"The Effects of Corporate Retentions on Personal Savings: Tests of Rival Hypotheses"

by

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.
ABSTRACT

Increases in corporate retentions do not necessarily increase private savings by an equal amount. They may be exactly compensated by reductions in other personal savings, partly compensated, or not compensated at all. The three hypotheses are examined along with the empirical support they have received. Empirical estimation is undertaken to test the three rival hypotheses by use of U.K. time-series data. Three frameworks are adopted; a Simple Linear model, both Static (SLS) and naive dynamic (SLD); a General Distributed Lag (GDL) model, and the Life Cycle Hypothesis in its General Distributed Lag Equivalence (LCHGDLE) form; a Simple Add-on (SAO) model derived from three identities and simple behavioural assumption(s).

It is shown that; both the GDL and the LCHGDLE models can be obtained from the SAO model. This highlights the underlying naiveté of their nature; all models can be nested in a general form, obtained by use of any of the GDL, LCHGDLE and SAO models. Support is found for the Add-on hypothesis; increases in corporate retentions will add-on, on a one-to-one basis, other personal savings. Doubt is cast upon the LCH prediction of perfect substitutability. Some macro-implications are briefly examined.
Introduction

The issue of 'contractual' savings (savings not subject to the discretion of all individuals) and their effects on personal savings has recently received a lot of attention. This is not surprising. Apart from providing a means of assessing the validity of alternative theories of consumption-savings behaviour and their implications, the issue bears on macroeconomic questions too; particularly theories of growth. Surprisingly, empirical studies are largely confined only to one form of contractual savings; namely those resulting from households' participation in pension plans schemes. Corporate retentions, the other major form of contractual savings, has received scant attention. This paper attempts to fill this gap. Section I examines rival hypotheses and the empirical support they have received. In section II empirical testing of the three rival hypotheses is carried out. Section III concludes with discussion.

I: Rival hypotheses and previous studies

Three rival hypotheses have been advanced in the literature. The first hypothesis posits an inverse substitutability between corporate and personal savings. In its extreme version substitutability must be perfect too. That is, no increase in private (personal plus corporate) savings is to be expected as a result of increases in corporate savings. Inter-temporal utility maximising households will manage to see "through the corporate veil", (Feldstein, 1973) and reach their preferred income-consumption (savings) position. In this form the hypothesis has been advanced and indeed is an immediate implication of the "life cycle hypothesis" (LCH) of Ando and Modigliani (1963).
The second hypothesis posits an imperfect substitutability. It rests on two different frameworks. In a frequently quoted argument advanced by Harrod (1948), the reasoning consists of two parts. In the first the author suggests that "corporate savings may not be additional to personal savings, but part of it"; (p.47). In the second, however, he finds it "conceivable that corporate savings might exceed the total that all individuals would be disposed to save ..." and calls the excess "surplus corporate savings"; (p. 48). Given the fundamentally individualistic life-cycle approach taken by Harrod, it appears reasonable to view his version of the imperfect substitutability hypothesis as a sub-case of the perfect substitutability hypothesis. This does not imply, however, that any imperfect substitutability should be viewed as relatively more advantageous to the Ando-Modigliani prediction. Managerialists share this prediction too; (e.g. Marris, 1964). The idea is as follows; managers control corporations and impose their will on households, capitalists and workers alike. For reasons associated with tax advantages, bonuses, and preference for internal expansion, managers will exhibit a preference for a higher retention ratio than the one of the rest of households. To the extent that managers have a dominant position in the firms the resulting "long-run variations in the retention ratio must cause long-run variations in the national propensity to save (in the same direction, of course, but smaller in magnitude)", (p. 295). In the absence of an overriding criterion to discriminate between the two versions of the imperfect substitutability hypothesis, it would appear more prudent to treat them as an independent-homogeneous hypothesis.

The third hypothesis, the add-on or complementarity or independence hypothesis, was originally advanced by Marglin (1974). His reasoning is similar to Marris' but it is also posited that households' personal savings are only a 'disequilibrium' phenomenon, due to the growth of incomes, and very low in magnitude; near zero. In such cases and in the face of constraints on borrowing, households' willingness to compensate for
increasing retentions by reducing personal savings will be frustrated; thus corporate savings will add on, on a one to one basis, personal savings.

Both Marris' version of the imperfect substitutability hypothesis and Marglin's version of the add-on hypothesis, rely heavily on a presumed managerial dominance of today's large corporations. In the absence of such a dominance these hypotheses need re-examining. The new evidence on the theory of the firm;—see e.g. Scott (1979) for a survey; also Francis (1980, 1980a), for more recent evidence,—suggest that a group of owners are mostly still in control of large corporations, and managers are either owners themselves or the latter's functionaries—employees, left to pursue their policies only to the extent that these policies are in accord with owners' interests. If not, managers are substituted by a more efficient managerial team.

In such a framework, Pitelis (1962) has developed a 'generalized' version of the add-on. With the "controlling group" comprising owners (who may be managers themselves) and high-level management (who are either owners or act in accord to owners' interests), it can be shown that for this group add-on type of behaviour will be observed by definition. That is, since this group control corporations, their ex-post savings in the form of retentions will be just a reflection of their ex-ante decision on the part of their income which they have decided neither to consume, nor to save in the form of personal savings; that is no substitutability is to be expected since an ex-ante preference for lower retentions on their part, would have simply been reflected in a lower ex-post retention ratio. With the 'controlling group' imposing its will for higher retentions on small-shareholders—workers, add-on type of behaviour will result on their part only if it can be shown that their personal savings are indeed too low to allow any substitutability: The existing evidence suggests that this may very well be the case, e.g. Klein et al (1956), Marglin (1974)
Pearce and Thomas (1981), Cuthbertson, (1982), Pitelis (1982), which implies that the add-on hypothesis should be expected to be supported by the data. We may, therefore, turn to the existing empirical work on the issue.

In their original articles neither Ando and Modigliani nor Marglin have provided any direct support to their hypotheses. Proponents of the perfect substitutability hypothesis have rather relied on an article by Denison (1958), for the US, where it was suggested that "the ratio of gross private savings to gross national product was about the same in 1929 and each of the years 1948 through 1953. This ratio continued to hold in 1954, 1955 and on the basis of preliminary estimates, in 1956" (p.261). The author attributed this stability to the observed tendency of the ratio of personal savings to GNP to be high when the corporate savings to GNP ratio was low, and vice versa. It is questionable however whether these results can be taken as support to the substitutability hypothesis. Marglin has criticised Denison's reliance on variations of savings to GNP ratios instead of variations to after-tax private income ratios. As Denison's own findings show, when ratios of corporate savings to after tax corporate income are considered, they are found not to move in opposite directions in most cases. Marglin, on the other hand, sought support for his hypothesis in the well known study by Cagan (1965) of responses of household personal savings to savings made by their participation in pension plans. In this study Cagan offered support to the independence hypothesis. Although, however, this finding contradicts the Ando-Modigliani implication, it can be questioned, whether it provides direct support to the same hypothesis for corporate savings.

In a subsequent cross-section study Modigliani (1970), regressed the ratio of corporate savings to private income, to the private savings-private income ratio, and for the twenty four countries covered, he found positive but insignificantly different from zero coefficients. From this he
concluded that his tests were inconclusive. More direct tests were under-
taken by Burmeister and Taubman (1969), Feldstein (1973), and Lambrinides,
(1972, 1974) for the US. For the UK there is, to our knowledge, only one
published study; by Feldstein and Fane (1973). Feldstein and Feldstein and
Fane, estimate consumption functions for the 1929-1966 (excluding 1942-1947),
and 1948-1969 periods respectively. Corporate retentions are included as a
separate explanatory variable. They find support for the imperfect substitut-
tability hypothesis: The coefficients for the corporate retentions variable
are approximately 0.50 and 0.25 for the US and UK respectively. Lambrinides
estimates private savings functions. In a time series study covering the
1919 through 1958 period, he finds support for the imperfect substitutability
hypothesis; in one case the obtained coefficient being 0.7. Theoretically
this implies a coefficient of minus 0.3 for a personal savings function, or
of 0.3 for a consumption function which is close to Feldstein and Fane's
results. In a cross section study Lambrinides (1972) examines the effects
of the ratio of organizational savings to national income, on the national
savings-national income ratio, for 18 developed countries. When corrected
for heteroscedasticity the obtained coefficient of the organizational savings
-national income ratio variable is 0.89 which is insignificantly different
from one at the 5% level of significance. This supports the add-on hypothesis.
More direct support to the latter is given by Burmeister and Taubman, (1969).
They estimate personal savings functions. Although testing rival hypotheses
is not the authors' direct aim and contrary to their expectations, in 12
equations where corporate retentions are included as a separate argument, a
positive and insignificant coefficient is found. Finally support for the
add-on hypothesis has been provided for the UK by the present author (1982).
In a consumption equation where corporate retentions were included as a
separate argument, a negative and insignificant coefficient was found which
supports Burmeister and Taubman's results.
We, therefore, find ourselves to the uncomfortable position of inconclusiveness. Three studies support the add-on hypothesis, three the imperfect substitutability, one is inconclusive and one is taken to support both. Obviously part of the observed differences may be explained in terms of differences in the data period used, different specification or/and differences in the country examined. In the next section we try to solve these problems by examining a fairly long period of time (1951-1981) for the UK and adopting a more comprehensive range of specifications than any of the previous studies. Examining, however, US data too, is not the aim of this paper.

II: Tests of Rival Hypotheses

Most empirical studies referred to in section I are either directly cast into the Life Cycle Hypothesis (LCH) framework, or into a framework very similar to it. In this section we try to examine a wide range of specifications to ensure comprehensiveness. The main issues examined here are: i) Whether the add-on hypothesis is supported by the data: ii) Whether it is the change rather than the level of the personal disposable income variable, which best explains the dependent variable. Three different frameworks are adopted to analyze the three rival hypotheses, and we further examine whether iii) these three frameworks may be cast into a general form in which they will be nested.

Thus we examine. A simple linear model in its static (SLS) form and in a naive, dynamic (SLD) form. A general distributed lag (GDL) model and the life cycle hypothesis in its general distributed lag equivalence (LCHGDLE) form. Finally, a simple add-on (SAC) model, derived from three identities and simple behavioural assumption(s). This proves to have the same estimated form as both the elaborate GDL and LCHGDLE models.
The SLS model: In its simplest form the hypothesis under examination can be written as

$$s^\text{prv}_t = s(y^\text{prs}_t, s^C_t, Z_t)$$  \hspace{1cm} (1)$$

where $y^\text{prs}_t$ represents personal disposable income at period $t$, $s^C_t$, corporate savings at period $t$, and where:

$$s^\text{prv}_t = s^\text{prs}_t + s^C_t$$  \hspace{1cm} (2)

$s^\text{prs}_t$ being personal savings in period $t$, and private income

$$y^\text{prv}_t = y^\text{prs}_t + s^C_t$$  \hspace{1cm} (3)

$Z$ is a vector of relevant explanatory variables; in this case the interest rate $(R_t)$. For estimation purposes, and including a constant term, equation 1 can be written as:

$$s^\text{prv}_t = \gamma_0 + \gamma_1 y^\text{prs}_t + \gamma_2 s^C_t + \gamma_3 R_t + u_t$$  \hspace{1cm} (4)

$$u_t = \text{NID}(0, \sigma^2)$$

The change of $y^\text{prs}_t$, $(\Delta y^\text{prs}_t)$, can be either introduced in equation 1 to replace $y^\text{prs}_t$, or as a separate explanatory variable, in which case equation 4 will be modified accordingly.

$$s^\text{prv}_t = \gamma_0 + \gamma_1 \Delta y^\text{prs}_t + \gamma_2 s^C_t + \gamma_3 R_t + u_t$$  \hspace{1cm} (4')

$$s^\text{prv}_t = \gamma_0 + \gamma_1 y^\text{prs}_t + \gamma_2 s^C_t + \gamma_3 R_t + \gamma_4 \Delta y^\text{prs}_t + u_t$$  \hspace{1cm} (4'')

The implications of the rival hypotheses in terms of equation 4 can be written as:

- a: Perfect substitutability $\gamma_1 = \gamma_2 < 1$
- b: Imperfect substitutability $0 < \gamma_1 < \gamma_2 < 1$
- c: Add-on $0 < \gamma_1 < \gamma_2 = 1$
On estimation equations 4', 4, and 4'' gave the results reported in table 1. Ordinary least squares were firstly used but first order serial correlation was found and a Maximum Likelihood Iterative technique was applied to account for it. For thirty one observations and the relevant number of degrees of freedom, the DW indicated that in all cases this technique was marginally successful in removing autocorrelation at the 5% level. In equation 4' the constant was insignificant and the equation was also run without it, (4'a). As it turned out, in both equations 4 and 4'', $\gamma_1$ is significantly smaller from $\gamma_2$ at the 5% level of significance of a one tailed 't' test, which contradicts the LCH prediction; moreover $\gamma_2$ is always insignificantly different from one in the 5% level of significance of a two-tailed 't' test, which supports the add-on hypothesis. In all equations the explanatory power is very high. Judging by the latter, it would appear from a comparison of equations 4' and 4 in table 1 that the level of the $y_{t}^{prs}$ variable provides a better explanation of the endogeneous variable than the difference of the $y_{t}^{prs}$ variable, ($\Delta y_{t}^{prs}$). This is dubious, however, for two reasons. Firstly, because $\Delta y_{t}^{prs}$ will be expected to be less trending with the dependent variable than $y_{t}^{prs}$. Secondly, since $\sum_{i=0}^{n} \Delta y_{t}^{prs} + y_{t}^{prs}$ as $n \to \infty$, it could be argued that $\gamma_1$ in 4 is a proxy for $\gamma_1$ in 4', while $\gamma_4$ in 4'' measures this period's differential impact of $\Delta y_{t}^{prs}$ on the dependent variable. As can be seen, equation 4'' has the highest explanatory power, is well specified and exhibits significant coefficients in all the explanatory variables; and with correct signs.

A formal discrimination of the two underlying behavioural hypotheses in the framework of the SLS model can be obtained by
A denotes difference.
* Indicates significance at the 5% level.
"w" refers to parentheses.

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**Table 1**

Tests of Partial Hypotheses in a Simple Linear Static (SLS) Model

Dependent Variable: Poultry Sales (SLS)
following the following procedure. Since for a homomorphic function

\[ f(\Delta y_{t}^{prs}) = f(y_{t}^{prs}) - f(y_{t-1}^{prs}) \]  

(5)

on substituting equation 5 in equation 4 we obtain the equation (in estimating form)

\[ S_{t}^{prv} = \gamma_{0} y_{t}^{prs} + \gamma_{1} S_{t}^{c} + \gamma_{3} R_{t} + \gamma_{4} y_{t}^{prs} + \gamma_{5} y_{t-1}^{prs} + u_{t} \]  

(6)

where the restriction \( \gamma_{4} = -\gamma_{5} \) should be satisfied. In equation 6, however, the two \( y_{t}^{prs} \) variables are perfectly collinear, therefore no estimation may be undertaken. On estimating, however, the equation;

\[ S_{t}^{prv} = \gamma_{0} y_{t}^{prs} + \gamma_{1} S_{t}^{c} + \gamma_{3} R_{t} + \gamma_{4} y_{t-1}^{prs} + u_{t} \]  

(6')

we may expect \( \gamma_{1} \) in equation 6' to capture the joint effect of \( \gamma_{1} \) plus \( \gamma_{4} \) in equation 6. From equation 5, moreover, the obtained sign of \( \gamma_{4} \) in equation 6' becomes a test of whether the underlying hypothesis is one in which \( \Delta y_{t}^{prs} \) rather than \( y_{t}^{prs} \) best explains the dependent variable. Namely a negative sign would result in the former case (from equation 5); while a positive sign would imply that the latter is the case. In this case the \( y_{t-1}^{prs} \) variable simply measures last period's impact of \( y_{t}^{prs} \) on the dependent variable which would be expected to be positive. On estimation equation 6' gave;

\[ S_{t}^{prv} = -6542.01 + 0.31 y_{t}^{prs} + 0.87 S_{t}^{c} + 307.43 R_{t} \]

\[- (5.10) \quad (7.29) \quad (12.69) \quad (4.50) \]

\[- 0.13 y_{t-1}^{prs} \quad + 0.76 u_{t-1} + \varepsilon_{t} \quad DW = 1.7581 \]

\[- (2.98) \quad (6.20) \]

\[ R^{2} = 0.9576 \]
As it turned out the sign of the coefficient of the lagged $\gamma^\text{PRS}_{t-1}$ variable is negative which supports the idea that $\Delta y^\text{PRS}_t$ best explains the dependent variable. As expected equation 6 exactly reproduces equation 4 with $\gamma^\text{L}_1$ in 6' equal to the sum of $\gamma^\text{L}_1$ and $\gamma^\text{L}_4$ in equation 4'.

1b: The SLD model: We may now proceed to introduce some more dynamics in equation 6' by including the lagged value of the $s^C_t$ variable as another explanatory variable; on estimation this gave;

$$s^\text{PRV}_t = -5966.08 + 0.29 s^\text{PRV}_{t-1} + 0.87 s^C_t + 0.29731 r_t - 0.15 s^\text{PRV}_{t-1}$$

(-6.55) (8.26) (15.71) (5.38) (-4.20)

$$+ 0.22 s^C_{t-1} + 0.72 u_{t-1} + \epsilon_t \quad R^2 = 0.9766$$

(3.92) (5.22) DW = 1.8570

We may firstly observe from this equation that the add-on hypothesis is supported, this time however only at the 1.0% level of significance. The equation further supports the idea of $\Delta y^\text{PRS}_t$ rather than $y^\text{PRS}_t$ being the underlying hypothesis. With a positive and significant coefficient of the lagged $s^C_t$ variable, a still higher value of the $R^2$ and the DW accepting the null hypothesis of no first order serial correlation, three conclusions may be drawn from these preliminary results: The add-on hypothesis is supported; the change of $y^\text{PRS}_t$ plays a crucial role in explaining the dependent variable, $s^\text{PRV}_t$. A more dynamic formulation might be a better means of examining our hypotheses; to which we now proceed.

2a: The GDL model: Starting from equation 1, disregarding at the moment the $z$ vector, introducing an infinite number of lags in the explanatory variables and assuming their effect to be of a geometrically declining form, we obtain; $s^\text{PRV}_t = y^0 + \sum_{i=0}^{\infty} \lambda^i (\gamma^\text{PRS}_t + \gamma^C s^C_{t-1}) + u_t$ (7)
Lagging the \( t \) subscripts by one period, multiplying both sides by \( \lambda \) and subtracting from the original formulation 7, gives:

\[
S_{t}^{\text{DPRV}} = (\gamma_{0} - \lambda \gamma_{0}) + \gamma_{t}^{\text{DPRV}} + \gamma_{z}S_{t}^{C} + \lambda S_{t-1}^{\text{DPRV}} + \varepsilon_{t}
\]

\[
\varepsilon_{t} = u_{t} - \lambda u_{t-1}
\]

Equations 8' and 8'' may be obtained in a way similar to the one by which 4' and 4'' were obtained, if it is assumed that \( \Delta y_{t}^{\text{DPRS}} \) has the same lag distribution as \( y_{t}^{\text{DPRS}} \). Thus we have the well known Koyck transformation with the one period lagged dependent variable appearing in the right hand side, and the error being a first order moving average (MA1) process of the original errors. As a result, if \( u_{t} \) in equation 7 is "white noise" \( \varepsilon_{t} \) in equation 8 will be necessarily autocorrelated. In face of the lagged dependent variable OLS estimates, will therefore be both biased and inconsistent. A strategy for detecting and taking care of autocorrelation must therefore be the cornerstone of our estimation procedure. In here the MA1 process will be approximated by a first order autorogressive (AR1) Markov process of the form

\[
u_{t} = \rho u_{t-1} + \varepsilon_{t}
\]

where \( \varepsilon_{t} \) will be a serially independent disturbance with zero mean.

A Maximum Likelihood Iterative technique will be applied to obtain values of the \( \rho \)'s. When significant values of \( \rho \)'s are found the ML estimates will be reported. When \( \rho \)'s values are insignificantly different from zero, equations will be reestimated with OLS and these estimates will be reported.

Some flexibility will be adopted with regard to the significance of the \( \rho \)'s, and ML estimates will be reported even when values of \( \rho \)'s are less than significant at the traditional levels of significance, since it was found that even relatively insignificant \( \rho \)'s can do damage to the obtained results. When
OLS estimates are reported, the values of $\rho$'s and their 't' statistics, obtained from the original ML equation are also reported. In view of the well known problems with the DW statistic in face of the lagged dependent variable, a Langrange Multiplier (LM) test will be applied in both cases to detect for higher (up to fourth order) autocorrelation. Significant values of the LM's as well as the DW's may also be taken as indicative of a misspecified equation (Harvey, 1981, Granger and Newbold, 1973). Sargent (1968) reported 'Monte Carlo' results of experiments in such a framework with use of five different techniques. A technique similar to ours, was found to perform better than Ordinary Least Squares, Two Stage Least Squares, and Three Pass Least Squares. In our case the procedure was generally successful too. Up to this point, the analysis was based on the assumption that the original disturbances $u_t$, were "white noise", something which "a priori there is no reason to suppose" (Malinvaud, 1970, p388). If the original errors exhibit positive first order autocorrelation, the application of the Koyck transformation will tend to remove it, which in the special case $\rho = \lambda$ will be exactly the case. When this happens OLS become obviously a less inappropriate technique and the Koyck transformation proves to be of further benefit. It is not surprising that as a result of the obvious problems associated with the "white noise" assumption, some researchers have preferred to use directly OLS in framework such as the above (e.g. Surrey, 1970). This is equivalent to assuming that the original errors are MA(1) or AR(1), not a less stringent assumption either.

Results from estimating equation 8 are reported in table 2. As in the case of the SLS and SLD models equation 8' is firstly estimated and it is followed by equations 8 and 8". In terms of explanatory power equation 8" ranks first and it is followed by equation 8' and 8 respectively. This supports earlier contentions. All three equations are well specified with all variables significantly different from zero and with expected signs. For the relevant degrees of freedom, in equations 8 and
8" the coefficients of the $y_{t}^{prs}$ and $S_{t}^{c}$ variables are significantly different from each other; while the coefficient of the latter is insignificantly different from one at the 5% level in equation 8; at the 1.0% level in equation 8' and supports the imperfect substitutability hypothesis in equation 8". Coming to more general considerations, the LM tests do not indicate the presence of significant autocorrelation in any of the equations. On balance, it appears that the GDL model, supports the add-on hypothesis. It clearly rejects the LCH predictions.

The reasons we have neglected the interest rate so far, will become clear later on; at the moment we may examine how our specifications 8, 8' and 8" will be modified in the face of its inclusion. Thus adding $R_{t}$ to equation 7 and applying the Koyck transformation gives;

$$s_{t}^{prv} = (\gamma_{0} - \lambda \gamma_{0}) + \gamma_{1} y_{t}^{prs} + \gamma_{2} S_{t}^{c} + \gamma_{3} R_{t}$$
$$- \gamma_{3}^{\lambda} R_{t-1} + \lambda S_{t-1}^{prv} + \epsilon_{t}$$
$$\epsilon_{t} = u_{t} - \lambda u_{t-1}$$

(9)

Similarly with equations 4' and 4" equations 9' and 9" may be obtained, under the assumption that $\Delta y_{t}^{prs}$ has the same distributed lag structure as $y_{t}^{prs}$. In equation 9 the interest rate appears along with its lagged value, the latter having a coefficient equal to minus the product of the coefficients of the $S_{t}^{c}$ and $S_{t-1}^{prv}$ variables a restriction which can be imposed and/or tested on estimation. As it turns out the introduction of the $-\gamma_{3}^{\lambda} R_{t-1}$ term has the effect of changing the shape of the geometric distribution of the $R_{t}$ variable since it can be shown that while in the case this term does not appear the stream of increments of $s_{t}^{prv}$ induced by a sustained increase of one unit in $R_{t}$ will be $\frac{\gamma_{2}}{1-\lambda}$, when it appears, the stream of increments is given by $\frac{\gamma_{2} + \gamma_{3}}{1-\lambda}$ (Surrey, 1970).
Depending on the sign of the coefficient $\gamma_3$, the lag will be flatter (if $\gamma_3 > 0$) or sharper (if $\gamma_3 < 0$). This particular feature of the GDL model introduces some flexibility to it, in that it allows the researcher to introduce, or not introduce the lagged values of the explanatory variables, according to his/her theoretical expectations. When a sharper or flatter distribution is expected this will immediately show-up in the sign of the lagged value, which may be taken to support or reject the a priori expectations. This procedure has considerable advantages in estimation, since testing coefficient constraints requires the use of a non-linear estimation program. It presupposes, however, that the GDL model is the correct one, and it is dangerous not to make this clear. Since in our case we only need to check the obtained results under three different frameworks, i.e. to check their robustness, we will not be testing the appropriateness or lack thereof of the models, themselves. Under this understanding the results of estimating $9'$, $9$ and $9''$ are reported in table 2. The negative sign restriction of the lagged interest rate variable is supported in equations $9$ and $9''$ but not in equation $9'$, which leaves some ambiguity with regard to the shape of the lag structure of the interest rate variable. Most previous considerations apply; the coefficient of the $S^C_t$ variable being insignificantly different from one at the 1.0% level of significance in equations $9$ and $9''$, and at the 5% level in equation $9'$.

The reasoning developed so far has interesting implications for our original assumption regarding the distributed lag structure of $\gamma^{prs}_t$ and $S^C_t$. It would be argued that $S^C_t$ being themselves savings should have a sharper lag distribution than $\gamma^{prs}_t$, in which case equation $9'$ would be modified to read

$$
S^{prv}_t = (\gamma_0 - \lambda \gamma_0) + \gamma_1 \gamma^{prs}_t + \gamma_2 S^C_t - \gamma_2 \lambda S^{c}_{t-1} + \gamma_3 R_t - \gamma_3 \lambda R_{t-1} + \lambda S^{prv}_{t-1} + \epsilon_t$$

$$
\epsilon_t = u_t - \lambda u_{t-1}
$$

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**DEPENDENT VARIABLE: PRIVATE SAVINGS (SPV) ; ANNUAL DATA: 1951-1981 U.K.**

"Tests of Prior Hypotheses in the Framework of the General Distributed Lag (GDL) model"

**Table 2**
and similarly 10' and 10'', retaining our previous assumption regarding the lag distribution of the $\Delta Y_t^{prs}$ variable. The results of estimating equations 10', 10, and 10'' are also reported in table 2. The negative sign restriction of the lagged $S_t^c$ variable, is supported in the two equations which include the $\Delta Y_t^{prs}$ variable, (10' and 10''), but not in equation 10. Moreover, in the last equation the imperfect substitutability hypothesis is supported while equations 10' and 10'' support the add-on at the 5% and 1.0% levels of significance respectively. Interestingly enough all equations are free from first order autocorrelation, and therefore OLS estimates are reported. We examine the implications of this phenomenon in the next page. As the LM tests indicate, our equations do not suffer from higher order autocorrelation either.

We now examine few interesting implications derived from the analysis so far. Firstly, unless we have a priori reasons to expect the $Y_t^{prs}$ variable to have, itself, a sharper distribution than the one implied by our estimated equations so far, and given the found insignificance of the lagged $S_t^c$ variable, equation 9 is the best which can be derived under a GDL model which ignores $\Delta Y_t^{prs}$. It gives

$$S_t^{Dyv} = -4528.35 + 0.11 Y_t^{prs} + 0.83 S_t^c + 220.84 R_t - 296.99 R_{t-1}$$

$$+ 0.26 S_{t-1}^{Dyv} - 0.05 u_{t-1} + \epsilon_t$$

$$DW = 2.0261, R^2 = 0.9964, LM = 5.96$$

Equation 9 has some very interesting features. Firstly, it has the highest explanatory variable from all equations which disregard $\Delta Y_t^{prs}$. Secondly it exhibits no autocorrelation of any order. Thirdly it provides a very interesting example of a case when the original assumption of "white noise" residuals is violated, and where the use of the Koyck transformation results in the correction of this problem. Indeed the non existence of first order auto-
correlation in equation 9 implies that the original residuals were positively autocorrelated with either an AR(1) or an MA(1) process, which was nearly exactly removed by the inducement of a negative MA(1) by the use of the Koyck transformation. Fourthly, the $\gamma_1 = \gamma_2$ prediction of the LCH is rejected at the 5% significance level. Fifthly, the add-on is supported at the 1.0% level. This, however, is the most we may expect if we disregard $\Delta y_t^{PRS}$. Its inclusion, as an explanatory variable, leads to a further increase in the explanatory power of the equations. Moreover, interestingly enough, in equation 10", the coefficient of the $y_t^{PRS}$ variable is insignificantly different from zero, which further supports the idea that $\Delta y_t^{PRS}$ best explains the dependent variable. When $y_t^{PRS}$ is dropped equation 10' obtains again.

For reasons to become clear when the Life Cycle Hypothesis will be tested, we may now assume again that the function $f(\Delta y_t^{PRS})$ is homomorphic write it as $f(y_t^{PRS}) - f(y_{t-1}^{PRS})$ and substitute it in equation 10'. This results in equation 11 which on estimation gives:

$$
S_{t}^{PRV} = -2264.34 + 0.31 y_t^{PRS} + 0.88 S_t^{C} + 215.52 R_t - 0.27 y_{t-1}^{PRS} \\
(-2.93) \quad (6.64) \quad (16.41) \quad (4.26) \quad (-4.43)
$$

$$
-0.29 S_t^{C} - 119.60 R_{t-1} + 0.57 S_{t-1}^{PRV} - 0.08 y_{t-1}^{PRS} + \epsilon_t \\
(-2.14) \quad (-1.79) \quad (5.48) \quad (-0.39)
$$

$$
R^2 = 0.9979 \quad DW = 2.0652 \quad LM = 1.35
$$

Equation 11, reproduces exactly equation 10", with the coefficient of the $y_t^{PRS}$ variable being equal to the sum of the coefficients of $y_t^{PRS}$ and $\Delta y_t^{PRS}$ in equation 10". As expected the sign of the lagged $y_t^{PRS}$ variable is negative; (note however that in this framework this may also be explained in terms of a sharper distributed lag structure of the $y_t^{PRS}$ variable which indicates the importance of the SLS model, on this respect). Equation 11 retains all the interesting properties of equation 9. Moreover it exhibits some more. Firstly a more general
form of the GDL model is supported. Secondly the LM test is further reduced which may be taken to imply a better specification (Harvey, 1981). The lagged $y^{prs}_t$ variable appears from direct substitution of $\Delta y^{prs}_t$ for its equivalent $y^{prs}_t - y^{prs}_{t-1}$, rather than any a priori expectations regarding the distributed lag structure of the $y^{prs}_t$ variable. Thirdly, all variables now are significantly different from zero at the 5% level, and the lagged $R_t$ at the 10% level, of significance. Fourthly, compared with equation 10, equation 11, provided an interesting example of a case where the addition rather than the exclusion of appropriate explanatory variables solves multicollinearity problems (Davidson, et al, 1978) as the coefficient of the lagged $S^c_t$ variable, is now markedly significantly different from zero. Finally, we may now observe that the constant term in equation 11, although significant under conventional standards, is fairly small quantitatively (15% of the mean value of the dependent variable). Moreover, given that OLS is the estimation technique in here, the probable existence of errors of measurement in the explanatory variables, will bias the constant term upwards. It would then be argued that more reliable estimates of the coefficients could be obtained, if the constant term was suppressed. In this case equation 11 bears another very interesting feature. Namely in its estimated form it becomes indistinguishable from the Life Cycle Hypothesis General Distributed Lag Equivalent (LCHGDE) model, if the interest rate variables are dropped. Alternatively, if it is posited that their effects are zero at steady state, or subsumed in the coefficients of the relevant explanatory variables out of steady state. To this, however, we now turn.

2: The LCHGDE model: The most recent version of the LCH model is given in Modigliani (1975) as;

$$C_t = a y^{prev}_t + (\delta - r)A_t$$  \hspace{1cm} (12)

where $C_t$ denotes consumers expenditure in period $t$ and $y^{prev}_t$ is defined as in
identity 2, but it also includes expected capital gains. \( A_t \) is end of period wealth, \( r \) the rate of return on assets and \( a \) and \( \delta \) are constants. This is an aggregate steady state form of the LCH and out of steady state \( a \) and \( \delta \) may vary with a number of factors including the rate of interest, the life profile of income and the expected productivity growth. Under fairly acceptable assumptions, exposted by Modigliani (p.16) equation 12 and the identity

\[
A_t = A_{t-1} + y_{t-1}^{PV} - C_{t-1}
\]

(13)

can be shown (Spiro, 1962, Davidson et al, 1978) to result in an equation of the form;

\[
C_t = a y_{t}^{PV} + (\delta - r - a) y_{t-1}^{PV} + (1 - \delta + r) C_{t-1} + \varepsilon_t
\]

(14)

\[
\varepsilon_t = u_t - u_{t-1}
\]

From equation 14 and the identity \( y_t^{PV} = s_t^{PV} + C_t \)

(15)

we may, on performing the relevant manipulations, obtain the equation;

\[
s_t^{PV} = (1-a) y_t^{PV} - (1-a) y_{t-1}^{PV} + (1+r-\delta) s_{t-1}^{PV} + \varepsilon_t
\]

(16)

\[
\varepsilon_t = u_t - u_{t-1}
\]

Equation 16 is one of the GDL form, and very similar in spirit to equation 11. Under the assumption of a correctly specified model, this equivalence might cast some doubt to the fairly common critique that the GDL model is rather ad-hoc. The problem resulting from having no constant in the equation is that both the \( R^2 \) and the LM tests (as based on the former) become unreliable, and we do not report them. Two important means of comparison are therefore lost. Some insight may be gained if the equation is firstly estimated with a constant term, and then
without it; then if the constant term is insignificant it could be said that the equation without constant will exhibit similar properties in terms of the $R^2$ and the LM test, to the ones of the equation which includes a constant term. This procedure, although not necessarily reliable all times, we judge to be much better than the alternative of reporting those statistics although their unreliability is proved beyond doubt. The results obtained from estimating equation 16 are given in table 3, As it stands, however, equation 16 provides no means by which rival hypotheses may be tested; but on substituting identity 3 in equation 16 we obtain

$$S_{t}^{prv} = (1-a) Y_{t}^{prs} + (1-a) S_{t}^{c} - (1-a) Y_{t-1}^{prs} - (1-a) S_{t-1}^{c}$$

$$+ (1+r^{-}a) S_{t-1}^{prv} + \epsilon_{t}$$

$$\epsilon_{t} = u_{t} - u_{t-1}$$

(17)

The obvious implication of equation 17 is that

$$f^{prv} Y_{t}^{prs} = f^{c} S_{t}^{c}$$

where $f^{i}$'s represent the first derivatives of $Y_{t}^{prs}$ and $S_{t}^{c}$ with respect to $S_{t}^{prv}$. Moreover, equation 17 in its estimated form is the exact equivalent of an equation derived from the GDL model where $\Delta Y_{t}^{prs}$ is taken as a separate explanatory variable and substituted for its equivalent $Y_{t}^{prs} - Y_{t-1}^{prs}$, and where a sharper distribution is assumed for the $S_{t}^{c}$ variable. Equation 17, is also a special case of equation 11 as explained. Its main interesting feature is that it provides a means of testing the perfect substitutability prediction of the LCH in its own framework. On estimating equation 17 firstly with a constant it gives;

$$S_{t}^{prv} = -2044.88* + 0.37* Y_{t}^{prs} + 0.89* S_{t}^{c} - 0.34* Y_{t-1}^{prs}$$

$$- 0.37* S_{t-1}^{c} + 0.67* S_{t-1}^{prv} + 0.03 u_{t-1} + \epsilon_{t}$$

(17)

$$(-2.05) (7.24) (13.34) (-5.03)$$

$$(-2.11) (5.12) (0.15)$$
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**TABLE 3**

**DEPENDENT VARIABLE: PRIVATE SAVINGS (SDY)**

The life cycle hypotheses, Generalized Partially Lag Equivariant (GPDE) and Partial Hypotheses.
\[ R^2 = 0.9961 \quad DW = 1.8860 \quad LM = 11.40 \]

Dropping the just significant at the 5\% level, and quantitatively fairly small (13.5 \% of the mean value of the dependent variable), constant, we obtain equation 17 in table 3. Ignoring any other considerations it may be seen in equations 17 and 17' that the equality of the coefficients of the \( v^{prs}_t \) and \( S^C_t \) variables restrictions is not satisfied at the 5\% level of significance: In contrast the add-on is supported in both cases; the \( S^C_t \) coefficient being always insignificantly different from one at the 5\% significance level. Surprisingly the \( S^C_t \) coefficient is even less significantly different from one in equation 17', which excludes the constant term, i.e. in the exact framework of the LCH GDLE model. The other drawback of equation 17 is that it appears to suffer from higher order autocorrelations (at the 2.5\% but not the 1\% significance level), which also implies a deficient specification. Including finally the interest rate in its level and its one period lagged form in equation 17 we obtain equation 11 and on dropping the constant term we obtain equations 18 and 18' in table 3. The consistently significant interest rate variable in the LCH framework amounts to positing that this variable has also a differential impact on the dependent variable, apart and above the one captured by the coefficients of the relevant explanatory variables. Regarding rival hypotheses, the comparison of equations 11 and 18' is hardly favourable for the LCH model since although all (apart from the lagged \( R_t \)) coefficients and 't' statistics are increased the coefficient of the \( S^C_t \) variable is now even less significantly different from one than in equation 11.

3\a: The S.A.O. model: We may now proceed to our third framework, the simple add-on model. We will be trying to show that the estimated forms of both the GDL and the LCHGDLE models can be obtained by simply substituting identities 2 and 3 in the equation

\[ S^{prs}_t = \gamma_1 S^{prs}_{t-1} + \gamma_2 \Delta Y^{prs}_t \quad (19) \]
Equation 19 can be derived either by assuming directly that

\[ S_{PrS}^t = f(S_{PrS}^{t-1}, \Delta_{PrS}^t) \]  

(20)

or alternatively by assuming

\[ S_{PrS}^t = f(\Delta_{PrS}^t) \]  

(21)

where \( \Delta_{PrS}^t \) affects the dependent variable by a geometrically declining lag. In its simplest form equation 19 is little more than an accounting identity. However it has some very interesting properties. Firstly it shares with the LCH the position that no constant term is required for the explanation of the dependent variable, although in equation 19 this is not due to any homogeneity postulate. Secondly on allowing its coefficients to vary freely equation 19 provides a direct test of the LCH, since under fairly innocent assumptions exosed in Modigliani (1975), equation 19 reduces to equation 12. Swamy (1968) has estimated equations of the form 19 for different countries and his tests Modigliani has named as "direct test" (p 17) of the LCH. On estimating equation 19 (and 19' with a constant term) we obtain the results reported in table 4. As expected the value of the constant is fairly small, and it is insignificantly different from zero at the 5% significance level. One is tempted to comment on these results since the LCH predicts a 0.9 coefficient for the lagged savings variable (reflecting the effects of the wealth variable, \( A_t \), in equation 12), a prediction markedly contrasted in equation 19, where the obtained coefficient is 0.97 and insignificantly different from one at the 5% level. This is not however the central aim of our work, to which aim we presently return. In its present form equation 19 has no implications for rival hypotheses of corporate savings. On substituting however identity 3 in equation 19 and rearranging we obtain an equation which for estimation purposes may be written as:
| Year | Value | (5'3) | (4'1) | (3'0) | (1'9) | (1'8) | (1'7) | (1'6) | (1'5) | (1'4) | (1'3) | (1'2) | (1'1) | (1'0) | (0'9) | (0'8) | (0'7) | (0'6) | (0'5) | (0'4) | (0'3) | (0'2) | (0'1) | (0'0) |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 2013 | 1.43  | 69.05  | 89.49 | 628.0 | 123.6 | 9.3   | 1.7   | 0.3   | 0.1   | -0.4  | -0.2  | -0.1  | -0.0  | -0.9  | -0.8  | -0.7  | -0.6  | -0.5  | -0.4  | -0.3  | -0.2  | -0.1  | -0.0  |
| 2014 | 1.0  | 69.05  | 89.49 | 628.0 | 123.6 | 9.3   | 1.7   | 0.3   | 0.1   | -0.4  | -0.2  | -0.1  | -0.0  | -0.9  | -0.8  | -0.7  | -0.6  | -0.5  | -0.4  | -0.3  | -0.2  | -0.1  | -0.0  |
| 2015 | 6.52  | 1.91   | 69.05 | 89.49 | 628.0 | 123.6 | 9.3   | 1.7   | 0.3   | 0.1   | -0.4  | -0.2  | -0.1  | -0.0  | -0.9  | -0.8  | -0.7  | -0.6  | -0.5  | -0.4  | -0.3  | -0.2  | -0.1  | -0.0  |
| 2016 | 1.75  | 69.05  | 89.49 | 628.0 | 123.6 | 9.3   | 1.7   | 0.3   | 0.1   | -0.4  | -0.2  | -0.1  | -0.0  | -0.9  | -0.8  | -0.7  | -0.6  | -0.5  | -0.4  | -0.3  | -0.2  | -0.1  | -0.0  |

**Dependent Variable:** Personal Services (PES)

**Intercept: 9.22, Private Services (PES) in 2014**
where the restriction \( \gamma_3 = 1 \) is implied. Equation 22 therefore provides a third framework for testing rival hypotheses, the coefficient of the \( S^C_t \) variable being again the crucial factor. On estimation equations 22 and 22' with a constant term gave the results reported in Table 4. Although the constant is significantly different from zero by conventional standards at the 5% level of significance it is very small (4.4% of the mean of the dependent variable) and it can be safely dropped. Moreover the coefficient of the \( S^C_t \) variable is insignificantly different from one which further supports the add-on hypothesis, and casts doubt on the LCH. From equation 22 to equations 18' and 11 is only a simple step. Thus, we firstly substitute identity 3 in equation 22 and obtain (in estimation form):

\[
S^{PRV}_t = \gamma_1 S^{PRV}_{t-1} + \gamma_2 S^C_{t-1} + \gamma_3 \Delta^{PRS}_t + \gamma_4 S^C_t
\]  
(23)

with the implied restriction \( \gamma_4 = -\gamma_2 = 1 \). Substituting in equation 23 the equivalent of \( \Delta^{PRS}_t \), \( \gamma^{PRS}_t \) \( \gamma^{PRS}_{t-1} \) we obtain (in estimation form)

\[
S^{PRV}_t = \gamma_1 S^{PRV}_{t-1} + \gamma_2 S^C_{t-1} + \gamma_3 \gamma^{PRS}_t + \gamma_4 \gamma^{PRS}_{t-1} + \gamma_5 S^C_t
\]  
(24)

i.e. the exact equivalent of equations 17 and 17' (with and without a constant term respectively.) The additional restriction being \( \gamma_3 = -\gamma_4 \). Moreover on adding the interest rate variables in 23' and 23 we obtain the equivalent of equation 10', of the GDL model which highlights the equivalence of the SAO with the LCHGDLE and GDL models. Adding the interest rate variable to the SAO model is equivalent to modifying our behavioural equations 21 and 20 to read:
\[ S_{t}^{prs} = f(\Delta Y_{t}^{prs}, R_{t}) \]  
\[ (21') \]

and 
\[ S_{t}^{prs} = f(S_{t-1}^{prs}, \Delta Y_{t}^{prs}, R_{t}) \]  
\[ (20') \]

respectively. If equation 21' is chosen as the underlying behavioural hypothesis, on following the previous steps, we obtain equations 10' and 11 as noticed, but if the alternative hypothesis 20' is chosen we obtain equation 18 of the LCHGDLE model (table 3), an equation which could be directly obtained by observing that equation 18' gave an insignificantly different from zero coefficient of the lagged interest rate variable. Estimating equation 24 as modified by assuming 20', with a constant gives the results reported in table 4 (equation 24') an equation which again could be derived from the GDL model (equation 11) by observing that the lagged \( R_{t} \) variable in this equation too failed to be significantly different from zero at the 5% level of significance. The obtained equations however are worse than the ones in equation 18 which appears to support the exclusion of both the constant and the lagged interest rate variable from the final equation; an idea more in accord to the SAO and LCHGDLE models, the latter augmented to include the interest rate as a separate explanatory variable. Regarding rival hypotheses, similar considerations apply for the results obtained from equation 24' as the ones obtained from the rest of our equations, which is not surprising given, as we have shown, the very close similarity (in terms of the estimated forms of the equations) of the three frameworks. It is worth stressing that the equations reported so far have far more interesting implications than the ones to which we have confined our attention. Namely, by testing the various restrictions implied from the three models, they themselves can be verified or falsified. This, however, is far beyond the scope of this paper. Suffice is to say here that regarding our aim, i.e. testing of rival hypotheses, it appears that the add-on hypothesis is overwhelmingly supported in all frameworks and in most cases. Support is moreover given to our second consideration, that is, the idea that it is the change rather than the level of the \( Y_{t}^{prs} \) variable which best explains savings behaviour.
Our third aim was rather methodological. We have seen that if appropriately treated all equations derived so far may be taken as special forms of equation 11 in which they may be nested. It would appear, therefore, that a general to specific approach could be followed. Since all three, SAO, LCHGDLE and GDL models can form a uniquely ordered set, a sequential testing procedure could be applied by testing the implied hypotheses in increasing order of restrictiveness. With maintained hypothesis equation 11, we could proceed to test down for the most specific model. Two are however the crucial factors which dissallowed such an undertaking. Firstly, version 11 of the GDL model was not a priori known. The appropriateness of using \( \Delta y_{t}^{prs} \) as separate explanatory variable should be first established. Secondly we will repeat that testing for the correct model was not our aim, at least in this paper. Rather, we attempted to test the rival hypotheses of section I by use of a comprehensive number of specification-models taking for granted the (clearly implausible) assumption that each one of them was the correct model. Interestingly enough it was found that the estimated form of the GDL and LCHGDLE models can be derived from simply substituting identities 2 and 3 in the simple equation 19. It appears that the apparently complicated forms of these models conceal their essentially simplistic nature.

III: Data and Discussion:

Throughout, the personal disposable income \( (y_{t}^{prs}) \), consumers expenditure \( (C_{t}) \), and corporate retentions \( (S_{t}^{C}) \) series used, are the usual series given in the National Income Accounts, i.e. Economic Trends, 1982. Annual Supplement and the National Income and Expenditure (Blue Book). The rest of the series, \( s_{t}^{prv}, s_{t}^{prs} \) and \( y_{t}^{prv} \), were constructed to conform with identities 2 and 3 in the text and the identity \( s_{t}^{prs} = y_{t}^{prs} - C_{t} \) (25). The interest rate series used is the Treasury Bill Rate, obtained from the International Monetary Fund (IMF) Financial Statistics, 1983. Thirty one annual observations
were used, covering the 1951-1981 period for the UK. Problems with post-war readjustments and the rationing of durables determined the starting period, and data availability the end period. Personal disposable income and corporate retentions are defined after tax and before providing for depreciation, stock appreciation and addition to tax reserves. All series were deflated by use of the Implied Consumers Expenditure Deflator. For what these data are, they provide outright support to the add-on hypothesis and we may now proceed to the examination of the main problems involved and the direction towards which our previous analysis would be modified, if it was possible to account for all the existing or potential problems.

The obvious problem is the compilation of the interest rate series from a different source. The Treasury Bill Yield series however, available from the Economic Trends Annual Supplements and the National Income and Expenditure Financial Statistics, do not go further back than 1955, and these four observations were the main reason why results obtained by the use of this series were not reported. Suffice is to say here, however, that the results which emerged when all previous regressions were run with the use of the Treasury Bill Yield series, were generally more favourable to the add-on hypothesis. All regressions were also run by using the short-run Government Bond Yield series and the long-run Government Bond Yield series obtained from the IMF Financial Statistics, 1983. By and large the same results were obtained which implies that there is something here to be explained.

There is a lot of controversy regarding the effects of the interest rate on consumption both in theory and the empirical evidence available. In terms of the former, two opposing effects are normally identified; the income effect, which tends to increase consumption and the substitution effect which tends to reduce consumption. It could be therefore argued that our findings support the substitution effect idea, an interpretation which we do not favour;
an alternative interpretation, however, is beyond the scope of this paper. In terms of the empirics, previous findings are generally inconclusive (see summary in Feldstein and Fane, 1973). Similar findings to ours are more recently reported in D. Peel (1975) for the UK and by use of the same interest rate series, (the Treasury Bill Rate). The final problem with regard to the interest rate is the lack of a real interest rate series which ideally should be used. To our knowledge none of the other studies have used such a measure, and in our case the problems associated with constructing one and/or choosing the 'appropriate' series by which the 'real' series would be constructed, deterred us from doing so. This may bias the results in case of accelerating inflation.

The rest of the problems which we will be considering in here may be divided in two categories, being firstly, data problems, including a) the philosophy underlying the compilation of the data by the official compilers, and the extent to which they conform with our theoretical (or even general consistency) requirements and b) our treatment of the given data. Secondly, econometric problems. It is convenient to start from the second part of the first category. The obvious critique to make is that, having not accounted for depreciation, the obtained coefficient of the \( S^c_t \) variable will be upwards biased. Although as a start, this criticism may be well taken there are various reasons why the "net" measure was not chosen. To start with, depreciation allowances far from being equal in value to the wear and tear of plants and machinery, are well known to be used either for profitable investment when opportunities arise, or for tax avoidance, according to the discretion of those who control them. As Feldstein (1973) observes, due to accounting conventions used to calculate capital consumption "net" \( S^c_t \) are bound to be an underestimate of the true net corporate savings. While the above do not contradict the fact that the "gross" measure is an overestimation, there are more considerations to come. Indeed by using both measures Feldstein
found similar results which led him to conclude that "further work to improve the measurement of retained earnings would not alter any conclusions" (p.165). Similarly in Lambrinides (1974) the respective coefficients obtained by use of a "gross" and "net" measure of the \( S^C_t \) variable were very similar.

Even if we accept, however, that our estimates will be upwards biased for this reason, it may be said that such a bias is desirable to a certain extent, if it is found that for reasons related to problems of a different type there are biases going towards the opposite direction. Indeed as it is widely known the official compilers treat differently the savings made out of corporations, that is as corporate income and retentions, while the savings done by unincorporated businesses are instead treated as personal disposable income, and personal savings. This treatment obviously introduces a downward bias on the coefficient of the \( S^C_t \) variable, which depending on the significance of the unincorporated sector, may very well be larger than the upward bias resulting by the use of the "gross" measure of \( S^C_t \). As our data show the ratio of the self-employment income to personal disposable income which was an average of 10.4% during the 1963-1970 period has increased to 11.9% in the 1970-1981 period. This increase is even bigger if the last two years are excluded because it is during these two years only that the ratio shows a tendency to decline. On its own this says that even if we assume that self-employed retain a constant proportion of their profits, the proportion of this source of "quasi" \( S^C_t \) included in the \( \nu^{prs}_t \) has gone considerably up in the last decade. If we make the more reasonable assumption, i.e. that their "quasi" retentions follow the movements of the corporate retentions series, a more impressive picture emerges since the business retentions ratio to both personal disposable, and private (\( \nu^{prs}_t \) and \( \nu^{prv}_t \) respectively) income, has shown a profound increase during the last decade; (again with a tendency to decline only in 1980 and 1981). Moreover, this is not the end, since there are not few people who could argue that having more severe problems with external finance, small
businesses are exactly the ones which need relying on their retained earnings, the
most; see for example Steindl, 1982, p.74). The inconsistent (although not
necessarily wrong) inclusion of "quasi" $S^C_t$ in the $y^{prs}_t$ instead of the $S^C_t$
series, will lead to an overestimation of the coefficient of the former and an
underestimation of the coefficient of the latter.

Another inconsistency in the National Accounts was firstly observed
and Cuthberston (1982) in the UK; it relates to the treatment of the savings
through private insurance (pension funds) as opposed to the national insurance ones.
Namely, contrary to the treatment of the latter, contributions to pension funds
are treated as personal savings and income, while benefits paid back to households are
not. In our framework this results in both the dependent variable, $S^{prv}_t$, and
the $y^{prs}_t$ variable being inflated from the inclusion of the difference of
contributions to benefits in them, since contributions are normally double than
benefits. The fact that the $y^{prs}_t$ variable includes this part of 'contractual'
savings will result in an upwards bias of its coefficient. No similar bias is
to be expected, however, in the coefficient of the $S^C_t$ variable from the
inclusion of savings through pension funds in the $S^{prv}_t$ variable, since being
contractual in nature, pension funds are not to be expected that they will be
affected by changes in corporate retentions. It may be added that the significance
of pension funds and therefore the associated biases, have increased in the recent
years. Indeed the difference of the contributions to pension funds over the
benefits have increased from an average of 4.8% in the 1963-1970 period, to a
8
6.6% in the 1971-1981 one.

The conclusion of the above analysis is that if correction of the
existing inconsistencies of the data was possible, the result would be expected
to be a higher coefficient of the $S^C_t$ variable and a lower coefficient for the
variable. On its own the first suggest that the add-on hypothesis would be supported even more profoundly. In connection with the second it implies that the LCH prediction of equal coefficients of the \( y_{t}^{prs} \) and \( S_{t}^{c} \) variables, would still be rejected.

We may now turn to the second source of problems, i.e. econometric problems. We have seen that as a result of the observed property of the Koyck transformation to remove positive first order autocorrelation when appearing in the original residuals, OLS were used to a substantial number of our equations, and most importantly to the most significant ones. This method now, is well known to lead to asymptotically biased and inconsistent estimates when a number of the explanatory variables is related to the endogenous variable with other simultaneous relationships. In a framework such as ours, this may be a very serious problem since \( S_{t}^{c} \) is related to \( S_{t}^{prv} \) through the identity 2. Moreover on substituting in 2 the variable \( S_{t}^{prs} \) with its equivalent (\( y_{t}^{prs} - C_{t} \)) we obtain

\[
S_{t}^{prv} = y_{t}^{prs} - C_{t} + S_{t}^{c}
\]  

(2')

which reveals another source of simultaneity through the interdependence of the endogenous variable \( S_{t}^{prv} \) with the other explanatory variable \( y_{t}^{prs} \). Various aspects of the problem may be examined starting from the direction of the bias in the parameter estimates. Indeed very few things can be said on this regard since it appears that the direction of the bias largely depends on our a priori assumptions. It is well established though, that under the conventional assumption of a less than one marginal propensity to save, the coefficient of the \( y_{t}^{prs} \) variable will be upwards biased. This reinforces the similar bias due to pension funds and "quasi" \( S_{t}^{c} \). The constant term, when included, will be either upwards or downwards biased depending on our a priori and generally rather ad hoc assumptions.
regarding the direction of the correlation between the residuals. Similar considerations apply for the coefficient of the $S^C_t$ variable. That the bias will exist in the face of simultaneity has been proved beyond doubt. Yet, the expectation that the parameter estimates will indeed be biased (in the limit too) and inconsistent, has been dismissed on various grounds. These include: the potential existence of recursive relationships in the model, the relative advantages and drawbacks of alternative techniques and finally, empirical observations that even when simultaneity is accounted for, the resulting estimates may not be different from the ones obtained by the use of OLS. The last two contributions are largely associated with the name of Modigliani. Thus Ando and Modigliani (1963) argue that, "the only adequate way of solving this difficulty would be to construct a complete model of the US economy" and that, even in this case "the efficiency of the estimates might be reduced", (p.411). Moreover when the alternative route is followed, "Modigliani has shown ... that an alternative simultaneous equation model yields about the same results" (Swamy 1968 p.115). Clearly, this does not need to be the case all times, but the very existence of such cases may be a result of recursive relationships being a more widespread phenomenon than it is usually thought. In any case, we think, it would be dogmatic to disallow the possibility of our estimates suffering from the results of simultaneity. There is a case however for arguing that this may not be a very serious problem.

The second serious problem, in frameworks such as ours, that is autocorrelation, did not prove to be a serious one in our case, largely due either to our successful approximation of the MA(1) process by an AR(1) one, or to the interesting feature of the Koyck transformation to remove autocorrelation from the originally positively autocorrelated, with MA(1) or AR(1) errors. Regarding the first issue, we need stress the possibility that such an approximation may not always be successful, which would necessitate the
use of a special program accounting for the MA(1) process, as in Wallis (1979). Either of the procedures however may be dangerous if the original residuals do not obey the non-autocorrelation assumption, which indicates the benefit of the use of a more flexible approach, such as the one in this paper.

Multicollinearity the other serious problem in time series did not appear to cause any problems to our estimates, judging from the very low standard errors. In few cases in which such a problem did turn up, it was found that contrary to conventional wisdom, its presence was eliminated by increasing rather than decreasing the dimensionality of the parameter space, that is by improving the specification of the model. This interesting observation was firstly emphasised by Davidson et al (1978).

The preceding, rather lengthy, discussion helped emphasising the idea that although econometric problems are of a significant nature, and may be very dangerous some times, it is the underlying data concepts which are more crucial in the final resort, since econometric problems can be accounted for, if appropriate care is taken. This problem as Cuthbertson's (1982) findings have revealed, was particularly acute in the recent discussion on the personal savings 'paradox'. Evidently great attention should be paid to the data series used.

As a conclusion we may say that the identifiable problems related to our results, on balance point out towards a direction more favourable to the add-on hypothesis. However since it is not possible to take care of all the potential problems, space must be left for the possibility of biased and/or inconsistent results. Regarding finally the data period covered, we feel that a sufficiently long period was examined to indicate that regarding at least our
analysis of the "controlling group's" behaviour, adding-on is a phenomenon hardly confinable to recent "managerialist" experience.

The following macro-implications can be derived from the previous results: Increases in corporate retentions will result in higher $s_{t}^{prv} / \gamma_{t}^{prv}$ ratios, therefore a lower $c_{t}^{v_{t}^{prv}}$ ratio. This will result in decreasing effective demand, which under conditions of excess capacity, will result in lower output, lower profit rates, lower investment and further reductions in effective demand; thus the economy will tend to be depressed. Policy measures should be directed towards a reduction of savings arising through corporate retentions and/or private pension funds. Measures such as the recent suggestions by the Government to increase workers' contributions to pension funds may have destructive results in the economy. The recent increase in the $c_{t}^{v_{t}^{prv}}$ ratio, which resulted from the dramatic fall in the $s_{t}^{C_{t}^{v_{t}^{prv}}}$ ratio, might be a sign of an upturn; which however may be thwarted by adverse policies.

SUMMARY

Three rival hypotheses have been examined along with the empirical support they have received. Empirical tests were designed and carried out to confront the rival hypotheses with UK time-series data. Three different frameworks were used: A Simple Linear model both Static (SLS) and Dynamic (SLD); a General Distributed Lag (GDL) model and the Life Cycle Hypothesis in its General Distributed Lag Equivalence (LCHGDLE) form; a Simple Add-on (SAO) model arising from simple behavioural assumption(s) and three identities. Support was found for the Add-on hypothesis. Evidence was given that it is the difference rather than the level of the personal disposable income variable which best explains the data. The GDL, LCHGDLE and SAO models were found to lead to the
same estimated form; this underlines the essential naïveté of the estimated forms of the GDL and LCH models (see also Marglin, 1974). All three frameworks can be nested in the obtained general form. These findings were obtained by use of private savings \( (S_{t}^{\text{prv}}) \) as dependent variable. Results obtained by use of personal savings \( (S_{t}^{\text{prs}}) \) instead, (Appendix I) have led to slightly different results. The latter do not contradict similar findings by Feldstein and Fane (1973). Some support is, however, given to the idea that \( S_{t}^{\text{prv}} \) should be the appropriate dependent variable.
Footnotes:

1. For a recent survey see Rose (1983).

2. Quantitatively corporate retentions are a much higher proportion of personal disposable income and private income (see below for definitions) than both pension funds, and personal savings taken together. Indeed in the 1963-1970 period the corporate retentions - personal disposable income ratio was an average of 13.7% which became 17.9% in the 1971-1981 period. For the same periods the personal savings ratio was 8.25% and 12.06% respectively, while the part of this ratio which was due to savings through pension funds was only 4.8% and 6.6% for the two periods respectively.

3. Modigliani (1970) seeks support for the perfect substitutability hypothesis by referring to this paragraph. As we will see, however, the second part of the reasoning is hardly favourable to this hypothesis, a fact already observed by Feldstein (1973) and Lambrinides (1972).

4. Organizational saving is defined as the sum of saving done by private and public corporations plus saving of the general government. National saving is the sum of personal plus organizational saving.

5. Tests of the add-on hypothesis may be undertaken by estimating consumption functions and/or savings functions (private or personal). In the latter case the coefficient of the \( S^*_t \) variable in the function \( S^*_t = f(Y^*_t, S^*_t) \), will be equal to one plus the coefficient of the \( S^*_t \) variable in the function \( S^*_t = f(Y^*_t, S^*_t) \), (definitions given in the text). Therefore an insignificantly different from zero coefficient of the \( S^*_t \) variable in an \( S^*_t \) function is support to the add-on hypothesis: See also Appendix I.

6. That is, equations including a lagged wealth variable along with the personal disposable income variable and other relevant explanatory variables. This equivalence, however, is more apparent than real since Ando and Modigliani (1963) and Modigliani (1970, 1975) favour the use of a private income variable, something nearly always overlooked in the existing literature.

7. Literally, private income \( Y^\text{PV} \) is defined as the sum of corporate income \( Y^c \) and personal disposable income \( Y^\text{prs} \) both after tax. Since, however, in the National Income Accounts \( Y^\text{prs} \) includes the part of \( Y^c \) due to dividends \( D_t \), and since \( Y^c = S^* + D_t \), by substituting we obtain identity 3 in the text.

8. The relevant figures were kindly provided by the Bank of England, and cover the 1963 I-1981 II period. We have readjusted the 1977-1981 years to conform with the figures given for these years by the Blue Book.
Appendix I:

Long Run Elasticities, Misspecification and Causality:

In this Appendix we try to show two things. Firstly that it is not a wise thing (although it is theoretically possible) to uncritically switch from private savings to personal savings or consumption functions; due to reasons related with the well known problem of spurious regression. Secondly that if this is done, and such problems do turn up, the back door mechanism of the LR elasticities can, at least in the GDL and LCHGDLE frameworks, provide a means of re-establishing consistent LR estimates of the elasticities. When this mechanism is not available spurious regression problems can only be avoided by making sure that our use of the dependent variable is the one suggested by our theoretical considerations. If this is not done, an undue degree of freedom is left to the researcher to reach results towards which (s)he might be prebiased.

Suppose we start from equation 9 in the text. With a 0.83 coefficient of the $S^c_t$ variable, insignificantly different from one at the 1.0% level, the formula given in p. 14 gives a LR elasticity of 1.12 for the $S^c_t$ variable, which implies that in the LR corporate retentions add on a more than one to one basis to the dependent variable, $S^{prv}_t$. On estimating now equation 9's equivalent personal savings function we obtain:

\[
S^{prs}_t = -41.09.06 + 0.11 S^{prs}_t + 0.05 S^c_t + 280.83 R_t - 232.88 R_{t-1} + 0.43 S^{prs}_{t-1} + 0.004 u_{t-1} + \varepsilon_t
\]

\[
(3.93) \quad (3.67) \quad (0.65) \quad (3.03) \quad (-2.04)
\]

\[
R^2 = 0.9711 \quad DW = 1.9684 \quad LM = 11.72
\]
In this case the insignificantly different from zero coefficient of the $S^C_t$ variable supports our previous findings. The introduction of the lagged $S^C_t$ variable in equation 9 gave equation 10 in table 2; the base year elasticity of $S^C_t$ being 0.82 and insignificantly different from one at the 1.0% level, and the LR elasticity being 1.26. Consider now its equivalent $S^{prs}_t$ function.

\[
S^{prs}_t = 4594.85 + 0.12y^{prs}_t - 0.18S^C_t + 222.34R_t + 0.37S^{prs}_{t-1} \\
(-6.04) (5.34) (-2.55) (3.27) (4.80)
\]

\[-292.64R_{t-1} + 0.35S^{prs}_{t-1} - 0.04u_{t-1} + \varepsilon_t \]  
(4.02) (2.83) (-0.21)  

(10A)

\[R^2 = 0.9849 \quad DW = 2.0055 \quad LM = 8.22\]

Equation 10A reproduces exactly both the SR and LR elasticities of equation 10; the first is found by taking 1-0.82, while the second can be obtained from the formula $-0.18 + 0.37/1-0.35 = 0.29$, i.e. the equivalent of 1.29 for equation 20 which is statistically the same to the one obtained from the latter. The use of the back door mechanism of the LR elasticities provides again outright support to the add-on hypothesis. There is now, however, a crucial difference. Namely the coefficient of the $S^C_t$ variable is negative and significantly different from zero, at the 5% level of significance. It is evident that if equation 10A is estimated instead of equation 10, the obtained coefficient can very well be taken to support the imperfect substitutability hypothesis. Indeed the coefficient of the $S^C_t$ variable in equation 10A is insignificantly different from the coefficient of $-0.25$ obtained by Feldstein and Fane (1973) for the postwar British experience. As already noted the above authors have estimated consumption functions and the $-0.25$ above is the equivalent for an $S^{prs}_t$ function of the 0.25 coefficient of the $S^C_t$ variable they have found in their
consumption functions.

More alarmingly when the personal savings equivalent of equation 10" in table 2 is estimated very similar results are obtained. The $S^C_t$ coefficient although smaller (0.12) this time is still significantly different from zero at the 5% level, giving support to the imperfect substitutability hypothesis. Although the LR elasticities are again equal and consistently support the add-on in both cases, the significance of the SR coefficient of the $S^C_t$ variable is certainly a disturbing phenomenon, which requires close examination. The significance of the problem becomes clearer when one realizes that traditional econometric techniques cannot do much to help us on this account. Indeed it cannot be argued that estimating a consumption or personal savings function will be a misspecification as Marglin (1975) and Pitalis (1982) have done, since such functions can be obtained directly from a private savings function, the only difference being the interpretation of the respective coefficients. Moreover traditionally by misspecification we imply a situation where a potential explanatory variable is inappropriately excluded from the estimated equation. When a variable is a priori known a specification test can be the conventional "t" test applied to the coefficient of this variable. All these are elementary. In our case, however, the estimation of personal as opposed to private savings function does make a difference. As already hinted, such phenomena can always be explained in terms of spurious correlation. Trending as they are our series may very well give rise to spurious significance; this however does not discriminate between the alternative specifications.

An alternative explanation can be given in terms of causality. Indeed it could be argued that although corporate retentions cause private savings, they are rather caused by consumption or personal savings. Regarding
Feldstein's (1973) findings, for example Marglin (1975) was arguing, "household consumption and retained earnings are determined simultaneously and the cause-effect mechanism running from retained earnings to consumption is considerably less plausible than the opposite mechanism" (p.11). This Marglin explains in a fashion very similar to the one of section III of this paper. Namely if consumption in period $t$ exceeds its predicted level, SR corporate profits will increase and since dividends behaviour in the SR is sticky, corporate retentions in the next period will increase more than proportionately. Although this sounds plausible, it is really difficult to provide a similar explanation for our findings when personal savings is the dependent variable. Indeed the only direct way this issue could be attacked, would be the application of causality test on the appropriate variables, such as the one proposed by Granger (1969). It has been suggested (Williams, Goodhart and Cowland, 1976) that tests of this kind will lead to unreliable results if there exists an intervening variable causing both variables under examination. Obviously in our case such a variable exists, that is the private income $y_{priv}^p$ one. On theoretical grounds, private income is the causing factor of all our three variables (private-personal savings and consumption), which disallows the application of causality testing. Indeed, and disappointingly enough, no definite conclusion can be drawn from the above discussion. It helps, however, to highlight the underlying danger of uncritically using seemingly equivalent specifications. To the extent that personal savings or consumption functions are chosen by the researcher as the appropriate dependent variable, our results allow for the existence of some imperfect substitutability. This however is of a very low order.

The danger of spurious results resulting from the use of different dependent variables does remain, however, and in our case it has provided a means of explaining the existence-finding of apparently conflicting results in cases where both the data series and the specifications used are of a similar nature.
For those who do not want such important issues being left to the discretion of the researcher, traditional criteria might be used to compare between the alternative specification. In our case such comparisons gave support to the use of private savings as the dependent variable. Indeed in both cases reported the explanatory power of the equation is higher when private savings are used as a dependent variable. Moreover taking the LM tests as a sign of either autocorrelated residuals and/or the existence of misspecification, or both, we observe that in one case the private savings equation performs better, while in the other both equations have exactly the same performance. These considerations do suggest that there is a case for arguing that private savings should be the dependent variable used in frameworks such as ours. This provides one explanation for the observed difference in our findings and in Feldstein and Fane's findings for the UK.
Bibliography:


