amount, even if he does not know θ individually. In other words, knowledge of function ε does not imply knowledge of θ .

The damage function is expressed by the twice differentiable function depending on z and denoted by $D(z)$. The assumptions related to the damage function are as follows:

$$D_z(0) = 0 \text{ and } D_{zz} > 0. \quad (8)$$

Note that assumptions (8) leads to $D_z > 0 \forall z > 0$.

Finally, the discount rate is denoted by δ , and the marginal cost of public funds is denoted by ρ . This last parameter enters the analysis when contractual incentives are taken into consideration.

In sum, the basic model has the following core elements:

- Heterogeneous farmers are differentiated according to the soil quality.
- Farmers’ profit is increasing and concave, and the marginal profit is increasing with soil quality.
- Aquifer pollution is due to the application of N-fertilizer.
- Aquifer pollution is viewed as a stock refuelled by topsoil agricultural activity. A time lag exists between the pollutant application and the impact on the aquifer.

The economic analysis that follows is based on a partial equilibrium approach with no price feedbacks from the rest of the economy.

3. Long run optimal trade-off between production and pollution in the complete information case

The social surplus is seen as the sum of producers' surplus (profit) less environmental damage. When information upon farmers is complete, the social planner's objective is as follows:

$$W = \int_0^\infty \left[\int_{\Theta} \pi(x(\theta, t), \theta) \gamma(\theta) d\theta - D(z(t)) \right] e^{-\delta t} dt. \quad (9)$$

Accordingly, the social planner's objective function is expressed below:

$$\max_{x(\theta, t)} W \text{ subject to (6), (7)}. \quad (10)$$

Differently from the usual optimal control program, the lag term appearing in the state dynamics (6) does not allow us to directly apply the Pontryagin's maximum principle. The solution arises when we consider the transformation of the command variable $y(\theta, t) = x(\theta, t - \beta)$. The objective function and the state evolution equation are transformed as follows:

$$\begin{aligned} W = & - \int_{-\beta}^0 \int_{\Theta} \pi(x(\theta, t), \theta) \gamma(\theta) d\theta e^{-\delta t} dt \\ & + \int_0^\infty \left[e^{\delta\beta} \int_{\Theta} \pi(y(\theta, t), \theta) \gamma(\theta) d\theta - D(z(t)) \right] e^{-\delta t} dt. \end{aligned} \quad (11)$$

$$\dot{z}(t) = -\tau z(t) + \int_{\Theta} a(\theta) y(\theta, t) \gamma(\theta) d\theta. \quad (12)$$

Thanks to the assumption (7), the first integral component of this last W expression can be taken out of the program. Aiming at the use of the maximum principle, we define the current value Hamiltonian in which the shadow price of the pollution stock is denoted $\lambda(t)$ and is designed to take a positive value:

$$H^c = e^{\delta\beta} \int_{\Theta} \pi(y(\theta, t), \theta) \gamma(\theta) d\theta - D(z(t)) - \lambda(t) \left[\int_{\Theta} a(\theta) y(\theta, t) \gamma(\theta) d\theta - \tau z(t) \right]. \quad (13)$$

According to our technical assumptions, the Pontryagin's maximum principle delivers the conditions holding the optimal solution: $\{y^*(\theta, t), z^*(t), \lambda^*(t)\}$:

$$y^*(\theta, t) \text{ maximises } H^c(y, z^*, \lambda^*). \quad (14)$$

$$\dot{\lambda}^*(t) - \tau \lambda^*(t) = H_z^c(y^*, z^*, \lambda^*). \quad (15)$$

Our redefined problem in $y^*(\theta, t)$ is convex and leads to the following equations:²

$$\pi_x(y^*(\theta, t), \theta) = a(\theta) \lambda^*(t) e^{-\delta \beta}. \quad (16)$$

$$\dot{\lambda}^*(t) - (\tau + \delta) \lambda^*(t) = D_z(z^*(t)). \quad (17)$$

The transversality condition is satisfied as:

$$\lim_{t \rightarrow \infty} \lambda(t) e^{-\delta t} z(t) = 0. \quad (18)$$

Condition (16) states that the θ farmer's profit provided by one additional unit of polluting input equals the discounted cost of the related marginal pollution evaluated at time $t + \beta$ and weighted by the individual polluting contribution $a(\theta)$. The solution in y to this equation is obtained from the expression (5) ($y^*(\theta, t) = \phi(\theta, a(\theta) \lambda(t) e^{-\delta \beta})$). The complete solution of the social planner's program is provided by the implicit relation between the command x and the shadow price λ , and by the two-dimension differential system, as summarised by the equation set (19):

$$\begin{aligned} \forall \theta, \forall t > 0 : x^*(\theta, t) &= \phi(\theta, a(\theta) \lambda(t + \beta) e^{-\delta \beta}) \\ z^*(t) &= -\tau z^*(t) + \int_{\Theta} a(\theta) x^*(\theta, t - \beta) \gamma(\theta) d\theta \\ \dot{\lambda}^*(t) - (\tau + \delta) \lambda^*(t) &= -D_z(z^*(t)) \end{aligned} \quad (19)$$

with (5) the conditions on ϕ and (18) the transversality condition.

There is only one steady state related to this system (proof in Appendix 7.1). The technical assumptions described above also produce a graph describing the paths related to this differential system (Figure 1).

The optimal amount of fertilizer, $x^*(\theta, 0)$, is determined by the shadow price, λ at time $t = \beta$ which is associated with the pollution stock, z , at the time $t = \beta$ (and not at time $t = 0$). This is one of the main reasons why the ambient tax as proposed in Fleming *et al.* (1995) is not optimal. Indeed, farmers are not sufficiently taxed when the stock pollution is on an increasing path (as during the first β years), and the tax is too high when the stock pollution is on a decreasing path. In the case of initial underpollution, that is

² The maximisation of (13) through to $y(\theta, t)$ is familiarly solved as $\max \int_U f(w(u), u) du$ which leads to the f.o.c.: $\frac{dw}{du} = 0$, for any $u \in U$.

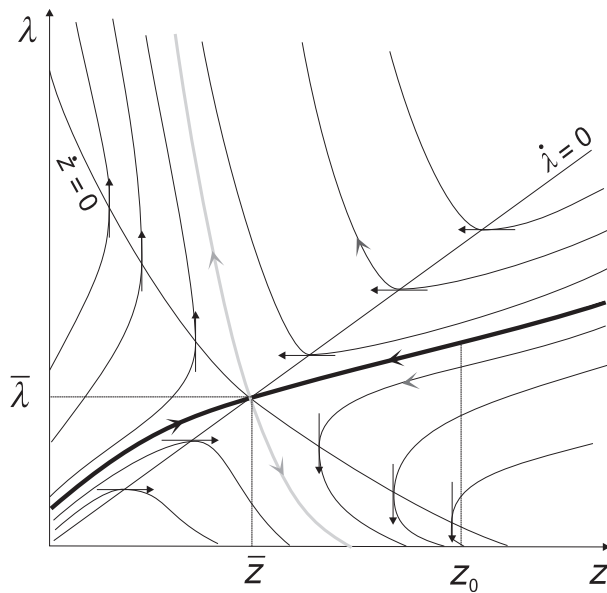


Figure 1. Phase diagram describing the paths linking the pollution state z and its shadow price λ . All lines drawn with directional indications refer to the general solution of the time differential equations describing the state and co-state evolution (named by the state variable, z , and its implicit price, λ). Among eligible paths, the bold black line refers to the optimal one converging towards the steady state, starting, respectively, from the right (left) in case of initial time over(under)-pollution.

$z(0) \leq \bar{z}$, and overpollution at $t = \beta$, that is $z(\beta) \geq \bar{z}$, the amount of fertilizer describes a decreasing path over time (Figure 2). The standard result in the case of instantaneous pollutant accumulation, that is an increasing path of the amount of fertilizer in the case of initial underpollution, may be inconsistent when the lag is taken into account. This creates various problems

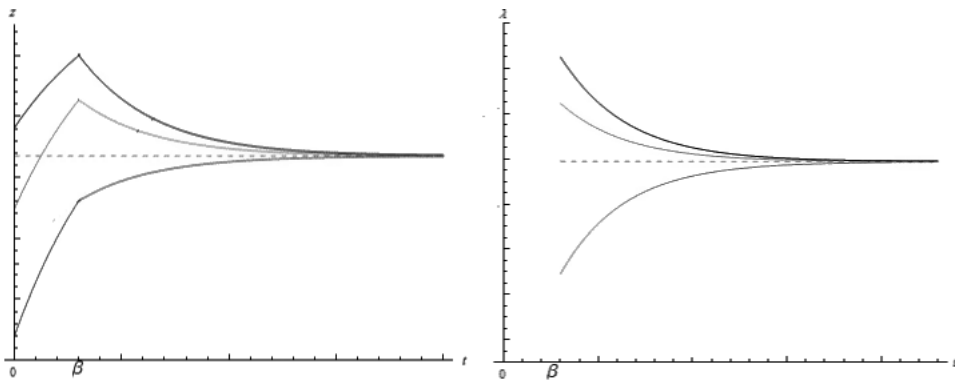


Figure 2. Time lag, β , impacts on the optimal dynamics of the pollution stock (z), on the left, and the shadow price (λ), on the right, for different initial states, z_0 . The x -axis refers to time, and the y -axis, respectively, refers to z and λ .

for the social planner. When the time lag is misevaluated, the chosen regulation path can differ, in qualitative terms, compared to what it should be, thus causing the social planner to miss the target. This is a key point because, due to the lag, the social planner cannot realise his mistake, and then readjust the environmental policy, until β years after the beginning of the regulation.

Proposition 3.1: The regulation path which determines the level of fertilizers, $x(\theta, t)$ depends on the level of pollution stock at $t = t + \beta$. In particular: if the initial stock is in underpollution at $t = 0$ but is in overpollution at $t = \beta$, a decreasing path of fertilizer level is required.

These results are illustrated by Figure 2:

Let us focus on the steady state $(\bar{z}, \bar{\lambda})$ defined by $\{\dot{y} = 0; \dot{z} = 0\}$. We are interested by the impact of the parameters β, δ, τ on the steady state, leading us to summarise the results in Propositions 3.2–3.4.

Proposition 3.2: At the steady state, the greater the delay between nitrogen-fertilizer application and its environmental impact, the greater the pollution level and the shadow price.

The two next propositions are expected and refer to more usual approaches. It is important to bear in mind the sensitivity of the main parameter.

Proposition 3.3: When the discount rate increases, the steady-state pollution level and the steady-state shadow price both increase.

Proposition 3.4: When the decline rate increases, that is more nitrates are absorbed by the aquifer, the steady-state pollution level and the steady-state shadow price both increase.

Proofs are given in Appendix 7.2.

The point now is to move from socially optimal input levels to incentive tools, when public funds are costly. To do this, we introduce the social planner's choice in supplying contracts to any θ farm. A contract is characterised by a two-dimensional function $(q(\theta, t), s(\theta, t))$ in which q refers to the upper limit of x -use of the polluting input and s refers to the lump-sum transfer as the counterpart of the profit decrease. Contracts are designed to be freely accepted by farms; consequently, the regulator has to prevent farmers from refusing contracts when their participation is viewed as socially beneficial.

The transfers call for costly public funds (i.e. one budget unit costs $1 + \rho$) and the social objective is now expressed as:

$$W = \int_0^\infty \left\{ \int_{\Theta} [\pi(q(\theta, t), \theta) - \rho s(\theta, t)] \gamma(\theta) d\theta - D(z(t)) \right\} e^{-\delta t} dt. \quad (20)$$

In the case of complete information, there is no place for informational rent. The reservation utility of the θ farm is the unconstrained profit characterised by the q -consumption equal to $\phi(\theta, 0)$ (constant over time). When public funds are costly, the individual discounted transfer is equal to the individual profit variation:

$$\int_0^\infty s(\theta, t) e^{-\delta t} dt = \int_0^\infty [\pi(\phi(\theta, 0), \theta) - \pi(q(\theta, t), \theta)] e^{-\delta t} dt. \quad (21)$$

The public objective can be rewritten by substitution of the transfer expressed above, so that the social planner's program is now:

$$\max_{q(\cdot, \cdot)} W = \int_0^\infty \left\{ \int_{\Theta} [(1 + \rho)\pi(q(\theta, t), \theta) - \rho\pi(\phi(\theta, 0), \theta)] \gamma(\theta) d\theta - D(z(t)) \right\} e^{-\delta t} dt, \quad (22)$$

arising with the unchanged dynamics of the state variable (still given by Eqn (12)). The implicit solution of this program is still provided through the change in the control variable with respect to the time lag parameter β . The contract (for any θ at any time for the quota q , and under an integral equation for any θ -transfer s) and the (z, λ) path are fully characterised by the system (23):

$$\begin{aligned} \forall \theta, \forall t > 0 : q^*(\theta, t) &= \phi\left(\theta, \frac{a(\theta)\lambda^*(t+\beta)e^{-\delta\beta}}{1+\rho}\right) \\ \forall \theta : \int_0^\infty s^*(\theta, t) e^{-\delta t} dt &= \int_0^\infty [\pi(\phi(\theta, 0), \theta) - \pi(q^*(\theta, t), \theta)] e^{-\delta t} dt \\ z^*(t) &= -\tau z^*(t) + \int_{\Theta} a(\theta) q^*(\theta, t - \beta) \gamma(\theta) d\theta \\ \dot{\lambda}^*(t) - (\tau + \delta)\lambda^*(t) &= -D_z(z^*(t)) \end{aligned} \quad (23)$$

Assumption (5); the transversality condition satisfied

When the parameter related to the shadow cost of public funds tends towards 0 (i.e. $\rho \rightarrow 0$), the system (23) tends towards the system (19). Noncostly transfers do not affect the solution (q, z, λ) .

The parameters θ , λ and δ have similar effects on the steady state as mentioned in system 19 analysis. Proposition 3.5 delivers the additional effect of the cost of public funds on the steady state (proof in Appendix 7.2).

Proposition 3.5: When the marginal cost of public funds increases, the pollution level and the shadow price in the steady state increase.

Put simply, to sum-up this section, if the social planner has perfect knowledge:

- At any time, the optimal amount of fertilizer is explicitly determined through the time-advanced implicit value of nitrate pollution, and it should decrease over time except in the case of permanent underpollution.
- At the steady state, the lag increases the level of optimal pollution as well as its implicit value.

4. The dynamic problem in the asymmetric information case

The social planner is now underinformed when he faces any individual θ farm, but he knows the statistical distribution. In others words, the social planner is unable to assess the nitrate losses related to each farm. We place our adverse selection problem in the framework of incentive theory, developed by Laffont and Tirole (1993) among others. More specially, our approach is in line with many others highlighting the implications of asymmetric information for the design of agricultural and environmental policies (Bourgeon *et al.* 1995; Fraser 2004; Gren 2004; Bontems and Bourgeon 2005).

We consider that the social planner offers a menu of contracts to each farm, and either the farmer θ selects one of the contracts or he/she refuses all of them. The menu is proposed at the initial time, binding farmers for the future. In other words, there is no place here for social gains through information learning. The problem for the social planner is to design the optimal menu regarding the social objective including farm profits, environmental damage, and regulation costs.

The menu of contracts is a two-dimensional function $(q(\theta, t), s(\theta, t))$. As in the previous complete information context, q denotes the ‘quota’ and s denotes the ‘subsidy’. Formally, the social planner acts by asking every farmer at time 0 whether or not he/she is contracting, and in the event of acceptance, asking for the characteristics of his/her θ farm. The participating farmer selects a contract through the report $\tilde{\theta}$. Acceptance by the farmer implies that he/she complies at time 0 with the upper bound $q(\tilde{\theta}, t)$ holding the q -input at any time t . He/She will receive the transfer $s(\tilde{\theta}, t)$.

The θ farmer’s program is to declare his/hers optimal report. Based on the revelation principle, the menu proposed by the social planner is a mechanism designed in such a way that the θ farmer’s dominant strategy is to report his/hers true characteristics θ . Theoretically, the social planner retains the possibility of designing the menu in such a way that the optimal set of participating farmers is a subset of Θ . This opportunity is explored in some studies on applications of incentive theory (Bourgeon *et al.* 1995). For simplicity, we do not keep this option here, even though the menu is possibly suboptimal.

Formally, we consider that the functions q and s have the requested mathematical properties allowing us to use derivatives as long as necessary. The first step of the analysis involves characterising the incentive-compatibility constraints and the participation constraint (the so-called rationality constraint). The starting point is the following θ farmer's program which defines the farmer's optimal report:

$$\max_{\tilde{\theta}} \int_0^{\infty} [\pi(q(\tilde{\theta}, t), \theta) + s(\tilde{\theta}, t)] e^{-\delta t} dt. \quad (24)$$

Note that the private discount rate is supposed to be equal to the public discount rate δ . Solving this program using first- and second-order conditions, and with the help of the revelation principle, we derive incentives constraints summarised by the relations 25 and 26:³

$$\int_0^{\infty} \left[\pi_x(q, \theta) \frac{\partial q}{\partial \theta} + \frac{\partial s}{\partial \theta} \right] e^{-\delta t} dt = 0 \text{ rel : IC1}. \quad (25)$$

$$\int_0^{\infty} \pi_{x\theta}(q, \theta) \frac{\partial q}{\partial \theta} e^{-\delta t} dt > 0 \text{ rel : IC2}. \quad (26)$$

The contract is supposed to be freely accepted by the θ farmer. When the regulator aims to induce the farmer to accept the contract, he/she has to ensure that the farmer does not lose by doing so.

Detailed explanations of the calculus are provided in Appendix 7.3. The characterisation of the full menu of contracts, the dynamic equations describing the evolution of the state z and the shadow price λ are summarised by the system (27):⁴

$$\begin{aligned} \forall \theta, \forall t > 0 : q^*(\theta, t) &= \phi \left(\theta, \frac{a(\theta)\lambda^*(t+\beta)e^{-\delta\beta}}{1+\rho} - \frac{\rho}{1+\rho} \pi_{x\theta}(q^*, \theta) \frac{\Gamma(\theta)}{\gamma(\theta)} \right) \\ S\bar{\theta} &= \int_0^{\infty} s(\bar{\theta}, t) e^{-\delta t} dt = \int_0^{\infty} [\pi(\phi(\bar{\theta}, 0), \bar{\theta}) - \pi(q(\bar{\theta}, t), \bar{\theta})] e^{-\delta t} dt \\ \forall \theta : \int_0^{\infty} s(\theta, t) e^{-\delta t} dt &= S(\bar{\theta}) + \int_0^{\infty} \int_{\bar{\theta}}^{\theta} \pi_x(q(u, t), u) \frac{\partial q}{\partial \theta}(u, t) du e^{-\delta t} dt. \quad (27) \\ \dot{z}^*(t) &= -\tau z^*(t) + \int_{\Theta} a(\theta) q^*(\theta, t - \beta) \gamma(\theta) d\theta \\ \dot{\lambda}^*(t) - (\tau + \delta) \lambda^*(t) &= -D_z(z^*(t)) \end{aligned}$$

eq. (18); the transversality condition satisfied

We assume that added technical conditions referring to the condition (26) hold and allow us to consider that the necessary conditions delivered by the

³ Usually in the field of mechanism design, the condition (26) is firstly supposed to hold, and has to be checked regarding the resulting contract (27). A third-derivative condition should be added to ensure that the contract (27) complies with incentive constraint (26).

⁴ Regarding hypotheses (4), it should be noted that the change of sign $\Pi_{x,\theta}$ changes $\Gamma(\theta)$ in $-(1-\Gamma(\theta))$. In addition, $\bar{\theta}$ is changed with θ when the sign of $\pi_{x,\theta}$ is negative (information rent increasing). We demonstrate that the qualitative results hold whatever the sign of $\Pi_{x,\theta}$.

system (27) describe the optimal solution. The optimal menu of contracts leads the regulator to design the quota q for any θ at any time t . The subsidy appears through an integral condition, which provides the social planner with flexibility regarding the support over time.

Compared to the system (23), the steady state related to the system (27) lets an additional negative term appear in the expression of the optimal quota, $q^* = \phi\left(\theta, \frac{a\lambda^* e^{-\delta\theta}}{1+\rho} - \frac{\rho}{1+\rho} \pi_{x\theta}(q^*, \theta) \frac{1}{\gamma}\right)$. This additional term does not allow us to deliver a general result in terms of the lag effect. The sign of third derivatives enters the conditions, which lead to the Proposition 4.2. Moreover, this sign plays a crucial role in the comparison between system (23) and system (27) (Proposition 4.1).

Proposition 4.1: In the case of asymmetric information, the pollution stock level, the shadow price and the total amount of instantaneous polluting input at the steady state are higher than in the case of perfect information when the third-derivative, $\Pi_{xx\theta}$, is negative. Otherwise, the effects are ambiguous.

Proof is delivered in Appendix 7.4.

Proposition 4.2: When the delay between the spreading of N -fertilizer on the farm and its impact increased, that is, the longer the lag, the greater the increase in the pollution level and the higher the shadow price in the steady state, if the third-derivative $\Pi_{xx\theta}$ is negative. Otherwise, the effects are ambiguous.

In other words, when the complementarity between topsoil quality and fertilizers decreases with respect to fertilizers, $\Pi_{xx\theta} < 0$, we find the same results as in the case of perfect information. Otherwise, when the complementarity between production factors increases ($\Pi_{xx\theta} > 0$) with respect to fertilizers, the effects on the shadow price and the pollution level are ambiguous.

Proof is given in Appendix 7.5.

Put simply, if the social planner is underinformed:

- At any time, the optimal amount of fertilizer implicitly depends on the time-lagged implicit value of nitrate pollution, on the cost of public funds, on the statistical distribution of the information-dependent parameter, and on the sign of the derivative of the marginal profit with respect to this parameter.
- Regarding the lag effect, findings drawn in the perfect information case may apply under condition relating to the third-order derivative of the profit function.

5. Discussion and perspective

To open the discussion, the analysis developed in this study is complemented by the numerical application presented below, thus paving the way for

real-world scenarios (see the model coupling chain for the French Seine river basin involving more than 7 million agricultural hectares and three large aquifers, when regulation is designed to target nitrate thresholds, in Bourgeois and Jayet (2012)).

Numerical simulations are based on the following additional elements : specification of the damage function, specification of the profit function, specification of the density function, and a set of values for parameters. The damage takes a standard quadratic form:

$$D(z) = \frac{k}{2}z^2, k > 0. \quad (28)$$

The profit function is normalised by prices and takes a form in accordance with usual *Nitrogen*-yield functions suitable for numerous crops:

$$\Pi(x, \theta) = 1 - e^{-\theta x} - x \text{ with } \theta \in [1, e]. \quad (29)$$

The function $1 - e^{-\theta x}$ links crop yield to the nitrogen fertilizer quantity denoted by x . The value of the soil quality index, θ , ranged within the interval $[1, e]$, consistent with assumption (4). The contribution of farmers to a stock of pollution is considered here not to be depend to the quality of soils ($a(\theta) = a$ for any θ). We assume that the density function follows a uniform distribution. The selected values a , k and ρ aim at clearly illustrating the different effects (noting that the opportunity cost of public funds, ρ , is in line with Laffont and Tirole (1993). The value of the discount rate, δ , matches the one recommended by regulatory bodies (Lebègue *et al.* 2005). According to hydro-geologists, a minimum of 10–60 years, depending on the aquifer, is necessary for N-fertilizer to leach into the groundwater (Legout *et al.* 2007; Gutierrez and Baran 2009). We set an intermediate value, $\beta = 30$ years, by default. Note that the U.S. Ogallala aquifer (covering eight U.S. states and providing 80 per cent of the drinking water of people living within the aquifer boundary) may fall within this category of groundwater, given the usual values of transfer velocity and depth of water. Finally, aquifers need up to several decades to eliminate traces of N-fertilizers. We thus deduce the decline rate, $\tau = 0.02$. Table 1 summarises the values of parameters.

Table 1 Parameter values

Parameters	a	k	$1+\rho$	δ	β	τ
Values	10^{-3}	10^{-5}	1.3	0.04	30	0.02
Units	dam^{-3}	$\text{€}/\text{kg N}^2/\text{dam}^6/\text{ha}/\text{year}$		Per year	year	Per year

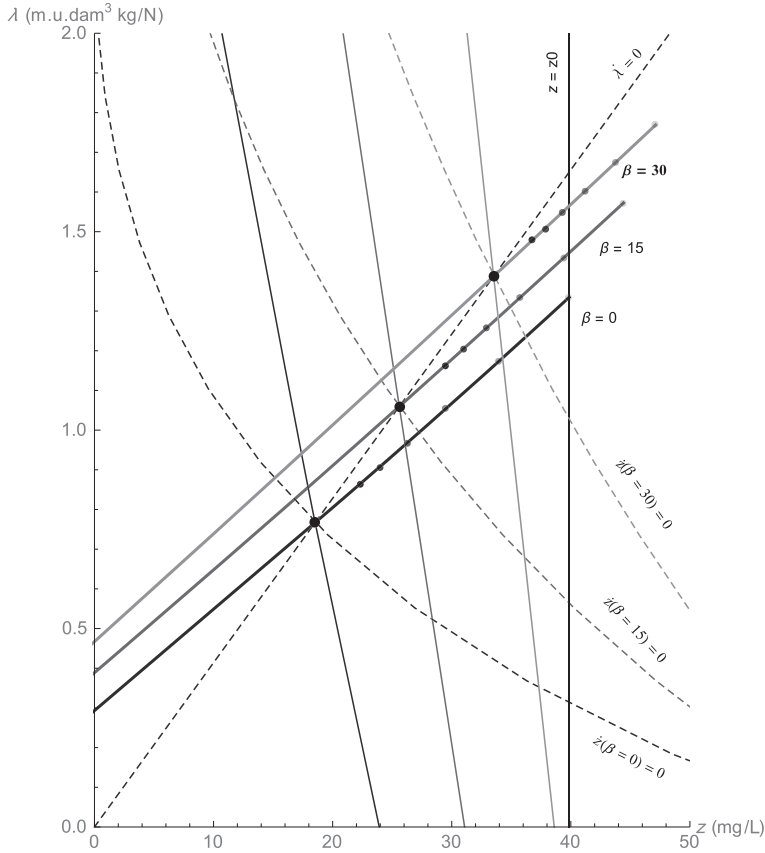


Figure 3. Comparison of both steady state and optimal dynamics regarding the pollution state z and its shadow price λ in the case of perfect information, for different values of the time lag β (respectively, 0, 15, 30 years). The vertical line refers to the initial value of z ($z = z_0$). Thick paths starting from the initial state $z_0 + \beta$ (i.e. the grey points on the right) match greyed points related to 10-year steps and converge to the steady states (the larger dots in the figure). The three other grey continuous curves refer to diverging paths.

After optimisation and solving under perfect information, we obtain the phase diagram illustrated by Figure 3 in line with Figure 1.⁵

Figure 3 illustrates the lag effect, when we focus on the steady state and on the optimal path for three values of β , including the case $\beta = 0$ (i.e. no lag) and the two other lags, respectively, $\beta = 15$ and $\beta = 30$ (years). In our example, the introduction of a time lag of 15 years increases the pollution stock by fifty per cent in the steady state. A time lag of 30 years would double the pollution stock. In general, as the time lag increases, the shadow price of the pollution becomes significantly higher.

An illustration of the lag influence on the dynamics of the pollution stock z is given in Figure 4, on the left. The lag obviously does not only impact the steady state. The pollution stock goes on increasing during the time interval

⁵ All computations and related graphs are obtained using Mathematica-7.

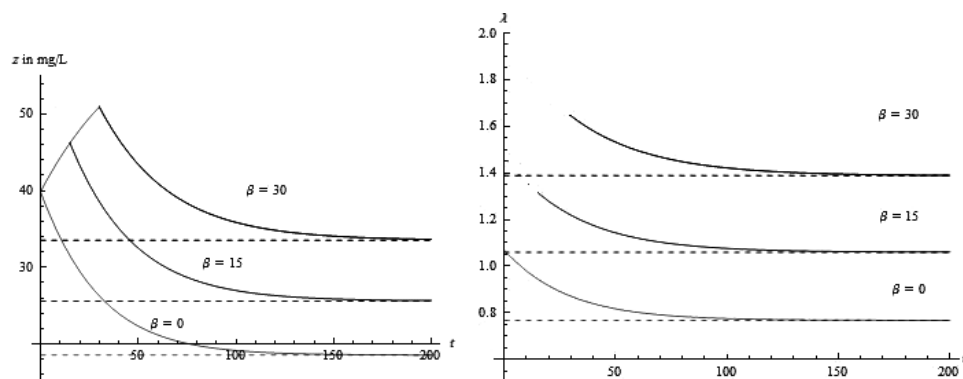


Figure 4. The dynamics of the optimal pollution stock, on the right, and the associated shadow price, on the left, when $\beta = 0$, $\beta = 15$ and $\beta = 30$.

$[0, \beta]$. In other words, the time lag modifies all the dynamics. Figure 4, on the right, shows that optimal management of pollution that takes into account the lag requires more effort (λ higher) for farmers, while the environmental results will be weaker.

Asymmetric information implies a cost on the regulatory side through the informational rent paid to farmers. The production each farmer is allowed is higher than in the case of perfect information. However, the best option for some farmers is not to produce and therefore receive a subsidy as compensation for their income loss. The overall effect on pollution stock is ambiguous (see Proposition 4.2). Regarding our profit function and its negative third-derivative Π_{xxx0} , the level of the pollution stock increases when we move from perfect information towards asymmetric information. The time lag effect is amplified in the case of asymmetric information. Figure 5 illustrates the steady state both in perfect and asymmetric information for different values of the time lag and for different values of the opportunity cost of public funds. Even when asymmetric information leads to an increase in the pollution stock and the shadow price in the steady state, its impact is smaller than that of the time lag.

6. Conclusion

We have developed a dynamic economic framework to assess the impacts of lag time on optimal NPS management. We analysed this impact with topsoil heterogeneity and an adverse selection problem. The solution takes the form of individual contract between the social planner and farmers. We have shown that the shadow price and the stock of pollutant at steady state increase with the lag. This result is important for the design of optimal policy by the social planner. Indeed, an optimal management of pollution which takes into account the lag requires more effort for farmers, and the environmental results will be weaker. In the case of asymmetric information,

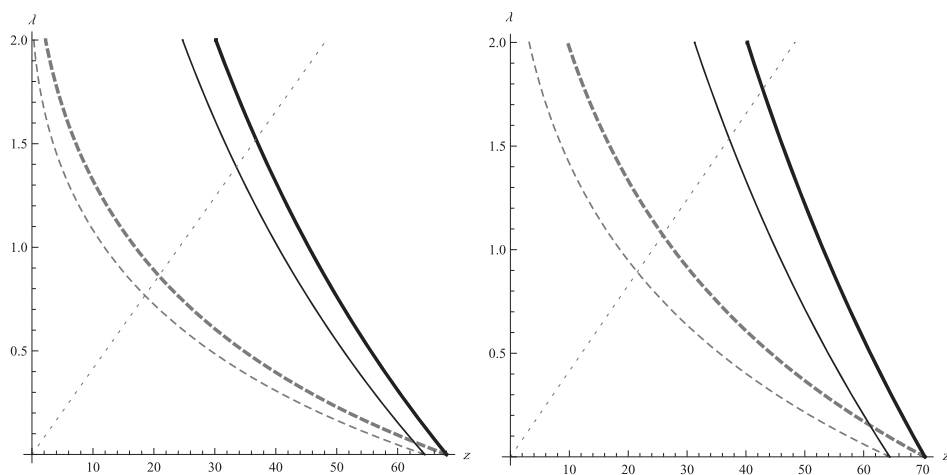


Figure 5. Impacts of the time lag β and of the opportunity cost of public funds ρ on the steady state, given perfect and asymmetric information: the steady state results from matching the increasing curve (i.e. the optimal path) and the dashed and solid curves which, respectively, relate to $\beta = 0$ in the case of perfect and asymmetric information, and $\beta = 30$ in the case of perfect and asymmetric information, when $\rho = 0.3$ is on the left and $\rho = 0.7$ on the right.

only a stringent condition on the profit function third-derivative sign could lead to unambiguous results.

However, for the standard functions (e.g. quadratic, Mitscherlich) used to represent agricultural activities, the asymmetric information strengthens the policy findings obtained under perfect information.

We have also shown that regulation does not depend both on present values of the shadow price and on the pollution stock but on their values at the present time plus the time lag, β . Moreover, we have shown that the shadow price can be decreasing, even if the initial stock is in an underpolluted state. This result is essential for the social planner. To ignore the lag can lead the planner to choose an inappropriate regulation path, and then to miss the target. When the policy maker acts without taking into account the β lag, he/she would realise he/she is wrong after β years, when he/she observes the deviation between the expected concentration level and the real one. To summarise, NPS pollution management is made more difficult when an apparently good status of an aquifer does not reflect its real status. Moreover, a bad assessment of water quality for the period to come will dramatically worsen the situation of European Union Member States which will probably face penalties imposed by the WFD, in addition to the welfare losses due to lag misvaluation.

The simulated results give an idea of the size of the lag effect. For example, the introduction of a 15-year time lag increases the pollution stock by fifty per cent in the steady state, while a 30-year time lag would double the pollution stock. These results show that longer time lags lead to larger divergence from the optimal policy settings when only information asymmetry is considered.

Regulation policy, viewed through a menu of contracts in our analysis, is appropriate for dealing with the heterogeneity of agents polluting an aquifer and with the diversity of aquifers regarding the lag time. Even if the problem of ex post control makes difficult any incentives mechanism per se, implementation is promoted by existing specifications related to public policies. In the European Union, farmers have to provide specified information when they benefit from CAP support, such as the amount of nitrogen used. This declaration could be used to control farmers' activity. Thanks to the wide range of real lag time, the policy would nevertheless have to be adapted to each aquifer. Even in the case of a homogeneous marginal contribution of pollution (i.e. the a parameter) and in the case of weak opportunity cost of public funds (equivalent to the case of complete information when $\rho = 0$), the lag time is one of the major physical drivers of environmental policy, leading the water agency to adapt solutions to individual aquifers as opposed to designing them at the scale of the river basin.

For further research, simulations based on coupling agro-economic and hydrological models, accounting for time lag effects, should considerably increase the capacity of regulators to implement policies in accordance with WHO recommendations.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1.