TECHNOLOGY, DIFFUSION, WAGES AND EMPLOYMENT

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.
Technology, diffusion, wages and employment

I. Introduction

The debate on the impact of technological change on employment has a long history. The current concern with the topic has been prompted by the realisation of the potential of microelectronics. This realisation has generated a number of commentaries (e.g. Freeman (1978), Barron and Curnow (1979), Jenkins and Sherman (1979)), the major predictions of which are that the introduction of new technology will lead to unemployment. A rider to this is that the faster new technology is introduced (ignoring international competitive aspects) the higher will be the unemployment resulting. Unfortunately much of this literature is either devoid of theory, or if theory is included it is implicit rather than explicit. The outcome of this is that much of the literature ignores what Beertje (1977) calls compensation effects. In essence, if a labour saving technology is introduced in one sector of the economy there may well be automatic responses in the economy that lead to increased employment elsewhere.

The purpose of this paper is two-fold:

1. To investigate the time path of employment after the introduction of new technology taking account of compensation effects and

2. To investigate, whether, once these compensation effects are allowed for, a higher speed of diffusion does mean lower employment.

The framework used is macro-orientated, firstly because although the paper is not specifically about micro-electronics, the debate on its
impact concerns the macro-economy and thus it is the macro-economy that is the main topic of interest. Secondly, it is only in a macro-orientated framework that one can really investigate compensation effects.

Compensation effects fall into three groups:

1. Technological multiplier effects. These can be exemplified as the employment creating effects in industry \( j \) resulting from the introduction of a new technology in industry \( i \), for a given final demand vector. Thus if new technology is wholly embodied the introduction of new technology will mean increased demand on the capital goods sector and thus employment effects in that sector.

2. Income effects. If new technology leads to changes in income levels or distribution then the pattern of final demand may change with consequent effects on employment.

3. Price effects. New process technology will probably mean lower costs and thus lower prices. New product technology should mean lower quality adjusted prices. The consequent demand increases should stimulate employment.

In this paper we wish to consider all three compensation effects. Technological multiplier effects will be introduced by using a two-sector framework, income effects are introduced by linking aggregate demands to wages and profits and price effects are introduced by linking prices to costs. Prices will not, however, be considered as perfectly flexible. It is well known, (see e.g. Neisser (1942)) that if prices react to notional excess demand and supplies in a Walrasian manner, in general, technological change will not cause unemployment. As believers in the Keynesian view of the world,
we will not allow prices to so react. Thus technological unemployment is a possibility. In Section 2 the theoretical model is detailed and in Sections 3 and 4 the time path of employment in the model is analysed under different time paths of wages. In Section 5 we consider the impact of higher diffusion speeds on employment and in Section 6 conclusions are drawn. The analysis is conducted within the context of a closed economy.

II. The Model

As one of the key factors to be analysed is the technology multiplier effect, the model is a two-sector one with consumption good and capital good sectors. We will consider only process innovations, and we will assume that all process innovations are wholly embodied. Thus to change technology a whole new set of capital goods is required. We will assume the economy is closed.

We consider two technologies, labelled 1 and 2, that are the old and new respectively. Both capital goods are made wholly by labour (although circulating capital could be allowed) but consumption goods are made by capital and labour. The technology of capital goods production is considered in this manner for it yields considerable simplification, however the omission of a capital goods input from capital goods production will bias any compensation effects we find. We represent the quantity relations of the two technologies as follows:

\[ K_{it} = \alpha C_{it} \quad i = 1, 2 \]  
\[ L_{it} = \beta C_{it} + b_i x_{it} \quad i = 1, 2 \]  
\[ x_{it} = \frac{dk_{it}}{dt} \quad i = 1, 2 \]
where $K_{it} =$ stock of capital good $i$ in time $t$

$C_{it} =$ output of consumption goods on capital good $i$ in time $t$

$L_{it} =$ labour employed producing or using capital good $i$ in time $t$

$x_{it} =$ output of capital good $i$ in time $t$.

We will consider that capital has an infinitely long physical life and there is thus no depreciation. Total labour supply will be given by (4)

$$L^S = L_0 e^{nt} \tag{4}$$

We will consider that at time $t$ the economy is in the process of making the transition from using technology 1 to using technology 2, and at any moment in time the two technologies are used in consumption goods production in the proportions $1 - \theta_t : \theta_t$. We specify that $\theta_t$ is the result of a diffusion process the time profile of which can be represented by a logistic curve (5).

$$\theta_t = \frac{1}{1 + \exp(-\eta - yt)} \tag{5}$$

This logistic diffusion process is assumed to be the result of forces outside the model. It summarises entrepreneurs' technique choice behaviour subsuming within it their expectations and anticipations with respect to future prices and profitability of the new technology. The theoretical underpinnings of such a relationship can be found in, for example, Mansfield (1968), Davies
(1979), Stoneman (1981). The apparent absence from this paper of expectations, profitability impacts on technique choice or explicit technique choice modelling, is thus something of a misapprehension for there are all presumed within (5). It is also further assumed that the policy makers can, if they wish, control \( y \), the speed of diffusion. They are assumed to be unable to influence the appearance of new technology (represented by \( n \)).

We will assume that at any time \( t \) the output of the relevant capital good is just sufficient to meet the demands of the consumer goods industry subject only to \( x_i > 0 \). Thus we have (6) for \( i = 1, 2 \), using a \( \dot{} \) to represent a time derivative.

\[
x_{it} = \alpha_i \dot{c}_{it} \quad \text{if } \alpha_i \dot{c}_{it} > 0
\]
\[
= 0 \quad \text{if } \alpha_i \dot{c}_{it} < 0
\]

(6)

This yields (7) and (8) for \( x_{it} > 0 \),

\[
x_{1t} = \alpha_1 ((1-\theta_t) \dot{c}_t - c_t \dot{\theta}_t)
\]

(7)

\[
x_{2t} = \alpha_2 (\theta_t \dot{c}_t + c_t \dot{\theta}_t)
\]

(8)

where \( c_t = c_{1t} + c_{2t} \). Defining \( g_t = \frac{\dot{c}_t}{c_t} \) we may state from (7) and (8) that
\[
x_{1t} > 0 \quad \text{if} \quad g_t > y^{\theta}_t \\
x_{2t} > 0 \quad \text{if} \quad g_t > y(\theta_t - 1)
\]

We will consider the transition process as the economy moves from using only technology 1 to only technology 2 in two stages. In stage 1 both capital goods 1 and 2 are produced and used. In stage 2 only capital good 2 is produced but both 1 and 2 are used. Thus we have

Stage 1 \quad g_t > y^{\theta}_t \quad \text{implying} \quad x_1 > 0, \quad x_2 > 0

Stage 2 \quad g_t < y^{\theta}_t \quad \text{implying} \quad x_1 = 0

We will show that in Stage 2, \( x_2 > 0 \). We will also show that Stage 1 always precedes Stage 2, and that once the economy is in Stage 2 it never reverts back to Stage 1.

These quantity relations assume that there is only one type of labour, and we are thus assuming no skill differentials. To the extent that these exist we will assume that on the job training can remove them and such training is taken account of in the technical coefficients. In this paper we are concerned with the aggregate demand for labour and not its skill breakdown and thus skills (and for that matter the geographical location of labour) are for all intents and purposes ignored.

Given the quantity relations we will now consider price relations. We assume that because there is no capital used in capital goods production, machines are priced at their labour cost. Let \( w \) be the wage rate which we will assume to be the same for all workers no matter what they are producing or which machines they are using. Let \( p_i \) be the price of machine \( i \) and then we have (9)
\[ p_i = w_b_i \quad i = 1, 2 \] (9)

Defining \( r_i \) as the rate of profit in consumption goods production from the use of machine \( i \), and letting the consumption good be the numeraire with a price of unity then (10) holds

\[ 1 = r_i p_i a_i + w b_i \quad i = 1, 2. \] (10)

For the new technology to be superior, and therefore for the entrepreneur's change of technique to be rational, technology 2 must be more profitable than technique 1. The superiority of technique 2 can be local or global, i.e. it can apply for one some or all wage rates. We will assume global superiority to avoid switchbacks from new to old technologies with changing wage rates. For technology 2 to yield a higher profit rate \( (r_2) \) than technology 1 \( (r_1) \) for all \( w \) we require that (11) holds

\[ b_1 a_1 > b_2 a_2 \quad \text{and} \quad p_1 > \beta_2 \] (11)

with at least one strict inequality holding. We will be assuming, therefore, that technology 2 is labour saving relative to technology 1 (i.e. it requires the same or less labour in both direct and indirect use). This would seem to fit both the micro-electronics case and be the most pessimistic assumption for unemployment implications. We have considered the behaviour of the model when technology 2 is only locally superior but the results add little.
Given the price and quantity relations we now close the model by specifying the demand environment. We assume that savings can be represented by a classical savings function with all wages consumed, all profits saved. We have experimented with a proportional savings function but as Hicks (1965) states, it sits much less happily in this framework. Defining 

\[ L_t = L_{1t} + L_{2t} \]

we may then write (13)

\[ C_t = w_t L_t \]  \hspace{1cm} (12)

Define \( \gamma_t \) as the ratio of the work force employed in time \( t \)

\[ \gamma_t = \frac{L_t}{L^*_t} \]  \hspace{1cm} (13)

This paper concerns the time path of \( \gamma_t \) as the economy makes the transition from using technology 1 to using technology 2.

From (13) we may derive (14)

\[ \frac{\dot{\gamma}_t}{\gamma_t} = \frac{\dot{L}_t}{L_t} - n \]  \hspace{1cm} (14)

From (12) we have (15) and thus (16)

\[ g_t = \frac{\dot{L}_t}{L_t} + \frac{\dot{w}_t}{w_t} \]  \hspace{1cm} (15)

\[ \frac{\dot{\gamma}_t}{\gamma_t} = g_t - \frac{\dot{w}_t}{w_t} - n \]  \hspace{1cm} (16)

Define \( z_t = \beta_1 (1 - \theta_t) + \beta_2 \theta_t \)
\[ q_t = a_1 b_1 (1 - \theta_t) + a_2 b_2 \theta_t \]

\[ \Delta \beta = \beta_1 - \beta_2 = \frac{-\Delta z_t}{\Delta \theta_t} > 0 \]

and \[ \Delta A = a_1 b_1 - a_2 b_2 = \frac{-\Delta q_t}{\Delta \theta_t} > 0 \]

Then \( z_t \) may be defined as the direct labour requirement per unit output of the consumption good and \( q_t \) the indirect labour (or capital) requirement, both varying over time as \( \theta_t \) changes.

Using these definitions we can derive from (2), (7) (8) and (12) that in Stage 1 (17) holds and in Stage 2 (18) holds

\[ \frac{C_t}{w_t} = L_t = c_t[z_t + q_t + \theta_t \Delta A] \]  

(17)

\[ \frac{C_t}{w_t} = L_t = c_t[z_t + a_2 b_2 (\theta_t \Delta A)] \]  

(18)

From which it is clear that in Stage 1 (19) holds and in Stage 2 (20) holds.

\[ g_t = \frac{1 - w_t z_t + w_t \Delta A \theta_t}{w_t q_t} \]  

(19)

\[ g_t = \frac{1 - w_t z_t}{w_t a_2 b_2 \theta_t} - \frac{\theta_t}{\theta_t} \]  

(20)
From (16), (19), (20) we may thus state that the time path of employment depends on

(1) The time path of consumption which in turn depends on the time path of wages
(2) The time path of $\theta_t$, the diffusion process and
(3) The technical coefficients of the two technologies.

The time path of $\theta_t$ and the technical coefficients have already been specified. With wages we have two possible approaches, (a) to model with wages determined by the excess supply of labour or (b) to consider wages determined exogenously. We feel that it is in the Keynesian (as opposed to the Walrasian) tradition to consider wages exogenously determined and thus we take this approach. In Section 3 the time path of $\gamma_t$ is investigated under a fixed wage assumption and in Section 4 wages are allowed to increase to keep the rate of profit in the economy constant. We have also investigated a case where wages move so as to maintain constant factor shares but the results do not differ materially from those in the second case. In Section 5 we investigate the impact in the two regimes of an increase in the diffusion speed.

In what follows we will assume that $t = 0$ represents the start of the transition process and at that date $\theta_0$ is given exogenously as the proportion of consumption goods output produced on the new technology. We assume that $\theta_0$ has been achieved without any impact on the economy. We will also allow that at time $0$, $\gamma_0 = 1$ which can be interpreted as that prior to time $0$ the economy has been growing on a full employment steady state growth path. We then ask whether the development of the economy generates increases or decreases in labour demand.
III. The Fix-Wage Path.

If the economy prior to time period $0$ has been growing on a full employment steady state growth path using only technique 1 and all wages were consumed, all profits saved, then the wage would have been given by (21)

$$w = \frac{1}{\beta_1 + \alpha_1 b_1 n}$$ (21)

On the fix wage path we assume that the wage stays at this level throughout the transition. We consider Stages 1 and 2 separately.

Stage 1

From (19) and (21), in Stage 1 $g_t$ is given by (22)

$$g_t = \frac{\alpha_1 b_1 n + \theta_t \Lambda \theta + \theta_t \Lambda A}{\theta_t}$$ (22)

Define $\theta_T$ as in (23)

$$\theta_T = \frac{\alpha_1 b_1 n}{\gamma_1 b_2 + \beta_2 - \beta_1}$$ (23)

We can show that if $\theta_T < \theta_T$, then $g_t > y\theta_T$ and we are in Stage 1. As $\theta_T$ is a monotonically increasing function of time this means that if $t < T$ we are in Stage 1. A sufficient condition for a Stage 1 to exist is that $\theta_T > 0$ which will hold if (24) holds

$$y > \frac{\beta_1 - \beta_2}{\alpha_2 b_2}$$ (24)

Define $r$ as in (25)

$$r = \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$ (25)
If $0_t > 0_T$, we are in a world where $q_t < y0_t$ and thus in Stage 2. Stage 2 will only exist if $\theta_2 < 1$, a sufficient condition for which is that (26) holds (given $\theta_2 > 0$)

$$a_1 b_1 n - a_2 b_2 y < \beta_2 - \beta_1$$  \hspace{1cm} (26)

Given $\beta_2 - \beta_1 < 0$, and $a_1 b_1 > a_2 b_2$ with one strict inequality holding, it is necessary, for a Stage 2 to exist, that $y > n$. Thus we may state that for both a Stage 1 and Stage 2 to exist it is necessary that both $y > \hat{r}$ and $y > n$. If $y < \hat{r}$ there is only a Stage 2, and if $y < n$ there is only a Stage 1. We will analyse the path of $\gamma_t$ as if both a Stage 1 and Stage 2 exist.

We can show that if $0_o < 0_T$, then the transition process starts in Stage 1. We will then enter Stage 2 if $q_t - y0_t$ falls with time. A sufficient condition for this is that (26) holds. Thus if a Stage 2 exists the economy will enter it. We will show later that if the economy enters Stage 2 it will not leave it.

Given (16) and the fixed wage, the growth rate of $\gamma_t$, the employment ratio, is given by (27)

$$\frac{\dot{\gamma}_t}{\gamma_t} = q_t - n$$  \hspace{1cm} (27)

Substituting for $q_t$ from (22) we achieve (28)

$$q_t - n = \frac{0_L \Lambda \beta + \Lambda \Lambda \left( \hat{r}_t + n \theta_t \right)}{q_t}$$  \hspace{1cm} (28)
Given \( \hat{\theta}_t = y(1-\hat{\theta}_t)\theta_t \) we may state that \( g_t - n > 0 \) if (29) holds

\[
(\beta_1 - \beta_2) + (\alpha_1 b_1 - \alpha_2 b_2)(y(1-\theta_t) + n) > 0
\]  

(29)

which must hold given \( \beta_1 > \beta_2 \) and \( \alpha_1 b_1 > \alpha_2 b_2 \) with at least one strict inequality. Thus in Stage 1 \( \gamma_t \) will increase with time. (Given that \( g_t - n \) also tells one of intertemporal movements in consumption per head, we see that consumption per head is rising over time in Stage 1.)

The mechanism at work here seems to be as follows. Prior to the new technology appearing sufficient profits were being generated to provide investment to support employment growth at rate \( n \). The new technology however generates a higher profit rate than the old. Thus as new technology is introduced in Stage 1, given that each machine in the capital stock is fully utilised (\( g_t > y\theta_t \)), and the wage rate is constant, a greater amount of profit must be realised in consumption goods production than previously. This greater profit is all invested, and given that investment only occurs to meet the demands of the consumption goods industry, it must mean that consumption goods output is growing faster than previously i.e. \( g_t > n \).

Stage 2

In Stage 2, \( \hat{\theta}_t > \theta_t \), \( t > T \) and \( g_t < y\theta_t \).

From (20) and (21) we may derive (30)

\[
g_t = \frac{a_1 b_1 n + \theta_t (\beta_1 - \beta_2)}{a_2 b_2 \theta_t} - y(1-\theta_t)
\]  

(30)

from which it is clear that \( g_t > y(\theta_t-1) \) and therefore \( x_2 > 0 \) for all of Stage 2. From (30) we may derive (31).

\[
g_t - y\theta_t = \frac{a_1 b_1 n}{a_2 b_2 \theta_t} + \frac{\beta_1 - \beta_2}{\alpha_2 b_2} - \gamma
\]  

(31)

1. An increase in \( \gamma_t \), given \( \gamma_0 \equiv 1 \), can only really be considered feasible if we resort to Hicks's device of the stockade economy or consider labour supply perfectly elastic at

\[
\frac{1}{w} = \beta_1 + b_1 u_1 n.
\]
from which it is clear that as time proceeds and \( g_t \) increases, \( g_t - y_0 \) falls. Thus once the economy enters Stage 2 it will not revert back to Stage 1. We note from above that \( y > r \) and \( y > n \).

To investigate the time path of \( Y_t \), we look at \( g_t - n \) which is given by (32)

\[
g_t - n = \frac{a_1 b_1 n}{a_2 b_2^2 t} + \frac{\beta - \beta_2}{a_2 b_2^2} - y + y_0 - n \tag{32}
\]

If we evaluate (32) for \( t = T \), i.e. \( q_T = 0 \), and for \( t = \infty \), i.e. \( q_\infty = 1 \), we find that \( g_t - n > 0 \) in each case. However taking the derivative of (32) with respect to \( q_T \) we obtain (33)

\[
\frac{d(g_t - n)}{dq_T} = y - \frac{a_1 b_1 n}{a_2 b_2^2 t} \tag{33}
\]

Define \( \hat{\vartheta} = \sqrt{\frac{a_1 b_1 n}{a_2 b_2^2 y}} \), i.e., as that \( \hat{\vartheta} \) where (33) equals zero. From (26) \( \hat{\vartheta} < 1 \). If \( \hat{\vartheta} > 0 \), we may show from (28) that \( (g_t - n) \) in Stage 1 is not only positive but increasing with \( t \). In Stage 2 \( (g_t - n) \) declines until \( \hat{\vartheta} = \hat{\vartheta} \) after which \( g_t - n \) rises again. At \( \hat{\vartheta} \) \( (g_t - n) \) is at a minimum but we have not been able to show whether at this point it is positive or negative. If \( \hat{\vartheta} < \theta_T \), then in Stage 2 \( (g_t - n) \) will increase for all \( t \), but in Stage 1 may decline with \( t \) for \( \hat{\vartheta} < \theta_T < \theta_T' \), (although \( g_t - n > 0 \)). In fact \( (g_t - n) \) will be at a minimum at \( \theta_T \), at which point \( g_t - n > 0 \), and thus if \( \hat{\vartheta} < \theta_T \), \( g_t > n \) for all \( t \) in both Stages 1 and 2 and the employment ratio will never fall.

Thus considering both stages of the transition, the employment ratio will rise in Stage 1 but in Stage 2 the time path of \( Y_t \) depends on whether \( \hat{\vartheta} > \theta_T \). If \( \hat{\vartheta} < \theta_T \), \( Y_t \) will rise for all \( t \). If \( \hat{\vartheta} > \theta_T \), \( Y_t \) may fall for some \( t \) but for \( t \) close to \( T \) and as \( t \to \infty \), \( Y_t \) will be increasing.
Given the imprecision of these results we have considered one other possibility in Stage 2. Because, in Stage 2, \( q_t < y_0 \), there is an absolute excess of capital of the old type. We might then argue that in a linear model such as this \( p_1 \) must fall to zero. In this case from (10) \( w \) must equal \( \frac{1}{p_1} \). If we allow \( w \) to go to this level in Stage 2, then in Stage 2 (34) holds

\[
g_t = \frac{p_1 - p_2}{b_2 \alpha_2} - y(1-\theta_t) \tag{34}
\]

From which it is clear that \( x_2 > 0 \) for all \( t \), and given \( y > \hat{r} \), once Stage 2 has been entered the economy will never revert to Stage 1.

From (34) we may derive (35)

\[
\log C_t = \hat{r}(t-T) - \log \theta_t + \log \theta_T + \log C_T \tag{35}
\]

and from (16) we get (36)

\[
\log \gamma_t - \log \gamma_T = \log C_t - \log C_T - n(t-T) - \log w_t + \log w_T \tag{36}
\]

and from (35) and (36) we get (37)

\[
\log \gamma_t - \log \gamma_T = (\hat{r}-n)(t-T) - \log \theta_t + \log \theta_T + \log \left( \frac{w_T}{w_t} \right) \tag{37}
\]

Interpreting \( w_t \) as the wage appropriate to Stage 2 \( \left( \frac{1}{p_1} \right) \) and \( w_T \) as that appropriate to Stage 1 \( \left( \frac{1}{b_1 \alpha_1 n} \right) \) we see that \( w_T < w_t \) and thus \( \gamma_t \).
falls at time $T$ as the wage increases.

From (37), we can see that if $\hat{r} < n$, $\gamma_t < \gamma_T$ for all $t > T$.

(However if $\hat{r} < n$, the economy cannot support a full employment growth path with $w = \frac{1}{\beta_1}$.) If $\hat{r} > n$, $\dot{\gamma}_t/\gamma_t > 0$ when (38) holds

$$0_t > 1 + \frac{n - \hat{r}}{\gamma}$$  (38)

and $\dot{\gamma}_t/\gamma_t < 0$ otherwise.

The problem with analysing the fixed wage path is that the workers are not getting any benefit from higher productivity and this is somewhat unrealistic. We turn then to consider the case where wages are rising at such a rate as to keep the overall rate of profit in the economy constant.

IV. The Fix Profit Rate Path

Assuming the economy was growing on a full employment steady state growth path prior to $t = 0$, the profit rate would have been $n$. To maintain this profit rate as the new technology is introduced (39) must hold

$$w_t = \frac{1}{\gamma_t + n\xi_t}$$  (39)

From (39) we get (40)

$$\frac{\dot{w}_t}{w_t} = \frac{(\xi_t + n\xi_t)\dot{\xi}_t}{\gamma_t + n\xi_t}$$  (40)
Thus if $\beta_1 > \beta_2$ ($\Lambda > 0$) and $b_2 a_2 > b_1 a_1$ ($\Delta > 0$) with at least one strict inequality, $\dot{w}_t / w_t > 0$.

Stage 1

From (19) we can now show that in Stage 1 $g_t$ is given by (41)

$$g_t = \frac{n + \delta_t \Lambda A}{q_t} \quad (41)$$

Define $\theta_T$ as in (42)

$$\theta_T = \frac{na_1 b_1}{n(a_1 b_1 - a_2 b_2)} + y a_2 b_2 \quad (42)$$

If $t < T$ and thus $\theta_t < \theta_T$, $g_t > y \theta_t$ and we are in Stage 1. Moreover $g_t - y \theta_t$ will fall over time, and thus we will eventually enter Stage 2. Stage 2 will exist if $\theta_T < 1$ i.e. if $y > n$. Stage 1 exists if $\theta_T > 0$ i.e. if $b_1 a_1 > b_2 a_2$. We will assume $y > n$ and $\theta_T < \theta_T$.

From (16), the rate of growth of the employment ratio will, after substitution from (41), be given by (43)

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{\theta}_t}{\theta_t} \left( \frac{\eta b_1}{a_1} - \frac{\Lambda \beta_1 b_1}{z_t + nq_t} \right) \quad (43)$$

From (43) we get (44)

$$\text{sign} \left( \frac{\dot{y}_t}{y_t} \right) = \text{sign} \left( b_1 a_1 \beta_2 - b_2 a_2 \beta_1 \right) \quad (44)$$
and thus
\[
\frac{\dot{r}_t}{r_t} > 0 \quad \text{if} \quad \frac{b_1a_1}{b_2a_2} > \frac{\beta_1}{\beta_2}
\]
\[
\frac{\dot{r}_t}{r_t} < 0 \quad \text{if} \quad \frac{b_1a_1}{b_2a_2} < \frac{\beta_1}{\beta_2}
\]

These conditions may be interpreted as follows. If the new technology has a higher ratio of indirect to direct labour inputs required to produce one unit of consumption goods than does the old technology, then the employment ratio will fall in Stage 1. If the new technology has a lower ratio, employment will rise. This may be interpreted further as that \( r_t \) will increase in Stage 1 if the new technology is less capital intensive and fall if the new technology is more capital intensive. \( r_t \) will equal unity for all \( t < T \) in the case of Hicks neutrality.

**Stage 2**

In Stage 2 \( g_t \) is given by (20) which may be reduced to (45)

\[
g_t = n - r(1 - \frac{\theta}{r}) + \frac{n\alpha_1b_1}{\alpha_2b_2} \left( \frac{1}{\theta} - 1 \right)
\]  
(45)

From (45) it is clear that \( g_t > r(\theta - 1) \) and \( x_2 > 0 \) for all \( t \).
Moreover \( g_t - r_0 \) will fall over time so we will never revert back to Stage 1.

From (45) we can immediately observe that as \( r \) grows, \( g_t \) grows.

and because \( \frac{\dot{w}}{w} \to 0 \) as \( r \to 1 \) the economy will eventually revert to growth at rate \( n \). From (16) substituting from (45) we get (46)

\[
\frac{\dot{r}_t}{r_t} = r(\theta - 1) + \frac{n\alpha_1b_1}{\alpha_2b_2} \left( \frac{1}{\theta} - 1 \right) - \frac{\dot{w}}{w}
\]  
(46)
(46) may be written as (47)

\[
\dot{\gamma}_t = \frac{\dot{\gamma}_t}{\gamma_t} - \frac{\alpha_1 n \beta_1 b_1}{a_2 b_2} \cdot \frac{1}{y_0} \cdot \dot{\gamma}_t - \frac{\alpha_1 (\Lambda \gamma + n \Lambda \dot{A})}{z_t + n \dot{q}_t} \cdot \dot{\theta}_t
\]

From which it is clear that \(\frac{\gamma_t}{\gamma_t} > 0\) if (48) holds

\[
\theta_t < \frac{nb_1 a_1 (\beta_1 + b_1 a_1 n)}{yb_2 a_2 (\beta_1 + b_1 a_1 n) + b_1 a_1 n (\beta_2 - b_2 a_1 n - b_2 a_2 n)} \equiv \bar{\theta}
\]

Define \(V \equiv (\beta_1 + b_1 a_1 n) (nb_1 a_1 - nb_2 a_2 + yb_2 a_2) > 0\)

\[
\text{then } \bar{\theta} = \frac{\theta_t V}{V + \beta_1 b_2 a_2 n - \beta_2 b_1 a_1 n}
\]

If \(\frac{b_1 a_1}{b_2 a_2} < \frac{\beta_1}{\beta_2}\) then \(\bar{\theta} < \theta_t\)

and if \(\frac{b_1 a_1}{b_2 a_2} > \frac{\beta_1}{\beta_2}\) then \(\bar{\theta} > \theta_t\).

Thus if \(\frac{b_1 a_1}{b_2 a_2} < \frac{\beta_1}{\beta_2}\), then \(\bar{\theta} < \theta_t\), but \(\theta_t > \theta_t\) for all \(t > T\), therefore \(\theta_t > \bar{\theta}\) and thus \(\frac{\gamma_t}{\gamma_t} < 0\) for all of Stage 2.

If \(\frac{b_1 a_1}{b_2 a_2} > \frac{\beta_1}{\beta_2}\), then in Stage 2 \(\frac{\gamma_t}{\gamma_t} > 0\) for \(\theta_t > \bar{\theta} > \theta_t\).

Combining the results on Stage 1 and Stage 2 we may therefore argue that

(i) If \(\frac{b_1 a_1}{b_2 a_2} < \frac{\beta_1}{\beta_2}\), then in Stage 1 \(\frac{\gamma_t}{\gamma_t} < 0\) and in Stage 2 \(\frac{\gamma_t}{\gamma_t} < 0\).

1/ One can show that if \(y > n\) then \(\bar{\theta} < 1\).
(ii) If \( \frac{b_1a_1}{b_2a_2} > \frac{\beta_1}{\beta_2} \), and the new technology is less capital intensive then in Stage 1 \( \frac{\dot{y}}{\dot{y}_t} < 0 \) and in Stage 2 \( \frac{\dot{y}}{\dot{y}_t} \) will initially be positive and then negative.

If we take two extreme examples of changes in technology
(a) \( \beta_1 = \beta_2, \ b_1a_1 > b_2a_2 \) and (b) \( b_1a_1 = b_2a_2, \ \beta_1 > \beta_2 \), we see that in example (a) the employment ratio will initially increase then fall and in example (b), the employment ratio will fall throughout Stages 1 and 2.

In the case where \( \frac{b_1a_1}{b_2a_2} = \frac{\beta_1}{\beta_2} \), the case of Hicks neutrality, the employment ratio will remain constant in Stage 1, then fall for all of Stage 2. (\( \theta_T = \bar{\theta} \)).

In addition to analysing the case where the profit rate remains constant we have also investigated the model under the assumption that factor shares remain constant (at the pre-innovation value). We find that the relative capital intensities determine \( \frac{\dot{y}}{\dot{y}} \) exactly as in the fixed profit rate case.

V. The Impact of Higher Diffusion Speeds

The purpose of this section is to investigate the impact on the time path of the employment ratio of variations in the diffusion speed \( y \). We will consider the two cases separately.

a) The fix wage path. On the fix wage path \( \theta_T \) is given by (24), and we may thus generate (49)

\[
\frac{\partial \theta_T}{\partial y} = - \frac{\alpha_{1 \beta_1 n y}}{(y a_2 b_2^2 + \beta_2 - \beta_1)^2} \quad < 0
\]

(49)

from which it is also obvious that \( \theta_T/\partial y < 0 \). Thus the higher is \( y \) the shorter is Stage 1.
To investigate the effect of changes in \( y \) on \( \gamma_t \) we can note that if \( w \) is constant then (50) holds for all \( t \)

\[
\frac{3y_t}{\partial y}. \frac{y}{\gamma_t} = \frac{3c_t}{\partial y}. \frac{y}{c_t}
\]  

(50)

From (19) we can derive (51)

\[
\log C_t - \log C_0 = \int_{0}^{t} \frac{y_1 e^{-yt} + y_2}{\delta_1 e^{-yt} + \delta_2} dt - \log \left[ \frac{w_b a_2 \theta_t}{w_b a_1 (1-\theta_t)} + \frac{w_b a_1 (1-\theta_t)}{} \right] \\
+ \log \left[ \frac{w_b a_2 \theta_0}{w_b a_1 (1-\theta_0)} + \frac{w_b a_1 (1-\theta_0)}{} \right]
\]  

(51)

where \( \delta_1 = w_b a_1 e^{-\eta} \)

\( \delta_2 = w_b a_2 \)

\( \gamma_1 = 1 - w \beta_2 \)

\( \gamma_2 = (1 - w \beta_1) e^{-\eta} \)

From (51) we get (52)

\[
\frac{\partial \log C_t}{\partial y} = \int_{0}^{t} \frac{(-\delta_2 y_1 + \delta_1 y_2) e^{-yt}}{[\delta_1 e^{-yt} + \delta_2]^2} dt - \frac{(b_2 a_2 - b_1 a_1) \frac{\partial \theta_t}{\partial y}}{w_b a_2 \theta_t + w_b a_1 (1-\theta_t)}
\]  

(52)

If \( b_2 a_2 < b_1 a_1 \), given \( \frac{\partial \theta_t}{\partial y} > 0 \), the second term is positive. The first term is positive if \( \delta_1 y_2 > \delta_2 y_1 \) i.e. if (53) holds

\[
\frac{b_1 a_1}{b_2 a_2} > \frac{1 - w \beta_1}{1 - w \beta_2}
\]  

(53)
Which will hold, given $\beta_1 > \beta_2$, $b_1 a_1 > b_2 a_2$ with at least one strict inequality. We can also look at $Y_T$. From (50) $\frac{\partial Y_T}{\partial y} \cdot Y_T = \partial C_T/\partial y \cdot Y / C_T$.

From (51) we can derive (54)

$$\log C_T - \log C_0 = \gamma_1 \log \left( \frac{b_1 a_1}{b_2 a_2} \cdot \frac{1 - \theta_0}{\theta_0} + 1 \right)$$

$$+ \frac{\gamma_2}{\gamma_2} \left( \log \left( \frac{b_1 a_1}{b_2 a_2} \cdot \frac{1 - \theta_0}{\theta_0} + 1 \right) \right)$$

$$- \log \left( \frac{wb_2 a_2 \theta_0 + wb_1 a_1 (1 - \theta_0)}{wb_2 a_2 \theta_0 + wb_1 a_1 (1 - \theta_0)} \right)$$

(54)

Without explicit derivation, we know that $\frac{d\theta_T}{dy} < 0$ thus $\frac{\theta_T}{1 - \theta_T} / dy < 0$. We may by inspection derive that if $b_1 a_1 > b_2 a_2$ all terms in (51) decline with an increase in $y$.

Thus in Stage 1 the impact of an increase in the diffusion speed is to reduce $T$, the length of Stage 1, to reduce $\theta_T$, to decrease $Y_T$, and to increase $Y_t$ for all $t < T$. Thus the faster is the speed of diffusion the higher is the employment ratio for all $t < T$.

In Stage 2, if $w$ stays at $\frac{1}{\beta_1 + b_1 a_1 n}$ we can show that (55) holds.

$$\frac{\partial \log Y_t}{\partial y} = \frac{\partial \log C_t}{\partial y}$$

(55)
From (30) we may derive (56)

\[
\log C_t - \log C_T = \left[ \frac{a_1 b_1 n + \beta - \beta_2}{a_2 b_2} \right] \left[ t - T \right] + \frac{a_{1\,1}}{a_2 b_2} \left( \frac{1}{\theta_t} - \frac{1}{\theta_T} \right)
\]

- \log \theta_t + \log \theta_T

Given (57) we get (58)

\[
\log \gamma_t - \log \gamma_T = \log C_t - \log C_T - n(t-T)
\]

\[
\log \gamma_t - \log \gamma_T = (t-T) \frac{(a_{1\,1} b_1 n - a_2 b_2 n + \beta_1 - \beta_2)}{a_2 b_2}
\]

\[
+ \frac{a_{1\,1}}{a_2 b_2} \left( \frac{1}{\theta_t} - \frac{1}{\theta_T} \right) - \log \theta_t + \log \theta_T
\]

As can be seen by inspection the first term is positive, the other terms negative. We may now derive (59)

\[
\frac{\partial \log \gamma_t}{\partial y} = - \left( \frac{1}{\theta_t} - \frac{1}{\theta_T} \right) \frac{a_{1\,1}}{a_2 b_2} \frac{n}{\theta_t^2} + \frac{a_{1\,1}}{a_2 b_2} \left( \frac{1}{\theta_t^2} + \frac{\partial \theta_t}{\partial y} + \frac{\partial \theta_T}{\partial y} \right)
\]

\[
- \frac{\partial \theta_t}{\partial y} \cdot \frac{1}{\theta_t} + \frac{\partial \theta_T}{\partial y} \cdot \frac{1}{\theta_T} + \frac{d \log \gamma_T}{dy}
\]

Despite having (59) just as it was not possible to find simple conditions to ensure \( g_t - n \) was positive in Stage 2, it is not possible to find
simple conditions to sign (59). If we take the alternative route and let \( w = \frac{1}{\beta_1} \) in Stage 2 one can show that if \( \hat{r} < n, \frac{\partial y_t}{\partial y} < 0 \) for all \( t > T \), but if \( \hat{r} > n \), simple conditions are not forthcoming.

(b) The fixed profit rate path

From the definition of \( \Theta_T \) we may derive that \( \frac{\partial \Theta_T}{\partial y} < 0 \) and thus \( \frac{\partial y_t}{\partial y} < 0 \). From (43) we get, by integration, that in Stage 1, (60) holds

\[
\gamma_t = \left( \frac{\hat{r}}{r(t)} + n \right) \gamma_0
\]

We can derive from (60) that (61) holds, given \( \gamma_0 = 1 \),

\[
\text{sign} \frac{\partial y_t}{\partial y} = \text{sign} \left( \beta_2 a_1 b_1 - \beta_1 a_2 b_2 \right) \frac{\partial \Theta_t}{\partial y}
\]

Given \( \frac{\partial \Theta_t}{\partial y} = t(1-0)\theta > 0 \) then \( \frac{\partial y_t}{\partial y} > 0 \) if \( \beta_2 a_1 b_1 > \beta_1 a_2 b_2 \)

and \( \frac{\partial y_t}{\partial y} < 0 \) if \( \beta_2 a_1 b_1 < \beta_1 a_2 b_2 \).

Given (60) and our expression for \( \Theta_T \) (42) we may derive (62)

\[
\gamma_T = \left( \frac{\alpha_1 \alpha_2 b_2 (y-r) + \beta_2 n a_1 b_1}{y a_1 b_1 a_2 b_2} \right) + n \right) \gamma_0
\]

From which it is clear that

\[
\text{sign} \frac{\partial y_T}{\partial y} = \text{sign} \left( \beta_1 a_2 b_2 - \beta_2 a_1 b_1 \right)
\]
Thus if \( \frac{a_1 b_1}{a_2 b_2} > \frac{\beta_1}{\beta_2} \) then in Stage 1 \( \frac{\dot{y}}{y_T} > 0, \frac{\partial y_T}{\partial y} < 0, \frac{\partial y_T}{\partial y} < 0 \)

and if \( \frac{a_1 b_1}{a_2 b_2} < \frac{\beta_1}{\beta_2} \) then in Stage 1 \( \frac{\dot{y}}{y_T} < 0, \frac{\partial y_T}{\partial y} < 0, \frac{\partial y_T}{\partial y} > 0 \)

\( \frac{\partial T}{\partial y} < 0, \frac{\partial T}{\partial y} < 0 \)

In Stage 2 we may derive from (46) that (64) holds (by integration)

\[
\log y_T - \log y_T = \log \theta_T - \log 0 + \frac{n a_1 b_1}{y a_2 b_2} \left( \frac{1}{T} - \frac{1}{t} \right)
\]

\[
\log \left[ \frac{2 \frac{\bar{z}}{T} + n q_T}{\bar{z}_T + n k_T} \right]
\]

(64)

However, we have not been able to find simple conditions to sign \( \frac{\partial y_T}{\partial y} \) for \( t > T \), and thus conclude that the impact may go either way. Thus although in Stage 1 the effect of an increase in the diffusion speed is to increase the employment ratio if the new technology is less capital intensive and decrease it if the new technology is more capital intensive, the effect in Stage 2 is much less clear.
VI. Conclusions and Implications

In this paper we have attempted to analyse the impact of technology on employment taking into account compensation effects. The model is a two sector one, closed by a classical savings function, with exogenously determined wage rates and a superimposed diffusion path. We have argued that the time path of the employment ratio will depend upon the time path of wages, the diffusion speed and the nature of the new and old technologies.

The transition path is analysed in two stages, in the first the old capital good continues to be used and produced alongside the new. In Stage 2 the old capital good is no longer produced but is still used alongside the new.

We have then shown that if the wage rate is fixed new technology will mean an increased demand for labour in Stage 1. If however the wage rate changes as new technology is introduced then this result only holds if the new technology is less capital intensive than the old. If the new technology is more capital intensive then the employment ratio will fall in Stage 1. We have also shown that on the fix wage path a higher diffusion speed will mean greater employment in Stage 1, and with wages changing a higher diffusion speed means higher employment if the employment ratio is increasing and lower employment if the employment ratio is falling.

In Stage 2 the results are less precise. With a fixed wage we have shown the conditions under which the employment ratio will rise and with a rising wage have generated similar results. We have also investigated the
impact of higher diffusion speeds. Although the results generated are not precise, they illustrate that there is no definite presumption that new technology or higher diffusion speeds mean lower employment.

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May 1981
References:


Technology, Diffusion, Wages and Employment

Summary

In this paper we analyse the impact of new technology on employment. The paper allows for compensation effects by using a two-sector representation of technology, linking prices to costs and demand to incomes. Technology is diffused slowly rather than immediately. The time path of employment is shown to depend on the time path of wages, the speed of diffusion and the characteristics of the technology. Two specific wage regimes are investigated in detail. The results suggest that there should be no presumption that new technology will always cause unemployment.

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