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# The impact of financial crises on the risk-return tradeoff and the leverage effect

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# The impact of financial crises on the risk-return tradeoff and the leverage effect\*

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## Abstract

We investigate the impact of financial crises on two fundamental features of stock returns, namely, the risk-return tradeoff and the leverage effect. We apply the fractionally integrated exponential GARCH-in-mean (FIEGARCH-M) model for daily stock return data, which includes both features and allows the co-existence of long memory in volatility and short memory in returns. We extend this model to allow the financial parameters governing the volatility-in-mean effect and the leverage effect to change during financial crises. An application to the daily U.S. stock index return series from 1926 through 2010 shows that both financial effects increase significantly during crises. Strikingly, the risk-return tradeoff is significantly positive only during financial crises, and insignificant during non-crisis periods. The leverage effect is negative throughout, but increases significantly by about 50% in magnitude during financial crises. No such changes are observed during NBER recessions, so in this sense financial crises are special. Applications to a number of major developed and emerging international stock markets confirm the increase in the leverage effect, whereas the international evidence on the risk-return tradeoff is mixed.

**JEL Classification:** C22, G01.

**Keywords:** FIEGARCH-M, financial crises, financial leverage, international markets, long memory, risk-return tradeoff, stock returns, volatility feedback.

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# 1 Introduction

Financial crises are times of simultaneous increases in risk and great losses in portfolio values. At face value, this basic observation may suggest that the risk-return relation during crisis periods is negative, and thus of opposite sign compared to the classical Merton (1973, 1980) positive risk compensation tradeoff. Negative volatility-return relations have been suggested in connection with the financial leverage and volatility feedback effects. The argument behind the financial leverage effect of Black (1976) and Christie (1982) is that an initial price drop increases the debt-equity ratio and hence expected risk. The volatility feedback effect is that increases in risk lead to higher discount rates and thus losses of value, e.g., Campbell & Hentschel (1992)—see also Black (1976, p. 179). More recently, Ang, Hodrick, Xing & Zhang (2006) have argued for a negative relation between volatility innovations and returns: Since volatility innovations are largest during crisis periods, stocks that comove with volatility pay off in bad states, and should thus require a smaller risk premium. The empirical evidence on these effects has been mixed, both regarding sign and significance, see, e.g., the discussion in Bollerslev & Zhou (2006) and the review by Lettau & Ludvigson (2010), and there has (to the best of our knowledge) been no systematic investigation of the possible changes in these effects during crisis periods.

In this paper, we show that the basic intuition described above appears to be wrong. Indeed, we show that the empirical relation between return and volatility turns positive exactly during financial crises, whereas it is negative or close to zero during normal periods. At the same time, the financial leverage effect increases by about 50% in magnitude during crisis periods. These changes are observed whether we focus on the recent subprime crisis or include all major financial crises starting with the Great Depression. On the other hand, the same changes in the financial effects (the risk-return relation and the leverage effect) are not observed during NBER recessions, suggesting that financial crises are somehow special.

We conduct our analysis in the framework of an extended version—with the financial parameters potentially changing during crises—of the FIEGARCH-M (or FIEGARCH-in-mean) model of Christensen, Nielsen & Zhu (2010), who generalize the FIEGARCH (fractionally integrated exponential generalized autoregressive conditional heteroskedasticity) model introduced by Bollerslev & Mikkelsen (1996). Many of the salient features of daily stock returns are well described by the FIEGARCH model. Thus, in addition to time-varying volatility and volatility clustering (the ARCH and GARCH effects, as in Engle (1982) and Bollerslev (1986)), and the resulting unconditional excess kurtosis or heavier than normal tails, the model accounts for both long memory in volatility (fractional integration, as in the FIGARCH model of Baillie, Bollerslev & Mikkelsen (1996)) and the leverage effect, i.e., asymmetric volatility reaction to positive and negative return innovations (the exponential feature as in Nelson’s (1991) EGARCH model). The FIEGARCH-M introduces a filtered volatility-in-mean generalization of the FIEGARCH model. The generalization allows a risk-return relation effect of changing conditional volatility on conditional expected stock returns, and generates unconditional skewness. Following recent literature (Ang et al. (2006) and Christensen & Nielsen (2007)), it is changes in volatility that enter the return equation. The filtering of volatility when entering it in the return specification implies that the long memory property of volatility (the fractionally integrated feature) does not spill over into returns, which would be theoretically and empirically unwarranted. Christensen et al. (2010)

show that the FIEGARCH-M model dominates the original FIEGARCH model as well as many other GARCH-type models (including EGARCH, GARCH-M, Spline-GARCH, etc.) according to standard criteria.

The extension in the present paper of the FIEGARCH-M model allows for a change in the financial parameters, in particular, the volatility-in-mean effect and the leverage effect, during financial crises. An application to CRSP value-weighted cum-dividend stock index return series from 1926 through 2010 for the U.S. shows that both financial effects increase significantly during crises. Strikingly, the risk-return tradeoff is significantly positive only during financial crises, and insignificant during non-crisis periods. The leverage effect is negative throughout, but increases significantly by about 50% in magnitude during financial crises. Again, since no such changes are observed during NBER recessions, financial crises are special in this sense. Applications to a number of major developed and emerging international stock markets confirm the increase in the leverage effect, whereas the international evidence on the risk-return tradeoff is mixed.

Our results suggest that a given increase in the debt/equity ratio leads to a greater increase in expected risk during crisis periods than during normal periods. Under the volatility feedback interpretation, the results suggest that a given increase in risk increases the discount rate by more during financial crisis than during normal periods. This is consistent with an increase in the (positive) risk-return relation during crises, which is what we also find.

It is noteworthy that our empirical results do not stem simply from the fact that financial crises are periods of negative returns and increased risk. Specifically, by itself, this basic empirical relation would suggest a negative risk-return relation, particularly during crisis periods, whereas we find the opposite. Of course, a naïve analysis, just regressing the return (or its sign) on the indicator variable for crisis periods, would yield a negative coefficient. So would a regression of the return (or its sign) on volatility measures not correcting for financial leverage or volatility feedback. This is the well-known identification issue that leverage or feedback may induce a negative bias in the measured risk-return relation. Our contribution is that the best-fitting model considered includes the interaction of a leverage or feedback effect in the volatility equation and a volatility-in-mean effect in the return equation, with both effects increasing during financial crises. In particular, as the coefficient on volatility changes in the estimated return equation goes from negative or near zero during normal periods to positive (consistent with the classical equilibrium asset pricing risk-return relation) during crisis periods, the result is opposite of that from the naïve analysis, or from the literature plagued by identification issues.

In statistical terms, as the interacting leverage and volatility-in-mean effects and the changes in these during crises are jointly significant in our preferred model, all these features appear to be identified. In economic terms, it is clear that, firstly, the basic observation that negative returns and increases in risk go hand in hand during financial crises is captured in our model by the leverage effect that furthermore increases during crisis periods, rather than by a negative risk-return relation. Secondly, when a negative return according to the leverage idea leads to increased debt/equity ratio and therefore increased risk and ultimately increased expected future return, or, according to the volatility feedback interpretation, when an increase in risk leads to an increased discount rate and hence lower price, i.e., a negative return, then under both interpretations the maintained economic rationale is in fact positive

risk compensation. This corresponds to our empirical finding that the estimated negative volatility-return relation in the volatility equation (interpreted as leverage or feedback) and the strengthening of this during crises is paralleled by a positive volatility-in-mean effect in the return equation, kicking in exactly during financial crisis periods.

In the next section, we present the FIEGARCH-M model with changing financial parameters, which incorporates all the above mentioned features. Section 3 describes the data and presents the empirical results, first for the U.S. and then for the other countries considered. Section 4 concludes.

## 2 The FIEGARCH-M model with changing financial parameters

That volatility exhibits long memory is well established in the recent empirical literature<sup>1</sup>, and financial theory may accommodate long memory in volatility as well, see Comte & Renault (1998). Many of the studies of long memory in volatility use GARCH-type frameworks, but to the best of our knowledge the only such model that includes a volatility-in-mean specification, i.e., a parametric relation across conditional means and variances, is the FIEGARCH-M model of Christensen et al. (2010). This model generalizes the FIEGARCH model of Bollerslev & Mikkelsen (1996) by introducing volatility into the return equation along the lines of the GARCH-M literature, following Engle, Lilien & Robins (1987). Since long memory in volatility introduced into the return equation in a linear fashion generates long memory in returns, which is neither theoretically nor empirically warranted, it is changes in volatility rather than volatility levels that enter the in-mean specification and induce a volatility-return relation. This follows Ang et al. (2006) and Christensen & Nielsen (2007).

In this section, we consider an extension of the FIEGARCH-M model to allow for changes in the financial parameters, in particular, the volatility-in-mean effect and the financial leverage effect, during financial crises.

### 2.1 Time-varying volatility-in-mean effect

Let the daily continuously compounded returns on the stock or stock market index be given by

$$r_t = \ln(P_t) - \ln(P_{t-1}), \quad (1)$$

where  $t$  is the daily time index and  $P_t$  the stock price or index level at time  $t$ . We use the conditional mean specification

$$r_t = \mu + \lambda_1 h_t + \lambda_{11} D_t h_t + \varepsilon_t, \quad (2)$$

where volatility changes enter in the form of  $h_t$ , defined in (5) below as the filtered (fractionally differenced) conditional variance, and  $D_t$  is an indicator variable taking the value 1 if a financial crisis is ongoing as of  $t - 1$  (when the conditional mean is formed), and 0 otherwise. In the original FIEGARCH-M model,  $\lambda_{11} = 0$ , and in the FIEGARCH model,  $\lambda_1 = \lambda_{11} = 0$ . Thus, the specification allows for a volatility-return relation through the parameter  $\lambda_1$ , and in the extended model of this paper,  $\lambda_{11}$  represents the change in this relation during financial crises. It is assumed that  $D_t$  is in the information set  $\mathcal{F}_{t-1}$  at time  $t - 1$ , i.e., it is known at  $t - 1$  whether a financial crisis is ongoing at this time, and  $\mathcal{F}_{t-1}$  is the  $\sigma$ -field

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<sup>1</sup>See, e.g., Baillie et al. (1996), Bollerslev & Mikkelsen (1996), Ding & Granger (1996), Breidt, Crato & de Lima (1998), Robinson (2001), Andersen, Bollerslev, Diebold & Labys (2003), and the references therein.

generated by  $\{D_t, r_{t-1}, D_{t-1}, r_{t-2}, D_{t-2}, \dots\}$ . In our empirical analysis, we experiment with changes in the start dates and end dates of financial crises, and document the robustness of our findings to such changes. Note that  $h_t$  is  $\mathcal{F}_{t-1}$ -measurable, so the return innovations are  $\varepsilon_t = r_t - E(r_t|\mathcal{F}_{t-1})$  with  $E(\cdot|\mathcal{F}_{t-1})$  denoting conditional expectation given  $\mathcal{F}_{t-1}$ . It follows that  $\varepsilon_t$  in (2) is a martingale difference sequence (with respect to  $\mathcal{F}_t$ ).

The conditional return variance is modeled as

$$\sigma_t^2 = \text{Var}(r_t|\mathcal{F}_{t-1}) = E(\varepsilon_t^2|\mathcal{F}_{t-1}). \quad (3)$$

As in the FIEGARCH-M model, the specification is

$$\phi(L)(1-L)^d(\ln \sigma_t^2 - \omega) = \psi(L)g_t, \quad (4)$$

with (fractional) volatility changes  $h_t$  in deviation from long run level defined as

$$h_t = (1-L)^d(\ln \sigma_t^2 - \omega) = \phi(L)^{-1}\psi(L)g_t, \quad (5)$$

where  $\omega$  is the mean of the logarithmic conditional variance,  $\phi(L)$  and  $\psi(L)$  are GARCH and ARCH polynomials in the lag operator,  $\phi(L) = (1 - \phi_1 L) \times \dots \times (1 - \phi_p L)$  and  $\psi(L) = (1 + \psi_1 L) \times \dots \times (1 + \psi_q L)$ ,  $g_t$  is the news impact function described in (7) below, and  $(1-L)^d$  is the fractional difference operator defined by its binomial expansion

$$(1-L)^d = \sum_{i=0}^{\infty} \frac{\Gamma(i-d)}{\Gamma(-d)\Gamma(i+1)} L^i, \quad (6)$$

where  $d$  is the order of fractional integration in log-variance and  $\Gamma(\alpha) = \int_0^{\infty} x^\alpha e^{-x} dx$  is the Gamma function. The fractional difference with  $0 < d < 1$  allows for stronger volatility persistence than that of the GARCH-type generated by the lag-polynomials  $\phi(L)$  and  $\psi(L)$ . To calculate the fractional differences  $h_t$ , we truncate the infinite sum in (6) at  $i = \min\{t-1, 1000\}$ , following Baillie et al. (1996) and Bollerslev & Mikkelsen (1996).

## 2.2 Time-varying leverage effect

The financial leverage (or exponential or asymmetry) effect is ensured by modeling  $\ln \sigma_t^2$  in (4), as opposed to  $\sigma_t^2$ , and by the definition of the news impact function  $g_t$  governing the manner in which past returns impact current volatility,

$$g_t = \theta_0 z_{t-1} + \theta_1 D_t z_{t-1} + \gamma(|z_{t-1}| - E|z_{t-1}|), \quad (7)$$

where  $z_{t-1} = \varepsilon_{t-1}/\sigma_{t-1}$  is the standardized innovation. For  $\theta_1 = 0$ , this is the news impact function from Nelson's (1991) EGARCH specification. Here,  $\gamma$  is the rate at which the magnitude of the normalized innovations in deviations from mean, i.e.,  $|z_{t-1}| - E|z_{t-1}|$ , enter into current volatility<sup>2</sup>, and  $\theta_0$  generates an asymmetry in news impact on volatility. Thus, if  $\theta_0 < 0$  then negative innovations induce higher volatility than positive innovations of the same magnitude. The asymmetric volatility reaction pattern may stem from a financial leverage effect, see, e.g., Black (1976), Christie (1982), Engle & Ng (1993), and Yu (2005).

<sup>2</sup>Note that if  $z_t$  is Gaussian, then  $E|z_t| = \sqrt{2/\pi}$ .

The standard argument from Black (1976) is that bad news decrease the stock price, hence increasing the debt-to-equity ratio (i.e., financial leverage). With equity carrying all asset risk, this makes the stock relatively riskier after the price drop and increases expected future volatility. Although asymmetric reaction to innovations of different sign does not in addition induce unconditional skewness in returns, the latter is instead produced by the in-mean feature (see He, Silvennoinen & Terasvirta (2008)) and hence also accommodated by the FIEGARCH-M specification. In the original FIEGARCH-M model,  $\theta_1 = 0$ , and in the extended model of this paper,  $\theta_1$  measures the change in the leverage or asymmetry effect during financial crises.

Following Bollerslev & Mikkelsen (1996) and Christensen et al. (2010), our empirical specifications actually allow for the effect of lagged returns in the conditional mean equation, as well as lagged volatility-in-mean effects. In addition, we allow for the possibility that it is the news impact itself rather than the volatility change that generates the volatility-in-mean effect. Thus, the FIEGARCH-M<sub>h</sub> model uses the return equation with volatility changes,

$$r_t = \mu_0 + \mu_1 r_{t-1} + \lambda_1 h_t + \lambda_{11} D_t h_t + \dots + \lambda_m h_{t-m+1} + \lambda_{m1} D_t h_{t-m+1} + \varepsilon_t, \quad (8)$$

and the FIEGARCH-M<sub>g</sub> model uses the return equation with news impacts,

$$r_t = \mu_0 + \mu_1 r_{t-1} + \lambda_1 g_t + \lambda_{11} D_t g_t + \dots + \lambda_m g_{t-m+1} + \lambda_{m1} D_t g_{t-m+1} + \varepsilon_t. \quad (9)$$

Since  $g_t$  is the most recent innovation to  $\sigma_t^2$ , and it is  $\mathcal{F}_{t-1}$ -measurable, the return innovations in (8) and (9) are again the martingale differences  $\varepsilon_t = r_t - E(r_t | \mathcal{F}_{t-1})$ , as in (2). The final FIEGARCH model in Bollerslev & Mikkelsen (1996) in fact has  $p = q = 1$  in the GARCH and ARCH polynomials. The final models in Christensen et al. (2010) use these values, as well as  $m = 3$  in the FIEGARCH-M<sub>h</sub> case, and  $m = 2$  in the FIEGARCH-M<sub>g</sub> case.

In our empirical work we exclude nontrading days due to weekends and holidays. Following Nelson (1991) and Bollerslev & Mikkelsen (1996), we include a variable  $N_t$  equal to the number of nontrading days between  $t - 1$  and  $t$  to account for the fact that volatility tends to be higher following weekend and holiday nontrading periods, but with each nontrading day contributing less to volatility than a trading day.<sup>3</sup> Thus, our volatility equation with  $p = q = 1$  becomes

$$h_t = (1 - L)^d (\ln \sigma_t^2 - \ln(1 + \delta N_t) - \omega) = \phi_1 h_{t-1} + g_t + \psi_1 g_{t-1}. \quad (10)$$

Here, the parameter  $\delta$  measures the contribution of each nontrading day to variance, as a fraction of the contribution from a trading day. Thus, the relevant measure of volatility changes  $h_t$  follows a special ARMA(1,1) process. The presence of  $h_{t-1}$  on the right hand side of (10) is a GARCH-effect, i.e., volatility (here, its fractional difference) depends on its own lag, whereas the pure ARCH-effect stems from past returns feeding into current volatility, namely, via the news impact  $g_t$  (and its lagged value) in (10).

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<sup>3</sup>We deleted zero returns for each country according to the following algorithm: (i) delete any three (or more) consecutive zero returns, (ii) delete any zero returns on a weekend, and (iii) delete any zero returns on days identified as holidays according to each country's official calendar.



### 2.3 Estimation of the model

Using (10) for volatility and either (8) or (9) to define the return innovations  $\varepsilon_t$ , the model is estimated by quasi-maximum likelihood (QML). The sample log-likelihood for return data  $r_t, t = 1, \dots, T$ , is

$$\ln L(\eta) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \left( \ln \sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2} \right), \quad (11)$$

where  $\eta = (\mu_0, \mu_1, \lambda_1, \lambda_{11}, \dots, \lambda_m, \lambda_{m1}, \omega, \delta, \theta_0, \theta_1, \gamma, \psi_1, \dots, \psi_q, \phi_1, \dots, \phi_p, d)$  is the unknown parameter vector to be estimated, of dimension  $p+q+2m+8$ . Thus, the additional parameters relative to the original FIEGARCH-M model are  $(\lambda_{11}, \dots, \lambda_{m1}, \theta_1)$ . Estimation is carried out by numerical maximization of  $\ln L(\eta)$ . To initialize the recursions on (10) and (8) respectively (9) we use the unconditional sample average and variance of  $r_t$  for the presample ( $t = 0, -1, \dots$ ) values of  $r_t$  and  $\sigma_t^2$ , and we use  $\varepsilon_t = 0$  for  $t = 0, -1, \dots$ . The distributional assumption behind the likelihood function is that the return innovations  $\varepsilon_t$  are conditionally normal. For robustness against departures from Gaussianity, we calculate robust standard errors based on the sandwich-formula  $H^{-1}VH^{-1}$ , where  $H$  is the Hessian of  $\ln L(\eta)$  and  $V$  the sum of the outer products of the individual quasi score contributions. Christensen et al. (2010) verify the validity of the QML robust standard errors using the wild bootstrap (Wu (1986)).

## 3 Empirical analysis

### 3.1 Data description

**Table 1 about here**

In our empirical work we consider both the U.S. and a number of developed and emerging economies. Table 1 shows information for each country about the stock market index used, start and end date, sample size, and summary statistics. The countries included are the G-7, the so-called BRIC countries (Brazil, Russia, India, China), and an additional four selected major emerging markets (Argentina, Mexico, South Korea, Thailand). The U.S. data are obtained from CRSP, the Russian data from the RTS Exchange, the data for Argentina, Brazil, France, Italy, Mexico, and South Korea from Global Financial Data, and the rest from Datastream. Argentina, Brazil, and Mexico have experienced periods of severe inflation, which is reflected in the large average annualized returns in the table. In the subsequent estimations, we apply a 20% truncation rule to the raw daily returns for all countries. This affects Brazil (6 days), Russia (3), China (4), Argentina (15), Mexico (2), and South Korea (11). Unreported estimations show that our results are robust to alternative truncation rules, or no truncation at all.

**Table 2 about here**

For the U.S. we consider seven financial crises during this period, namely, the Great Depression, the 1937-1938 stock market crash, the 1973 oil crisis, the 1987 stock market

crash, the 2000 dotcom bubble burst, the 9-11 terrorist attack in 2001, and the subprime crisis 2007-2009. Table 2 lists these crises and their start and end dates. The set of relevant financial crises and their approximate start and end dates are based on Kindleberger & Aliber (2005), Longstaff (2010), and Afonso, Kovner & Schoar (2011). The exact start date used in the empirical analysis is identified as a day with a large drop in the index (typically more than 5%) as close as possible to the approximate start date from the literature. Similarly, the exact end date used is defined as the local minimum of the index nearest to the approximate end date. Thus, we define  $D_t = 1$  during the crisis periods from Table 2, and  $D_t = 0$  otherwise. For robustness to misspecification of the exact start and end dates of crises, we compare below with results obtained by extending each crisis period by 10% (symmetrically, shifting both start and end date), and also by similarly shortening the crisis by 10%.

### Figure 1 about here

Figure 1 shows the U.S. index and return series, with financial crisis periods indicated by shaded bars. The declines in the index during crisis periods are evident in the top panel, whereas it is difficult to discern a generally increased volatility during these periods from the bottom panel.

### 3.2 Empirical results for the U.S.

#### Table 3 about here

Estimation and test results appear in Table 3. The results in the first two columns are for the exact specifications of the final models from Christensen et al. (2010), with  $m = 3$  volatility changes  $h_t$  in-mean in the first column, following (8), and  $m = 2$  news impacts  $g_t$  in-mean in the second column, following (9). The results are similar to those from Christensen et al. (2010) who used the shorter period ending in 2006. Thus, both the volatility-in-mean and financial leverage effects are generally significant at conventional levels (robust asymptotic standard errors are in parentheses). In particular,  $\theta_0$  is negative and strongly significant in both columns. With  $h_t$  in-mean (first column), the effect of the most recent volatility change,  $\lambda_1$ , is negative. Here, it is the next two lags of  $h_t$  that are significant (with opposite signs). With news impact  $g_t$  in-mean (second column), again the leading term enters negatively, and the second term is significantly positive. All other parameters (the FIEGARCH parameters) are significant, including the memory parameter  $d$ .

The last two columns of Table 3 show the results from the extended model specification allowing for changes in the financial parameters during crisis periods. Both  $\lambda_{11}$ , the change in the leading volatility-in-mean term, and  $\theta_1$ , the change in the financial leverage effect, are statistically significant at conventional levels. The robust  $t$ -statistics on  $\lambda_{11}$  and  $\theta_1$  exceed 3, both for the case with volatility changes in-mean (third column of the table) and with news impact in-mean (fourth column). The change in the volatility-in-mean effect is positive in both specifications. Indeed, the change is so great that the in-mean effect at the first lag,  $\lambda_1 + \lambda_{11}$ , turns positive during financial crises, whereas it is negative (and insignificant) during

noncrisis periods, as it is in the model with constant parameters (Table 3). Furthermore, the financial leverage effect is strengthened during crisis periods in both specifications. The effect is always present, i.e.,  $\theta_0$  is negative, but the combined leverage effect  $\theta_0 + \theta_1$  during financial crises is stronger, i.e.,  $\theta_1 < 0$ . From the point estimates, the leverage effect is about 50% greater in magnitude during financial crises, which is considerable, and the difference is significant.

### 3.3 Robustness and model fit for the U.S.

Table 3 also shows the maximized log-likelihood and the Akaike and Schwartz (Bayesian) information criteria, reported as AIC and SIC. The log-likelihood increases by more than 30 in the extended model specifications allowing for changes in the financial parameters during crisis periods, compared to the corresponding specifications without changing parameters. Clearly, this is a large gain, with only three and four additional parameters in the FIEGARCH- $M_g$  respectively FIEGARCH- $M_h$  models. Between the two, SIC favors the former and AIC the latter.

The Ljung-Box portmanteau statistics for serial correlation in the standardized return innovations,  $\hat{z}_t = \hat{\varepsilon}_t / \hat{\sigma}_t$ , are reported as  $Q_{10}$  and  $Q_{100}$  for 10 and 100 lags, respectively. In GARCH-type models,  $p$ -values from standard  $\chi^2$ -distributions are not reliable, but the statistics are still useful for model comparison. So are the similar Ljung-Box statistics for absolute standardized return innovations  $|\hat{z}_t|$ , indicated with a superscript  $A$  in the table, since absolute returns are serially correlated in GARCH models even when raw returns are not. Generally, the FIEGARCH- $M_h$  model gets slightly better  $Q$ -statistics than the FIEGARCH- $M_g$  model, both with and without changing financial parameters.

The final four rows of Table 3 report the Engle & Ng (1993) sign bias and size bias misspecification tests, for which one and two asterisks denote rejection at the 5% and 1% level, respectively. These tests examine whether the squared normalized residual,  $\hat{z}_t^2$ , can be predicted by variables in the information set which are not included in the volatility model, in which case this is misspecified. The sign bias test examines whether  $\hat{z}_t^2$  can be predicted by information on the sign of return shocks. The negative (positive) size bias test examines whether large and small negative (positive) return shocks have different effects on  $\hat{z}_t^2$ . Both the three separate tests and a joint test are reported. The test results do not show strong signs of misspecification, with only two statistics out of 16 significant at the 5% level.

Christensen et al. (2010) considered the same CRSP value-weighted stock index return series as in this paper, but for a slightly shorter time period ending in 2006, and compared the FIEGARCH- $M$  model with many alternative models spanning a broad spectrum of volatility specifications. In particular, the alternative models considered included the GARCH, IGARCH, Spline-GARCH, FIGARCH, A-FIGARCH, EGARCH, and FIEGARCH, see their Table 1, as well as in-mean variants of the same models, see their Table 2. For the time period considered, it was shown that the FIEGARCH- $M$  models are superior to all these alternative specifications in terms of model fit, i.e., in terms of log-likelihood, the AIC and BIC criteria, the  $Q$  tests, and the Engle & Ng (1993) sign bias and size bias misspecification tests. For the latter tests, it was found that specifications without the exponential feature rejected in 35 out of 36 of these tests (nine specifications and four versions of the tests) at the 1% level, while the EGARCH and EGARCH- $M$  models each rejected in two of the four tests at the 1% level. On the other hand, the FIEGARCH- $M_g$  and FIEGARCH- $M_h$  specifications

had only one rejection, which was at the 1% level for the FIEGARCH- $M_g$  model. Based on these findings, we proceed with the FIEGARCH-M models in this paper, and we report only results for the specification with  $h_t$  in-mean, but similar results are obtained in the alternative specification with  $g_t$  in-mean.

#### **Table 4 about here**

To verify the robustness of our findings from Table 3, we carry out a number of additional investigations, with results reported in Table 4. Again, the Engle & Ng (1993) sign bias and size bias misspecification tests show no signs of misspecification, with only one rejection out of 16 tests.

The first two columns of Table 4 use the alternative definitions of the crisis indicator  $D_t$ , with crisis periods extended and shortened by 10% in columns one and two, respectively. It is clear from the table that the exact definition of the start and end dates of each crisis are not important for the overall conclusion, namely that the volatility-in-mean and financial leverage effects increase during crisis periods. Indeed, the volatility-in-mean effect is insignificant outside crisis periods, and is an order of magnitude larger and significantly positive during financial crises.

The third column of Table 4 instead sets  $D_t = 1$  during official NBER recessions, and 0 otherwise. From the results, there are no significant changes in the financial parameters  $\lambda$  and  $\theta$  during NBER recessions. This verifies that there is something special about financial crises. It is during financial crises, as opposed to general economic downturns, that the risk-return tradeoff and leverage effects change—indeed, with the risk-return tradeoff insignificant outside financial crisis periods.

Finally, out of current interest, the last column of Table 4 shows the results of including only the recent subprime crisis, i.e.,  $D_t = 1$  from December 3, 2007, to March 9, 2009, and  $D_t = 0$  otherwise. Again, the change parameters are large in magnitude and strongly significant, with robust asymptotic  $t$ -statistics of 8.2 for the increase  $\lambda_{11}$  in the volatility-in-mean effect, and  $-6.8$  for the strengthening of the financial leverage effect. This shows that the changes are not specific to the earlier crises in the data period.

### **3.4 Financial crises for other countries**

#### **Table 5 about here**

Next, we investigate to which extent the results carry over to other countries. We consider in turn the remaining G-7 countries, the BRIC countries, and the four additional major emerging markets. Of course, for each country analyzed, the set of financial crises should be reconsidered. Table 5 shows the list of crises included for each country. In addition to the previous literature references, we also consulted Radelet & Sachs (1998), Desai (2000), and Reinhart & Rogoff (2009) for selection and dating of the country specific crises. Due to the shorter time series of daily returns available for these countries, we report only the results for a parsimonious FIEGARCH-M specification with  $m = 1$  volatility-in-mean term, but

similar results (although not always significant) are obtained for larger values of  $m$ . Figures 2-3 show the index levels and returns for the remaining G-7 countries, Figures 4-5 for the BRIC countries, and Figures 6-7 for the additional emerging markets.

**Figures 2-7 about here**

### **3.5 Empirical results for other countries**

**Table 6 about here**

Estimation and test results for the remaining six G-7 countries appear in Table 6. In this table, the Engle & Ng (1993) sign bias and size bias misspecification tests show some signs of misspecification for Germany and Italy, but not for the remaining four countries. Thus, we interpret the parameter estimates for Germany and Italy cautiously.

For all G-7 countries, the basic FIEGARCH parameters are similar to those for the U.S.. Regarding the special financial parameters  $\lambda$  and  $\theta$ , the positive sign of the change in the former during crises, as seen in the U.S. results, extends to all countries except Italy and Japan, although the increase is statistically insignificant. The strengthening of the financial leverage effect during crises extends to all countries and is significant for France, Germany, Italy, and Japan. The leverage effect is present during normal periods, as well, i.e.,  $\theta_0$  is negative for all countries, and it is significant for all countries except Italy. Thus, as the only country, Italy has no leverage effect during noncrisis periods, but this could be a consequence of the possible misspecification in the model for Italy. Furthermore, the change during crisis periods,  $\theta_1$ , is negative and much larger in magnitude than for the other countries, so that the combined effect during crises,  $\theta_0 + \theta_1$ , is similar (more than 0.1 in magnitude) for all countries, including Italy.

**Table 7 about here**

Results for the BRIC countries appear in Table 7. Here, the Engle & Ng (1993) sign bias and size bias misspecification tests show signs of misspecification for Brazil, but not for the other three countries, so again estimates for Brazil must be interpreted with care.

For the BRIC countries, the evidence on the risk-return tradeoff is mixed and mostly insignificant. On the other hand, by the point estimates, the leverage effect is present both during and outside crisis periods, but it is stronger during crises, i.e., both  $\theta_0$  and  $\theta_1$  are negative throughout. The leverage change parameter  $\theta_1$  is large in magnitude and significant for China and Russia. Also  $\theta_0$  is significant for Russia. Thus, the results so far suggest that the leverage effect is always negative, and stronger during financial crises.

**Table 8 about here**

To further explore this hypothesis, we finally consider in Table 8 the four additional major emerging markets, namely, Argentina, Mexico, South Korea, and Thailand. In this table, there are no indications of misspecification since none of the Engle & Ng (1993) sign bias and size bias misspecification tests reject.

Except for an insignificant point estimate of  $\theta_0$  for Argentina, all  $\theta$  estimates are negative in Table 8. This is generally consistent with the hypothesis that the leverage effect is always negative, and stronger during financial crises, although not all estimates are significant ( $\theta_0$  for Mexico,  $\theta_1$  and  $\lambda_1$  for Thailand, and  $\lambda_{11}$  for South Korea are significant).

## 4 Concluding remarks

In this paper, we introduce an extension of the fractionally integrated exponential GARCH-in-mean (FIEGARCH-M) model for daily stock return data with long memory in return volatility of Christensen et al. (2010). The extended model allows for a change in the financial parameters, in particular, the volatility-in-mean effect and the leverage effect, during financial crises. We show that this extension delivers interesting and novel empirical results regarding financial crises.

Our application to CRSP value-weighted cum-dividend stock index return series from 1926 through 2010 for the U.S. shows that both financial effects increase significantly during crises. Strikingly, the risk-return tradeoff is significantly positive only during financial crises, and insignificant during non-crisis periods. The leverage effect is negative throughout, but increases significantly by about 50% in magnitude during financial crises. No such changes are observed during NBER recessions, so in this sense financial crises are special.

Further conclusions emerge from comparing with results from a number of major developed and emerging international stock markets, although the results are generally stronger for the U.S. than for each of the other countries considered, perhaps due to more crises and better data availability. Regarding the risk-return tradeoff, the  $\lambda$  parameters are mainly positive in the Asian economies, whereas they are insignificant in Latin America. For the leverage parameters,  $\theta$ , the results are very strong and show that the leverage effect is negative throughout, and considerably stronger during financial crises—as in the U.S., again by about 50% or more in magnitude in all countries.

It is conceivable that our estimated leverage effect in fact measures a volatility feedback effect. Like the leverage effect, the volatility feedback effect induces a negative relation between risk and price, provided risk compensation is positive: Increased risk in the presence of a positive risk-return relation increases the discount rate and hence induces a price drop. This is consistent with what happens during crisis periods, and with our findings that the negative relation (leverage, or volatility feedback) is markedly stronger when the risk-return relation sets in—exactly during financial crises.

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Table 1: Summary statistics

Country	Index	Start date	End date	Sample size	Average return	Standard deviation	Normality test
U.S.	CRSP value-weighted	1926/01/02	2010/12/31	22,528	10.38%	18.21%	42,302**
Canada	TSX Composite	1969/01/02	2010/12/31	10,495	6.03%	14.94%	12,586**
France	CAC 40	1968/09/17	2010/12/31	10,455	6.80%	17.95%	8976**
Germany	DAX	1965/01/05	2010/12/30	11,524	5.77%	19.41%	8139**
Italy	MIBTEL	1957/01/02	2010/12/30	13,288	4.83%	19.62%	6952**
Japan	Nikkei 225	1950/04/04	2010/12/30	15,052	7.77%	19.03%	15,169**
U.K.	FTSE All Shares	1969/01/02	2010/12/31	10,521	6.82%	17.22%	8405**
Brazil	Bovespa	1972/01/03	2010/12/30	9654	81.08%	44.53%	32,809**
Russia	RTX	1995/09/04	2010/12/30	3757	19.12%	45.16%	2174**
India	BSE 30	1979/04/04	2010/12/31	7041	18.13%	27.58%	4537**
China	Shanghai Composite	1991/01/03	2010/12/31	4896	15.74%	40.56%	15,807**
Argentina	MERVAL	1967/01/02	2010/12/30	10,839	65.59%	46.51%	86,590**
Mexico	IPC	1985/01/02	2010/12/31	6400	35.80%	29.76%	10,848**
South Korea	KOSPI Composite	1962/01/05	2010/12/30	13,707	11.94%	31.77%	184,370**
Thailand	Bangkok SPI	1975/05/02	2010/12/30	8657	6.73%	23.72%	7194**

Note: This table reports summary statistics for the market index used for each country. For each index, we provide the index name, the start and end dates, as well as the sample size (the number of daily observations). We also report the annualized average return, the annualized standard deviation (both in nominal terms), and the JB normality test statistics. \*\* denotes significance at 1% level.

Table 2: Crisis list for the U.S. market

Crisis	Start date	End date	Duration (trading days)
The Great Depression	1929/08/01	1933/02/28	1063
The 1937-1938 Stock Market Crash	1937/05/03	1938/04/01	273
The 1973 Oil Crisis	1973/10/29	1974/10/03	235
The 1987 Stock Market Crash	1987/10/19	1988/12/30	304
The 2000 Dotcom Bubble Burst	2000/03/10	2001/04/16	276
The 2001-9-11 Terrorist Attack	2001/09/11	2002/10/09	268
The Subprime Crisis	2007/12/03	2009/03/09	317

Note: This table presents the crisis list for the U.S. market. For each crisis we provide the start and end dates and the duration in trading days.

Table 3: FIEGARCH-M models including all seven crises

Parameter	FIEGARCH-M <sub>h</sub>	FIEGARCH-M <sub>g</sub>	FIEGARCH-M <sub>h</sub>	FIEGARCH-M <sub>g</sub>
$\mu_0$	$4.773 \times 10^{-4}$ ( $5.234 \times 10^{-5}$ )	$4.945 \times 10^{-4}$ ( $4.987 \times 10^{-5}$ )	$4.760 \times 10^{-4}$ ( $5.451 \times 10^{-5}$ )	$4.961 \times 10^{-4}$ ( $5.014 \times 10^{-5}$ )
$\mu_1$	0.09471 ( $9.587 \times 10^{-3}$ )	0.09288 ( $7.712 \times 10^{-3}$ )	0.1055 ( $7.379 \times 10^{-3}$ )	0.1045 ( $8.855 \times 10^{-3}$ )
$\lambda_1$	$-7.947 \times 10^{-4}$ ( $5.024 \times 10^{-4}$ )	$-8.520 \times 10^{-4}$ ( $4.695 \times 10^{-4}$ )	$-7.513 \times 10^{-4}$ ( $4.756 \times 10^{-4}$ )	$-8.130 \times 10^{-4}$ ( $4.889 \times 10^{-4}$ )
$\lambda_{11}$	—	—	$8.883 \times 10^{-3}$ ( $1.580 \times 10^{-3}$ )	$8.657 \times 10^{-3}$ ( $2.500 \times 10^{-3}$ )
$\lambda_2$	$1.558 \times 10^{-3}$ ( $3.907 \times 10^{-4}$ )	$1.327 \times 10^{-3}$ ( $3.441 \times 10^{-4}$ )	$1.526 \times 10^{-3}$ ( $4.338 \times 10^{-4}$ )	$1.305 \times 10^{-3}$ ( $3.652 \times 10^{-4}$ )
$\lambda_{21}$	—	—	$-1.213 \times 10^{-3}$ ( $1.466 \times 10^{-3}$ )	$-1.235 \times 10^{-3}$ ( $2.623 \times 10^{-3}$ )
$\lambda_3$	$-7.830 \times 10^{-4}$ ( $3.715 \times 10^{-4}$ )	—	$-7.979 \times 10^{-4}$ ( $3.949 \times 10^{-4}$ )	—
$\lambda_{31}$	—	—	$-2.027 \times 10^{-3}$ ( $1.147 \times 10^{-3}$ )	—
$\omega$	-8.915 (0.1438)	-8.925 (0.1433)	-9.026 (0.1490)	-9.037 (0.1419)
$\delta$	0.1960 (0.03568)	0.1969 (0.03557)	0.1923 (0.03490)	0.1927 (0.03481)
$\theta_0$	-0.1195 (0.01320)	-0.1198 (0.01318)	-0.1115 (0.01471)	-0.1116 (0.01420)
$\theta_1$	—	—	-0.06206 (0.01759)	-0.06202 (0.01986)
$\gamma$	0.2065 (0.01503)	0.2065 (0.01500)	0.2006 (0.01496)	0.2006 (0.01506)
$\phi_1$	0.7457 (0.06773)	0.7402 (0.07049)	0.7607 (0.06472)	0.7512 (0.06834)
$\psi_1$	-0.4760 (0.1101)	-0.4719 (0.1141)	-0.4939 (0.1077)	-0.4834 (0.1116)
$d$	0.5369 (0.02714)	0.5380 (0.02694)	0.5287 (0.02753)	0.5316 (0.02754)
$\ln L(\eta)$	75, 272.02	75, 270.68	75.306.25	75, 303.58
AIC	-150, 520.05	-150, 519.36	-150, 580.51	-150, 579.16
SIC	-150, 423.78	-150, 431.11	-150, 452.15	-150, 466.85
$Q_{10}$	19.90	21.74	21.07	22.54
$Q_{100}$	125.18	127.26	125.73	127.40
$Q_{10}^A$	38.88	38.47	35.57	35.30
$Q_{100}^A$	215.96	216.31	198.95	199.74
Sign bias	1.990*	1.923	1.990*	1.834
Negative size bias	-1.090	-1.061	-0.7762	-0.7564
Positive size bias	-1.131	-1.110	-1.062	-1.032
Joint test	3.990	3.735	4.051	3.431

Note: QML estimates are reported with robust standard errors in parentheses. Also reported are  $\ln L(\eta)$ , the value of the maximized log-likelihood function, and the Akaike and Schwarz (or Bayesian) information criteria, respectively. The values of the Ljung-Box portmanteau statistic for up to  $K$ 'th order serial dependence in the standardized residuals,  $\hat{\varepsilon}_t/\hat{\sigma}_t$ , and the absolute standardized residuals,  $|\hat{\varepsilon}_t/\hat{\sigma}_t|$ , are denoted  $Q_K$  and  $Q_K^A$ , respectively. \* and \*\* denote rejection at the 5% and 1% level, respectively.

Table 4: FIEGARCH-M models with extended and shortened crisis periods

Parameter	Extended	Shortened	NBER	Subprime
$\mu_0$	$4.941 \times 10^{-4}$ ( $5.588 \times 10^{-5}$ )	$4.720 \times 10^{-4}$ ( $5.305 \times 10^{-5}$ )	$4.895 \times 10^{-4}$ ( $5.370 \times 10^{-5}$ )	$4.775 \times 10^{-4}$ ( $5.789 \times 10^{-5}$ )
$\mu_1$	0.1061 (0.01085)	0.1011 ( $8.186 \times 10^{-3}$ )	0.09503 ( $7.640 \times 10^{-3}$ )	0.09629 ( $7.068 \times 10^{-3}$ )
$\lambda_1$	$-7.832 \times 10^{-4}$ ( $5.515 \times 10^{-4}$ )	$-8.235 \times 10^{-4}$ ( $4.724 \times 10^{-4}$ )	$-7.299 \times 10^{-4}$ ( $4.668 \times 10^{-4}$ )	$-8.244 \times 10^{-4}$ ( $4.574 \times 10^{-4}$ )
$\lambda_{11}$	$8.388 \times 10^{-3}$ ( $2.137 \times 10^{-3}$ )	$7.580 \times 10^{-3}$ ( $1.899 \times 10^{-3}$ )	$-3.238 \times 10^{-4}$ ( $1.155 \times 10^{-3}$ )	0.01513 ( $1.850 \times 10^{-3}$ )
$\lambda_2$	$1.704 \times 10^{-3}$ ( $4.076 \times 10^{-4}$ )	$1.548 \times 10^{-3}$ ( $4.036 \times 10^{-4}$ )	$1.368 \times 10^{-3}$ ( $3.830 \times 10^{-4}$ )	$1.610 \times 10^{-3}$ ( $3.796 \times 10^{-4}$ )
$\lambda_{21}$	$-3.147 \times 10^{-3}$ ( $1.542 \times 10^{-3}$ )	$-9.429 \times 10^{-5}$ ( $2.054 \times 10^{-3}$ )	$1.236 \times 10^{-3}$ ( $8.999 \times 10^{-4}$ )	$-3.762 \times 10^{-3}$ ( $1.763 \times 10^{-3}$ )
$\lambda_3$	$-8.719 \times 10^{-4}$ ( $4.073 \times 10^{-4}$ )	$-8.105 \times 10^{-4}$ ( $3.745 \times 10^{-4}$ )	$-5.535 \times 10^{-4}$ ( $3.598 \times 10^{-4}$ )	$-7.988 \times 10^{-4}$ ( $3.492 \times 10^{-4}$ )
$\lambda_{31}$	$-1.159 \times 10^{-3}$ ( $2.051 \times 10^{-3}$ )	$-4.744 \times 10^{-4}$ ( $1.505 \times 10^{-3}$ )	$-1.360 \times 10^{-3}$ ( $8.060 \times 10^{-4}$ )	$-1.127 \times 10^{-3}$ ( $1.539 \times 10^{-3}$ )
$\omega$	-9.012 (0.1416)	-8.959 (0.1434)	-8.966 (0.1408)	-8.960 (0.1718)
$\delta$	0.1890 (0.03403)	0.1940 (0.03522)	0.1956 (0.03497)	0.1962 (0.03648)
$\theta_0$	-0.1068 (0.01286)	-0.1160 (0.01396)	-0.1114 (0.01626)	-0.1180 (0.01307)
$\theta_1$	-0.05301 (0.01928)	-0.04010 (0.02165)	-0.03489 (0.01991)	-0.09579 (0.01402)
$\gamma$	0.1966 (0.01367)	0.2035 (0.01494)	0.2056 (0.01569)	0.2031 (0.01332)
$\phi_1$	0.7435 (0.06778)	0.7431 (0.06358)	0.7540 (0.06421)	0.7503 (0.04558)
$\psi_1$	-0.4602 (0.1149)	-0.4715 (0.1048)	-0.4849 (0.1085)	-0.4777 (0.04325)
$d$	0.5370 (0.02678)	0.5357 (0.02655)	0.5319 (0.02694)	0.5328 (0.02714)
$\ln L(\eta)$	75,304.94	75,291.05	75,279.68	75,286.26
AIC	-150,577.87	-150,550.10	-150,527.35	-150,540.33
SIC	-150,449.51	-150,421.74	-150,398.99	-150,411.97
$Q_{10}$	23.45	20.31	19.05	20.42
$Q_{100}$	129.29	125.48	123.88	124.55
$Q_{10}^A$	33.54	37.64	36.71	37.41
$Q_{100}^A$	202.27	208.02	209.35	212.11
Sign bias	1.959	1.891	1.925	1.977**
Negative size bias	-0.9055	-0.9066	-0.9975	-0.9755
Positive size bias	-0.8519	-1.025	-1.037	-1.048
Joint test	3.875	3.589	3.715	3.915

Note: QML estimates are reported with robust standard errors in parentheses. Also reported are  $\ln L(\eta)$ , the value of the maximized log-likelihood function, and the Akaike and Schwarz (or Bayesian) information criteria, respectively. The values of the Ljung-Box portmanteau statistic for up to  $K$ 'th order serial dependence in the standardized residuals,  $\hat{\varepsilon}_t/\hat{\sigma}_t$ , and the absolute standardized residuals,  $|\hat{\varepsilon}_t/\hat{\sigma}_t|$ , are denoted  $Q_K$  and  $Q_K^A$ , respectively. \* and \*\* denote rejection at the 5% and 1% level, respectively.

Table 5: Crises for other countries

Country	1973 oil crisis	1987 crash	2000 dotcom	2001-9-11 attack	Subprime	Country-specific
Canada	X	X	X	X	X	–
France	X	X	X	X	X	–
Germany	X	X	X	X	X	–
Italy	X	X	X	X	X	–
Japan	X	X	X	X	X	Bubble collapse 1990/02/05-1992/02/04
U.K.	X	X	X	X	X	–
Brazil	X	X	–	–	X	Financial crisis 1999/01/20-2002/10/17
Russia	–	–	–	–	X	Economic crisis 1998/08/18-1999/02/09
India	–	X	–	–	X	–
China	–	–	–	–	X	Asian financial crisis 1997/05/13-1997/09/24
Argentina	–	X	–	–	X	Economic crisis 2000/04/14-2001/01/02
Mexico	–	X	–	–	X	Peso crisis 1994/12/21-1995/03/22
South Korea	–	X	–	–	X	Asian financial crisis 1997/10/24-1998/10/01
Thailand	–	X	–	–	X	Asian financial crisis 1997/07/07-1998/09/04

Note: This table presents the crisis list for other countries. The symbols “X” and “–” denote the inclusion and exclusion, respectively, of a crisis. We do not include the 2000 dotcom and the 2001-9-11 attack crises for the BRIC and other emerging markets, since these two crises mainly affect the developed markets. In some cases, inclusion of a crisis is precluded by the length of the time series for the given country. We also list country-specific crises for each country, if any. For the first five crises (1973 oil crisis, 1987 stock market crash, 2000 dotcom, 2001-9-11 attack, and subprime) the crisis start and end dates are the same as for the U.S. market.

Table 6: FIEGARCH-M models for G-7 countries

Parameter	Canada	France	Germany	Italy	Japan	U.K.
$\mu_0$	$3.635 \times 10^{-4}$ ( $9.466 \times 10^{-5}$ )	$2.227 \times 10^{-4}$ ( $1.215 \times 10^{-4}$ )	$2.262 \times 10^{-4}$ ( $9.107 \times 10^{-5}$ )	$4.254 \times 10^{-4}$ ( $9.178 \times 10^{-5}$ )	$4.427 \times 10^{-4}$ ( $8.080 \times 10^{-5}$ )	$3.898 \times 10^{-4}$ ( $8.436 \times 10^{-5}$ )
$\lambda_1$	$-2.745 \times 10^{-3}$ ( $7.479 \times 10^{-4}$ )	$-2.498 \times 10^{-3}$ ( $1.401 \times 10^{-3}$ )	$7.064 \times 10^{-4}$ ( $1.298 \times 10^{-3}$ )	$1.762 \times 10^{-3}$ ( $8.949 \times 10^{-4}$ )	$9.966 \times 10^{-4}$ ( $6.305 \times 10^{-4}$ )	$-2.681 \times 10^{-3}$ ( $6.788 \times 10^{-4}$ )
$\lambda_{11}$	$1.994 \times 10^{-3}$ ( $1.307 \times 10^{-3}$ )	$6.546 \times 10^{-4}$ ( $2.761 \times 10^{-3}$ )	$9.790 \times 10^{-4}$ ( $1.966 \times 10^{-3}$ )	$-4.293 \times 10^{-3}$ ( $2.120 \times 10^{-3}$ )	$-4.147 \times 10^{-4}$ ( $1.209 \times 10^{-3}$ )	$2.507 \times 10^{-3}$ ( $1.885 \times 10^{-3}$ )
$\omega$	-9.105 (0.2574)	-8.785 (0.1746)	-9.052 (0.1804)	-8.662 (0.1874)	-8.708 (0.2134)	-9.142 (0.2638)
$\delta$	0.1519 (0.03501)	0.1684 (0.04873)	0.1920 (0.05241)	0.2540 (0.03876)	0.3163 (0.04527)	0.1064 (0.05031)
$\theta_0$	-0.07952 (0.01574)	-0.05571 (0.01516)	-0.04081 (0.01701)	$-5.955 \times 10^{-3}$ (0.01054)	-0.1012 (0.02663)	-0.07283 (0.01064)
$\theta_1$	-0.03617 (0.03946)	-0.04951 (0.02520)	-0.06640 (0.01650)	-0.1272 (0.03243)	-0.1002 (0.02727)	-0.03203 (0.02884)
$\gamma$	0.2945 (0.03881)	0.1906 (0.02398)	0.1570 (0.03977)	0.2742 (0.02272)	0.3155 (0.04914)	0.2205 (0.02488)
$\phi_1$	0.9410 (0.03705)	0.8680 (0.02777)	0.6423 (0.1077)	0.8432 (0.07010)	0.6967 (0.08551)	0.7569 (0.1328)
$\psi_1$	-0.8342 (0.07333)	-0.4011 (0.1323)	0.2009 (0.3181)	-0.5461 (0.1165)	-0.4102 (0.1455)	-0.5464 (0.2043)
$d$	0.4588 (0.05908)	0.4111 (0.06137)	0.5086 (0.04769)	0.4579 (0.07557)	0.4827 (0.03281)	0.5920 (0.04931)
$\ln L(\eta)$	36,486.77	33,779.47	36,554.80	41704.65	48,451.40	34,594.24
AIC	-72,951.53	-67,536.94	-73,087.60	-83,387.30	-96,880.79	-69,166.48
SIC	-72,871.69	-67,457.14	-73,006.73	-83,304.86	-96,796.98	-69,086.61
$Q_{10}$	256.58	196.10	91.06	402.89	150.93	120.59
$Q_{100}$	350.97	293.78	193.08	572.86	265.71	247.42
$Q_{10}^A$	12.63	4.440	27.86	21.39	28.93	15.76
$Q_{100}^A$	128.64	119.20	146.68	149.88	126.85	112.98
Sign bias	-0.5939	-0.5531	3.055**	2.281*	1.741	0.3756
Negative size bias	0.5430	0.7067	-1.880	-2.460**	-0.3648	0.6569
Positive size bias	2.098*	-0.9370	-3.778**	-3.326**	-2.143*	0.4950
Joint test	4.914	2.872	15.47**	13.33**	5.478	2.003

Note: QML estimates are reported with robust standard errors in parentheses. Also reported are  $\ln L(\eta)$ , the value of the maximized log-likelihood function, and the Akaike and Schwarz (or Bayesian) information criteria, respectively. The values of the Ljung-Box portmanteau statistic for up to  $K$ 'th order serial dependence in the standardized residuals,  $\hat{\varepsilon}_t/\hat{\sigma}_t$ , and the absolute standardized residuals,  $|\hat{\varepsilon}_t/\hat{\sigma}_t|$ , are denoted  $Q_K$  and  $Q_K^A$ , respectively. \* and \*\* denote rejection at the 5% and 1% level, respectively.

Table 7: FIEGARCH-M models for BRIC countries

Parameter	Brazil	Russia	India	China
$\mu_0$	$1.839 \times 10^{-3}$ ( $2.151 \times 10^{-4}$ )	$1.488 \times 10^{-3}$ ( $3.843 \times 10^{-4}$ )	$9.063 \times 10^{-4}$ ( $2.199 \times 10^{-4}$ )	$1.018 \times 10^{-3}$ ( $5.784 \times 10^{-4}$ )
$\lambda_1$	$1.242 \times 10^{-3}$ ( $2.055 \times 10^{-3}$ )	$-2.122 \times 10^{-3}$ ( $2.100 \times 10^{-3}$ )	$1.637 \times 10^{-3}$ ( $1.401 \times 10^{-3}$ )	$7.249 \times 10^{-3}$ ( $1.664 \times 10^{-3}$ )
$\lambda_{11}$	$4.902 \times 10^{-3}$ ( $3.579 \times 10^{-3}$ )	$-0.01525$ ( $5.984 \times 10^{-3}$ )	$2.137 \times 10^{-3}$ ( $6.163 \times 10^{-3}$ )	$-9.157 \times 10^{-3}$ ( $6.384 \times 10^{-3}$ )
$\omega$	$-7.228$ (0.1920)	$-7.221$ (0.2546)	$-8.114$ (0.1790)	$-7.214$ (0.3745)
$\delta$	$0.08028$ (0.02957)	$0.1707$ (0.04062)	$0.3562$ (0.04867)	$0.2041$ (0.04354)
$\theta_0$	$-0.01360$ (0.01085)	$-0.03903$ (0.01634)	$-0.01836$ (0.01565)	$-1.409 \times 10^{-3}$ (0.01303)
$\theta_1$	$-0.02728$ (0.01701)	$-0.1397$ (0.04194)	$-0.06861$ (0.05139)	$-0.1018$ (0.03831)
$\gamma$	$0.2469$ (0.02842)	$0.3134$ (0.04580)	$0.3084$ (0.04867)	$0.2757$ (0.04837)
$\phi_1$	$0.7045$ (0.1107)	$0.7537$ (0.1325)	$0.8450$ (0.05793)	$0.6487$ (0.1567)
$\psi_1$	$-0.2368$ (0.2097)	$-0.2699$ (0.1777)	$-0.4560$ (0.1488)	$0.06879$ (0.2424)
$d$	$0.5224$ (0.04661)	$0.4272$ (0.08844)	$0.3603$ (0.08817)	$0.4631$ (0.1084)
$\ln L(\eta)$	23, 229.81	8901.00	19, 665.51	12, 855.10
AIC	-46, 437.62	-17, 780.01	-39, 309.02	-25, 688.20
SIC	-46.358.69	-17, 711.46	-39, 233.57	-25, 616.75
$Q_{10}$	446.17	79.37	124.46	93.24
$Q_{100}$	829.44	154.47	218.75	295.75
$Q_{10}^A$	11.92	4.202	6.677	13.46
$Q_{100}^A$	123.69	85.18	90.31	131.33
Sign bias	3.090**	0.2525	1.530	1.312
Negative size bias	-3.803**	-0.6606	-1.260	-1.258
Positive size bias	-3.084**	1.716	-2.072*	0.02408
Joint test	18.36**	5.320	4.701	2.717

Note: QML estimates are reported with robust standard errors in parentheses. Also reported are  $\ln L(\eta)$ , the value of the maximized log-likelihood function, and the Akaike and Schwarz (or Bayesian) information criteria, respectively. The values of the Ljung-Box portmanteau statistic for up to  $K$ 'th order serial dependence in the standardized residuals,  $\hat{\varepsilon}_t/\hat{\sigma}_t$ , and the absolute standardized residuals,  $|\hat{\varepsilon}_t/\hat{\sigma}_t|$ , are denoted  $Q_K$  and  $Q_K^A$ , respectively. \* and \*\* denote rejection at the 5% and 1% level, respectively.

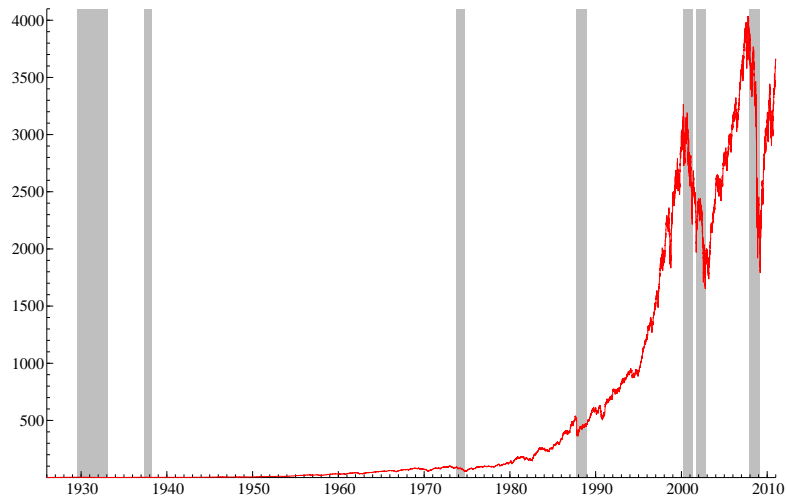
Table 8: FIEGARCH-M models for other emerging markets

Parameter	Argentina	Mexico	South Korea	Thailand
$\mu_0$	$1.440 \times 10^{-3}$ ( $3.203 \times 10^{-4}$ )	$1.222 \times 10^{-3}$ ( $1.895 \times 10^{-4}$ )	$5.947 \times 10^{-4}$ ( $2.142 \times 10^{-4}$ )	$3.236 \times 10^{-4}$ ( $1.319 \times 10^{-4}$ )
$\lambda_1$	$3.129 \times 10^{-3}$ ( $1.654 \times 10^{-3}$ )	$-1.758 \times 10^{-3}$ ( $1.579 \times 10^{-3}$ )	$2.034 \times 10^{-3}$ ( $2.105 \times 10^{-3}$ )	$1.283 \times 10^{-3}$ ( $6.302 \times 10^{-4}$ )
$\lambda_{11}$	$-2.696 \times 10^{-3}$ ( $3.423 \times 10^{-3}$ )	$-4.679 \times 10^{-3}$ ( $9.583 \times 10^{-3}$ )	$5.186 \times 10^{-3}$ ( $2.085 \times 10^{-3}$ )	$1.965 \times 10^{-4}$ ( $2.686 \times 10^{-3}$ )
$\omega$	-6.886 (0.2389)	-7.530 (0.2528)	-6.561 (0.4917)	-7.810 (0.3340)
$\delta$	0.1518 (0.02701)	0.1221 (0.04430)	0.3978 (0.07292)	0.1736 (0.05040)
$\theta_0$	$2.109 \times 10^{-3}$ (0.01482)	-0.08640 (0.01675)	$-8.252 \times 10^{-3}$ (0.02202)	$-2.513 \times 10^{-3}$ (0.02670)
$\theta_1$	-0.02771 (0.03493)	-0.06287 (0.05058)	-0.03552 (0.02811)	-0.09568 (0.04688)
$\gamma$	0.3819 (0.04179)	0.2991 (0.02947)	0.2613 (0.01017)	0.4670 (0.05512)
$\phi_1$	0.6686 (0.1916)	0.7165 (0.1224)	0.2492 (0.03550)	0.3748 (0.9881)
$\psi_1$	-0.3772 (0.2750)	-0.5114 (0.1795)	0.3953 (0.06473)	-0.08945 (1.114)
$d$	0.4891 (0.06475)	0.5351 (0.05095)	0.5405 (0.03207)	0.4989 (0.08146)
$\ln L(\eta)$	27, 222.28	17, 736.55	40, 594.48	26, 548.98
AIC	-54, 442.56	-35, 451.09	-81, 166.96	-53, 075.95
SIC	-54, 342.36	-35, 376.69	-81, 084.18	-52, 998.23
$Q_{10}$	382.26	208.54	156.38	377.25
$Q_{100}$	606.90	373.77	291.07	574.57
$Q_{10}^A$	12.69	11.54	28.63	17.21
$Q_{100}^A$	128.69	123.56	140.00	115.98
Sign bias	1.585	1.667	0.5417	0.7495
Negative size bias	-0.8912	-0.9685	-0.5339	-0.2605
Positive size bias	0.8313	0.4579	-0.03485	-0.5379
Joint test	5.996	5.530	0.4989	0.6118

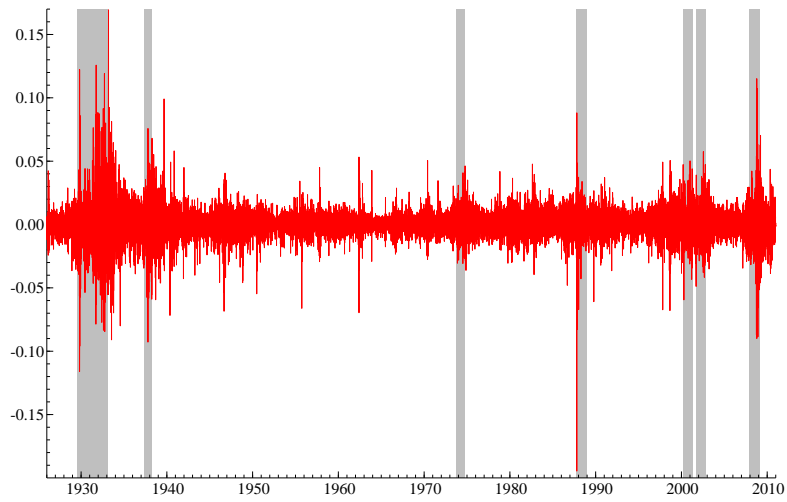
Note: QML estimates are reported with robust standard errors in parentheses. Also reported are  $\ln L(\eta)$ , the value of the maximized log-likelihood function, and the Akaike and Schwarz (or Bayesian) information criteria, respectively. The values of the Ljung-Box portmanteau statistic for up to  $K$ 'th order serial dependence in the standardized residuals,  $\hat{\varepsilon}_t/\hat{\sigma}_t$ , and the absolute standardized residuals,  $|\hat{\varepsilon}_t/\hat{\sigma}_t|$ , are denoted  $Q_K$  and  $Q_K^A$ , respectively. \* and \*\* denote rejection at the 5% and 1% level, respectively.

Figure 1: Time series plots of U.S. CRSP value-weighted index level and returns

(a) CRSP value-weighted index level



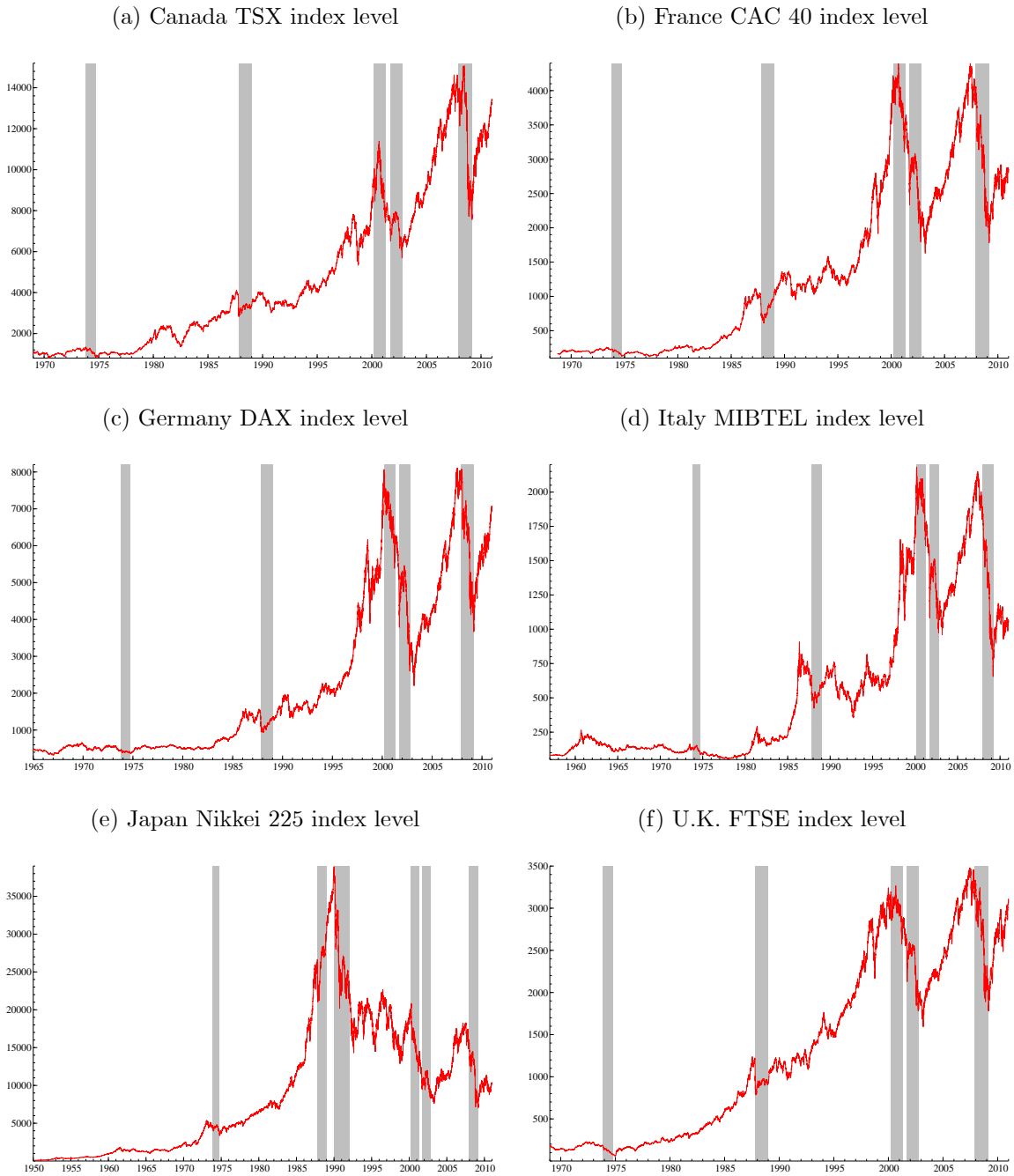
(b) CRSP value-weighted index returns



Note: The shaded bars indicate periods of financial crisis.



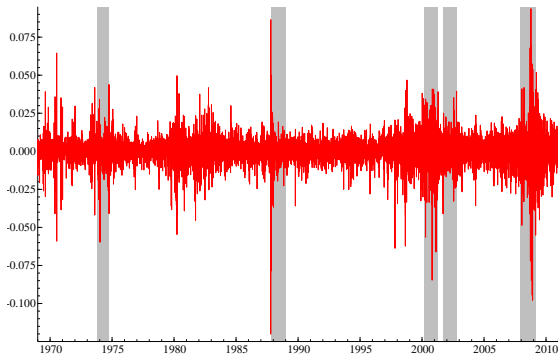
Figure 2: Time series plots of G-7 index levels



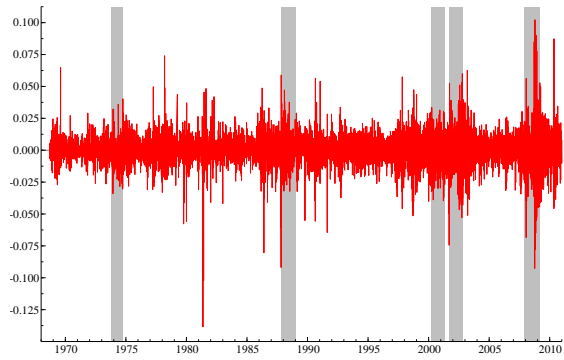
Note: The shaded bars indicate periods of financial crisis.

Figure 3: Time series plots of G-7 index returns

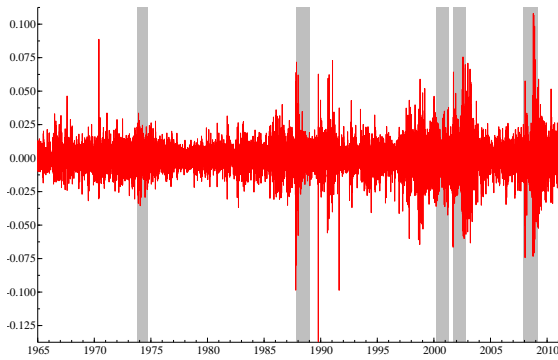
(a) Canada TSX returns



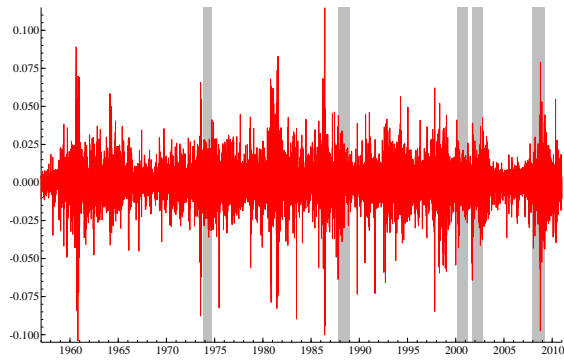
(b) France CAC 40 returns



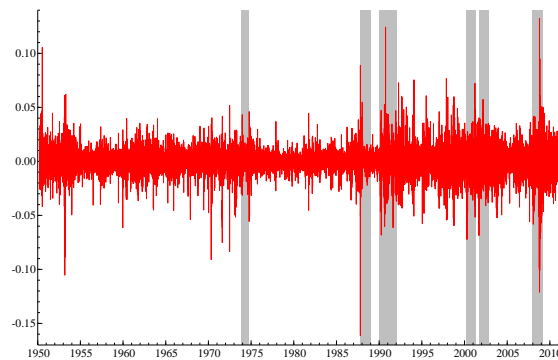
(c) Germany DAX returns



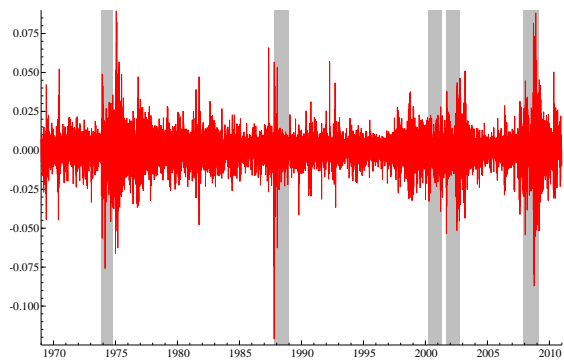
(d) Italy MIBTEL returns



(e) Japan Nikkei 225 returns

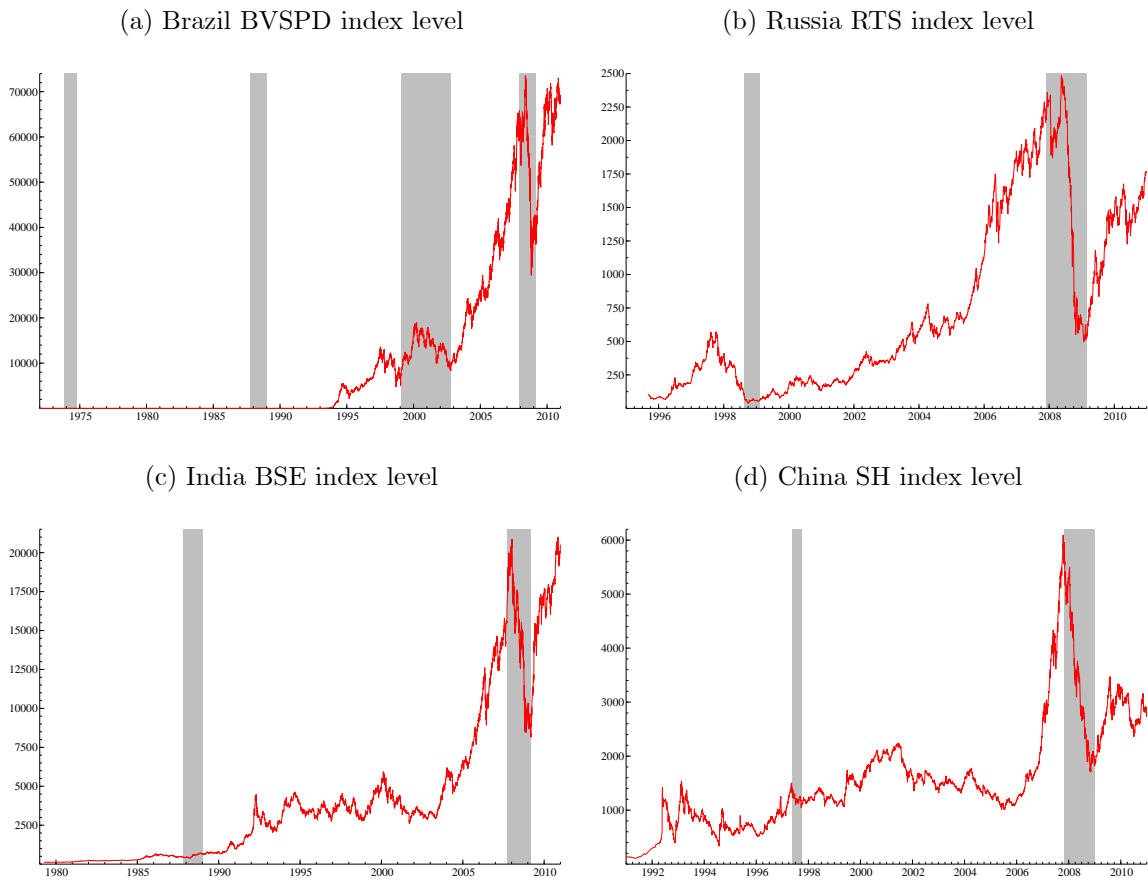


(f) U.K. FTSE returns



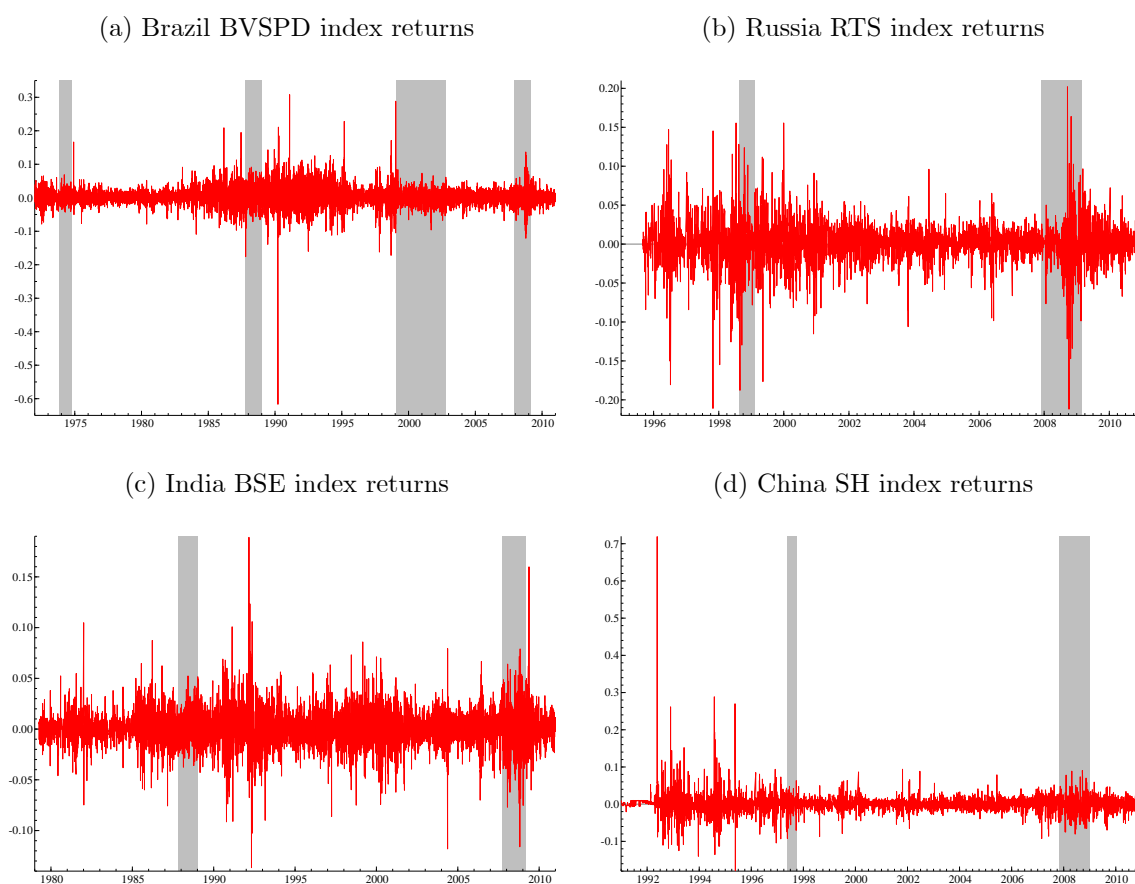
Note: The shaded bars indicate periods of financial crisis.

Figure 4: Time series plots of BRIC index levels



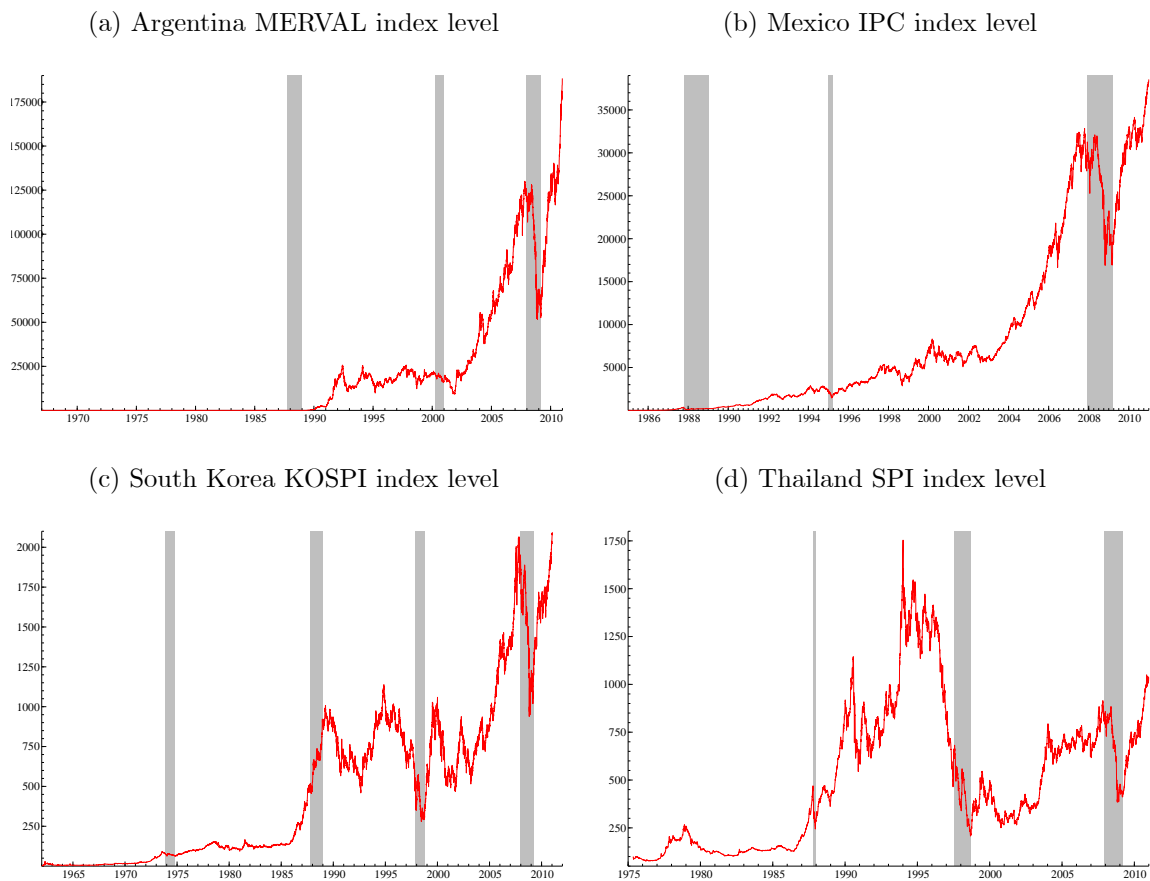
Note: The shaded bars indicate periods of financial crisis.

Figure 5: Time series plots of BRIC index returns



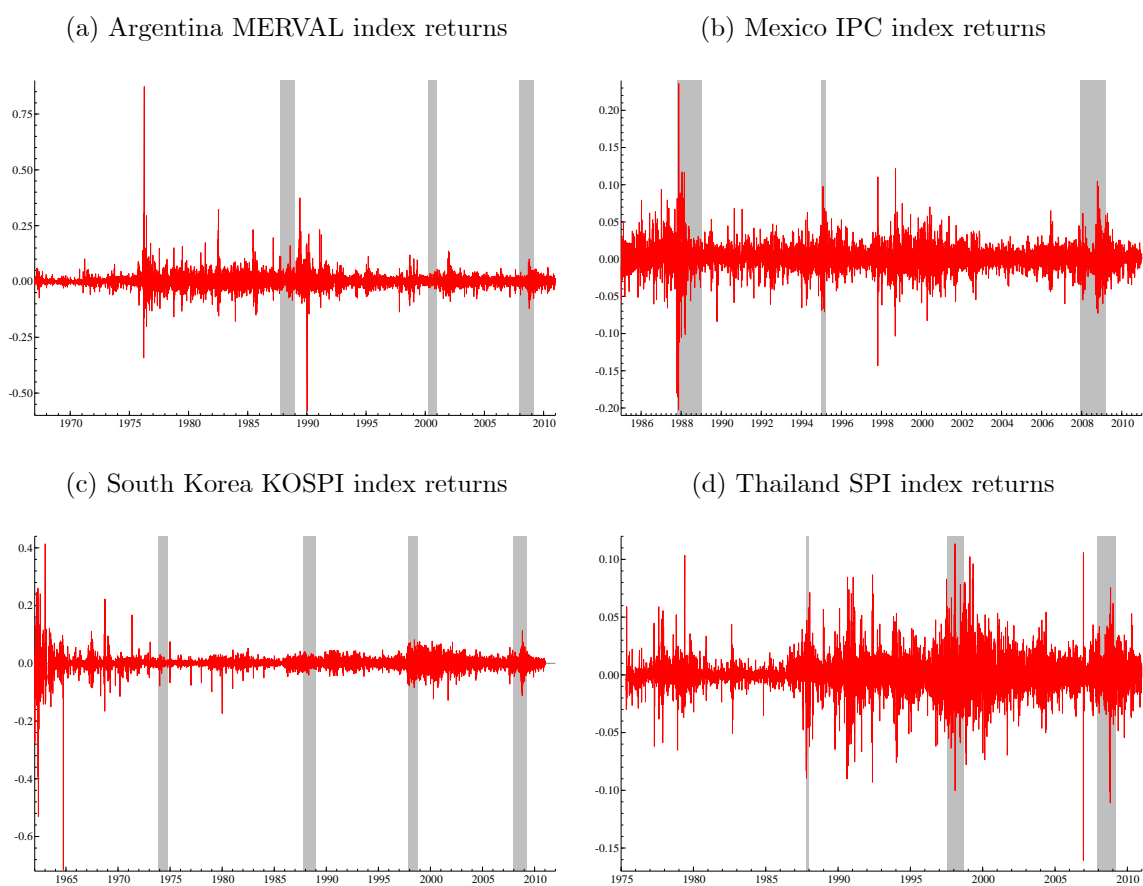
Note: The shaded bars indicate periods of financial crisis.

Figure 6: Time series plots of emerging markets index levels



Note: The shaded bars indicate periods of financial crisis.

Figure 7: Time series plots of emerging markets index returns



Note: The shaded bars indicate periods of financial crisis.