QUANTIFYING LONG RUN AGRICULTURAL RISKS AND EVALUATING FARMER RESPONSES TO RISK

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Elliptical Symmetry and Mean Variance Portfolio Choice

by

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Abstract

Results of Chamberlain and Meyer are combined to extend Meyer's location-scale condition to portfolio choice models where the distribution of returns is elliptically symmetric. This extension implies that mean-variance choice is consistent with expected utility maximizing choice for such models. All expected utility maximizing portfolios lie on the mean-variance efficiency frontier which can be generated with quadratic risk programming. A test for elliptical symmetry is also described. This test enables one to determine whether a given set of portfolio data satisfies the conditions which make mean-variance choice consistent with expected utility maximization.

Elliptical Symmetry and Mean Variance Portfolio Choice

Quadratic risk programming and mean variance models of decision making under uncertainty have been fruitful tools in economic research on behavior under uncertainty. In recent years the use of these methods has diminished because of criticisms of the restrictive assumptions (normality of returns or quadratic utility function) under which such methods are consistent behavioral models of rational choice under uncertainty. Meyer showed that such consistency is present in a wider class of models with univariate risk than was previously believed to be available. Unfortunately many interesting problems contain multivariate risks, and Meyer's results do not directly apply to such cases. In this paper we show that Meyer's results can be generalized to a large class of models with multivariate risk. The distinguishing characteristic of this class of models is that the distribution of the multivariate risks be elliptically symmetric. In the following sections we describe elliptical symmetry, show how Meyer's results can be generalized, and we present a test for elliptical symmetry that is easy to implement.

Elliptical Symmetry

The family of multivariate elliptically symmetric distributions is a large family of distributions whose members include the multivariate normal, multivariate t, Pearson Type II, Pearson Type VII, and certain mixtures of normal distributions. The distinguishing characteristic of members of the family is that their contours of equal density have the same elliptical shape. The family contains distributions that are long-tailed and short-tailed relative to the normal, and the support of distributions in the family can be infinite or finite.

Assuming that the random vector X has a density function elliptical symmetry of X can be defined by: the m x 1 random vector X follows an elliptically symmetric distribution with parameters μ (m x 1) and V (m x m), written $X \sim E(\mu, V)$, if its density function is of the form:

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$$f(X) = C\,(\det V)^{-1/2}\, g\,[(X-\mu)'V^{-1}(X-\mu)] \tag{1.1}$$

for some one dimensional density g. The matrix V is positive definite.

The characteristic function, $\phi(t)$, of elliptically symmetric random variables has the form:

$$\phi(t) = e^{i\mu' t}\,\psi(t'Vt) \tag{1.2}$$

And, provided they exist, $E(X) = \mu$ and $Cov(x) = \alpha V$, where $\alpha = -2\psi'(0)$. It follows from this that all distributions that are $E_m(\mu, V)$ have the same mean vector, μ, and correlation matrix, $P = (\rho_{ij})$.

All marginal and conditional distributions of elliptically symmetric distributions are elliptical symmetric (Muirhead, pp.34-36). Univariate elliptically symmetric distributions are symmetric. Any linear combination, $Y = b'Z$, of any subset, Z, of of the components of an elliptical random vector is elliptically symmetric (Kelker). And, if $X \sim E_m(\mu, V)$ and all of the elements of X are independent, then X is normal.

An important subset of the family of elliptically symmetric distributions is the family of spherically symmetric distributions. This subset of distributions is characterized by mean vectors of zero, $\mu = 0$, and scalar covariance matrices, $V = \sigma^2 I$. Note that the result about independence implying normality does not imply that spherically symmetric distributions are normal because independence is not the same as absence of correlation. This family of distributions is called spherically symmetric because for all members of this family contours of equal density are spheres centered at the origin.

Clearly, if $X \sim E_m(\mu, V)$ and L is an m x m lower triangular Cholesky factor of V (ie. $V = L L'$), then $Y = L^{-1}(X - \mu)$ is $E_m(0, I)$ or spherically symmetric. If Y has an m-variate spherically symmetric distribution with $P(Y=0) = 0$ and $r = ||Y|| = (Y'Y)^{-1/2}$, $T = Y/||Y||$, then T is uniformly distributed on the m dimensional unit hypersphere, and r and T are independent (Muirhead, p.38). This result is central to the test for elliptical symmetry that we propose and investigate.

Elliptical Symmetry and the Location Scale Condition

Meyer and Sinn have shown that, in models with a univariate source of risk, expected utility maximization is entirely consistent with mean-standard deviation choice if decisions only change the location and scale of the risk. And Meyer has derived a rich set of comparative static results for such models. All of Meyer and Sinn's results can be applied to the portfolio choice problem with elliptically symmetric returns. Direct application of Meyer and Sinn's results can take place because changes in the composition of a portfolio only change the location and scale of total returns.

This claim contradicts Sinn who argues that the only distribution class, with finite variance, that satisfies the requirement that the distribution of final wealth be invariant to changes in the portfolio structure is the class of normal distributions. In fact all members of the elliptical family share with the normal distribution the reproduction property that Sinn refers to in justifying his argument. In other words any linear combination of individual components of an elliptical random vector will have a distribution in the same linear class. A particularly important aspect of this generalization is the fact that there are elliptical distributions with bounded support, so that the arguments against the normal distribution because it allows unbounded negative returns do not apply to all members of the elliptical family.

To see that changes in the composition of a portfolio whose individual components come from an elliptically symmetric distribution only change the location and scale of total returns consider the portfolio choice problem where the n-vector X is the vector of returns on individual elements of the portfolio, and X has an elliptically symmetric distribution with mean vector μ and covariance matrix Σ. The vector of weights on the elements of the portfolio is b. Thus total return on the portfolio is $b'X$. Since X is elliptically symmetric we know that $Y = b'X$ is also elliptically symmetric, which means that the centered variable $Y - b'\mu$ is symmetric about the origin. The mean of Y, which will be denoted by ξ, is $b'\mu$, and the variance of Y, denoted by σ^2, is $b'\Sigma b$.

Let $Z = (Y - \xi)/\sigma$; Z is a symmetric random variable with mean 0 and variance 1, and $Y = \xi + \sigma Z$. Changes in the vector b will not change the random variable Z. Thus the portfolio choice problem can be written as:

$$\underset{b}{\text{maximize}}\ E[U(\xi + \sigma Z) \mid \text{subject to } \xi = b'\mu \text{ and } \sigma^2 = b'\Sigma b \qquad (2.1)$$

With the portfolio choice problem formulated in this manner it can be seen that changes in the composition of the portfolio will only change the location, ξ, or the scale, σ, of the risk. This is a direct result of the fact that Y is a univariate elliptically symmetric random variable. Applying Chamberlain's theorem on the consistency between expected utility maximization and mean-variance representation of preferences, if X is not elliptically symmetric then changes in the weights on the elements of the portfolio, b, will change moments of the distribution of Y, other than the mean and the variance. Thus , the representation (3.2) is invalid if returns are not elliptically symmetric because the random variable Z is not invariant to changes in b.

The recognition that Meyer and Sinn's results apply to the portfolio choice problem with elliptically symmetric returns makes available a rich set of comparative static results. Following Tobin, the portfolio choice problem can be analyzed by examining preferences in mean-standard deviation space.

The first properties of preferences that can be derived are that expected utility increases in mean returns, ξ, if $U'(\xi + \sigma Z) \geq 0$ for all $(\xi + \sigma Z)$, and that expected utility decreases in the standard deviation of returns, σ, if $U''(\xi + \sigma Z) \leq 0$ for all $(\xi + \sigma Z)$. Together these two properties can be used to sign the slope of indifference curves in (σ,ξ) space for risk averse agents. The slope of an indifference curve $S(\sigma,\xi)$ in (σ,ξ) space is:

$$S(\sigma,\xi) = -(\partial E\, U[(\xi + \sigma Z)]/\partial\sigma)/(\partial E[U(\xi + \sigma Z)]/\partial\xi. \qquad (2.2)$$

So $S(\sigma,\xi) \geq 0$ for all $(\xi + \sigma Z)$ if $U'(\xi + \sigma Z) \geq 0$ and $U''(\xi + \sigma Z) \leq 0$.

Thus, in mean-standard deviation space preferences can be represented by positively sloped indifference curves for risk averse investors, and optimal portfolios can be determined by finding tangencies between these indifference curves and the collection of mean-variance efficient portfolios. The optimal choices found by means of these arguments will be entirely consistent with optimal choices according to expected utility analysis.

From second degree stochastic dominance arguments it is known that if risky return A is preferred to risky return B by all risk averse individuals then $E[A] = E[B]$, and $Var[A] < Var[B]$. But the converse is not true in general. However, if the returns are from portfolios composed from elliptically symmetric returns the converse is true. If $E[A] = E[B]$ and $Var[A] < Var[B]$ then A is preferred to B by all risk averse investors. Thus, the E-V efficient frontier is the complete set of alternatives from which risk averse investors will make their choices.

Further, the less obvious properties of preferences in mean-standard deviation space derived by Meyer can be applied to the portfolio choice problem. First, $\partial S(\sigma,\xi)/\partial\xi \leq (=,\geq)\ 0$ for ξ and $\sigma > 0$ if and only if the utility function exhibits decreasing (constant, increasing) absolute risk aversion for all $\xi + \sigma Z$. Second, $\partial S(t\sigma,t\xi)/\partial t \geq (=,\leq)\ 0$ if and only if the utility function displays increasing (constant, decreasing) relative risk aversion. And third, $S_1(\sigma,\xi) \geq S_2(\sigma,\xi)$ for all (σ,ξ) if and only if $U_1(\xi + \sigma Z)$ is more risk averse than $U_2(\xi + \sigma Z)$ for all $(\xi + \sigma Z)$.

These properties provide information about the nature of optimal portfolios without exact information on utility functions or risk aversion coefficients. For example, given decreasing absolute risk aversion a ceteris paribus increase in mean returns will lead to an optimal portfolio with a higher mean and a higher variance. Or, the higher the degree of risk aversion the lower the mean and variance in the optimal portfolio. In essence, both of these statements mean that decision makers with lower degrees of risk aversion will choose portfolios futher from the minumum variance portfolio than decision makers with higher degrees of risk aversion.

To summarize, if portfolio returns are elliptically symmetric then expected utility maximizing investors will restrict their choice to portfolios on the mean-variance efficiency frontier. And individuals with lower degrees of risk aversion will choose portolios on the frontier that are further from the minimum variance portfolio than choices made by individuals with higher degrees of risk aversion.

Of course, as Freund showed the assumptions of constant absolute risk aversion and normal returns produce the more specific result of identifying the optimal portfolio. So the additional analytic power provided by the results presented here occur in cases when returns are elliptical but not normal. These results should have wide applicability because there are many elliptical distributions that are not normal.

For example, the use of the normal distribution in portfolio analysis is frequently criticized because it allows infinitely large negative or positive returns on investments. Elliptical distributions provide a way to maintain the analytic convenience of the normal distribution while eliminating this criticism because there are many elliptical distributions with bounded support. An example of such a distribution is presented below.

The Nearest Neighbor Test for Ellipticity

Let F = the collection of all elliptically symmetric distributions which are absolutely continuous. The hypothesis to be tested is:

$$H_0 : f \in F \quad \text{against} \quad H_1 : f \notin F. \tag{3.1}$$

Let $\{X_1, X_2, \ldots, X_n\}$ be a sample of observations on an m-dimensional random vector. Let $\hat{\mu},\hat{\Sigma}$ be estimates of the mean vector and covariance matrix of the data. If Σ is positive definite it has a Cholesky factorization $\hat{\Sigma} = LL'$, where L is lower triangular. Using L and μ to transform the sample into standardized deviations from means, $Y_i = L^{-1} (X_i - \mu)$, $i=1,2,\ldots,n$. If the observed data X_i comes from an elliptically symmetric distribution, then Y_i is a sample from a spherically symmetric distribution.

From the properties of spherically symmetric distributions it is known that normalized values of Y_i, $Z_i = Y_i/||Y_i||$, are uniformly distributed on the unit hypersphere and are independent of the random variables $r_i = ||Y_i||$. The inverse of this transformation provides an easily implemented method for generating samples from a spherically symmetric distribution. First generate n variables, Z_i, uniformly distributed on the unit hypersphere; then, generate a random sample from some univariate distribution with nonnegative support to obtain the radius r_i, $i=1,2,\ldots,n$; then the spherically symmetric sample is obtained by setting $Y_i = r_i * Z_i$, $i=1,2,\ldots,n$.

Let $R_i = \text{rank}\,(||Y_i||)/n$. The nearest neighbor test for elliptical symmetry exploits the property that the pairs (R_i, Z_i) are approximately uniformly distributed on the product of $\{1/n, 2/n, \ldots, 1\}$ with the unit hypersphere. Uniformity means that the radial distance between nearest neighbors (nodes) that form the shortest path around this space should be similar for all pairs of nodes. This is in contrast to a case of a nonuniform density where nodes will tend to be clustered in groups that correspond to regions of high density. In this case, distances between nodes in a cluster will be small, but distances that must be traversed in moving from one cluster to the next cluster will be relatively larger. There will also tend to be isolated nodes in low density regions which will have large nearest neighbor distances.

Based on previous investigations, we propose the test statistic

$$V = -4\,\bar{D} + .5\,S^2 \tag{3.2}$$

where

$$\bar{D} = 1/n \sum_{i=1}^{n} D_i$$

$$S^2 = 1/n \sum_{i=1}^{n} (D_i - \bar{D})^2$$

and $D_1, D_2, \ldots, D_n$ are the nearest neighbor distances obtained from unique nondiagonal elements of a matrix S composed of elements $S = ||(R_i,Z_i) - (R_j,Z_j)||^2$. The null hypothesis is rejected for large values of V. It should be emphasized that we only consider $m=6$ and $n=21$. Other dimensions and sample sizes will probably requires different weightings of $\bar{D}$ and S^2.

Critical values for the test statistic are determined by simulating the null distribution of the test statistic. This simulation requires that a particular member of the null family, such as the multivariate t-distribution, be chosen to generate the null distribution. In order to investigate whether the null distribution is sensitive to the member of the null family that is chosen, the null distribution is calculated under 6 distributions that are in the null family.

Application to Portfolio Data

The observed portfolio return data that was tested consists of returns on two equity instruments: 1) the Standard and Poors 500 common stock index, and 2) a portfolio of small company stocks; returns on three debt instruments: 3) a municipal bonds index, 4) a certificate of deposit rate, 5) passbook account rate; and 6) returns to grain farming on 500-640 acre farms in Northern Illinois. This is considered to be a standard array of instruments that grain farmers can use to diversify their financial risk. There were 21 annual observations on each of these 6 returns. The data are given in Table 1. The sample mean vector and covariance matrix of the return data are reported in Table 2.

The data was centered and scaled with the estimated mean vector and the lower triangular Cholesky factor of the estimated covariance matrix, as described above. Then these sphered residuals were normalized to lie on the unit hypersphere by dividing each observation by its norm and its rank computed and normalized. The test statistic is the linear combination of mean and variance of the squared distances between nearest neighbors as described above.

The value of the test statistic from the data set is -.040616. The p-value of this statistic is 0.0000 under all of the null distributions. None of the Monte Carlo samples of size 1000 from the null distributions produced a test statistic value as large as this. Based on the upper confidence limit formula for small proportions we can say $p < .003$ with 95% confidence. The members of the null family that were used as null distributions and the differences in their null distributions are explained in the next section.

The p-value of approximately 0.000 is strong evidence that the data does not come from an elliptically symmetric distribution. This implies that the optimal portfolio cannot be characterized entirely in terms of its mean and variance. The mean variance-efficient set does not correspond to the expected utility efficient set of portfolios that can be constructed from this return data. Identification of the the expected utility efficient set requires more information about the true distribution of the return data. The result of the nearest neighbor test is a strong reason to rule out the possibility that the true distribution is elliptically symmetric. Unfortunately, the task of identifying a reasonable candidate for the true distribution of the data is difficult because the set of tractable, parameterized, non-elliptical, multivariate distributions is small. This suggests that methods which do not require knowledge of the exact distribution of the data such as stochastic dominance based on empirical cdf's should be used to try to identify optimal portfolios.

Monte Carlo Results

All Monte Carlo results are for a sample size of 21 and a 6 dimensional random vector because these were the dimensions of the data in the portfolio analysis problem which prompted the development of this test for elliptical symmetry. The sample size should not be a limitation because annual data on portfolio returns is likely to contain around this many observations. And the performance of the test is likely to be better for larger data sets because there will be more observations to distinguish uniform from non-uniform distributions over the unit hypersphere. In this sense the results of these Monte Carlo experiments can be considered to provide a lower bound on the empirical p-values, and a lower bound on power. The test statistic is likely to perform better with larger samples. The test statistic is likley to perform more poorly if random vectors of higher dimensions are analyzed with the same small number of observations.

In the Monte Carlo results the p-value of the test statistic is approximated by the proportion of Monte Carlo samples from the null distribution whose test statistic values are greater than that of the data set being analyzed. In the experiments on the power of the test, power is approximated by the proportion of samples generated from a non-ellipitcal distribution that are rejected at a given significance level. The test is designed to be insensitive to the type of elliptical distribution which is used to generate the null distribution. Therefore 6 different elliptical distributions have been used to examine the robustness of the null hypothesis to differences in the members of the null family used to generate the null distribution.

Each null distribution was generated from a Monte Carlo sample of size 1000. The null distributions were generated in the manner discussed above. First, variables uniformly distributed on the unit hypersphere were obtained by generating independent standard normal random variables and normalizing the resulting random vectors to unit length. Then, spherically symmetric random variables were obtained by multiplying the random vectors obtained in the first step by random variables obtained from distributions with non-negative supports. Six different distributions were used for the distribution of r^2, the square of the radius. If r^2 is distributed as a chi-squared random variable, then the resulting spherical distribution is normal. If r^2 is distributed as a Beta(m/2,p+1) the resulting distribution is a multivariate Pearson Type II. If r2 is distributed as U/(U-1) where U follows a Beta(m/2,p-m/2) a multivariate Pearson type VII or multivariate t distribution is obtained. Finally call the distribution that results when r^2 follows a lognormal(μ,σ^2), "multivariate lognormal", and the distribution that results when r^2 follows an exponential(θ) distribution, "multivariate exponential".

The results of the examination of the p-value of the test statistic are reported in Table 3 . The first row of this table reports the .05 critical value of the test statistic under each of the null distributions. Rows two through seven contain the proportion of rejections for each critical value under each null distribution. By construction the diagonal elements of rows two to seven are .05. The off-diagonal elements can be used to examine the consistency of significance levels. In Table 3 the off-diagonal elements range from .022 to .075.

The multivariate t distribution with 5 degrees of freedom produces the critical value largest in absolute value, and the smallest percentage of true null hypotheses rejected. The multivariate t distribution with 8 degrees of freedom produces a slightly smaller critical value and larger percentage of true nulls rejected. The Pearson type II distribution produces the smallest critical values and the largest number of true nulls rejected. In general these results indicate an acceptable level of agreement between the null distributions, with the Pearson type II distribution exhibiting slightly more Type I error than the significance level.

To aid the interpretation of the results in Table 3, histograms of the null distributions are presented in figures 1 through 6. These histograms are constructed from the Monte Carlo simulation with 1000 observations. The similarity of the histograms of the null distribution is striking. Each of the null distributions has non-zero frequency on the interval -13 to -8, and each distribution appears to be fairly symmetric. The mode of each distribution lies in the interval -11 to -10, and the distributions appear to have similar dispersion. The similarity in the shape of the null distributions provides limited evidence that the null distribution is nearly independent of the member of the null family that is used to generate the null distribution. The robustness of the shape implies that inferences drawn from the hypothesis test should be nearly the same irrespective of which member of the null family is chosen to generate the null distribution.

Questions about power are investigated by constructing two sequences of alternative distributions. One sequence, called the "inflated normal", was constructed by distorting the normal distribution. These alternative distributions were constructed by multiplying coordinates of the normal distribution by 10 or 20 when the value of the coordinates were positive. This causes the distributions to be stretched in the positive orthant in the direction of the "inflated" coordinates. We also used inflation factors of 4 or 6 with 4 inflated coordinates. The other sequence was constructed from the multivariate Burr family of distributions with parameters ranging from 1 to .125 (Johnson).

The results of this study of the power of the test are presented in tables 4 and 5. The entries in both tables are the percentage of null hypotheses that are rejected at a significance level of .05. The results for the distorted normal distributions are contained in table 4. One sees that the power is generally quite good with one exception.

When two or more coordinates are inflated the power ranges from .309 to 1.00 with the majority of values concentrated from .93 to 1.00. The exception is the alternative distributions where only one coordinate is inflated. This is probably a reflection of the fact that the contours of equal probability are similar to ellipses for these distributions.

Table 5 contains results for the Burr family alternative. The power of the test against these alternatives ranges from .298 to .999. The alternative against which the test has the lowest power is the Burr with parameter equal to 1. The low power in this case is due to the fact equal probability contours are close to ellipses for this distribution. When the parameter increases to 2 the contours are indistinguishable from ellipses.

The general result of these power studies is that the nearest neighbor test has good power against alternatives with equal probability contours that differ from ellipses. It is likely that expected utility maximizing portfolio choices from distributions with contours close to ellipses are close to the mean-variance efficiency frontier, implying that the loss from failure to identify nonelliptical distributions close to ellipticity is small. Verification of this conjecture is left to future research.

Conclusions

Results derived by Meyer have been extended to multivariate random variables and portfolio choice problems. If a set of random variables are elliptically symmetric and they are combined in some affine manner then expected utility is solely a function of the mean and variance of returns. Risk averse decision makers will make choices from the mean-standard deviation efficiency frontier. Meyer's comparative statics concerning changes in mean, variance, and degree of risk aversion can be applied directly. A statistical test has been presented which is able to discriminate between elliptical and nonelliptical distributions. Application of this test and failure to reject ellipticity should provide a firm basis for the use of quadratic risk programming models to analyze optimal choice in the presence of uncertainty. Rejection of ellipticity is an indication that variance is an inadequate characterization of the risks that are relevant to a rational economic decision maker.

References

Agnew,R. "Counter-examples to an Assertion Conerning the Normal Distribution and a New Stochastic Price Fluctuation Model," Review of Economic Studies 38(1971): 381-383.

Beran,R. "Testing for Ellipsoidal Symmetry of a Multivariate Density,"The Annals of Statistics, 7(1979):150-162.

Borch,K. "A Note on Uncertainty and Indifference Curves," Review of Economic Studies 36(1969): 1-4.

Cass,D. and J. Stiglitz. "The Structure of Investor Preferences and Asset Returns, and Separability in Portfolio Theory: A Contribution to the Pure Theory of Mutual Funds," Jour. of Economic Theory 2(1970):122-160.

Chamberlin,G. "A Characterization of Distributions that Imply Mean Variance Utility Functions," Jour. of Economic Theory, 29(1983):185-201.

Eaton,M. and T. Kariya. "Robust Tests for Spherical Symmetry," The Annals of Statistics, 5(1977): 206-215.

Feldstein,M. "Mean Variance Analysis in the Theory of Liquidity Preference and Portfolio Selection," Review of Economic Studies, 36(1969): 5-12.

Kelker,D. "Distribution Theory of Spherical Distributions and a Location-Scale Parameter Generalization," Sankhya, ser.A, 32(1970):419-430.

Markowitz,H. Portfolio Selection (New York: Wiley), 1959.

Meyer,J. "Two Moment Decision Models and Expected Utility Maximization," American Economic Review 77(1987): 421-430.

Muirhead,R. Aspects of Multivariate Statistical Theory (New York: Wiley), 1982.

Romano,J. "A Bootstrap Revival of Some Nonparametric Distance Tests", Jour. of the American Statistical Association, 83(1988): 698-708.

Samuelson,P. "A General Proof that Diversification Pays," Jour. of Financial and Quantitative Analysis, 2(1967): 1-13.

Tobin,J. "Liquidity Preference as Behavior Towards Risk," Review of Economic

Table 1: Portfolio Data

X1	X2	X3	X4	X5	X6
11.13	22.80	23.57	-0.14	0.0	4.00
9.72	16.48	23.52	5.71	3.91	4.00
17.51	12.45	41.75	-2.52	4.35	4.00
19.18	-10.06	-7.01	-0.16	5.47	4.00
10.31	23.98	83.57	-4.04	5.02	4.00
7.78	11.06	35.97	-0.19	5.86	4.00
11.71	-8.50	-25.05	-14.56	7.77	4.00
3.70	4.01	-17.43	24.50	7.56	4.50
7.54	14.31	16.50	11.31	4.99	4.50
17.13	18.98	4.43	10.36	4.67	4.50
26.72	-14.66	-30.90	4.74	8.41	5.00
47.80	-26.47	-19.95	-12.32	10.27	5.00
29.49	37.20	57.82	8.59	6.43	5.00
33.07	23.84	57.38	27.18	5.27	5.00
39.02	-7.18	25.38	8.19	5.58	5.00
15.36	6.56	23.46	-1.92	8.25	5.00
18.27	18.44	23.46	0.72	11.22	5.25
11.71	32.42	43.46	-13.43	13.07	5.25
8.10	-4.91	13.88	3.72	15.91	5.25
-5.31	21.41	58.71	28.01	12.57	5.25
-6.58	22.51	39.67	20.28	9.07	5.25

X1: Returns on Farm Assets; X2: Standard and Poors 500; X3: Small Company Stocks
X4: Municipal Bonds; X5: Certificates of Deposit; X6: Passbook Savings.

All series are annual from 1963 to 1983.

Sources: X1: Illinois Farm Business-Farm Management Association;
X2-X4: Stocks, Bonds, Bills, and Inflation Ibbotson and Sinquefield (1984);
X5-X6: Federal Reserve Bulletin.

Table 2 - Sample Mean and Covariance of Portfolio Data

	X1	X2	X3	X4	X5	X6
			Mean			
	15.87	10.22	21.57	6.42	7.41	4.65
			Covariance			
X1	179.87	-91.16	-56.13	-78.57	-6.67	1.01
X2		277.33	380.25	79.56	-10.43	0.49
X3			764.67	63.62	-16.08	0.57
X4				267.44	6.14	2.84
X5					13.44	1.41
X6						0.28

X1: Return on Farm Assets; X2: Standard and Poors 500; X3: Small Company Stocks; X4: Municipal Bonds; X5: Certificates of Deposit; X6: Passbook Savings

Sources: Illinois Farm Business-Farm Managment Records; Ibbotson, Stocks, Bonds, Bills, and Inflation, 1984 Yearbook; Standard and Poors Security Index; Federal Reserve Bulletin.

Table 3

Consistency of Null Distributions

1000 Monte Carlo Samples

		Null Distribution					
		N1	N2	N3	N4	N5	N6
.05 Critical Value		-9.2633	-9.3277	-9.2415	-9.2727	-9.2072	-9.0622
	N1	.050	.045	.053	.048	.054	.070
Percent of	N2	.065	.050	.060	.057	.062	.075
Null Hypotheses	N3	.046	.041	.050	.045	.052	.067
Rejected	N4	.051	.045	.054	.050	.055	.070
	N5	.041	.036	.043	.038	.050	.060
	N6	.022	.024	.029	.028	.030	.050

N1: normal; N2: t with 5 degrees of freedom; N3: t with 8 d. of f.
N4: spherical lognormal; N5: spherical exponential;
N6: Pearson Type III.

Table 4

Power of Nearest Neighbor Test Against Alternatives

of Distorted Normal Distributions

			Null Distribution					
			N1	N2	N3	N4	N5	N6
Alternative Distribution	X1	: 10	.065	.073	.064	.066	.057	.043
	X1	: 20	.088	.095	.086	.088	.083	.059
	X1-X2	: 10	.447	.471	.437	.449	.428	.359
	X1-X2	: 20	.621	.652	.613	.626	.593	.511
	X1-X3	: 10	.959	.964	.957	.959	.955	.930
	X1-X3	: 20	.997	.998	.997	.998	.997	.994
	X1-X4	: 10	.999	1.000	.998	1.000	.998	.996
	X1-X4	: 20	1.000	1.000	1.000	1.000	1.000	1.000
	X1-X5	: 10	.999	.999	.999	.999	.999	.998
	X1-X5	: 20	.999	1.000	.999	1.000	.999	.999
	X1-X6	: 10	.381	.409	.372	.387	.357	.309
	X1-X6	: 20	.486	.506	.479	.487	.462	.417
	X1-X4	: 4	.453	.474	.441	.460	.426	.373
	X1-X4	: 6	.893	.901	.884	.895	.878	.847

N1: normal; N2: t with 5 degrees of freedom; N3: t with 8 d. of f.
N4: spherical lognormal; N5: spherical exponential;
N6: Pearson Type II.

Table 5

Power of Nearest Neighbor Test Against Alternatives of the Burr Distribution

Null Distribution

Alternative Distribution	N1	N2	N3	N4	N5	N6
$\alpha = 1.0$	.346	.365	.344	.349	.333	.298
$\alpha = 0.75$	.553	.567	.549	.555	.541	.507
$\alpha = 0.625$	.678	.696	.673	.680	.665	.624
$\alpha = 0.5$	.821	.838	.817	.823	.810	.778
$\alpha = 0.375$	.934	.941	.932	.936	.930	.918
$\alpha = 0.25$	.988	.991	.988	.989	.988	.984
$\alpha = 0.125$	.999	.999	.999	.999	.999	.999

N1: normal; N2: t with 5 degrees of freedom; N3: t with 8 degrees of freedom; N4: "spherical lognormal"; N5: "spherical exponential"; N6: Pearson Type II.

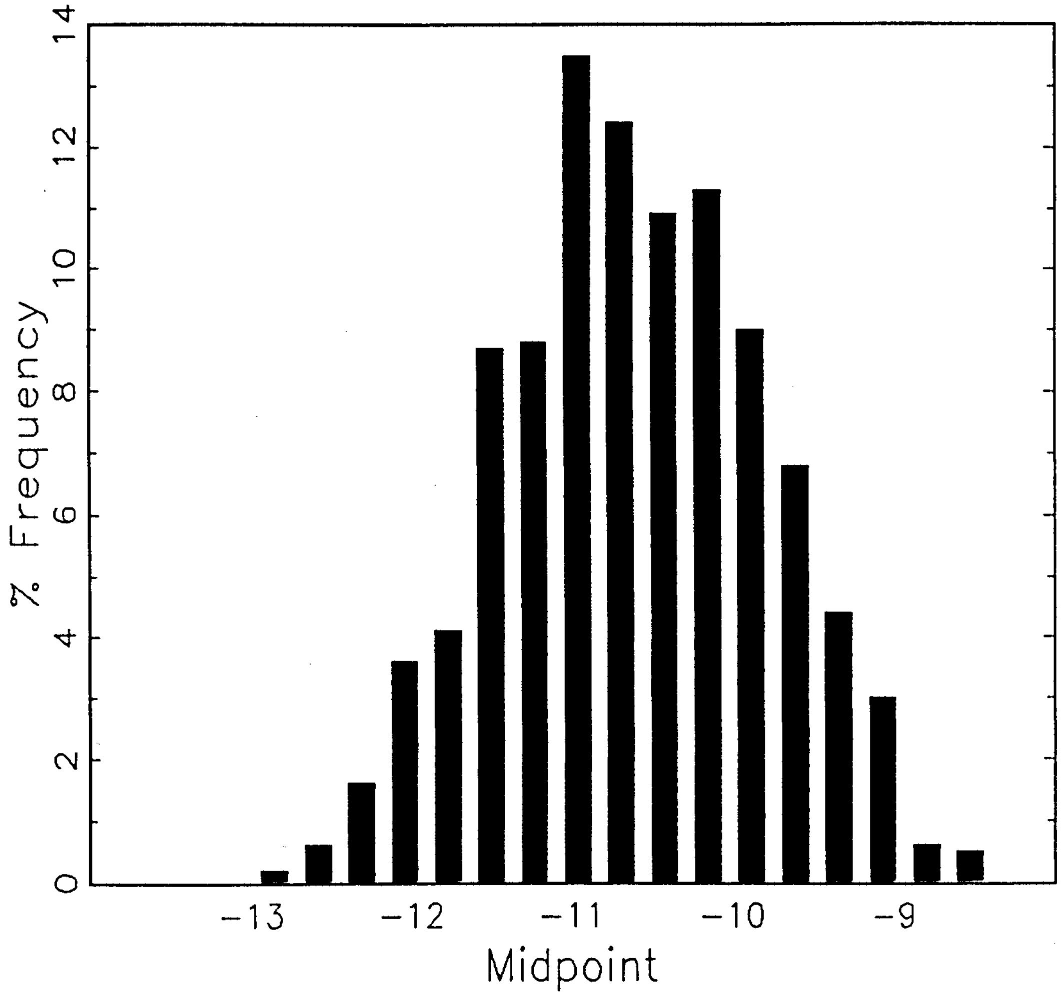

Figure 1

Normal Null Distribution

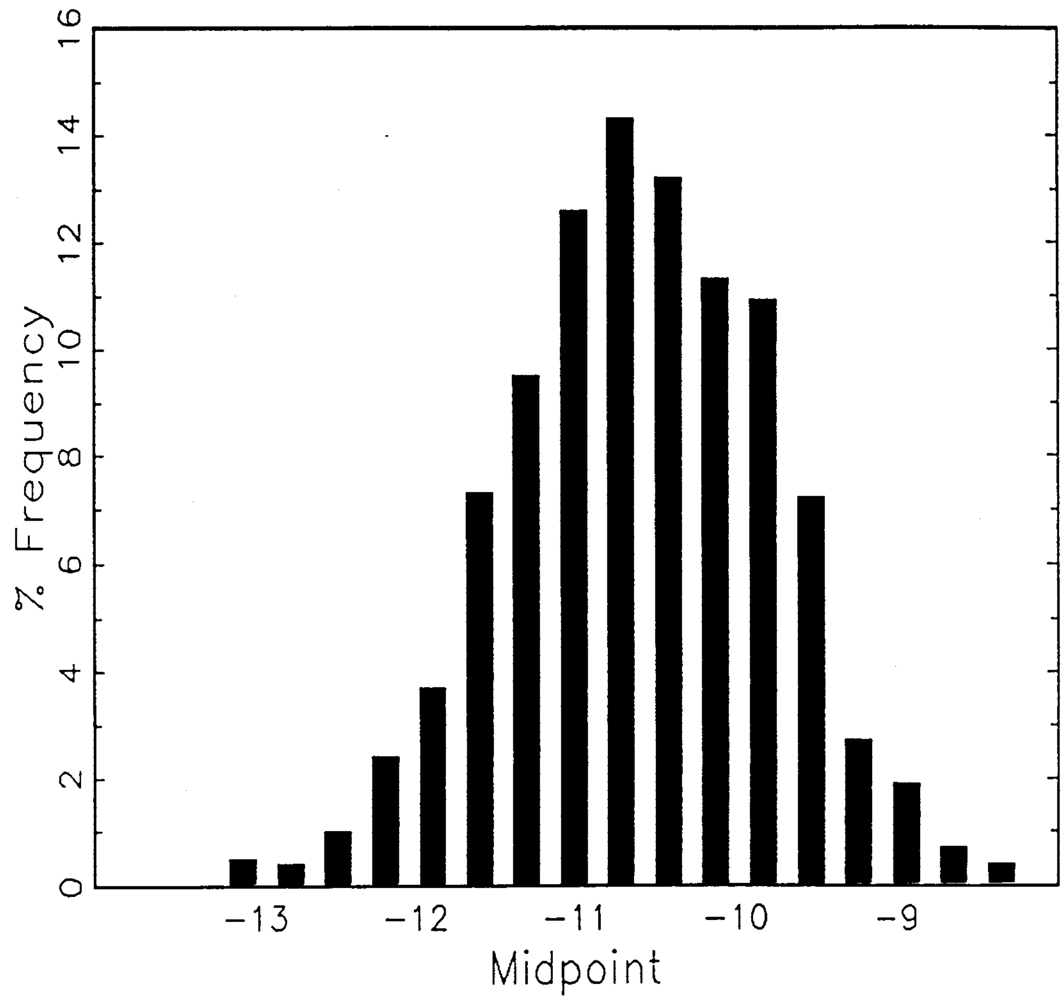

Figure 2

Students' T with 5 d.f. Null Distribution

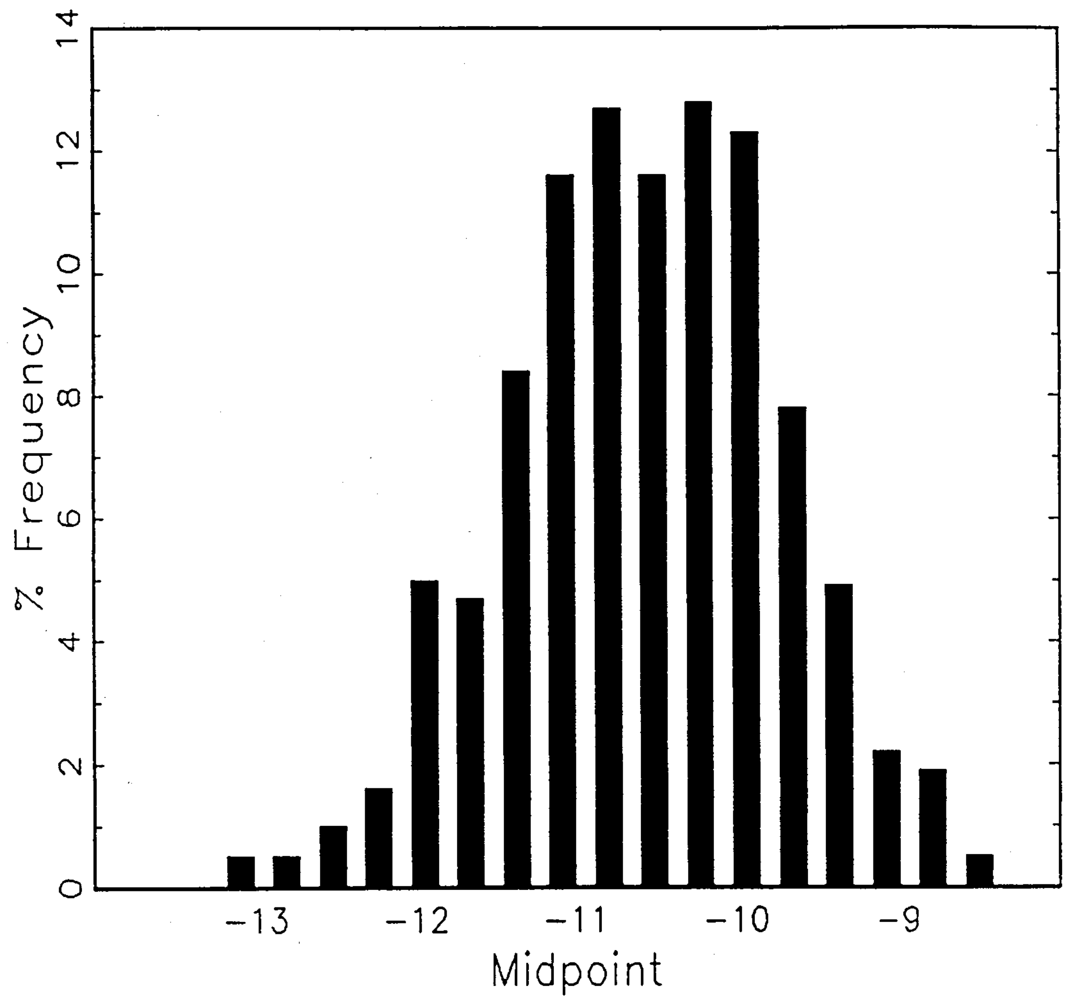

Figure 3

Students' T with 8 d.f. Null Distribution

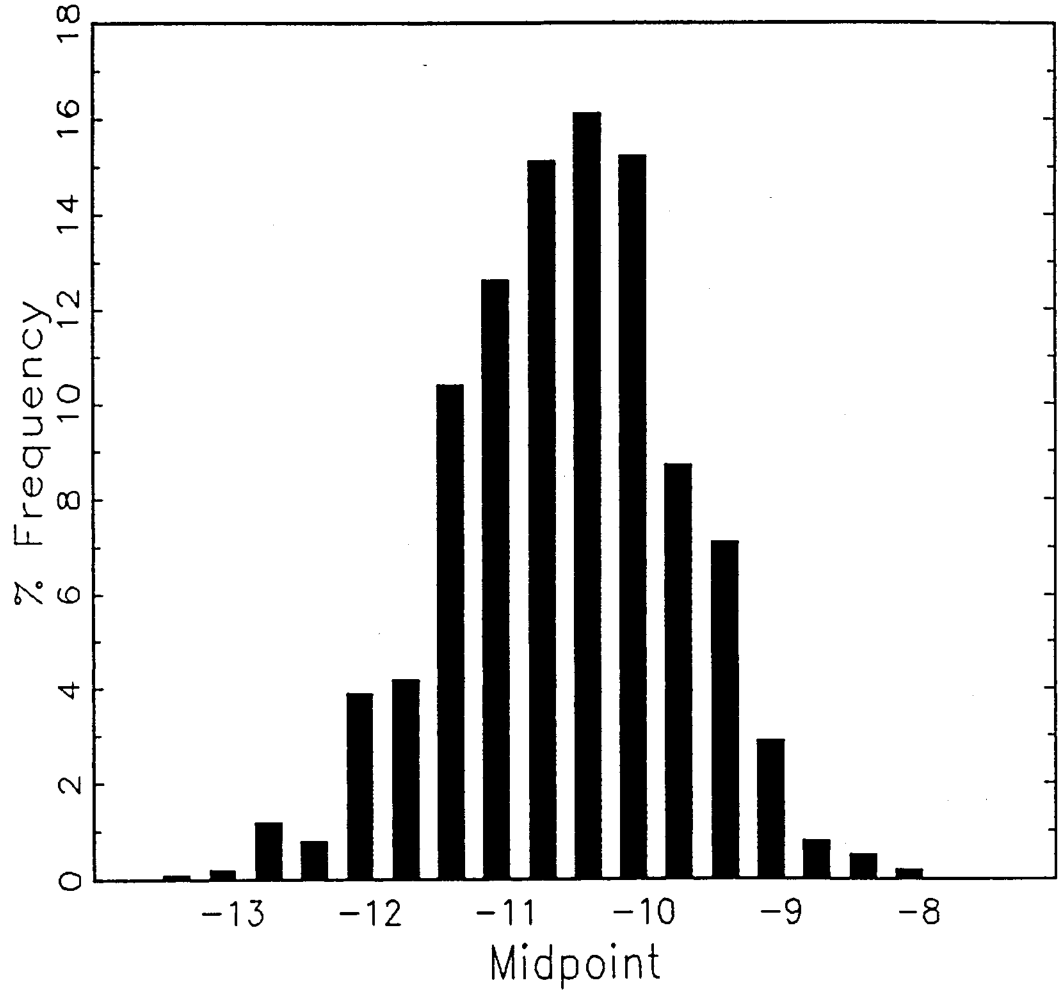

Figure 4

"Lognormal" Null Distribution

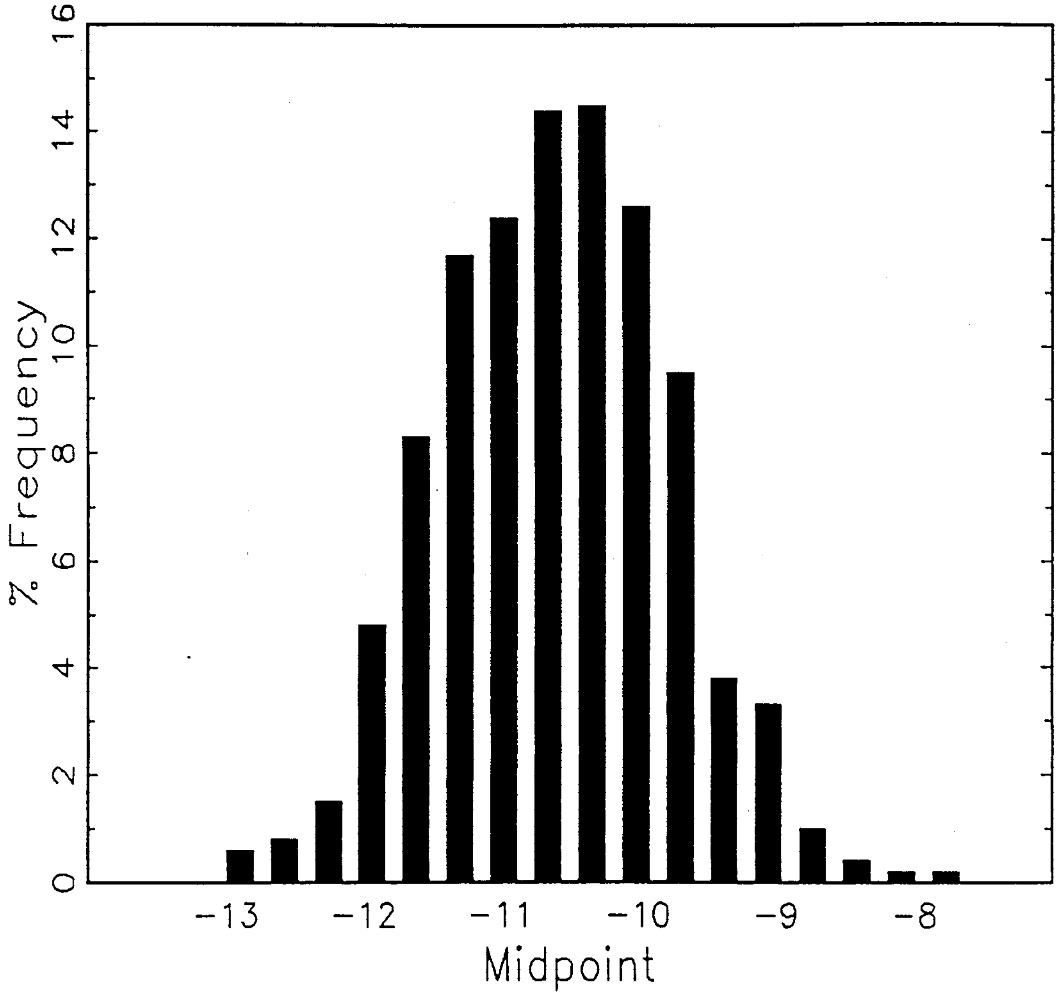

Figure 5

"Exponential" Null Distribution

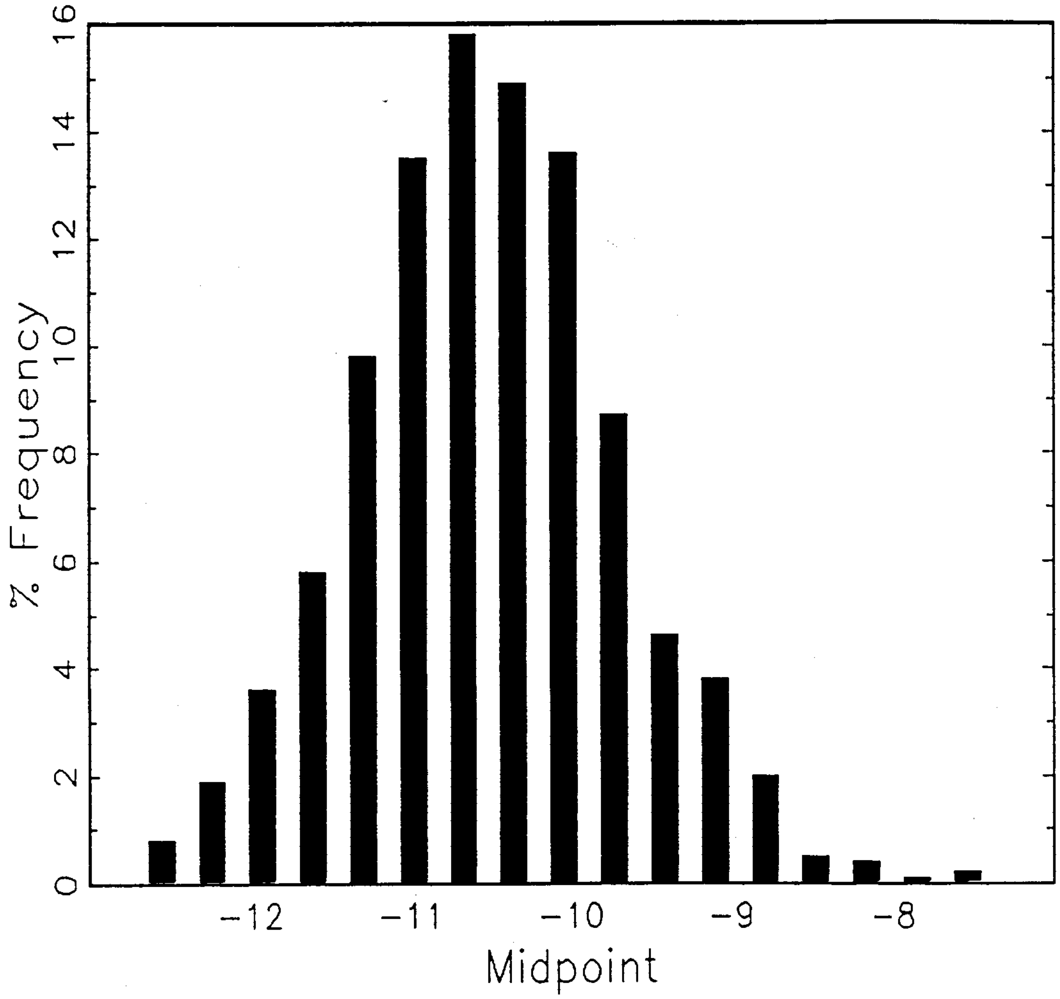

Figure 6

Pearson Type II Null Distribution