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Has Canadian House Price Growth been Excessive?

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Abstract

The dramatic rise in the ratio of Canada's average house price to average rent has led to speculation that there is a bubble in the Canadian housing market. Others have argued, however, that the currently high level of house prices may be rationalized by the low cost of financing, given the decline in interest rates over the last two decades. In this article, we assess these arguments through the lens of a simple asset pricing model applied to city-level data. We quantify the extent to which excess growth in Canadian house prices depends on the nature of the current regime governing *real* interest rates, expections of rent growth in different cities and variations in property taxes.

Journal of Economic Literature Classification: R10, R21, G12
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1 Introduction

In this paper we use a canonical asset pricing framework to consider the extent to which the growth of house prices in major Canadian cities since 1987 is explained by changes in rents, real interest rates, and property taxes. We are interested specifically in the extent, if any, to which current prices indicate over-valuation of residential housing. Overall, for the cities we study, we find over-valuations, relative to the predictions of our model, ranging from -12 percent for Edmonton (indicating *under-valuation)* to 31 percent for Vancouver, with a population weighted average of 11 percent.

The extent to which house prices have appreciated by less than or in excess of the predictions of our theory depends significantly on the way participants in the housing market view (currently low) real interest rates: Are they here to stay or a transitory phenomenon? Specifically, we find that if market-participants place a high probablity on Canada having returned to a regime of lower "normal" real interest rates, then our measures of the over-valuation of residential real estate in major Canadian cities are significantly reduced relative to those implied under the assumption that real interest rates follow a simple autoregressive process which implies rapid reversion to the sample mean.

Our work is motivated by recent observations that Canada's average price-rent ratio has risen dramatically since the 1990s. We focus on the growth of the *price-rent ratio* as several factors which drive house *prices* may be expected to have a similar effect on *rents*. For example, increases in either construction or land costs (possibly due to more stringent development regulations in the face of population growth) would tend to increase both purchase prices and rents. Moreover, in many settings (including the model we study) these would have little effect on the price-rent ratio.[1] Similarly, increases in either income or population which drive up house prices will also raise rents.

Growth of the price-rent ratio is commonly advanced as an indicator of the potential extent of over-valuation of owner-occupied housing (*e.g.*, see OECD, 2014). *The Economist* magazine publishes an index of the aggregate price-rent ratio relative to its long-run average for a number of countries. According to this indicator, the aggregate price-rent ratio in Canada at the end of 2014 was 89% higher than its historical average.[2] This rapid and sustained rise (by comparison, at its peak in 2006 the same indicator for the U.S. was 52%

[1] This statement holds for prices and rents on identical units. To the extent that rental and owner occupied units require different quantities and/or types of land systematically, changing land prices could lead to changes in the price-rent ratio through a composition effect.

[2] For Canada the index is computed relative to its mean over the period 1975 -2014.

above its historical average) has induced some commentators to argue that a speculative bubble is under way; the collapse of which may have a calamitous effect on the Canadian economy (*e.g.* see O'Brien, 2013 and Roubini, 2013).

While increases in the price-rent ratio are commonly treated as evidence of housing being overvalued, indexes like that employed by the OECD and *The Economist* are subject to a number of criticisms. In particular, questions have been raised recently regarding the appropriateness of the aggregate, quality-adjusted rent price index produced by Statistics Canada and used as the denominator in these indices. The rent index has grown at a much slower rate since 1990 than average rents for various types of accommodation measured in survey data by the Canadian Mortgage and Housing Corporation (CMHC).[3] Moreover, this measure implies that real rents *declined* on average by over 50% during the 1970s, and have continued to decline, though at a less dramatic pace, since 1980. In contrast, quality-adjusted real rents in the US have grown on average by 1% a year since 1970. While improvements should, of course, result in an index of quality adjusted rents growing more slowly than average market rents, we argue that the difference has likely been overstated.[4]

We address concerns regarding the rent data by taking two different approaches. First, we develop an alternative index of aggregate rents using available data from Statistics Canada and the CMHC. We are able to construct an unadjusted real rent index that goes back to 1970. When we compare it with the US real rent price index we find it has very similar long-run properties. Using this index, we find that the aggregate repeat-sale price-rent ratio in 2014 was 55% above its average over the period prior to 1996. While informative, the use of aggregate indices ignores considerable variation across locations. We therefore also use CMHC rental survey data, which is compiled for relatively large Canadian cities at the Census Metropolitan Area (CMA) level since 1987 and study the behaviour of price-rental ratios at the city level.

A second criticism of the use of simple price-rent ratios as indicators of excess valuation of owned housing is that they ignore the role of variation in the expected costs of mortgage finance, fluctuations in expected rent growth, and changing property taxes. Recent mortgage rates (both nominal and real) have been substantially lower than during the 1970's and 1980's and some market observers have argued that this may be sufficient to rationalize high house prices in Canada (see e.g. Wiebe, 2014 and Arseneau, 2015). Also, rent growth has

[3]The difference is substantial; as Dunning (2014) notes, the Statistics Canada index grows at 1.4% per year on average, while CMHC reports average rent growth of 2.2%, more than 50% higher.

[4]Indeed, "excess quality adjustment" was partly addressed by Statistics Canada when their methodology changed in 2009.

varied substantially across cities and over time, reflecting in part variation in overall housing demand. Finally, although effective property taxes are difficult to compute, it is clear that they have also varied substantially both across cities and over time. For these reasons, we develop a tractable analytical framework that attempts to account for these important factors in an index of the relative valuation of owned to rental housing.

Under a variety of assumptions, the most important determinant, quantitatively, of the price-rent ratio is the real interest rate. A marked decline in real interest rates over the last two decades could, in principle, rationalize substantial increases in price-rent ratios through the lowering of the cost of financing household investment in housing. As Glaeser, Gottlieb and Gyourko (2011) point out, however, relatively low interest rates justify high observed price-rent ratios only if they are expected to be very persistent. Indeed, an assumption along these lines was made by Himmelberg, Mayer and Sinai (2005), who argued that rising prices at the start of the recent U.S. housing boom might largely reflect fundamentals. If one were to take account of mean reversion in interest rates, house prices might be much less sensitive to interest rate movements.

In our analysis, we consider the possibility that the interest rate has shifted over time between two regimes: A high interest rate regime that obtained in the 1980's and early 1990's, and a *new normal* low interest regime which characterized the economy earlier and to which it has returned since the mid 1990's. To this end, we estimate a regime-switching model of interest rates. When agents use this process to forecast the interest rate (*i.e.* when they believe that the long-run or "normal" real interest rate is highly likely to have fallen since the 1980s and 1990s) we estimate the extent of excess valuation to be considerably lower (up to forty percentage points lower on average) than that implied by simple price-rent ratios with the interest rate fixed at its constant long-run value. Moreover, once we incorporate variation in the effective property tax rate, excess valuations are reduced even further for some cities.

For comparison purposes, we also consider two other possibilities for agents' expectations of real interest rate movements. First, we posit a simple behavioural expectations strategy, in which agents extrapolate forward using a moving average of past and current rates. This approach yields predicted house prices in 2014 which are for most cities *higher* than their actual observed values. Second, we take a rational expectations approach by estimating a simple autoregressive process for the real interest rate using historical data on real mortgage rates in Canada beginning in 1951. This approach results in estimated excess valuations only ten percentage points lower on average than those implied by simple price-rent ratios.

Ours is related to several other papers in the literature. Verbrugge (2008) provides a detailed analysis of the deviation of the user cost of ownership from rents in US cities during their housing boom for housing units of observationally equivalent quality. Although our analysis is based on much less detailed, aggregate city-level data, we emphasize the important role of expectations regarding interest rates and rent growth. Granziera and Kozicki (2012) study US houses price fluctuations from the perspective of a Lucas-tree asset pricing model and consider the implications of alternative assumptions regarding expectations. In contrast to their analysis, we focus on the comparison of owning and renting a house of a given quality, rather than marginal increments to housing. Moreover, we do not impose a tight relationship between the discount factor and rent growth as they do.[5]

Sommer at al. (2013) develop a dynamic equilibrium model of owning and renting and calibrate it to match aggregate US data. They argue that lower interest rates and relaxed lending standards can account for approximately 50% of the increase in the U.S. house price–rent ratio between 1995 and 2006. Han, Han and Zhu (2014) study the extent to which price-income and price-rent ratios in Beijing in 2014 can be rationalized by expectations of future macroeconomic changes along a long-run balanced growth path. In the end, the main drivers of the price-rent ratio in all of these models are expected rent growth, lending conditions including expected interest rates and taxes. Rather than modelling them explicitly, we take rent growth to be a summary statistic of the main underlying fundamentals driving supply and demand conditions at the city level. This approach allows us to focus on the role of alternative stochastic process for interest rates and rent growth.[6]

The remainder of the paper is organized as follows. In Section 2, we document and discuss the implications of price-rent ratios for Canada and the US and a group of 12 major Canadian cities. Then, in Section 3, we develop a generalized framework for measuring the relative value of owned housing, incorporating variation in interest rates, local rent growth and property taxes. Section 4 considers alternative empirical models of interest rates and estimates their parameters. Sections 5 uses the theoretical framework together with our empirical estimates to measure implied excess valuations under different assumptions regarding forecasts of future interest rates, treatment of property taxes, rent growth and the benchmark period. Section 6 offers some conclusions and appendices provide more details on both the data sources and our calculations.

[5] We find no evidence of such a correlation in Canadian data.

[6] We focus less on lending standards as this appears to have been less of an issue in Canada. In fact, the role of changing lending standards in driving the US housing boom has been called into question recently (see as Foote, Gerardi and Willen, 2012).

2 Price-Rent Ratios

2.1 Aggregates: Canada vs. the United States

Although our primary focus is on Canadian cities, it is interesting to compare first the behaviour of aggregate indices of the price-rent ratio in Canada and the U.S., for two reasons. First, much of the current interest in Canadian house prices is inspired by the recent U.S. experience and the concern that a similar price decline may eventually occur in Canada. Second, it is possible to construct a longer time series at the aggregate level than at the CMA level.

In making cross-country comparisons it is common to measure average rents using rent price indices that are constructed as part of the overall consumer price index (*e.g.* Girouard at al. 2006). This data is typically quality-adjusted in various ways and the methodology used varies across countries and over time. As a result, some care must be taken in interpreting the aggregate data and, consequently, in comparing price-rent ratios across countries.

Until the end of 1977, the U.S. price index for rents omitted most rent increases that took place when units had a change of tenants or were vacant. This was seen as biasing inflation estimates downward, and between 1978 and 1985 the Bureau of Labor Statistics (BLS) implemented a series of methodological changes that reduced this "nonresponse bias". Recently, Crone, Nakamura and Voith (2006) have made efforts to correct for the bias that occurred *prior* to 1985 in order to construct a consistent series for the price rent index.[7] The right-hand panel of Figure 1 shows the U.S. real rent index measured using their corrected rent price index relative to the US consumption deflator. According to this measure, US real rents grew at an average rate of 1% per year between 1970 and 2014.[8]

For Canada, the quality-adjusted rent price index constructed by Statistics Canada appears to have grown much more slowly than other measures of market rents (such as those produced by the CMHC) and has some peculiar properties that are inconsistent with those of rent price indices of other countries. In particular, it implies that average *real* rents (i.e. rents relative to the consumption deflator) have declined dramatically in Canada since 1970. While it is certainly possible for real rents to fall, it is not clear why the behaviour of real

[7] Specifically, they set up a model of nonresponse bias, parameterize it, and test it using a BLS microdata set for rents.

[8] Using the uncorrected BLS index and deflating by the CPI would imply much slower real rent growth (e.g. Sommer, Sullivan, and Verbrugge, 2013). Using census micro data, Davis and Ortalo-Magné (2011) estimate quality-adjusted real rent growth to be 1% per year between 1980 and 2000.

Canadian rents is so starkly different from that of the US.[9] Moreover, if we believe the share of expenditure on shelter to be roughly constant over time, we should expect average real rents to grow on average with renters' incomes.

To address these concerns we construct an alternative index for average rents paid in Canada.[10] Figure 1 compares our constructed index relative to the household consumption deflator to the real rent price in the U.S. As may be seen, its long-run properties are very similar, even though it is not quality-adjusted: real rents grow on average over the period by about 1% a year since 1970.[11] There are, of course, some significant differences over shorter sub-periods. In particular, while real rent growth appears to have slowed in the US since the mid-2000s, it has continued upward in Canada, with no sign of slowing.

Figure 2 depicts price-rent ratios using the rent indices illustrated in Figure 1 for each country. These ratios are computed relative to their average over the period 1971-1996.[12] The solid line for Canada uses the average price of existing houses sold through the MLS, whereas the dashed lines for each country use repeat-sales price indices.[13] A repeat-sales price index should, in principle, adjust for some aspects of quality in the housing stock resulting from new additions. In the case of Canada the repeat-sales index has been produced for 11 major cities since 1999, when we have normalized it to equal the MLS index. We are effectively assuming here that its average over the benchmark period is the same as that of the MLS index.[14]

Since the Canadian rent data is not quality-adjusted, it is not clear which of the price-rent ratios is most appropriate for our purposes. If, on the one hand, the average quality of the rental stock has grown at the same rate as the housing stock, then the calculation using the MLS index is more meaningful. If, on the other hand, the quality of the rental

[9]This likely reflects in part "excess quality adjustment" similar to the nonresponse bias in the BLS data discussed earliers. For Canada, until 2009 any increment in rent that occurred when the tenant changed was counted as resulting from an increase in quality. This methodology was corrected in July 2009 (see http://www23.statcan.gc.ca/imdb-bmdi/document/2301_D41_T9_V1-eng.pdf).

[10]See Appendix B for details of our calculations and an extended discussion of the Statscan aggregate rent index.

[11]Crone et al. (2006) estimate that that the annual difference between median rent growth (based on the American Housing survey) and the CPI rent price index was only 0.1 percentage points between 1985-2001. During this period, at least, quality adjustment had only a minor impact on average.

[12]In our view, benchmarking relative to the average over the full sample period is not appropriate. If prices continually rise relative to rents for an extended period of time, the long-run average will rise as well. If there is an over-valuation or bubble in the price of owner-occupied housing, this approach would thus yield an underestimate of it.

[13]See Appendix A for details.

[14]For some cities the Teranet index can be computed for earlier years. As we will see below, for those cases, the index does in fact move very closely with the MLS index prior to 1999.

stock has not grown as fast as that of owner-occupied housing, then the Teranet index may be more appropriate.[15] Although the two indexes move together fairly closely after 1999, throughout our discussion (of both aggregate and city level data) we will present results using both indices.[16]

Table 1: Price-rent ratio (% deviation from 1971-96 average)

CMA (year)	Average resale price	Repeat-sales index
	(Not quality adjusted)	(Quality adjusted)
Canada (2006)	39	33
U.S. (2006)	–	36
Canada (2014)	62	55
U.S. (2014)	–	6

Table 1 documents the deviation of the price-rent ratio at the end of 2014 from its average over the period 1971-96. In 2014 the Canadian MLS price-rent ratio was 62% greater than this long-run average. Using the repeat sales indices, the Canadian price-rent ratio was 55% higher, whereas that for the US it was 6% greater. At the height of the U.S. housing boom in 2006, the U.S. repeat-sales price-rent ratio was 32% higher than its long-run average, and the Canadian ratio stood slightly below that level relative to its respective average.

2.2 City-Level Price-Rent Ratios in Canada

Given the significant variation in rent growth and other factors across cities, we view our subsequent quantitative analysis as being more applicable to the CMA level. Unlike the aggregate data, however, we have not been able to construct meaningful *city-level* price-rent ratios prior to 1987.[17] We therefore use the average over the first decade for which data is available (1987-96) as our "benchmark decade". While this may seem somewhat arbitrary,

[15] In the US case both prices and rents are ostensibly quality adjusted.

[16] A third source of house price data is the New Housing Price Index published by Statistics Canada, which is quality-adjusted. For all cities and in aggregate this index has grown more slowly than those considered here. We have chosen not to present results based on this data for two reasons: (1) New homes are mostly built on peripheral land around cities and are not as easily comparable with rental units which are more commonly located centrally. (2) We are uncertain about the implications of the quality-adjustment in this data too. For example, according to this data, prices for (quality-adjusted) new homes in Vancouver appear to be lower in 2014 than they were in 1993.

[17] Although MLS price data by city goes back to 1982, the rent data is only available from 1987. Quality-adjusted indices of the total cost of renting are available by city for earlier years from Statistics Canada. They appear, however, to suffer from similar problems to those of the aggregate index.

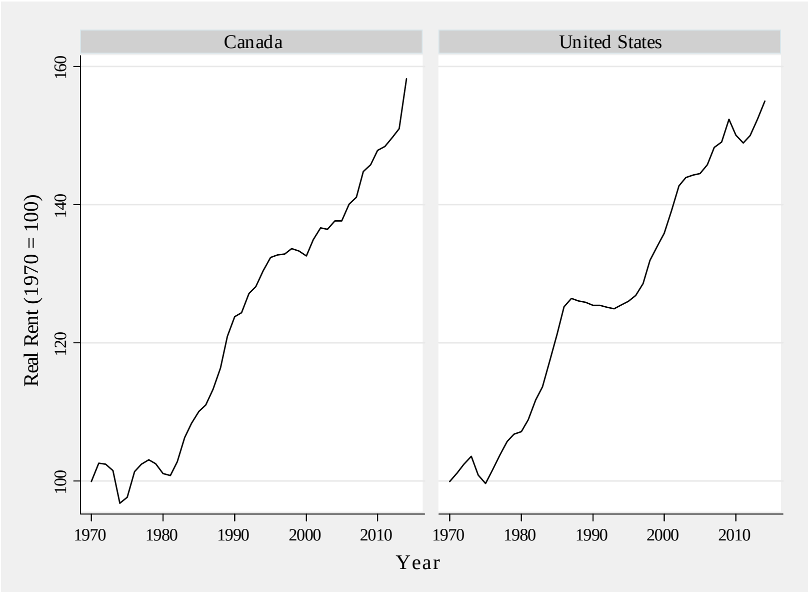

Figure 1: Real rents in Canada and the US

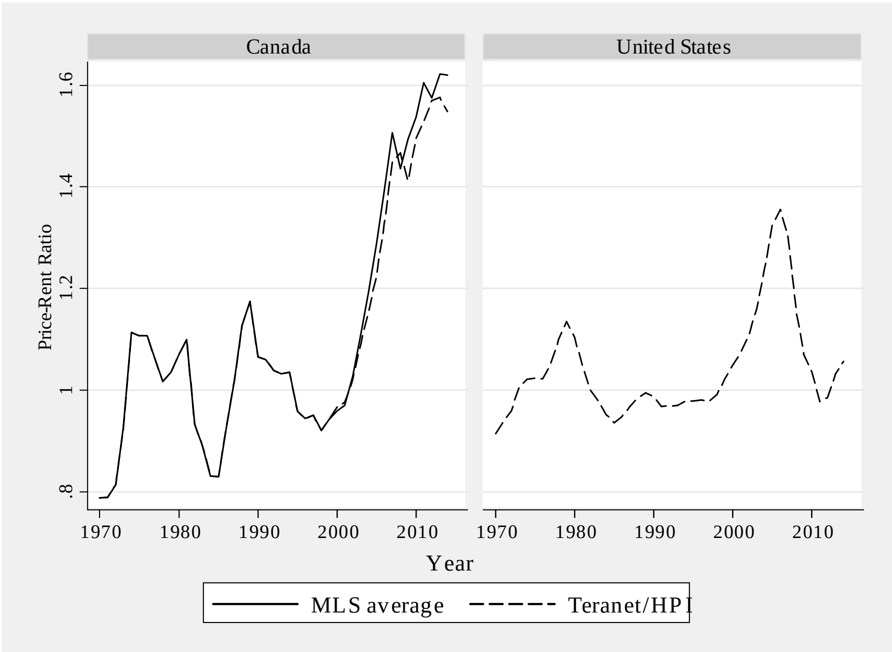

Figure 2: Aggregate price-rent ratio relative to 1971-1996 average

as we will see below the resulting average excess valuation across cities is consistent with the aggregate data for Canada computed above for a longer benchmark period.[18]

Table 2: Price-rent ratio in 2014 (% deviation from 1987-96 average)

CMA	MLS average	Teranet
	(Not quality-adjusted)	(Quality-adjusted)
Calgary	59	51
Edmonton	46	36
Gatineau*	80	48
Halifax	60	40
Hamilton	59	36
Montreal	96	69
Ottawa*	55	39
Quebec	102	81
Toronto	56	45
Vancouver	91	79
Victoria	63	66
Winnipeg	76	64
Average	70	55
Weighted	72	56

* Teranet index is computed for combined Ottawa-Gatineau region

Figure 3 documents the price-rental ratio for 12 large CMAs relative to the average over 1987-96. Here, rents are the (unadjusted) average rents for two-bedroom apartments published by the CMHC.[19] As before the solid lines correspond to the average prices of existing homes sold through the MLS and the dashed lines correspond to the Teranet repeat-sale price index. Although this index is available for all cities from 1999, for some cities it was available earlier. As may be seen, the two indexes move together closely for most cities, especially prior to 2001. There are, however, several exceptions. For Gatineau in particular, the Teranet index has grown much more slowly than the MLS index. Unfortunately, interpreting this is complicated by the fact that the Teranet index is actually computed for the combined Ottawa-Gatineau region.[20] During the benchmark period both indices (where they are available) fluctuate relatively tightly around one for most cities. The main exceptions

[18] Extending the benchmark period by a few years (e.g. 1987-2000) makes little difference to the results.

[19] Although average rents for other types of accommodation are available, they tend to move together. We use those for two-bedroom apartments as these appear to be the most common and their rents are the least volatile.

[20] Note, however, that average prices have in fact grown at very similar rates in the two regions since 1999.

are Toronto and Hamilton which experienced much larger fluctuations around the average during that decade.

Table 2 documents the extent to which price-rent ratios in each city have grown by 2014 relative to their respective averages during the benchmark decade for both the MLS average price and the Teranet resale price. The population-weighted averages are 72% and 56% respectively, but there is considerable variation across cities.[21] In all cases but one (Victoria), measures of the price-rent ratio using the Teranet index imply *lower* price-rent ratios in 2014. Overall, price-rent ratios have grown dramatically in all cities, with Quebec at 102%, representing the maximum. These numbers reflect simply the fact of growth in the city-level price-rent ratios. To ask the question of whether this growth has been in some sense excessive, we now develop a theory of the price of owned relative to rental housing.

3 A Model of the Price-Rent Ratio

In studying the evolution of price-rent ratios it is common to compare movements to those predicted by the user-cost model of Poterba (1984). The central idea in this model is that in equilibrium the marginal home-owner should be indifferent between renting and buying. Versions of this framework have been applied recently to US data by Himmelberg, Mayer and Sinai (2005) and Glaeser, Gottlieb and Gyourko (2011) and to OECD data by Girouard *et al.* (2006). In these studies and others, deterministic versions of the theory are typically parameterized using long-run or moving averages of key variables as proxies. Here, we take the stochastic elements of the model more seriously in order to consider and compare alternative models of expectation formation and to incorporate real-time data into forecasts.[22] In so doing, we provide a relatively simple and tractable framework that does not require significant computational analysis.

We assume that asset markets are complete and there are no trading frictions. Let the rent associated with a housing unit of quality q (determined by size, closeness to amenities, etc.) in city c at time t can be expressed as

$$R_{ct}(q) = x_{ct}q. \tag{1}$$

[21] The average excess value for the Teranet index in 2014 is very similar to that implied by the aggregate data using a longer benchmark period. Those for the MLS index are not comparable because the aggregate value is an average over 35 cities, not just the 12 considered here.

[22] In a working paper version of their 2011 article, Glaeser et al. (2010) also discuss the potential implications of mean-reverting stochastic interest rates, but take a somewhat different appraoch.

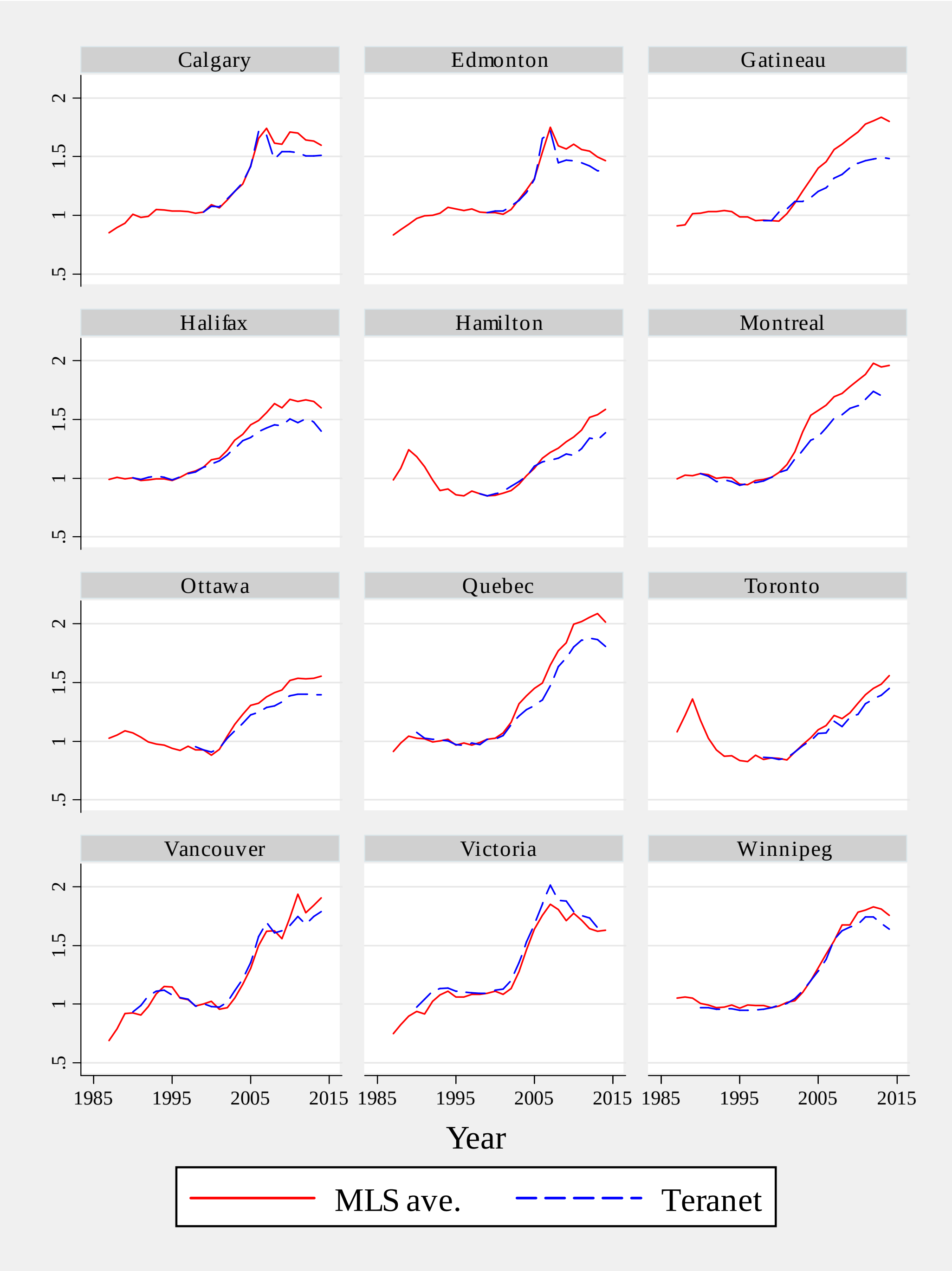

Figure 3: Price-rent ratios relative to benchmark decade

Here, x_{ct} represents a summary index of the all the factors, possibly stochastic, that effect the supply and demand for housing in city c. For example, if the housing stock is endogenous, it will include the unit cost of producing a housing unit of quality q, which in turn may depend on the number of houses built, land prices and wages. If the housing stock is perfectly inelastic, x_{ct} will represent the marginal benefit of a housing unit of quality q, which may depend on the distribution of incomes and demographic factors. If the distribution of the quality of rented housing units is $F_{ct}^R(q)$, the average rent in city c is

$$R_{ct} = \int R_{ct}(q)dF_{ct}^R(q) = x_{ct} \int q dF_{ct}^R(q) \tag{2}$$

where the integration is over the range of qualities, q.[23]

For a renter, the expected outlays from renting over the duration of his/her tenancy is the present discounted value of current and expected future rental payments. We assume that discount rates and rent growth evolve over time according to stationary stochastic processes. Under these assumptions one can express the present discounted cost of renting a housing unit of quality q in city c at time t as

$$C_{rct}(q) = Z_{ct}R_{ct}(q) \tag{3}$$

where

$$Z_{ct} = 1 + E_t\left[\sum_{j=1}^{\infty} D_{t+j}\frac{R_{ct+j}}{R_{ct}}\right] \quad \text{and} \quad D_{t+j} = \prod_{s=1}^{j}\left(\frac{1}{1+r_{t+s}}\right). \tag{4}$$

Z_{ct} thus depends on the forecasted means and variances of future interest, r_{t+j} and rental growth rates, $g_{ct+j} = \frac{R_{ct+j}}{R_{ct}}$, conditional on information available at time t.

Let $C_{pct}(q)$ denote the present value of the cost of becoming the owner of a house of quality q purchased at price $P_{ct}(q)$. This consists of a down-payment, $\psi P_{ct}(q)$, the present discounted value of future mortgage payments, current and future property taxes and current and future maintenance costs. We assume that the real mortgage rate faced by the representative household is a multiple ζ of the real rate at which they discount the future, r_t. In the United States, where interest payments on *residential* mortgages are deductible from income taxes, ζ could be one minus the income tax rate. In Canada, such interest payments are not tax deductible, so $\zeta = 1$.[24]

[23] It is not important that quality, q, is treated as continuous in (2). Quality can be discrete and it will have no effect on the properties of the model on which we focus.

[24] While Canada does offer mortgage interest deductability for mortgages taken for investment purposes, we do not allow for it here. The reason for this is that in our model, the marginal home-owner equates the

Computation of property taxes at the city level is, in general, complicated. The effective property tax rate depends both on the mill rate and the evolution of property assessments which, typically, both differ significantly from transactions prices and vary both across cities and over time. To begin with, we simplify the analysis by assuming that the mill rate, τ, and the ratio of initial assessment to purchase price, α, are both constant.[25] We assume also that property assessments are expected to grow at the same rate, g_{ct}, as rents. Similarly, maintenance costs are assumed to be a constant proportion δ of expected housing value.

Under these assumptions, the cost of owning a representative housing unit may be expressed as

$$C_{pct}(q) = P_{ct}(q)\left[\psi + \zeta(1-\psi) + (\tau\alpha + \delta)\, Z_{ct}\right] \tag{5}$$

where Z_{ct} is defined above (see Appendix D for the derivation of 5). If all costs of renting and owning are accounted for correctly, then the costs of owning a house of quality q will equal the cost of renting it, $C_{pct}(q) = C_{crt}(q)$. Let $F_{ct}^{H}(q)$ denote the distribution of quality amongst owned housing in city c. It then follows that the average price of owner-occupied house can be expressed as

$$P_{ct} = \int P_{ct}(q)dF_{ct}^{H}(q) = \frac{\theta_{ct} Z_{ct} R_{ct}}{\psi + \zeta(1-\psi) + (\tau\alpha + \delta)\, Z_{ct}} \tag{6}$$

where

$$\theta_{ct} = \frac{\bar{q}_{ct}^{H}}{\bar{q}_{ct}^{R}} = \frac{\int q dF_{ct}^{H}(q)}{\int q dF_{ct}^{R}(q)}. \tag{7}$$

When considering movements in the ratio of the average price to the average rent, we effectively assume that the relative average qualities of the two groups of housing remain constant over time in a given location, $\theta_{ct} = \theta_c$. When using the Teranet index for house prices, the numerator of (7) is time varying and we are thus effectively adjusting the price to allow for this quality adjustment. Of course, the premium could change over time for reasons other than quality. For example, if there is a pure ownership premium, rising incomes might raise the demand for owned versus rental housing, thereby causing θ_{ct} to rise over time and across cities.[26] Unless, however, the relative costs of producing owned versus rental housing change in a secular fashion over time, it is not clear why θ_{ct} would have a substantial trend.[27]

cost of renting and owning their *residence*. We are grateful to an anonymous referee for pointing this out. We also abstract from a number of other effective subsidies to ownership (in both countries) which may play likely a minor role.

[25] Below we consider the potential implications of effective property tax rates that vary over time.

[26] This premium could also be viewed as a reduced-form way to represent differences in the risk-characteristics and/or differences in the borrowing costs associated with owning and renting.

[27] One possibility could be that such a trend arises due to changes in the compostion of either the housing stock or households rent vs. own choices.

4 Expectations

From (6), it is clear that the predictions of the model depend crucially on expectations regarding interest rates and rent growth. In this section we describe the different cases we consider for the evolution of these variables, and agents' expectations regarding them, over time.

4.1 Interest Rates

For mortgage interest rates we use the CMHC's average 5-year conventional mortgage lending rate for Canada and the FHFA's average terms on conventional single family mortgages for the U.S. These series are available on an annual basis from 1951 and 1963, respectively. The ex-post real mortgage interest rate, r_t, is then computed as the mortgage rate at date $t-1$ minus the inflation rate between $t-1$ and t. We consider three alternative specification of interest rate expectations:

4.1.1 Extrapolative Discounting

A simple "behavioural" approach to discounting future rents is to assume that recent real interest rates will persist indefinitely. More precisely, we suppose that households discount using a simple moving average of the last four years of real mortgage interest rates; effectively assuming that the rate will not change at all over their lifetimes. We use a four-year moving average to capture the idea that it may take a few years for households to become confident that interest rates will persist near their current levels.

While expectations based on this approach are, of course, inconsistent with statistical forecasts based on historical data, we view this "deterministic" user-cost approach as a useful behavioural benchmark. Also, the assumption that rates will not change may be thought of as capturing, in some sense, the effect of 25 and 30-year fixed (nominal) rate mortgages, which are common in the U.S.[28]

4.1.2 Rational Expectations: A simple autoregressive process

While interest rates experience persistent fluctuations, history suggests that they tend to be mean-reverting. That is, a lower than average rate today may not imply particularly

[28] Although the average rate on long term fixed-rate mortgages is typically higher than that on the 5-year mortgage in Canada.

low rates in a few years time. To address this issue we now assume that the real mortgage interest rate follows a simple stationary autoregressive process given by

$$\hat{r}_t = \bar{r} + \sum_{s=1}^{k} \mu_s (\hat{r}_{t-s} - \bar{r}) + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{8}$$

where $\hat{r}_t = \ln(1 + r_t)$ and k denotes the lag-length.

The parameters of this process were estimated for various lag lengths. A single lag was determined to be optimal based on both the Akaike Information Criterion (AIC) and the Schwarz Criterion (Bayesian Information Criterion). The estimation results for $k = 1$ and the values of the AIC and BIC for $k > 1$ are provided in Table 3.[29] These estimates imply that the real mortgage interest rate should be expected to revert to an unconditional mean of 4.9%. Similar estimates are obtained for the U.S. and are reported in the Appendix C.

Table 3: Interest rate process parameter estimates: Simple Autoregressive

Parameter	Value	Std. error		k	AIC	BIC
$(1-\mu_1)\bar{r}$	0.014	(0.005)		1	-320.78	-316.53
μ_1	0.714	(0.092)		2	-315.17	-308.84
σ_ε	0.018			3	-307.34	-298.96
R^2	0.502			4	-299.16	-288.77
Log-likelihood	162.7					

Note: Robust standard errors in parenthesis

4.1.3 Rational Expectations: Regime-switching

There are reasons to believe that global long-run, "normal" real interest rates have, in the last decade, fallen permanently relative to their levels in the 1980s and early 1990s. Thus, Canadian mortgage rates may be rationally forecast to remain low. Beaudry and Bergevin (2013), for example, identify several factors that are expected to result in low global real interest rates over the next decade or so. These include slower growth of labour forces and aging populations in developed countries which are expected to reduce investment demand and increase savings, high and rising savings by households in China and other emerging economies and the persistent after-effects of the Great Recession which continues to dampen investment demand and has induced less borrowing and greater saving by US households.

Along similar lines, several recent papers have considered the possibility of "secular stagnation" heralding a prolonged era of low real interest rates. For examples, consider Eichen-

[29] The optimal lag length is taken to be that for which both the AIC and BIC are minimized.

green (2015), Gordon (2015), and Summers (2015).[30] Hamilton *et al.* (2015) are more skeptical regarding the evidence. For a dynamic stochastic general equilibrium theory of increased housing demand in response to low interest rates caused by secular stagnation, see Thwaites (2015).

For our purposes, it makes little difference *why* or even *if* we have entered a new regime of real interest rates. What matters is the extent to which market participants place some likelihood on this possibility. According to our simple user-cost model, if they do there will be implications for house prices and rents.

To allow for the possibility of a new normal characterized by low real interest rates, we estimate a simple two-regime switching process for the real interest rate in which the likelihood of being in one regime rather than the other depends only on recent observations of the interest rate.[31] Specifically

$$\hat{r}_t = \begin{cases} (1-\mu_h)\bar{r}_h + \mu_h \hat{r}_{t-1} + \sigma_h \varepsilon_t & \text{if } \alpha + \beta r_t^* + u_t > 0 \\ (1-\mu_l)\bar{r}_l + \mu_l \hat{r}_{t-1} + \sigma_l \varepsilon_t & \text{otherwise} \end{cases} \tag{9}$$

where $\varepsilon_t \sim N(0,1)$, $u_t \sim N(0,1)$ and $r_t^* = \sum_{s=1}^{4} \hat{r}_{t-s}$. This process allows for both mean-reversion and changes to the permanent component of the interest rate. We are not, of course, the first to model interest rates in this way. Ang and Beckaert (2002), for example, consider a number of alternative regime-switching models and provide compelling evidence of their superiority over simple autoregressive specifications. This set up provides a parsimonious way of allowing for a time-varying long-run or "natural" rate of interest.

The estimation results are provided in Table 4. All of the estimated parameters ($\bar{r}_i$, μ_i and σ_i) are allowed to vary across the two possible regimes, $i \in \{l, h\}$. Not suprisingly, a likelihood ratio test (with two degrees of freedom) confirms that this model is preferred to the simple AR(1). The estimates imply that the two potential regimes consist of a high long-run real interest rate regime with $\bar{r}_h = 5.1\%$ and a low long-run rate regime with $\bar{r}_l = 3.4\%$. Figure 4 depicts the estimated probability assigned to being in the low interest rate regime at each date. This procedure generates a probability of being in the low rate regime which varies from a low of 0.09 in 1987 to a value of 0.82 in 2014. Similar estimates are obtained for the U.S. and are reported in Appendix C. At each date, we assign the estimated probabilities to being in each of the two regimes when calculating present discounted values. In effect this implies that it gradually becomes increasingly likely that the long run rate to which the real interest rate is expected to revert is the lower value (i.e. $\bar{r}_l = 3.4\%$).

[30] These papers constituted a session at the January 2015 meeting of the American Economics Association.
[31] Again we use a four-year moving average of past real interest rates.

Table 4: Interest rate process estimates: Canada, Regime-switching

High regime			Low regime		
$(1-\mu_h)\bar{r}_h$	0.017	(0.008)	$(1-\mu_l)\bar{r}_l$	0.004	(0.002)
μ_h	0.645	(0.155)	μ_l	0.882	(0.034)
σ_h	0.024		σ_l	0.006	
R^2	0.359		R^2	0.920	
Switching equation					
α	1.95	(0.17)			
β	-34.5	(3.01)			
R^2	0.70				
Log-likelihood	219.7				

Note: Robust standard errors in parenthesis

4.2 Rent Growth Rates

Throughout most of our analysis we assume that real rent growth in city (or country) c follows a first-order autoregressive process given by

$$g_{ct+1} = (1-\rho_c)\bar{g}_c + \rho_c g_{ct} + e_{ct+1} \qquad e_{ct} \sim N(0, \sigma_{ec}^2) \tag{10}$$

where $\rho_c \in (0,1)$ and $\bar{g}_c$ represents the average or long-run rent growth for country or city c. The parameter estimates for each country and each city for 1987-2014 are provided in Table 5. We specify the processes to have a single lag-length because of limited data. There is, however, little if any persistence in rent growth. We could, in principle, have also allowed for correlation between real interest rates and rent growth rates. For example, we could have estimated a VAR system in r_t and g_{ct} for each city. In our data, however, there appears to be no such correlation, so we ignore it for simplicity. In Section 5.4 we consider alternative specifications of (10) for some cities.

Table 5: Estimates of Real Rent Growth Processes

Country/CMA	$\bar{g}$	ρ	σ_e	R^2
Calgary	.019 (.016)	.428 (.186)	.046	.180
Edmonton	.022 (.015)	.521 (.172)	.038	.276
Gatineau	.000 (.006)	.396 (.185)	.018	.160
Halifax	.006 (.005)	.315 (.196)	.016	.107
Hamilton	.009 (.004)	-.044 (.194)	.015	.002
Montreal	.001 (.005)	.240 (.182)	.020	.068
Ottawa	.006 (.007)	.322 (.198)	.024	.099
Quebec	.002 (.005)	.364 (.188)	.017	.136
Toronto	.009 (.005)	.231 (.198)	.019	.054
Vancouver	.010 (.005)	.125 (.202)	.021	.016
Victoria	.009 (.008)	.706 (.156)	.014	.499
Winnipeg	.011 (.011)	.719 (.146)	.015	.502
Canada	.010 (.003)	.333 (.157)	.015	.010
U.S.	.010 (.003)	.499 (.136)	.011	.247

Notes: (1) Estimation by non-linear least squares

(2) Standard errors in parenthesis

4.3 Present value of future rents

In Appendix D we show that for the simple autoregressive processes for real interest and rent growth rates specified above, the expected present value of rental payments can be expressed as

$$Z_{ct}(\bar{r}, \mu, \sigma_\varepsilon) = 1 + \sum_{j=1}^{\infty} \exp\left(-\left(M^r_{t,j} + \frac{V^r_{t,j}}{2} - M^g_{ct,j} - \frac{V^g_{ct,j}}{2}\right)\right) \tag{11}$$

where

$$M^r_{t,j} = j\bar{r} + \frac{\mu\left(1-\mu^j\right)}{1-\mu}(r_t - \bar{r}), \tag{12}$$

$$V^r_{t,j} = \frac{\sigma^2_\varepsilon}{(1-\mu)^2}\left(j - 2\mu\left(\frac{1-\mu^j}{1-\mu}\right) + \mu^2\left(\frac{1-\mu^{2j}}{1-\mu^2}\right)\right) \tag{13}$$

and similar expressions hold for $M^g_{ct,j}$ and $V^g_{ct,j}$. We use our point estimates from Tables 3 and 5 to parameterize these expressions and, at each date, given r_t and g_t we solve forward and approximate the sum by truncating it to 1000 periods.

For the extrapolative discounting case, we simply replace $M^r_{t,j}$ with the four-year moving average of r_t and set $V^r_{t,j} = 0$. For the regime-switching model, we compute a present value for each regime, $Z_{ct}(\bar{r}_l, \mu_l, \sigma_l)$ and $Z_{ct}(\bar{r}_h, \mu_h, \sigma_h)$, and use the estimated parameters from Table 4 and the implied probabilities depicted in Figure 4 to compute the overall expected present value conditional on recent observations of the real interest rate. To capture the fact that these mortgage rates are locked-in for five years, in each case we also replace the first five years of expected mortgage rates with the initial rate. This has very little impact on our present value calculations.

5 Applications

5.1 Canada vs. The U.S. in Aggregate

We start by applying our framework to aggregate data and comparing the implications for the US and Canada. As noted earlier, it is possible to construct *national* price and rent indices going back to 1970 for both countries. We calibrate the asset pricing model to be consistent with aggregate facts for each country. For Canada we set $\zeta_{can} = 1$ (which renders the value of ψ irrelevant) whereas for the U.S. we set $\zeta_{us} = 0.75$ and $\psi = 0.2$, reflecting a marginal income tax rate of 25% and an average down-payment ratio of 20%, respectively. In fact, as long as these parameters are roughly constant over time, their exact values have only very small effects on our results. We set θ for each country so that the price-rent ratio implied by the theory (*i.e.* given by (6)) is equal to the average of the observed price-rent ratio between 1971 and 1996. We fix the effective property tax rate and δ equal to 0.008 for both countries.

Figure 5 shows the price index for each country together with the prices predicted by our user cost model under both rational discounting assumptions.[32] For the U.S. (right hand panel) the model performs reasonably well until the 1990s under both sets of assumptions. It captures the main movements in average prices with relatively small deviations. The model does predict an upswing in average prices in the mid 2000s but one nowhere near as large as the observed increase in actual prices. By 2010, a large "correction" in average house prices brought the data back in line with the predictions of the model. In the case of Canada, the model does not do quite as well. It captures the general direction of movements in house prices prior to 1990, but significantly understates the increase in house prices during the late 1980's and early 1990's. By 2000, the data appears to be back in line with the model's fundamentals under both sets of discounting assumptions. Since 2002, although real house prices have been predicted to rise, actual prices have increasingly diverged from those

[32] The predicted price under extrapolative discounting is not depicted since it is volatile and swamps movements in the other variables.

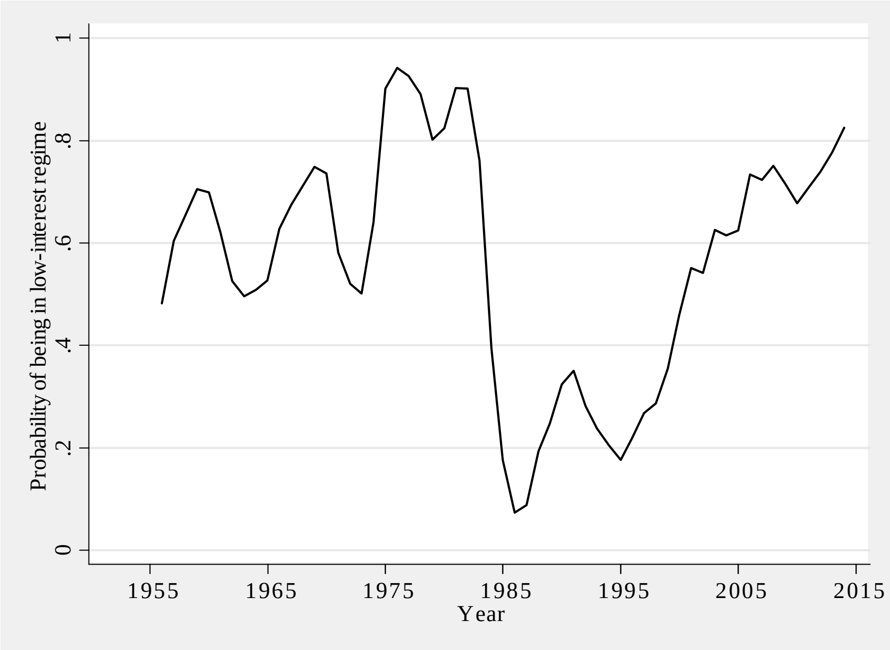

Figure 4: Probability of being in the low-interest regime

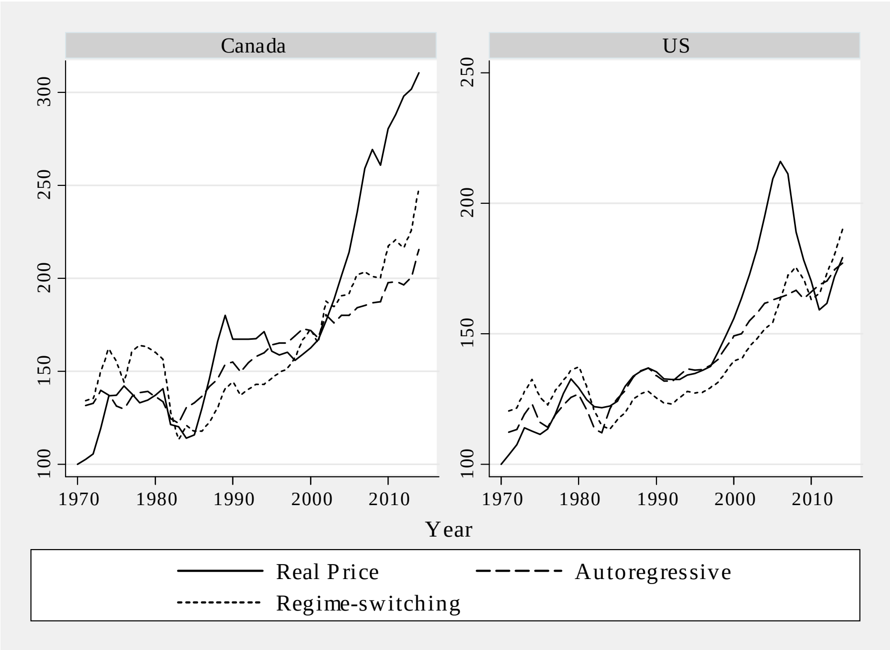

Figure 5: Actual and predicted aggregate prices (1970 = 100)

implied by the model.

Table 6 contains the implied *cumulative excess valuation* of housing on average at the end of 2014. Here and below, we define the excess valuation as the difference between the average observed price-rent ratio during the benchmark period (measured by either the MLS average or the repeat-sale index described above) and either the actual 2014-end price-rent ratio or the prediction of the model under each of the three discounting assumptions. In the table it can be seen that the regime-switching model implies that average prices are either 30% or 24% higher than predicted, depending on whether the MLS average or repeat-sale index is used to measure price growth.

Table 6: Cumulative excess valuation by 2014 (% deviation from 1971-96 average)

Country	2014 Price-rent ratio		Extrapolation		Autoregressive		Regime switch	
	MLS	Rep. sale	MLS.	Rep. sale	MLS	Rep. sale	MLS	Rep. sale
Canada	62	55	19	14	51	44	30	24
United States		6		-10		1		-6

5.2 Canadian Cities

As for the national Canadian case, we keep $\zeta = 1$ and set θ_c for each city so that the price-rent ratio implied the theory (*i.e.* given by (6)) is equal to the average of the observed price-rent ratio during the benchmark decade.[33] Initially, we fix the effective property tax rates at $\tau\alpha = 0.008$ in every city, reflecting the national average over the sample period. In Section 5.3, we allow property tax rates to vary across cities and over time. Finally we also set $\delta = 0.008$, reflecting the fact that for the cities considered property taxes consistently account for about 50% of all non-mortgage costs of home-ownership.

Figure 6 compares the observed time path of house prices to those implied by our theory under alternative assumptions regarding expected real interest rates described above.[34] For each city, the solid line depicts the actual price as measured by the MLS index. In each case, the particular interest rate forecast makes a big difference for the implied house price series. These differences account for significant variation in the implied cumulative excess valuation (if any) of owned housing in each city by the end of 2014. These are contained in Table 7.

The importance of the interest rate forecast is perhaps most strikingly apparent for the case of *extrapolative discounting*. In this case, for all cities, the price at the end of 2014

[33] Consequently, the values of θ_c vary with each of the cases described below.
[34] Again we do not depict the case of extrapolative discounting.

implied by the theory *exceeds* the actual price. Thus, this behavioural approach suggests that there is *no* excess valuation of houses for the cities in our sample. Rather, this approach to interest rate expectations may be interpreted as implying that houses in most cities were significantly *undervalued* at the end of 2014.

Such an approach to discounting is, of course, inconsistent with rational expectations given the historical evolution of real interest rates. Figure 6 shows clearly that forecasting rationally based on this experience makes a big difference. The short-dashed lines in the figure depict the implied theoretical price levels under the assumption that agents forecast using the *simple autoregressive* process estimated above. In this case it is clear that when agents' expectations reflect the observed mean reversion in the historical data, the fact that interest rates are, and have recently been, low has a much smaller impact on present value calculations and, hence, on the predicted path of prices.

Table 7: Excess valuation in 2014 (% deviation from 1987-96 average)

CMA	Price-rent ratio		Extrapolation		Autoregressive		Regime switch	
	MLS av	Teranet	MLS av	Teranet	MLS av	Teranet	MLS av	Teranet
Calgary	59	51	-47	-50	43	35	8	2
Edmonton	46	36	-56	-59	30	20	-3	-10
Gatineau*	80	48	-10	-26	63	35	34	10
Halifax	60	40	-25	-35	45	27	16	2
Hamilton	59	36	-29	-37	45	27	15	1
Montreal	96	69	-2	-16	78	53	46	25
Ottawa*	55	39	-27	-35	42	28	14	2
Quebec	102	81	-2	-12	82	63	48	33
Toronto	56	45	-31	-35	43	33	13	5
Vancouver	91	79	-16	-21	74	63	37	29
Victoria	63	66	-28	-26	50	52	18	20
Winnipeg	76	64	-30	-35	51	41	18	10
Average	70	55	-25	-32	54	40	22	11
Weighted	72	56	-24	-31	56	41	24	12

According to the estimated process, the real mortgage interest rate reverts to it's estimated long-run level ($\overline{r} = 4.9\%$) sufficiently quickly that variation in interest rates has only small effects on the user costs of owning over an extended horizon. Compared to growth of the raw price-rent ratio, the excess valuations in 2014 implied by this assumption are lower by between 9 to 25 percentage points, depending on the city and price series used. Excess valuation remains high, however, 56% or 41% on average, for the MLS average and Teranet

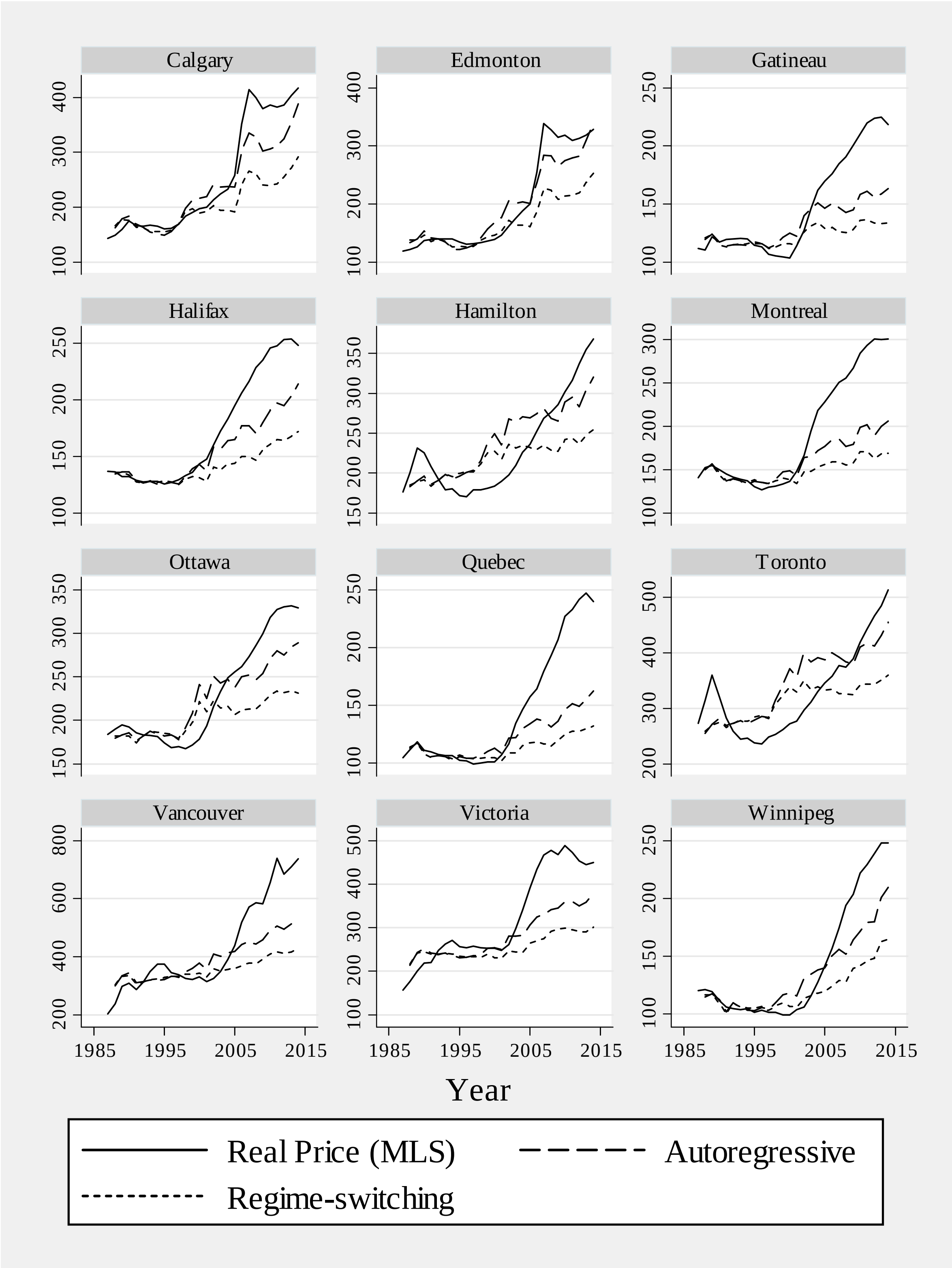

Figure 6: City-level price predictions under alternative discounting assumptions

series, respectively. Thus, if interest rates follow the *simple autoregressive* process, the fact that they are currently low cannot account for the majority of the increase in the price-rent ratios for these cities over the sample period.

The long-dashed lines in Figure 6 depict the theoretical price implied by the *regime switching* process. In this case the prices implied by the theory increase by much more than if the interest rate is assumed to follow the simple autoregressive process. In this case, estimated price growth implies excess valuations of less than half that based on the growth of the raw price-rent ratio. These excess valuations are still substantial for many cities, and average 24% and 12% by the end of 2014 for the MLS and Teranet indices, respectively.

5.3 Allowing for property tax variation

Accounting for effective tax rates which are heterogeneous across cities but time-invariant makes little difference for our calculations, since they affect price-rent ratios both in the benchmark decade and later in the same way. Rather, what matters for our estimates of cumulative over-valuation is variation in property taxes over time. Effective property tax rates vary differentially across cities over time both because mill rates, τ_{ct}, and property assessment ratios, α_{ct}, vary. Unfortunately, computing effective property tax rates at the city-level, $\tau_{ct}\alpha_{ct}$, is difficult because 1) mill rates are set by individual municipalities and not recorded by any central agency and 2) average assessment values are not published by Statistics Canada.

Here we make use of calculations by Murrell (2008), who estimates average effective property tax rates at the provincial level for three time periods: 1981-83, 1997-99 and 2005-07; by computing the ratio of average residential property taxes paid by homeowners to average property values. Murrell finds that after rising somewhat during the 1980s, effective property tax rates fell in every province between 1999 and 2006. This should not be surprising: even if mill rates rose somewhat, assessed values did not generally rise as rapidly as actual house prices.[35] We replicate Murrell's calculations for 2010-12 and find that while the tax rate continued to fall in some cities, it *rose* in most after 2006.

We now attempt to capture the effects of the observed effective tax rate decline in, admittedly, a rather crude fashion. Specifically, for each province we assume the tax rates in 2013-14 are equal to those estimated for 2010-12. We then linearly interpolate to obtain the rates between 1982, 1999, 2007 and 2011. The implied tax rate time paths are assumed to

[35] Murrell's results suggest that assessed values were less than half of actual property values by 2005.

be the same for each city in a given province. At each date we assume that agents expect property tax rates to remain constant.

While these calculations are admittedly simple, what matters for the excess valuation calculations is their impact in 2014 relative to the benchmark decade. Table 8 records the effect of variable property taxes in the case of regime switching in interest rates. In the theory, to the extent that effective property tax rates declined in any of the cities, this would have contributed to predicted price growth and thus *lower* excess valuation in 2014. As may be seen in the table, for those cities in which effective tax rates declined significantly (specifically, Calgary, Edmonton, Winnipeg, Vancouver and Victoria) the implied excess valuations decrease substantially. In the other cities there is little (in some cases no) effect. Only in Halifax did accounting for property tax changes *raise* the implied excess valuation. Overall, the population-weighted average excess valuation falls by two or three percentage points, depending on the price series used.

Table 8: Excess valuation in 2014: Effects of property tax changes:

CMA	No Property Tax Changes		Variable Property Taxes	
	MLS ave.	Teranet	MLS ave.	Teranet
Calgary	8	2	-2	-7
Edmonton	-3	-10	-12	-18
Gatineau*	34	10	33	10
Halifax	16	2	18	3
Hamilton	15	1	14	0
Montreal	46	25	45	25
Ottawa*	14	2	13	1
Quebec	48	33	48	32
Toronto	13	5	12	4
Vancouver	37	29	31	23
Victoria	18	20	12	14
Winnipeg	18	10	11	4
Average	22	11	19	8
Weighted	24	12	21	10

5.4 Accounting for atypical rent growth in Quebec

In all of the cases considered above, the three Quebec cities stand out as having experienced excessive price-rent growth relative to that predicted by the model. A glance at real rents over the sample provides one possible reason for this: real rent growth over the sample in these cities is close to zero on average, whereas in all other cities it was positive. This low overall average masks the fact, illustrated in Figure 8, that real rents declined on average until the late 1990s before growing (quite rapidly in Montreal and Quebec City) subsequently. We now consider the possibility that the *process* for rent growth in Quebec changed in the late 1990's as a result of external factors. For example, the potential for separation of Quebec from the rest of Canada may have been perceived as becoming increasingly likely until after the Quebec referendum of October 30, 1995. Subsequently, confidence that separation would not occur (in the near future, at least) gradually grew. To the extent that separation could have increased the probability of outward migration and/or economic instability, its likelihood could be negatively related to rent growth. For our purposes, however, this is only one possible story. All that matters for our estimates of cumulative over-valuation is the estimated change in the rent growth process.

To capture the change in the process of rent growth in as simple a way as possible we introduce a dummy term in the rent growth equations (10) for the cities in Quebec that allows the unconditional mean to adjust before and after 1996. Table 9 documents the implications for the estimated cumulative over-valuation of housing in the three Quebec cities and the overall averages for *all* cities. Property tax changes are included here, so the averages should be compared to those in Table 8.

Table 9: Excess valuation in 2014: Accounting for rent growth in Quebec

CMA	Base regime switching		With post-96 Dummy	
	MLS ave.	Teranet	MLS ave.	Teranet
Gatineau*	33	10	17	-3
Montreal	45	25	7	-8
Quebec	48	32	12	1
Average (all cities)	19	8	11	1
Weighted (all cities)	21	10	11	1

Figures for all other CMA's remain as in Table 8

Unsurprisingly given Figure 8, the impact of the inclusion of the structural break in the rent processes has a major effect on the path of predicted house prices, especially for Montreal and Quebec. For these cities, the implied excess valuations are cut by 50% or more. The overall simple average for Canada declines by eight or nine percentage points (depending of the price series) and the weighted average by even more (ten or 11 percent), reflecting the particularly large adjustment for Montreal.

Figure 9 depicts the actual (MLS ave.) and predicted price paths when we incorporate variation in tax rates across cities and time, with rents for the Quebec cities adjusted for the

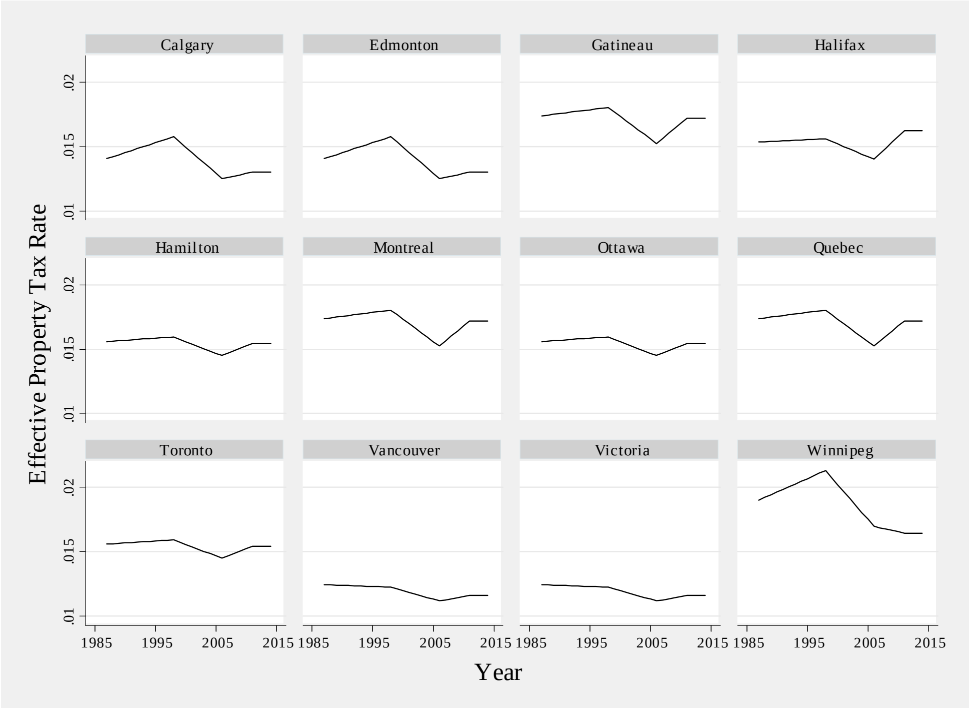

Figure 7: Property tax rates (Murrell (2008) and authors' calculations)

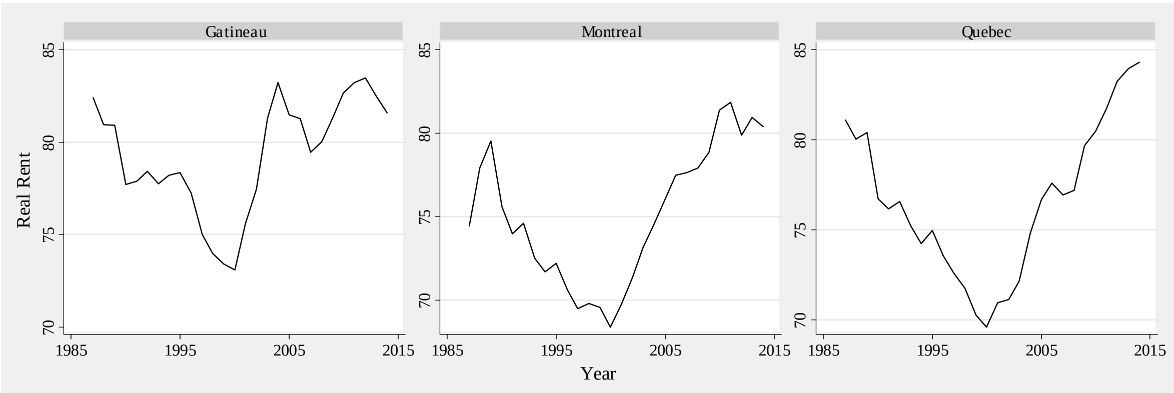

Figure 8: Real Rents in Quebec

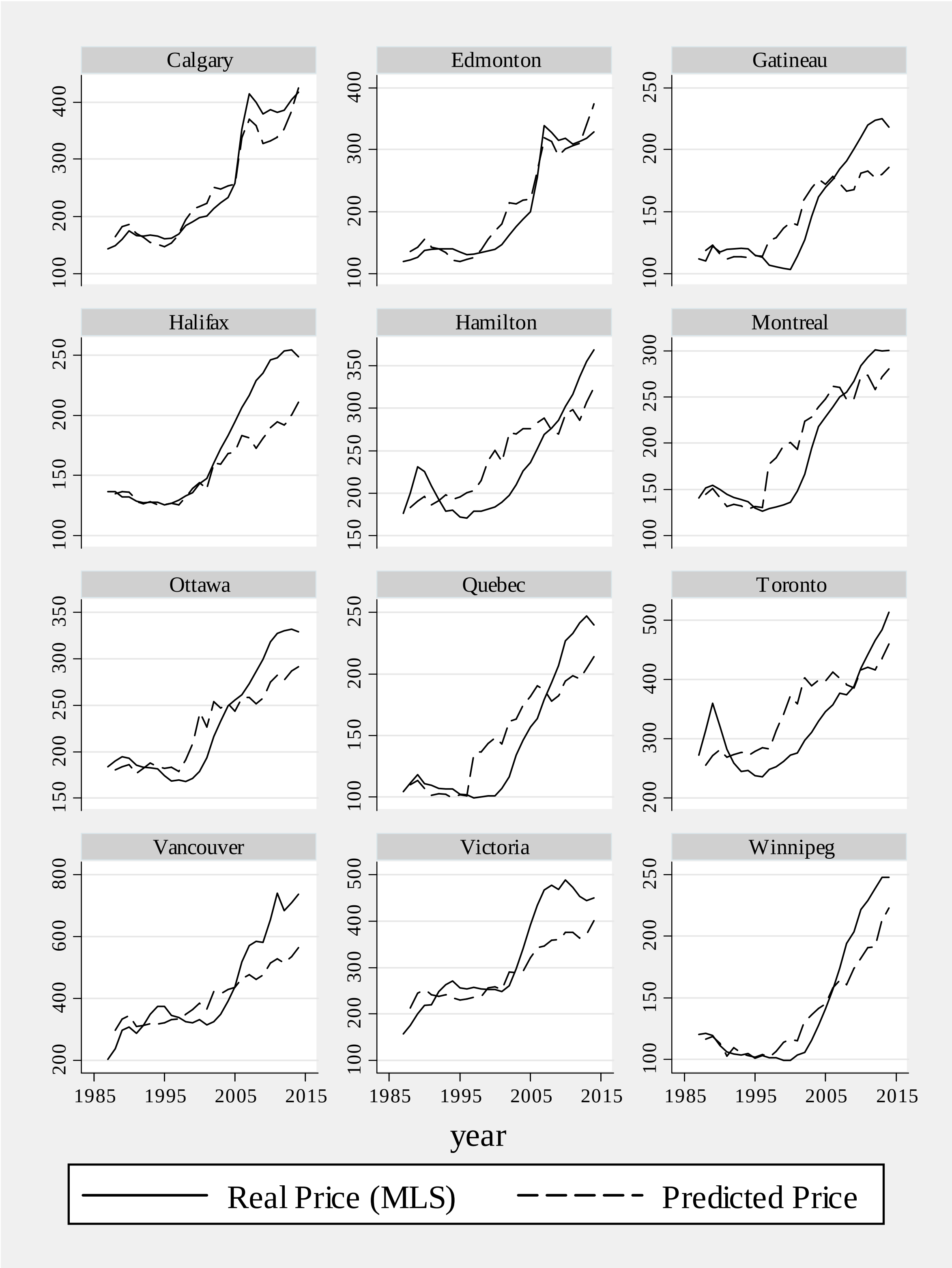

Figure 9: Prices with regime switching, property taxes and 1996 rent dummies for Quebec

observed mid-1990's break. Comparing to Figure 6, it is clear that adjusting for property taxes and the particulars of rent growth in Quebec makes a significant difference, but still leaves house prices in most CMA's exhibiting significant excess valuation by the end of 2014.

5.5 An alternative benchmark period

It is possible that our choice of the benchmark period (1987-96) may have biased the excess valuations. We are particularly concerned that it may have biased them *upwards*. This would be the case if owned housing were *undervalued* in some or all cities during the benchmark period. Note, however, that the average national price-rent ratio during this period was actually very close to its average over the period 1971-1996. Nevertheless, in Table 10 we reproduce the results using the average over the entire available sample (1987-2014) as a benchmark.[36] In this case, the implied weighted average excess valuation lies in a slightly higher range between 3 and 13%. However, for some cities (most notably Victoria and Vancouver), the excess valuation is considerably lower when using this benchmark period.

Table 10: Excess valuation in 2014: Benchmarked to sample average, 1987-2014

	Relative to 1987-1996		Relative to 1987-2014	
CMA	MLS ave.	Teranet	MLS ave.	Teranet
Calgary	-2	-7	-4	-9
Edmonton	-12	-18	-8	-15
Gatineau*	17	-3	16	-2
Halifax	18	3	6	-8
Hamilton	14	0	19	4
Montreal	7	-8	12	-4
Ottawa*	13	1	11	-1
Quebec	12	1	15	3
Toronto	12	4	18	10
Vancouver	31	23	21	14
Victoria	12	14	0	2
Winnipeg	11	4	7	0
Average	11	1	9	-1
Weighted	11	1	13	3

[36] We continue to adjust for the atypical rent in Quebec.

6 Concluding remarks

We assess the valuation of owner-occupied residential housing in Canada using a model in which house prices equate the costs or renting and owning for the marginal buyer. Quantitatively, such an assessment for housing markets in Canada poses some serious challenges. First, as we and others have documented, the quality-adjusted rent price index constructed by Statistics Canada has some peculiar properties that make it inconsistent with those of other countries. Second, the relative costs of owning and renting depend crucially on expectations regarding the path of future interest and rental growth rates, and it is not clear how these expectations should be modelled. Finally, the extent of over-valuation is also potentially sensitive to the choice of a benchmark period against which one compares current realizations, especially when one has limited historical data.

In this paper we have addressed problems with the rent data by constructing an alternative, unadjusted index of average rents for Canada, the long-run properties of which seems more closely aligned with the experiences of the U.S. than implied by the rent-price index, and by focussing on city-level market rent data. We then develop a simple user cost model that takes seriously the stochastic properties of real interest and rental growth rates and which allows us to consider the implications of alternative assumptions regarding expectations, property taxes and other factors. We also argue that it is reasonable to use the first decade for which city-level price data is available (1987-96) as a benchmark period.

Alternative assumptions regarding expected future interest rates, expected rent growth and property taxes have large effects on indicators of the relative valuation of owned versus rental housing. We argue that while it is reasonable to allow for mean-reversion in real interest rates, there is substantial evidence that the long-run real interest has fallen significantly relative to its level in the late 1980's and early 1990's. We compute an indicator of the cumulative excess valuation that takes this possibility into account, as well as variation in rental growth across cities, property taxes, and quality improvements. We also attempt to account for characteristics of rent growth which are peculiar to cities in Quebec, by distinguishing the periods before and after the 1995 Quebec sovereignty referendum.

We find that the excess valuation of owned versus rental housing has increased substantially in some cities, though not nearly as much as would be implied by standard indicators of the price-rent ratio. In some cities (*e.g.* Calgary and Edmonton), accounting for these other factors can *more* than rationalize the observed price movements, indicating the possibility of significant *under-valuation*.[37] In others (especially Vancouver and Victoria) the excess valuation remains substantial even when we account for these factors. After accounting for change in the interest rate regime, property taxes and the potential impact of the Quebec sovreignty referendum on rents (Tables 7 and 8), we find that the implied excess valuations, relative to the benchmark decade, range between -12% (Edmonton) and 31% (Vancouver) and the weighted average is between 1% (quality-adjusted) and 11% (unadjusted). In all cases our measure of excess valuation is significantly lower than that implied by considering only the growth of the price-rent ratio.

Whether these calculations reflect "over-valuation" of residential housing in Canadian cities depends on whether or not the *ownership premium*, θ_{ct}, for residential housing has increased substantially for fundamental reasons. In most cases, not much of this can be accounted for by relative quality improvements, but it is possible that rising real incomes could have increased the households' marginal utility from (and hence desire for) home-ownership. Unless, however, the relative costs of producing owner-occupied (as opposed to rental) housing change in a secular fashion over time, it is not clear why θ_{ct} would have a substantial trend. Another possibility is that there have been significant composition effects due to rising incomes or rising inequality.

[37] Although prices in these cities grew substantially so did rents. This may reflect a broad increase in demand rather than "overvaluation."

7 Appendices

Appendix A: Data Sources

Canadian Data

The average aggregate resale house price index after 1990 is the MLS® Average Residential Price for existing houses for 35 cities. Prior to 1990 we use Bank for International Settlements (BIS) data for the average price of existing homes obtained from the OECD (http://www.oecd.org/eco/outlook/Focus%20on%20House_Prices_indices.xls).

The **aggregate repeat-sales index** is the Teranet/National Bank 11 city average (http://www.housepriceindex.ca/).

Average aggregate rent from 1970 to 2000 is computed as the annual gross paid rent paid in current prices by Canadian households from the National Accounts: Statistics Canada (Table 380-0024) divided by the total occupied stock of rental units from Statistics Canada, Table 030-0001, CANSIM Series V227376. After 2000 we use the national average rent for two bedroom apartments from CMHC's Housing Market Indicators adjusted by the ratio of the two series in 2000 (http://www.cmhc-schl.gc.ca/en/corp/about/cahoob/data/data_001.cfm).

Average city-level resale house prices are the MLS® Average Residential Price for existing houses by city (1987-2014) taken from the Canadian Housing Observer (https://www03.cmhc-schl.gc.ca/catalog/productList.cfm?cat=122&lang=en&fr=1414002320325)

Teranet-National Bank House price index by city (1999-2014) is taken from the Canadian Housing Observer, Housing Market Indicators (http://www.cmhc-schl.gc.ca/en/corp/about/cahoob/data/index.cfm)

Average rents by city refer to the average rent for two bedroom apartments taken from the Canadian Housing Observer, Housing Market Indicators (http://www.cmhc-schl.gc.ca/en/corp/about/cahoob/data/index.cfm)

The **mortgage rate** is annual average of the CMHC's 5-year conventional mortgage lending rate from Statistics Canada Table 027-0015. This is also described as the "average residential mortgage lending rate: (5 year)" in Statistics Canada Table 176-0043.

The **price level** is the implicit price index for household final consumption from Statistics Canada Table 384-0039. The inflation rate is the growth rate in this index.

Average effective tax rates by province are based on calculations by Murrell (2008) prior to 2006. For later dates, data on average property taxes are from Statistics Canada, Survey of Household Spending (Tables 203-0003 and 203-0021) and property values per dwelling are from a Statistics Canada report on Residential Property Values in 2011 (http://www.statcan.gc.ca/daily-quotidien/140528/dq140528b-eng.htm).

Average maintenance and other costs of home-ownership are also taken from Statistics Canada, Survey of Household Spending (Tables 203-0003 and 203-0021). We find that property taxes consistently account for about 50% of the total non-mortgage costs of home-ownership.

US Data

The **aggregate repeat-sales price index** is constructed using the Housing Price Purchase Index from the FHFA from 1991 and the adjusted all-transaction index from the OECD prior to 1991 (http://www.oecd.org/eco/outlook/Focus%20on%20House_Prices_indices.xls).

The **average aggregate rent index** from 1970 to 2000 is taken from Crone, Nakamura and Voith. (2006), Appendix Table 1. After 2000, we use the rent component of the CPI-W from the BLS, adjusted so that the indices are equal in 2000.

The **mortgage rate** is the FHFA's "Terms on conventional single family mortgages, annual national averages, all homes" (http://www.fhfa.gov/DataTools/Downloads/Pages/Monthly-Interest-Rate-Data.aspx)

The **price level** is the implicit price deflator for personal consumption expenditures from FRED® Economic Data.

Appendix B: An Alternative National Rent Index for Canada

Figure 10 shows the real rent price implied by the OECD data, calculated by dividing real housing prices by the price-rent ratio.[38] As can be seen, this measure implies that real rent prices declined by over 50% during the 1970s, before declining at a less dramatic pace on average since 1980. In contrast in the US, for example, real rents measured this way grew on average with the economy.

The consequence of using the rent price index from Statistics Canada as a benchmark to measure the extent of over-valuation in owned housing markets is illustrated in Figure 11. This figure shows the price-rent ratios relative to the sample average for Canada and the United States. While for the US, the price-rent ratio can be argued to vary around its long run average, the Canadian price-rent ratio appears to have had an upward trend

[38] Computing the real rent price using the price index data directly from Statistics Canada generates an index with similar proeprties.

even before the current housing boom. In particular, the figure suggests that the price-rent ratio in Canada during the 1970s was dramatically below its long-run value. Based on this measure, the Canadian price-rent ratio in 2014 exceeded its long-run value by 89%. At the height of the US housing boom in 2006, the price-rent ratio was less than half of this.

To address these concerns we construct an alternative index for average rent paid in Canada. We make use of aggregate rent paid by households from the national accounts, which we divide by estimates of the aggregate stock of dwellings that were occupied and rented. Unfortunately, annual collection of the housing stock data was terminated by Statistics Canada after 2000. After this date we use the average rent on two bedroom apartments in metropolitan areas from the CMHC. This data is available from 1990 until 2014. Although, these data are not exactly the same, during the overlapping period (1990-2000) they move together quite closely. We therefore adjust the CMHC data down in proportion to the ratio of the two series in 2000 two create a single index for nominal rents.

Appendix C: Additional Estimates

Table 11: Interest rate process estimates: US, Single regime

Parameter	Value	Std. error		k	AIC	BIC
$(1-\mu_1)\bar{r}$	.010	(.003)		1	-336.50	-332.22
μ_1	.710	(.080)		2	-329.90	-323.52
σ_ε	.016			3	-326.53	-318.08
R^2	.564			4	-318.64	-308.16

Note: Robust standard errors in parenthesis

Table 12: Interest rate process estimates: US, Two regimes

High regime			Low regime		
$(1-\mu_h)\bar{r}_h$	.010	(.007)	$(1-\mu_l)\bar{r}_l$	.007	(.005)
μ_h	.774	(.218)	μ_l	.808	(.090)
σ_h	.021		σ_l	.009	
R^2	.379		R^2	.746	
Switching equation					
α	6.654	(.122)			
β	-173.5	(3.09)			
σ	.1622				
R^2	0.998				

Note: Robust standard errors in parenthesis

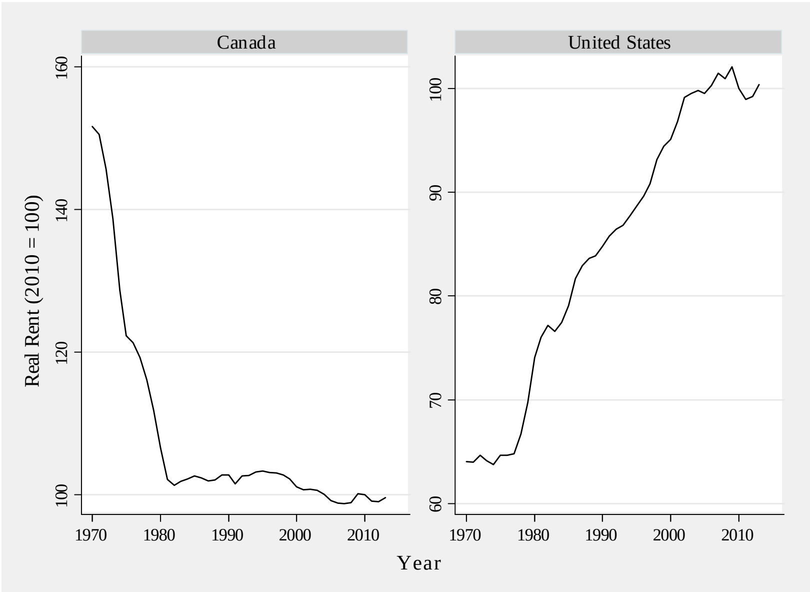

Figure 10: Real Rent Price implied by OECD and Statistics Canada CPI data

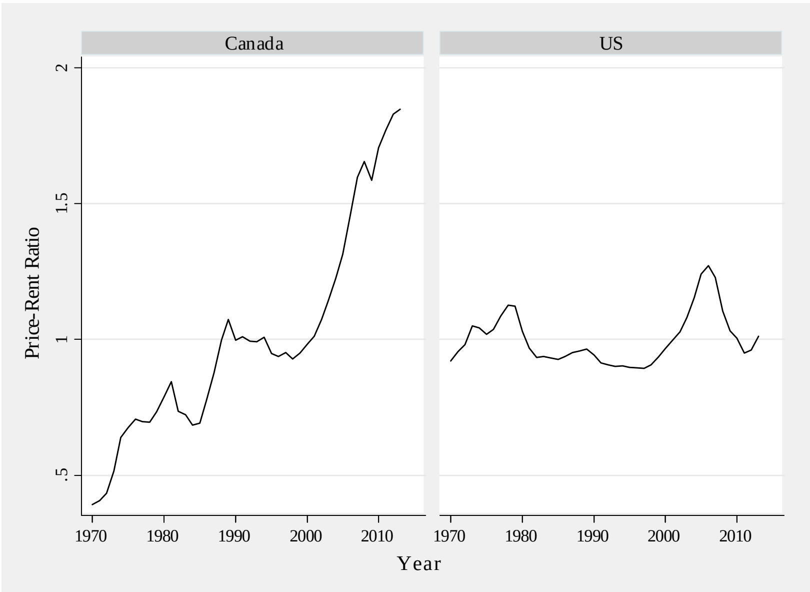

Figure 11: Price-Rent Ratio relative to long run average: OECD data

Appendix D: Present Value Calculations

The present value of rental payments in city c at time t can be expressed as

$$Z_{ct}R_{ct} = R_{ct} + R_{ct}E_t \sum_{j=1}^{\infty} \exp\left(-\left(\sum_{s=1}^{j} \hat{r}_{t+s} - \sum_{s=1}^{j} g_{ct+s}\right)\right) \tag{14}$$

where $\hat{r}_{t+s} = \ln(1 + r_{t+s})$ and g_{ct} denotes the growth rate of rents.

The cost of housing is

$$\psi P_{ct} + T_{ct} + E_t \sum_{j=1}^{\infty} \prod_{s=1}^{j} \left(\frac{1}{1 + r_{t+s}}\right) (\beta r_{t+j}(1-\psi)P_{ct} + T_{ct+j}) \tag{15}$$

This simplifies to[39]

$$(\psi + \beta(1-\psi))\, P_{ct} + T_{ct} + E_t \sum_{j=1}^{\infty} \exp\left(-\sum_{s=1}^{j} \hat{r}_{t+s}\right) T_{ct+j} \tag{16}$$

Under the assumptions described in the main text, the time $t + j$ value of property taxes with constant mill rates, τ_c, and assessed value to price ratios, α_c, is given by

$$T_{ct+j} = \tau_c \alpha_c P_{ct} e^{\sum_{s=1}^{j} g_{ct+s}} \tag{17}$$

It follows that cost of owning is given by (5).

Using (8), the total interest accumulated between t and j is

$$X_{t+j}^{r} = \sum_{s=1}^{j} \hat{r}_{t+s} \tag{18}$$

is normally distributed with conditional mean

$$M_{t,j}^{r} = E_t X_{t+j}^{r} = \sum_{s=1}^{j} \left[\bar{r} + \mu^s(\hat{r}_t - \bar{r})\right] \tag{19}$$

$$= j\bar{r} + (\hat{r}_t - \bar{r}) \sum_{s=1}^{j} \mu^s \tag{20}$$

$$= j\bar{r} + \frac{\mu\,(1-\mu^j)}{1-\mu}(r_t - \bar{r}) \tag{21}$$

[39] This follows from the fact that

$$\sum_{j=1}^{\infty} \prod_{s=1}^{j} \left(\frac{1}{1 + r_{t+s}}\right) r_{t+j} = 1.$$

and variance

$$\begin{aligned}
V^r_{t,j} &= \mathrm{Var}_t\left(X^r_{t+j}\right) = \mathrm{Var}_t\left(\sum_{s=1}^{j}\varepsilon_{t+s}\sum_{i=0}^{j-s}(1+\mu+...+\mu^{j-s})\right) && (22)\\
&= \mathrm{Var}_t\left(\sum_{s=1}^{j}\varepsilon_{t+s}\frac{1-\mu^{j-s+1}}{1-\mu}\right) = \sigma^2_\varepsilon\sum_{s=1}^{j}\left(\frac{1-\mu^{j-s+1}}{1-\mu}\right)^2 && (23)\\
&= \frac{\sigma^2_\varepsilon}{(1-\mu)^2}\sum_{s=1}^{j}\left(1-2\mu^{j-s+1}+\left(\mu^2\right)^{j-s+1}\right) && (24)\\
&= \frac{\sigma^2_\varepsilon}{(1-\mu)^2}\left(j-2\mu\sum_{s=1}^{j}\mu^{j-s}+\mu^2\sum_{s=1}^{j}\left(\mu^2\right)^{j-s}\right) && (25)\\
V^r_{t,j} &= \frac{\sigma^2_\varepsilon}{(1-\mu)^2}\left(j-2\mu\left(\frac{1-\mu^j}{1-\mu}\right)+\mu^2\left(\frac{1-\mu^{2j}}{1-\mu^2}\right)\right) && (26)
\end{aligned}$$

Similarly, using (10), the conditional mean of rent growth between t and j is

$$M^g_{ct,j} = j\bar{g}_c + \frac{\rho_c\left(1-\rho_c^j\right)}{1-\rho_c}(g_{ct}-\bar{g}_c) \qquad (27)$$

and its variance is

$$V^g_{ct,j} = \frac{\sigma^2_{ce}}{(1-\rho_c)^2}\left(j-2\rho_c\left(\frac{1-\rho_c^j}{1-\rho_c}\right)+\rho_c^2\left(\frac{1-\rho_c^{2j}}{1-\rho_c^2}\right)\right) \qquad (28)$$

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