WARWICK

#298

POVERTY, INCENTIVES AND LINEAR INCOME TAXATION

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No. 298

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

June 1988

UNIVERSITY OF WARWICK,
COVENTRY

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* This paper and its companion piece, Kanbur and Keen (1988), were presented at the IFS conference on "The Economics of Social Security" held on April 15 1988 to mark the introduction of major social security and income maintenance reforms by the United Kingdom Government. We are grateful to participants at that conference for their comments, and to Tim Besley for helpful discussions.

1. INTRODUCTION

The reform of social security and income maintenance programs is now on the policy agenda of many OECD countries, there being a widespread view that these systems are rife with inefficiencies - that they involve excessive leakages of benefits to the non-poor and severe disincentive effects. The UK Government began a review of the system in 1985. This culminated in the legislation that came into effect in 1988, billed as the most radical reform of the social security system for forty years. Summarising its merits, the Secretary of State for Social Services claimed that:

> This new, more coherent and better targeted structure will direct help more clearly where it is most needed and foster incentives to work. (John Moore, Hansard 121,179)

A social security and income maintenance system may serve many purposes. It might be used, for instance, to help households achieve a preferred allocation of resources over the lifecycle, or to provide them with insurance of a kind that private markets are thought unable to offer. The central objective, however, is clear enough: it is the alleviation of poverty. This was certainly a major plank of the Beveridge proposals that launched the modern social security system in the UK; and the other main plank - the insurance principle - is arguably now little more than illusion (Dilnot, Kay and Morris, 1984). Illusion aside, it is poverty alleviation that lies at the heart of the current debate. This is clear both from the claims made by John Moore and other advocates of the Fowler reforms and from many of the counter-claims made by their opponents. John Moore's defence of the reforms also raises the other recurrent theme: a concern with incentive effects. One may of course believe that social security is (or should be) about more than the relief of poverty (see for instance Stern (1987)); one might also take the view that labour supply responses - at least those of principal earners - are for practical purposes rather insignificant (though it would be difficult to believe that they place no ultimate constraint on the relief that can be provided). Nevertheless, it is the two issues of targeting and incentives that have come to dominate policy discussion in the UK; and it is these policy concerns that determine the focus of this paper.

On targeting, two aspects of the strategy associated with the Fowler reforms are especially noteworthy. The first is a tendency towards increased means testing (income testing). The second is a redirection of resources towards low paid working families at the particular expense, many believe, of pensioners. Can these policies be justified in terms of their own stated objective of reducing poverty? The purpose of this paper and its companion piece, Kanbur and Keen (1988), is to develop a framework within which questions of this kind can be addressed. In the context of a theoretical model in which individuals can freely vary their labour supply, we attempt to make precise the trade-offs to be faced when the objective is to minimise poverty and incentive effects on both poor and non-poor have to be taken into account. In subsequent work we hope to apply this framework in greater empirical detail to the UK and to the US.

In the present paper attention is confined to linear tax-benefit systems, that is ones which are simply equivalent to the imposition of a poll tax or subsidy (demogrant) combined with the taxation of all other income at a constant marginal rate. This includes, of course, many of the most familiar proposals for radical social security reform: the simplest negative income tax, social dividend and tax credit schemes, for instance, are all of precisely this form. Section 3 of the paper considers, from an explicit perspective of poverty alleviation, the design and evaluation of such schemes when a single linear tax structure is to be applied to a homogeneous population; the use of non-linear taxation for poverty alleviation is analysed in Kanbur and Keen (1988). Section 4 moves on to the case in which contingent information can be used to apply different linear tax structures to different groups of the population. This provides a simple setting within which many of the current issues of retargeting can be considered. By way of illustration, we provide some preliminary calculations bearing on the sort of retargeting from pensioners to working families that is currently emerging in the UK.

Before proceeding to a detailed consideration of poverty alleviation strategies, however, we need to consider how exactly poverty is to be measured once incentive effects are recognised. The next section begins the analysis by addressing this question.

2. LABOUR, LEISURE AND THE MEASUREMENT OF POVERTY

There is of course a vast literature on the concept and measurement of poverty (recently reviewed by Atkinson (1987)). The almost invariable strategy in applied work, however, is simple enough: someone is regarded as poor if and only if their income is below some specified poverty line, and measures of aggregate poverty are then constructed from the existence and (perhaps) extent of such income shortfalls. Measures of poverty thus rest, in practice, on measures of income. Empirical studies have indeed emphasised a variety of issues in the detailed construction of income variables, such as the nature of any adjustment for family composition and the relative merits of short- and longer-term income definitions. Their broad objective, nevertheless, remains that of measuring the income that a household actually receives. The larger question that this raises becomes especially evident when - as here - labour supply responses are at issue: for in attaching significance only to income, poverty measures of the usual kind attach no importance at all to the effort made to generate that income. They attach no value, that is, to leisure.

The difficulties that consequently arise in gauging poverty in terms of income received are obvious. A household or individual - terms we shall take as synonymous - could have enormous potential earnings and yet, in preferring instead to remain idle, be counted as poor. Conversely, such measures would treat as non-poor someone who managed to raise their income above the poverty line only by working inordinately hard. One response to such problems would be to assess poverty by reference not to the income that a household actually does receive but to that which it could receive by working some 'standard' number of hours. This of course opens up the question as to what should constitute standard hours. Nevertheless, such a formulation in terms of 'standard income' does have some appeal in capturing the commonplace view that by working a reasonable number of hours a household ought to be able to secure a reasonable income, and locates the policy interest firmly in the low wage rates of the poor. This approach also corresponds to a notion of poverty as the absence of 'capabilities', advanced in a more general setting by Sen (1985).

From the usual welfare-theoretic perspective, however, neither received not standard income is a satisfactory summary statistic: on either definition, it is perfectly possible for a change in the environment to lead (for instance) to a reduction in a household's income and yet leave that household, in its own view, better off than before. Consider for instance Figure 1, and suppose that the budget constraint is initially AA'; the household then works AH hours, which we can imagine to be exactly the amount deemed acceptable under the standard income approach. The tax-benefit system is now reformed, changing the budget constraint to ABB'. As a result, the household's received income falls (since b is below a) and so does its standard income (since c also lies below a). But the household also now attains a higher level of utility.

This points to a third and explicitly welfaristic approach to the treatment of leisure, focussing not on income gaps but on shortfalls between the utility levels households actually attain and some poverty level u_z. Denoting by $e(w,u)$ the virtual income needed to achieve utility u at a marginal wage w, choice of some reference wage w_r leads to

$$z(w_r) = e(w_r, u_z)$$

as a corresponding poverty line level of equivalent income (in the sense of King (1983)). The poverty of a household that achieves utility u could then be gauged by comparing its equivalent income $e(w_r,u)$ to this poverty line. In this way it would be straightforward to ensure that a reform which raised a household's welfare could not also increase its

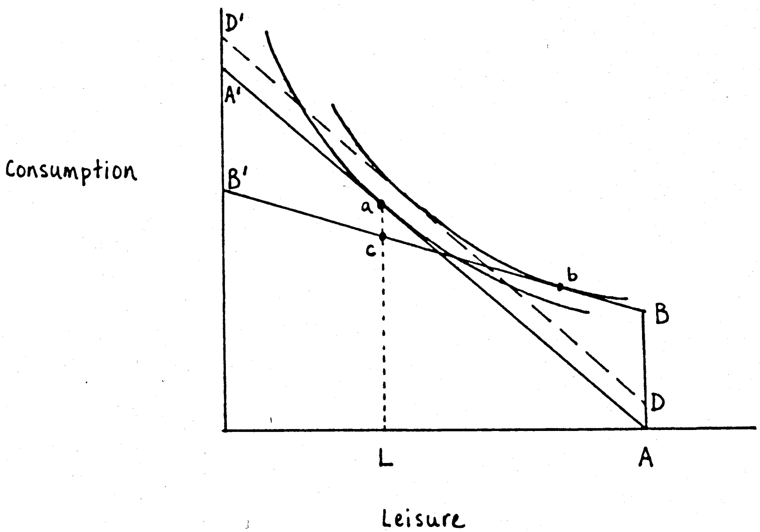

Figure 1

measured poverty. In Figure 1 for instance, and taking as the reference wage that corresponding to the initial budget constraint AA′, the welfare improvement generated by the reform is reflected in an increase in equivalent income of AD (the line DD′ being parallel to AA′).

There are thus (at least) three distinct concepts of income that might be used to define and measure poverty: received, standard and equivalent. These lead in turn to alternative criteria for the design and evaluation of policies for the alleviation of poverty, each of which has its merits. The equivalent income approach embodies the welfarism characteristic of the optimal tax literature. Indeed it amounts to little more than imposing additional structure on a social welfare function of the usual kind. For this reason, we shall explore this approach no further. Those who view poverty purely in terms of household welfare may consequently be inclined to stop reading at this point. Yet the alternative approaches sketched out above do seem to capture concerns central to the discussion of policy in this area. The formulation in terms of standard income makes some concession, albeit a crude one, to Sen's influential critique of welfarism. That in terms of received income corresponds directly to the procedures that dominate empirical work and underly everyday debate. Whilst recognising their limitations, and certainly without meaning to discard the lessons from existing welfaristic analyses, it therefore seems worthwhile exploring the implications of these alternative approaches for the construction of policies to relieve poverty. That is the purpose in the remainder of the paper.

To formalise issues of poverty alleviation it is necessary to put more structure on the poverty index than we have yet had to do. Though it will be clear that some of the conclusions below are of more general relevance, attention will here be confined to the P_α-class of measures developed by Foster, Greer and Thorbecke (1984). For a non-negative income variable y distributed with density $f(y)$, and with a poverty line of z, these take the form

$$P_\alpha = \int_0^z \left(\frac{z-y}{z}\right)^\alpha f(y)dy \qquad , \qquad \alpha \geq 0. \tag{2.1}$$

Also used in related contexts by Besley and Kanbur (1987) and Kanbur (1986, 1987), the P_α-class has the merit of combining analytical convenience and - through the parameter α - ethical flexibility. Taking $\alpha = 0$, 1 and 2, for instance, one has

$$P_0 = H \tag{2.2}$$

$$P_1 = H\left[\frac{z-\bar{y}_p}{z}\right] = HI \tag{2.3}$$

$$P_2 = H[I^2 + (1-I)^2 V_p^2] \tag{2.4}$$

where H is the simple headcount measure (the proportion of the population in poverty), $\bar{y}_p = E[y|y \leq z]$ is the mean income of the poor (so that I is just the income-gap ratio) and V_p is the coefficient of variation of income amongst the poor. More generally, as α increases the P_α measure becomes increasingly sensitive to the incomes of the very poorest households; hence α is sometimes described as parameterising 'poverty aversion'.

3. LINEAR INCOME TAXATION WITH A HOMOGENEOUS POPULATION

A linear income tax is one characterised by a constant marginal tax rate t and a poll subsidy G. As noted in the introduction, schemes of this kind have - in various guises - occupied a central position in diuscussions of social security reform. But while linear taxes have been widely studied in the context of social welfare maximisation (see for instance Dixit and Sandmo (1977)), the poverty-oriented literature has gone little beyond the simple arithmetic of the trade-off between providing a decent basic benefit and maintaining reasonably low marginal tax rates. This leaves unanswered the obvious question: When labour supply effects are explicitly recognised, how would one design a linear income tax to achieve the maximal reduction of poverty? This section addresses that question in the setting of a homogeneous population, with a single tax schedule to be applied to all households.

Suppose then that households have identical preferences $u(c,1-h)$ defined on consumption c of a composite good and on leisure, the latter being the time endowment (normalised at unity) less hours worked h. Households differ only in their (non-negative) wage rates, which are denoted by w and distributed with a density $f(w)$ that is unaffected by the tax system. Given its wage rate, each household chooses the hours that it works to maximise its utility subject to the budget constraint

$$c = G + (1-t)wh$$

implied by the linear tax structure, leading to a labour supply function of the form $h[(1-t)w,G]$.

As discussed in the preceding section, the objective of poverty alleviation might be formulated either in terms of the income that a household actually receives, now given by

$$y(w) = G + (1-t)wh[(1-t)w,G], \tag{3.1}$$

or in terms of that it would receive if it worked some standard number of hours L (assumed to be constant across households), which is given by

$$y_L(w) = G + (1-t)wL. \tag{3.2}$$

We begin with the first of these approaches.

Denoting by h_w the derivative of labour supply with respect to the net wage, the assumption that

$$h + (1-t)wh_w > 0 \tag{3.3}$$

(equivalently, that the wage elasticity be no less than -1) implies that net income strictly increases with the wage rate. There then exists some poverty line wage ω, defined by

$$y(\omega) = z, \tag{3.4}$$

such that a household is poor if and only if it faces a wage lower than ω. This enables the poverty index corresponding to the received income approach to be written as

$$P_\alpha = \int_0^\omega \left(\frac{z-G-(1-t)wh[(1-t)w,G]}{z} \right)^\alpha f(w)dw \tag{3.5}$$

(and we henceforth assume $\alpha > 0$). The problem of poverty alleviation thus becomes that of choosing the tax parameters t and G to minimise P_α in (3.5), subject to the constraint that

$$t\int_0^\infty wh[(1-t)w,G]f(w)dw - G = R, \tag{3.6}$$

where R denotes the revenue (per capita) required by the government. The first-order conditions for this program are easily derived. It is more instructive, however, to cast the problem explicitly as one of reform: starting from an arbitrary initial tax structure, how would poverty be affected by a revenue-neutral increase in the marginal tax rate?

The benchmark case is that in which labour supply is completely inelastic, say at h^*. From the revenue constraint (3.5), an increase in the marginal rate then enables the poll subsidy to be increased by

$$\frac{dG}{dt} = \bar{w}h^*, \tag{3.7}$$

where $\bar{w}$ denotes the mean wage in the population. Differentiating (3.5), the effects through ω vanish (as a consequence of (3.4)) to leave

$$\frac{dP_\alpha}{dt} = \int_0^\omega J[z,y(w)]\left[\frac{dG}{dt} - wh^*\right]f(w)dw \tag{3.8}$$

where

$$J[z,y(w)] \equiv \frac{-\alpha}{z}\left[\frac{z-y(w)}{z}\right]^{\alpha-1} \tag{3.9}$$

is strictly negative for $w < \omega$. Combining (3.7) and (3.8) gives

$$\frac{dP_\alpha}{dt} = \int_0^\omega J[z,y(w)]\{\bar{w} - w\}h^*f(w)dw. \tag{3.10}$$

So long as the poverty line wage is less than the mean wage, one thus has the expected conclusion: in the absence of incentive effects, a revenue-neutral increase in the marginal tax rate unambiguously reduces poverty (unless there are no poor households to begin with), with the increase in the basic benefit that it allows more than offsetting a directly adverse effect. One is thus led to continue increasing the marginal rate until either it reaches 100%, or the basic benefit is raised to the poverty line, or at least some of those with above-average wages are impoverished.

No such simple prescriptions are available when labour supply responses are admitted. Denoting by h_G the derivative of hours worked with respect to the poll subsidy, the analogue to (3.8) is then

$$\frac{dP_\alpha}{dt} = \int_0^\omega J[z,y(w)]\left[\frac{dG}{dt}[1 + (1-t)wh_G] - w[h + (1-t)wh_w]\right]f(w)dw, \tag{3.11}$$

where dG/dt is again derived from the revenue constraint (3.6) but now also depends on supply responses. Comparing (3.11) with (3.8), the assumption in (3.3) implies that the direct effect of increasing the marginal tax rate is still to worsen poverty. Against this, a higher marginal rate may again generate increased revenue and so finance a more generous poll subsidy; but since $h_G \leq 0$ (so long as leisure is normal) the beneficial effects of this on measured income may now be dampened by an induced reduction in labour supply. At this level of generality there is little more to be said: few enlightening and perfectly general formulae seem to be available.

Consider then the special case in which preferences are of the Cobb-Douglas form

$$u(c,1-h) = (1-\delta)\mathrm{Ln}[c] + \delta \mathrm{Ln}[1-h] \quad , \quad \delta \epsilon [0,1],$$

generating the labour supply function

$$h[(1-t)w,G] = (1-\delta) - \frac{\delta G}{(1-t)w} \tag{3.12}$$

and hence net income

$$y(w) = (1-\delta)\{(1-t)w + G\}. \tag{3.13}$$

Higher values of δ thus imply more sensitive labour supply behaviour; and $1-\delta$ is simply the proportion of full income that each household allocates to the consumption good. Suppose also that $R = 0$, so that the government merely requires the tax-benefit system to break even. Using these simplifications in (3.11) and the revenue constraint (3.6), and recalling too the definitions of $J(z,y)$ and P_α, one finds after lengthy manipulations that the effect of a balanced-budget increase in the marginal rate is given by

$$\left\{\frac{(1-t)z}{\alpha}\right\}\frac{dP_\alpha}{dt} = (P_{\alpha-1} - P_\alpha)z - \sigma(\delta,t)\left\{\frac{GP_{\alpha-1}}{t}\right\} \tag{3.14}$$

where

$$\sigma(\delta,t) = \frac{(1-\delta)(1-t)}{1-t(1-\delta)} \epsilon [0,1] \tag{3.15}$$

(and we are assuming $t \epsilon (0,1)$). The first term on the right hand side of

(3.14) is unambiguously positive (by the definition of P_α), and can be thought of as capturing the harmful direct effect of increasing the marginal rate. (This interpretation, and more generally the significance here of $P_{\alpha-1}$, will become clearer in the next section). The second term captures both the opposing effect of an increased poll subsidy and the mitigating consequences of labour supply adjustments. Note that one simple implication follows immediately from (3.12): since $\sigma(\delta,t)$ is decreasing in δ, a revenue-neutral decrease in the marginal rate is more likely to reduce poverty, cet. par., the more responsive is labour supply. Perhaps surprisingly, however, the implications of alternative degrees of poverty aversion seem to be ambiguous: both $P_{\alpha-1}$ and the difference $P_{\alpha-1} - P_\alpha$ are readily shown to be decreasing in α.

The formula (3.14) provides a convenient framework for illustrative calculations. Take for instance the case $\alpha = 1$. Denoting by $K \equiv \bar{y}_p/G$ (>1) the ratio of the mean (received net) income of the poor to the poll subsidy, (3.14) then becomes

$$\left\{\frac{t(1-t)z}{HG}\right\}\frac{dP_1}{dt} = tK - \sigma(\delta,t). \tag{3.16}$$

Note from the definition in (3.15) that $\sigma(\delta,t)$ is strictly decreasing in t, with $\sigma(\delta,0) > 0$ and $\sigma(\delta,1) = 0$. Applied to (3,16), these observations imply the existence of a critical marginal tax rate $t^* \epsilon(0,1)$, defined by

$$t^*K = \sigma(\delta,t^*), \tag{3.17}$$

such that a small revenue-neutral increase in the marginal rate will reduce poverty (conversely, increase it) if and only if the rate is initially less (greater) than t^*. Table 3.1 reports this critical marginal rate for a range of values of δ and K. The observation following (3.14) above implies that t^* must decrease with δ, and this emerges strongly from the calculations. Rather less obviously, it also emerges that t^* decreases with

K; that is, increasing the marginal rate is less likely to reduce poverty the higher is the average income of the poor relative to the basic benefit. The intuition here is that when K is low the pre-tax earnings of the poor are so insubstantial that reducing the marginal rate has little impact on their net incomes while raising the poll subsidy has a considerable one. Clearly the critical tax rate varies widely across the circumstances shown in the Table. Bearing in mind, however, that the tax system is not being required to raise any revenue, the rates that emerge tend perhaps to be rather higher than might have been anticipated: even when the poor receive, on average, twice the basic benefit and δ is as high as 0.6, poverty alleviation points to a marginal tax rate of nearly 20%. Of course it must be emphasised that these numbers - and others to follow - are no more than suggestive, relating as they do to a particular choice of α and a very special preference structure.

It remains to consider the second approach, in which poverty is defined not in terms of the income that households <u>do</u> receive but that which they <u>could</u> receive by working standard hours. The relevant income concept is then that of (3.2) above, and the object of policy becomes that of minimising

$$P^L_\alpha = \int_0^\omega \left[\frac{z-G-(1-t)wL}{z}\right]^\alpha f(w)dw, \tag{3.18}$$

subject again to the revenue constraint (3.6). (Typically of course both the poverty line wage ω and the poverty line income z will now differ from their counterparts in (3.5); to save notation, this is not made explicit). Differentiating (3.18) for the effect of increasing the marginal rate one now finds

$$\frac{dP^L_\alpha}{dt} = \int_0^\omega J[z,y_L(w)]\left[\frac{dG}{dt} - wL\right]f(w)dw. \tag{3.19}$$

This is identical in form to the corresponding expression (3.8) for the received income approach with inelastic labour supply; the number of hours that the government considers 'reasonable', L, appears in the same way as did the actual hours, h^*. What is different, of course, is that dG/dt is here typically not a constant; labour supply responses, while no longer directly affecting the minimand, retain importance through their implications for revenue.

Taking again the special case of Cobb-Douglas preferences, further manipulations now yield

$$\left\{\frac{(1-t)z}{\alpha}\right\}\frac{dP^L_\alpha}{dt} = (P^L_{\alpha-1} - P^L_\alpha)z - \frac{\sigma(\delta,t)}{(1-\delta)}\left\{\frac{GP^L_{\alpha-1}}{t}\right\}. \tag{3.20}$$

Comparing this with the analogous expression (3.14) for the received income approach, it is clear that - other things being equal (including the initial values of the poverty indices for α and $\alpha-1$) - a revenue-neutral increase in the marginal tax rate is now more likely to reduce poverty. For that under the received income approach labour supply responses are likely to diminish the attractions of financing a higher poll subsidy by raising the marginal tax rate: so long as leisure is normal in demand and hours worked increasing in the marginal wage (as in the Cobb-Douglas case), both of these tax changes reduce labour supply, tending on that account to reduce the incomes received by the poor and so to increase their measured poverty. When poverty is gauged in terms of standard income, however, these considerations are absent: since the revenue implications are unchanged, increasing the marginal tax rate is therefore more likely to prove beneficial.

Further simplification can again be achieved for the case $\alpha = 1$. Arguing as before, and denoting by K^L the ratio of the mean standard income of the poor to the poll subsidy, the condition

Table 3.1: The critical marginal tax rate t^*

	Ratio of mean received income of poor to poll subsidy (K)		
δ	1.1	1.5	2.0
0.2	.57	.46	.36
0.4	.42	.33	.26
0.6	.26	.23	.18

Note: Equations (3.15) and (3.17) in the text lead to a quadratic equation with solutions, in obvious notation,

$$t^+, t^- = \{K + 1-\delta \pm [(K + 1-\delta)^2 - 4K(1-\delta)^2]^{½}\}/2(1-\delta)K.$$

With $K > 0$ and $\delta \in (0,1)$, it is readily shown that $t^+ > 1$. The critical value t^* is therefore t^-, and this is the figure reported.

$$t^L(1-\delta)K^L = \sigma(\delta,t^L) \tag{3.21}$$

then defines a critical marginal tax rate $t^L \epsilon (0,1)$ such that a small revenue-neutral increase in the marginal rate reduces (increases) poverty if and only if t is initially below (above) t^L. Some calculations are reported in Table 3.2. Interestingly, the numbers that emerge are not only uniformly higher than those in Table 3.1 but also considerably less sensitive to the strength of labour supply responses.

Table 3.2: The critical marginal tax rate t^L

	Ratio of mean standard income of poor to poll subsidy (K^L)		
δ	1.1	1.5	2.0
0.2	.66	.54	.43
0.4	.58	.48	.40
0.6	.54	.45	.37

Note: By the same arguments as in the Note to table 3.1, t^L is calculated by using $(1-\delta)K^L$ instead of K in the formula given there.

4. TAXATION AND TARGETING BY CONTINGENCIES

It is now widely recognised that one way of easing the tension between the provision of a decent level of support and the avoidance of high marginal tax rates may be by using non-income ('contingent' or 'categorical') information to impose different tax-benefit schedules on different groups of the population. This idea underlies, for instance, both the modified Social Dividend scheme of the Meade Committee (1978) and the more thorough-going proposals of Dilnot, Kay and Morris (1984). At a more formal level, the potential advantages of 'tagging' have also been emphasised by Akerlof (1978). And indeed it is obvious that the British social security system has long made considerable use of contingent information. No less obvious is the general principle that the enlightened use of such information can only improve matters (assuming, heroically, that its collection and exploitation are costless). What is not clear, however, is the precise form that, from the perspective of poverty alleviation, an enlightened usage would take. How might one decide, for instance, whether poverty in the UK will be reduced by the policy of retargeting resources away from the elderly and towards low paid families with children? More generally, how should one design the tax structures to be imposed on distinct groups in order to have the maximum impact on aggregate poverty? That is the issue explored in this section.

Suppose then that the population can be divided into two mutually exclusive and exhaustive groups, A and B, to which distinct linear tax systems may be applied. The underlying contingencies are assumed to be absolute, so that households are unable to switch between groups. Denoting by $\theta \in (0,1)$ the proportion of the population in group A, the results of Foster, Greer and Thorbecke (1984) imply that the P_α measure of aggregate poverty now decomposes as

$$P_\alpha = \theta P_\alpha^A + (1-\theta)P_\alpha^B, \qquad (4.1)$$

where P_α^i is the corresponding index of poverty within group i. For brevity, attention will be restricted in this section to the case in which poverty is defined in terms of received income and preferences are Cobb-Douglas. Assuming these preferences to be identical within groups but perhaps different between them, the relevant contingency-specific poverty indices are then, in obvious notation,

$$P_\alpha^i = \int_0^{\omega_i} \left[\frac{z_i - G_i - (1-t_i) w h^i[1-t_i)w, G_i]}{z_i} \right]^\alpha f_i(w) dw, \quad i = A,B \quad (4.2)$$

where

$$h^i[(1-t)w, G] = (1-\delta_i) - \frac{\delta_i G}{(1-t)w} \quad , \quad \delta_i \epsilon [0,1] \quad (4.3)$$

(and note that the poverty lines of the two groups may differ).

Using (4.2) and (4.3) in (4.1) gives the minimand for a policy of poverty alleviation; the revenue constraint now takes the form

$$\theta \left\{ t_A \int_0^\infty w h^A f_A(w) dw - G_A \right\} + (1-\theta) \left\{ t_B \int_0^\infty w h^B f_B(w) dw - G_B \right\} = R, \quad (4.4)$$

with the h^i given by (4.3).

The trade-offs to be faced are clearly now much more complex than in the homogeneous population case. It is helpful to begin by considering just one: that between the poll subsidies paid to the two groups. Holding marginal tax rates constant at their initial (arbitrary) levels, under what circumstances would aggregate poverty be reduced by cutting the poll subsidy given to one group in order to finance an increase in that paid to the other? From (4.1)-(4.3), perturbing the poll subsidies in such a way causes aggregate poverty to change by

$$dP_\alpha = -\theta \left(\frac{\alpha(1-\delta_A)}{z_A} \right) P_{\alpha-1}^A dG_A - (1-\theta) \left(\frac{\alpha(1-\delta_B)}{z_B} \right) P_{\alpha-1}^B dG_B \quad (4.5)$$

while revenue-neutrality requires, from (4.4) and (4.2), that

$$-\theta\left\{\frac{(1-\delta_A)}{\sigma(\delta_A,t_A)}\right\} dG_A - (1-\theta)\left\{\frac{(1-\delta_B)}{\sigma(\delta_B,t_B)}\right\} dG_B = 0 \qquad (4.6)$$

where $\sigma(\delta,t)$ is as in (3.15) above. Combining (4.5) and (4.6) gives

$$dP_\alpha = -\left\{\frac{\alpha(1-\theta)(1-\delta_A)}{\sigma(\delta_A,t_A)}\right\}\left[\frac{\sigma(\delta_A,t_A)P^A_{\alpha-1}}{z_A} - \frac{\sigma(\delta_B,t_B)P^B_{\alpha-1}}{z_B}\right] dG_A. \qquad (4.7)$$

Thus a retargeting of resources away from group B and towards group A - in the sense of a small reduction in the poll subsidy to the former accompanied by a balanced-budget increase in that to the latter - will reduce aggregate poverty if and only if

$$\frac{\sigma(\delta_A,t_A)P^A_{\alpha-1}}{z_A} > \frac{\sigma(\delta_B,t_B)P^B_{\alpha-1}}{z_B} . \qquad (4.8)$$

This reduces to the simple condition

$$P^A_{\alpha-1} > P^B_{\alpha-1} \qquad (4.9)$$

if the two groups have the same poverty line and either $\delta_A = \delta_B = 0$, so that labour supply is perfectly inelastic (as in Kanbur (1986)), or $\delta_A = \delta_B$ and $t_A = t_B$, so that the two groups have the same preferences and face the same marginal tax rate (in which case differential treatment may nevertheless be desirable as a response to different wage distributions).

The appearance in these conditions of the terms $P^i_{\alpha-1}$ emphasises that the reduction of aggregate poverty measured in some particular way is typically not best pursued by redirecting resources towards whichever group is poorest in terms of that same measure. What matters, of course, if the marginal effect on the measure of interest. The structure of the P_α index is such that the implied rule takes an especially simple form: to minimise the aggregate index for some specific choice of α, look first at the within-group indices for $\alpha-1$. Suppose for instance that we have chosen $\alpha = 1$ and that the poverty lines of the two groups are the same. Recalling (2.3), our objective is then simply to maximise the net income of the poor. Imagine now that we have some fixed sum to spend on increasing the poll subsidy to one group or the other, and assume for simplicity both that labour supply is inelastic and that the groups are of the same size. Which group should we favour? The disadvantage of having to spend this money as a poll subsidy is that some of it will be wasted on the non-poor; giving it to group i, the proportion of our fixed sum that will reach the poor is just the proportion of that group which is in poverty. To achieve the largest possible increase in the total income of the poor, we should therefore allocate the funds to whichever group has the higher headcount ratio. Recalling (2.2), we thus arrive at the comparison implied by (4.9): that between P_0^A and P_0^B.

Returning to the general case in which labour supply may vary and poverty lines differ, it is immediate from (4.8) that retargeting towards group A is more likely to be desirable, <u>cet. par.</u>, the higher is $\sigma(\delta_A, t_A)$ and the lower is z_A. As already noted, $\sigma(\delta, t)$ is decreasing in both δ and t. Thus group A is more likely to be favoured the less responsive is its labour supply behaviour and the lower is the marginal tax rate it faces. The intuition here is straightforward. When δ_A is relatively low the income effect of increasing the poll subsidy to group A - which points towards a reduction in net income, dampening the beneficial impact on poverty - is relatively weak; conversely a high δ_B indicates a relatively powerful income effect acting to mitigate the impact of reducing the poll subsidy to group B. And when t_A is relatively low so too is the revenue cost of the reduction in the hours worked by members of group A as a result of their receiving a higher poll subsidy; conversely a high t_B is helpful in recouping revenue from the increased labour supply in group B. The reason why group A is more likely to be favoured the lower its poverty line is simpler still: the smaller is z_A the larger is the effect of a unit increase of income on the proportionate poverty gaps used to calculate P_α.

Consider, by way of example, the inferences that might be drawn from the UK statistics shown in Table 4.1. Suppose first that labour supply is completely inelastic in both groups and the two poverty lines the same, so that the condition in (4.9) applies. Then since both P_0^i and P_1^i are higher for pensioner couples, reduction of aggregate poverty measured with either $\alpha = 1$ or $\alpha = 2$ calls for a retargeting of lump sum payments <u>towards</u> this group and away from the non-pensioner families. Nor is this conclusion plausibly overturned by allowing for different poverty lines or the operation of incentive effects. Since the Supplementary Benefit levels underlying the figures in Table 4.1 were higher for the non-pensioners than the pensioner, (4.8) implies that the first of these considerations unambiguously favours retargeting towards the pensioner group. And it is equally clear that, under the natural approximation of $\delta_i = 0$ for the pensioners, any responsiveness of labour supply amongst the non-retired -however small - only strengthens still further the case for retargeting away from them. Suppose for example that the two groups both face a marginal tax rate of 0.4, that $\delta_i = 0$ for the pensioners and $\delta_i = .25$ the non-pensioners; measuring aggregate poverty with $\alpha = 1$ (and even ignoring the difference in poverty lines), the headcount ratio of 11.1% for the pensioners implies that retargeting away from this group would only be justified if the proportion of the non-pensioners in poverty were not 2.9 per cent but more than 17.3% per cent.

So far we have looked only at the balance between the poll subsidies paid to the two groups, taking as given the marginal tax rates that they face. Turning now to the full optimisation problem, the necessary conditions for poverty minimisation (with $t_i \neq 1$, $i = A,B$) can be written as

Table 4.1: Poverty indices for 1983

	P_{α}^{i}	
	$\alpha = 0$	$\alpha = 1$
Pensioner couples	.111	.010
Non-pensioner couples with two children	.029	.006

Source: Taken from Table 3.2 in Morris and Preston (1986), where further details may be found. Calculated from Family Expenditure Survey data, in terms of 'normal net income', and with poverty lines set at Supplementary Benefit levels.

$$G_i: \qquad P^i_{\alpha-1} - \left(\frac{\lambda z_i}{\alpha}\right)\frac{1}{\sigma(\delta_i,t_i)} = 0 \quad , \quad i = A,B \tag{4.10}$$

$$t_i: \qquad (P^i_{\alpha-1} - P^i_{\alpha})z_i - (1-\delta_i)G_iP^i_{\alpha-1}$$

$$- \left(\frac{\lambda z_i}{\alpha}\right)\left\{(1-t_i)(1-\delta_i)\bar{w}_i - \frac{\delta_i G_i}{(1-t_i)}\right\} = 0, \; i = A,B \tag{4.11}$$

where λ is the Lagrange multiplier attached to the revenue constraint (4.4) and $\bar{w}_i$ denotes the mean wage of group i. From (4.10) one finds that an optimum

$$\frac{P^A_{\alpha-1}}{P^B_{\alpha-1}} = \frac{\sigma(\delta_B,t_B)z_A}{\sigma(\delta_A,t_A)z_B} \quad , \tag{4.12}$$

as indeed was evident from (4.7) above. Substituting from (4.10) into the second term of (4.11) and recalling (3.13), optimal deployment of both sets of instruments yields the further simple rule

$$\frac{P^A_{\alpha-1} - P^A_{\alpha}}{P^B_{\alpha-1} - P^B_{\alpha}} = \frac{y^A(\bar{w}_A)}{y^B(\bar{w}_B)} \tag{4.13}$$

where $y^i(w)$ denotes the net income of a household in group i facing gross wage w. And since Cobb-Douglas preferences imply that net income is linear in the wage rate (again recalling (3.13)) the right hand side of (4.13) is just the ratio of the mean net incomes of the two groups. Suppose for instance that we have chosen $\alpha = 1$, and consider once more the figures in Table 4.1: (4.13) implies that, in a world of linear taxes, the situation they reveal could be optimal only if the net incomes of pensioner couples were on average more than four times larger than those of the non-pensioners. Pursuing the case $\alpha = 1$, the condition in (4.13) reduces further to

$$\frac{S^A}{S^B} = \frac{z_A}{z_B} \tag{4.14}$$

where

$$S^i = \int_0^{\omega_i} y^i(w) f_i(w)\,dw \left[\int_0^{\infty} y^i(w) f_i(w)\,dw\right]^{-1}. \qquad (4.15)$$

The minimisation of aggregate poverty P_1 thus requires that the within-group income shares of the poor stand in the same ratio as their poverty lines. Such information is typically not produced in empirical work. Yet it is crucial for ascertaining the optimality or otherwise of current retargeting strategies. It is our intention to conduct such detailed empirical analysis in future work.

5. CONCLUDING REMARKS

The object of this paper has been to develop a framework for analysing the efficacy of alternative tax-benefit structures in terms of the commonly stated objective of poverty reduction. The analysis has led to some appealingly simple rules. In addition, preliminary and illustrative calculations have been given that suggest a prima facie case against the kind of retargeting currently being attempted in the UK, away from the old and towards the young. Of course, even if one puts aside both the restrictiveness of the assumptions from which they derive and the narrowness of the single objective - that of poverty minimisation - to which they relate, rules of this kind cannot be expected to eliminate all controversy. What they can do, however, is identify the critical issues of disagreement or doubt. It may be, for instance, that a policy of retargetting away from the retired and towards low-paid working families appears desirable in terms of aggregate poverty alleviation only at high degrees of poverty aversion or when using equivalence scales very different from those implicit in official benefit rates. As in other areas of optimal tax theory, the essential purpose is not to close debate but to inform it.

While they may offer administrative advantages, there is no compelling reason to confine attention to the linear tax structures studied here. The use of non-linear income taxation for poverty relief is considered in Kanbur and Keen (1988), where it is shown that minimisation of a poverty index defined in terms of received income requires that the poorest of the poor (assuming that they work) face a strictly negative marginal tax rate. One implication of this is worth noting here. For it is well-known that maximisation of any orthodox social welfare function requires that the marginal rate of tax be everywhere non-negative

(Mirrlees (1971)), so that such a situation would then be precluded. The consequences of pursuing a non-welfaristic programme of poverty alleviation are thus not merely quantitative; the policy implications may be qualitatively different to any that could be generated by a welfaristic approach.

REFERENCES

Akerlof, G. A. (1978) 'The economics of "tagging" as applied to the optimal income tax, welfare programs and manpower planning', American Economic Review 68, 8-19.

Atkinson, A. B. (1987), 'On the measurement of poverty', Econometrica 55, 749-64.

Besley, T. and S. M. R. Kanbur (1987), 'Food subsidies and poverty alleviation', forthcoming in the Economic Journal.

Dilnot, A. W., Kay, J. A. and C. N. Morris (1984), The Reform of Social Security, Oxford University Press, Oxford.

Dixit, A. K. and A. Sandmo (1977), 'Some simplified formulae for optimal income taxation', Scandinavian Journal of Economics 79, 417-423.

Foster, J., Greer, J. and E. Thorbecke (1984), 'A class of decomposable poverty measures', Econometrica 52, 761-766.

Kanbur, S. M. R. (1986), 'Budgetary rules for poverty alleviation', Institute for International Economic Studies, University of Stockholm, Discussion Paper No. 363.

Kanbur, S. M. R. (1987), 'Transfers, targeting and poverty', Economic Policy 4, 112-136 and 141-147.

Kanbur, S. M. R. and M. J. Keen (1988), 'Non-linear income taxation for poverty alleviation', mimeo.

King, M. A. (1983), 'Welfare analysis of tax reforms using household data', Journal of Public Economics 21, 183-214.

Meade Committee (1978), The Structure and Reform of Direct Taxation, Allen Allen and Unwin, London.

Mirrlees, J. A. (1971), 'An exploration in the theory of optimum income taxation', Review of Economic Studies 38, 175-208.

Morris, C. N. and I. Preston (1986), 'Inequality, poverty and the redistribution of income', Bulletin of Economic Research 38, 277-344.

Sen, A. K. (1976), 'Poverty: An ordinal approach to measurement', Econometrica 44, 219-231.

Sen, A. K. (1985), Commodities and Capabilities, North-Holland, Amsterdam.

Stern, N. H. (1987), Discussion of Kanbur (1987), Economic Policy 4, 136-140.

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