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Fixed Effects and Stochastic Frontier Estimates of Firm-Level Technical Efficiency

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ABSTRACT

Parameter estimates from OLS, fixed effects and stochastic production frontier models are compared using panel data for a sample of Vermont dairy farms. The results confirm that OLS production function models without firm effects lead to biased estimates. The estimates from the production frontier model area also deemed to be biased.

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FIXED EFFECTS AND STOCHASTIC FRONTIER ESTIMATES OF FIRM-LEVEL TECHNICAL EFFICIENCY

For many years, the production function has played a key role in applied economic analysis (Douglas). Throughout this long period, there have been many criticisms as well as improvements of production function methodology. The purpose of this paper is to bring together two strands of applied production function analysis which have attracted considerable interest in recent years. Specifically, this paper compares, using a real world data set, the performance of a stochastic production frontier with a fixed effects production model. In the next section we present a brief overview of the methodologies used followed by a section containing the data and empirical models. Then the results and analysis are presented and we close with some concluding comments.

Methodological Framework

The use of production functions to analyze firm level efficiency dates back at least to a paper written by Heady and published over 40 years ago. Since this early work, a great deal of progress has been made in efficiency measurement via production functions. Two specific methodologies which have been developed and used for this purpose, and which are the focus of this paper, are the fixed effects model, also known as the least squares with dummy variables or covariance model, and the frontier production function.

The fixed effects model was introduced by Hoch in 1955 and extended by Mundlak, and Hoch (1958 and 1962). Recently, there has been a resurgence in interest on the fixed effects model as evidenced by the studies conducted by Hoch (1976a and 1976b), Dawson and Lingard, Lingard, Castillo and Jayasuriya, Turvey and Lowenberg-DeBoer, Seale, and Myers.

In the fixed effects model, which requires panel data for estimation (i.e., a cross section of firms observed over several years), dummy variables are introduced to incorporate time and individual firm effects. The purpose of the time effects is to account for upward shifts of the production function due to technological change while the firm effects account for management. Mundlak has shown that introducing firm effects to a production function leads to parameter estimates that are free of management bias, hence overcoming the omitted variable problem discussed by Griliches. In addition, Hoch (1962) has shown that the fixed effects model reduces and might even avoid the simultaneous equation bias associated with single equation production function models (Marschak and Andrews). In sum, two potentially serious problems associated with single equation production function models, namely simultaneous equation bias and the omitted management variable problem, are resolved by using the fixed effects model.[1] Of course, this requires panel data which often is not available.

The production frontier methodology was initiated by Farrell in a path-breaking paper published in 1957. After a decade of little activity in frontier work, Aigner and Chu introduced a deterministic parametric (Cobb-Douglas) production frontier and employed linear and quadratic programming for estimation. These authors also proposed the use of a probabilistic frontier function which was later applied by Timmer. A major shortcoming of the deterministic parametric model is that no explicit statistical distribution for the efficiency term is made which yields estimators with unknown statistical properties. Various ways of overcoming this problem have been proposed by Afriat, Richmond, Schmidt (1976), and Greene.

A deficiency characterizing all deterministic frontier models is their sensitivity to outliers. A more recent approach for measuring efficiency, which seeks to alleviate the outlier problem, is the stochastic frontier model developed by Aigner, Lovell and Schmidt, and by Meeusen and van den Broeck. The stochastic frontier model assumes an error term that has two additive

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1 Zellner, Kmenta and Drèze have shown that if managers are assumed to maximize the mathematical expectation of profits then a single equation production function model is also free of simultaneous equation bias.

components -- a symmetric component which accounts for pure random factors, and a one-sided component which captures the effects of inefficiency relative to the stochastic frontier. A recent extension by Jondrow et al. has solved the previous inability of deriving firm efficiency measures from stochastic frontiers.[2]

The use of frontier production functions relying on cross-sectional data has been extensive. Much of the published work has dealt with agricultural firms, primarily dairy farms (e.g. Bravo-Ureta; Bravo-Ureta and Rieger; Quiroga and Elterich; Tauer and Belbase; and Grisley and Mascarenhas).

As mentioned previously, the firm specific dummy variables in the fixed effect model have been interpreted as a management index. Hoch (1976a), however, has provided support for the view that the firm effects can be interpreted as a measure of technical efficiency. Moreover, Russell and Young have argued that "... while there may be a conceptual difference between management expertise and efficiency, their effects on output cannot be distinguished using existing techniques" (p. 141). Consequently, there is a clear link between the fixed effects model and the stochastic production frontier methodology recently introduced by Schmidt and Sickles, and refined by Battese and Coelli, which makes use of panel data. Both methodologies offer a direct way of obtaining a measure of technical efficiency which can be used as a proxy for management. However, the assumptions and the techniques used to estimate the two models are very different.

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[2] For a detailed discussion of frontier function methodology see Forsund, Lovell and Schmidt, and Schmidt (1985-86).

Data and Empirical Model

The data used in this study was obtained from farms participating in the Electronic Farm Record Keeping System (ELFAC). The data set includes 33 Vermont dairy farms for the period 1971-1984. Thus, a total of 462 observations are used in the analysis.

The production function estimated, expressed in general form, is

$$(1)\quad Y = f(X_1, X_2, X_3, X_4, X_5, X_6, D_i, D_t) \qquad (i = 1, 2,...33;\ t = 1,2,...14)$$

where Y and all the Xs are annual figures measured in natural logs. The variables are defined as follows:

- Y = annual milk output per cow measured in lbs.;
- X_1 = worker equivalents per cow including operator, family and hired labor;
- X_2 = purchased dairy concentrates per cow in lbs.;
- X_3 = capital services per cow, measured in 1977 dollars, including repairs and maintenance on machinery and equipment, gas and oil, electricity, and depreciation on buildings and equipment set at 3 and 15 percent of current value, respectively;
- X_4 = crop expense per cow, measured in 1977 dollars, including fertilizer, seed, spray, lime and miscellaneous items;
- X_5 = animal expense per cow, measured in 1977 dollars, including veterinary, breeding and animal supplies;
- X_6 = number of cows;
- D_i = dummy variable equal to 1 for the ith firm and zero otherwise; and
- D_t = dummy variable equal to 1 for the tth year and zero otherwise.

Four versions of the production function model shown in equation (1) are estimated. In all cases the Cobb-Douglas functional form is used. The models estimated are:

$$\textit{Model I:}\quad Y_{it} = \alpha_o + \Sigma\beta_j X_{ij} + e_{it} \qquad (j=1,2,...6)$$

This model restricts the firm and time effects to zero and is estimated using least squares.

Model II: $Y_{it} = \alpha_o + \Sigma\beta_j X_{ij} + \gamma_i D_i + \delta_t D_t + e_{it}$

This is the fixed effects model which incorporates both firm (D_i) and time (D_t) effects; hence, as discussed above, this formulation is not subject to simultaneous equation bias nor to omitted management bias. Model II is also estimated using least squares.

Model III: $Y_{it} = \alpha_o + \Sigma\beta_j X_{ij} + \delta_t D_t + e_{it}$

This specification excludes the firm effects but includes the time effects.

Model IV: $Y_{it} = \alpha_o + \Sigma\beta_j X_{ij} + \delta_t D_t + \epsilon_{it}$ *and*

$\epsilon_{it} = v_{it} - u_i$.

In this model, e_{it} is the composed error term where v_{it} captures statistical noise and u_i represents the average technical efficiency of the i^{th} firm (Aigner, Lovell and Schmidt; and Meeusen and van den Broeck). The two components v and u are assumed to be independent of each other, where v is the two sided normally distributed random error ($v \sim N(0,\sigma_v^2)$), and u is the one-sided component with a half-normal distribution ($u \sim |N(\mu,\sigma_u^2)|$). Following Battese and Coelli, the estimated technical efficiency for the i^{th} firm is given by $\exp(-\hat{u}_i)$ where

$$\hat{u}_i = M_i^* + \sigma_* \left[\frac{f(-M_i^*/\sigma_*)}{1-F(-M_i^*/\sigma_*)} \right] \qquad (2)$$

and $M_i^* = (-\sigma_u^2 e_j + T^{-1}\mu\sigma_v^2)/(\sigma_u^2 + T^{-1}\sigma_v^2)$, $e_i = \sum e_{it}/T$, $\sigma_*^2 = \sigma_u^2\sigma_v^2(\sigma_v^2 + T\sigma_u^2)^{-1}$, T is the number of years, and f and F represent the normal density and distribution functions, respectively.[3] Model IV is similar to the one estimated by Battese and Coelli, except that in this paper we include time effects.

Results and Analysis

Table 1 presents estimates for the four models discussed in the previous section. It should be noted that given our specification, a measure of returns to size is obtained from the parameters on cows (X_6) (Salem; and Brown and Medoff). Thus, models I and II suggest that the hypothesis of constant returns to size cannot be rejected for this sample. By contrast, models III and IV show a slightly decreasing returns to size equal to approximately 0.96 (1- the coefficient of X_6). Hence, based on the results concerning economies of size we observe no major differences across the various models. We also compute the quasi function coefficient (Beattie and Taylor), defined here as the percent increase in output per cow, keeping the number of cows constant, resulting from a one percent increase in all of the five inputs expressed on a per cow basis (X_1 through X_5). The value of the quasi function coefficient is obtained by summing the parameters for all explanatory variables except cows (X_6). As Table 1 shows, the quasi function coefficient for Model I, which excludes both time and firm effects is 0.616. The value for Model II, which includes both time and firm effects is 0.339 and for Model III, which incorporates time effects only, is 0.576. Finally, the quasi function coefficient for the frontier model (IV) which also includes time effects is 0.567. Therefore, the values of

[3] For a detailed explanation see Battese and Coelli.

Table 1. Production Function Estimates for a Panel Data Set of Vermont Dairy Farms Using Four Alternative Models. [1]

Variable	Mean (Std. Dev.)	Model I Parameter (Std. Error)	Model II Parameter (Std. Error)	Model III Parameter (Std. Error)	Model IV Parameter (Std. Error)
Intercept	---	5.615***	7.885***	5.597***	5.757***
		(.149)	(.279)	(.154)	(.156)
X_1	0.039	-0.007	0.024	-0.041*	-0.039*
	(.01)	(.019)	(.021)	(.021)	(.021)
X_2	5310.40	0.253***	0.127***	0.252***	0.248***
	(1616.30)	(0.18)	(.019)	(.018)	(.013)
X_3	220.50	0.118***	0.052**	0.112***	0.113***
	(61.70)	(.019)	(.021)	(.020)	(.022)
X_4	134.80	0.120***	0.0606***	0.130***	0.123***
	(48.00)	(.015)	(.014)	(.015)	(.016)
X_5	69.60	0.132***	0.076***	0.123***	0.122***
	(26.80)	(0.014)	(.016)	(.014)	(.0159)
X_6	64.50	-0.019	-0.033	-0.035**	-0.039***
	(38.60)	(.014)	(.038)	(.015)	(.016)
Quasi Function Coefficient[3]	---	0.616	0.339	0.576	0.567
SSE[2]	---	4.663	2.132	4.417	---
$\bar{R}^2$	---	0.694	0.845	0.701	---
F	---	175***	50***	58***	---
λ	---	---	---	---	1.251***
σ	---	---	---	---	0.125***
Log Likelihood	---	---	---	---	420

[1] For model and variable definition see the text.
[2] Error sum of squares.
[3] The quasi function coefficient is calculated by adding the parameters for all explanatory variables except X_6 (cows).
*** Significant at 1% level.
** Significant at 5% level.
* Significant at 10% level.

the quasi function coefficients suggest that Models I, III and IV are neutrally scaled versions of each other, while Model II is considerably different from the other three.

A close inspection of Table 1 shows that the value of all individual slope parameters for a given variable is remarkably close for Models I, III and IV. By comparison, the regression estimates for β_2, β_3, β_4 and β_5 in Model II are consistently about one-half the value of the respective parameters in the other three models. The labor coefficient (β_1), which is negative in Models I, III and IV, is positive, although not significantly different from zero, in Model II. This pattern is consistent with Griliches' argument that when management (proxied here by technical efficiency) is omitted from the model the resulting parameters are biased. In particular, we find an upward bias for all parameter estimates except in the case of labor where this bias appears to be negative.

Table 2 presents the average firm level technical efficiency (TE) over the 1971-84 period computed from Model II (Fixed Effects) and Model IV (Prod. Frontier). As mentioned earlier, the firm dummies in the fixed effects model are interpreted as a measure of TE (e.g. Hoch 1955 and 1976b). The technical efficiency index from the fixed effects models is computed simply by taking the antilog of the coefficient of each firm dummy. In general, some values will be higher than one and some less than one while the TE index for the excluded firm will be equal to one. Firms with values larger than one are considered more efficient than the reference firm, while those with values lower than one are less efficient. It is possible, however, that all firms might have a value lower (higher) than one if the reference firm turns out to be the most (least) efficient one. In our case, we chose the firm with the lowest ID number as the reference firm. Coincidentally, this turned out to be the most efficient firm in the sample. Thus, the average TE index for our sample ranges from 65 to 100 percent, with a mean of 84.3 percent. It should

Table 2. Average Technical Efficiency Estimates for a Sample of 33 Vermont Dairy Farms for 1971-84.

Farm ID	Technical Efficiency: Fixed Effects (Model II)	Technical Efficiency: Prod. Frontier (Model IV)
01	100.0	94.3
02	87.7	89.6
03	98.6	88.0
04	88.7	89.7
05	78.0	86.7
06	84.5	90.8
07	81.0	92.3
08	86.5	90.0
09	90.4	91.4
10	91.8	90.4
11	93.5	91.4
12	78.1	86.5
13	99.1	94.0
14	92.2	89.3
15	82.8	91.3
16	90.0	90.9
17	80.3	87.0
18	91.5	92.2
19	80.8	89.9
20	83.5	89.0
21	72.4	81.6
22	90.6	89.8
23	83.9	90.8
24	93.1	91.5
25	74.8	81.5
26	71.4	82.9
27	71.4	88.1
28	75.3	89.9
29	87.8	88.2
30	69.4	86.4
31	65.0	82.2
32	84.2	89.7
33	82.7	89.4
Mean	84.3	89.0
Minimum	65.0	81.5
Maximum	100.0	94.3

be noted that of the 32 firm effects parameters estimated 28 are significant at the five percent level or higher. This by itself is evidence in support of the fixed effect model.

The TE index for the stochastic production frontier, as explained earlier, is equal to the antilog of the one sided component u_i. As shown in Table 2, the average TE index computed from the production frontier is 89.0 percent with a minimum of 81.5 percent and a maximum of 94.3 percent. A comparison of the TE indexes obtained from Models II and IV suggest that the fixed effects model, at least for our data set, yields a much wider distribution of efficiency than the frontier model. A simple correlation between the two efficiency vectors yields a value of 0.75 suggesting that the two models yield a fairly consistent ranking of firms according to their level of TE. We also calculated the correlation coefficient between TE and herd size, which gives a value of 0.068 for the stochastic frontier and 0.262 for the fixed effects model.

Given that Models I and III are nested in Model II we used the 'classical model selection' criterion, discussed by Kmenta (pp. 593-594), to determine which is the best model in statistical grounds. By simple inspection of the data presented in Table 1, Model II appears to be superior based on the significance of individual parameters and on the adjusted R^2s. The statistical results of the 'classical model selection' tests indicate that, at the 0.01 level, Model II dominates Model I, Model I dominates Model III, and Model II dominates Model III. Thus, these tests indicate that Model II is superior to Models I and III. Unfortunately, a similar set of tests cannot be conducted with reference to Model IV since it is not nested within the other models.

Concluding Comments

In this paper we have evaluated the performance of the fixed effects model vis à vis the stochastic production frontier model recently introduced by Battese and Coelli using a panel of 33 Vermont dairy farms for the period 1971-1984. Our results are consistent with the view that OLS production function models that exclude firm effects lead to biased parameter estimates. In addition, given the great similarity between the biased parameters and those estimated from the production frontier, we can infer that the latter set of parameters is also biased. An important conclusion that can be derived from these results is that whenever panel data is available the model of choice is the fixed effects. The extent to which this finding carries over to other data sets remains to be determined.

In closing we should note that a basic assumption of the models used in this paper is that efficiency remains constant over the period of analysis. Although this is an obvious limitation, there appears to be no clear way of resolving this problem at the present time. One approach that can be used to investigate this issue is to divide the data into shorter sub-periods. This was not done here due to time constraints but will be pursued in the future.

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