Sorting into Contests: Evidence from Production Contracts

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Abstract

In this paper we investigate sorting patterns among chicken contract producers. We show that the sub-game perfect Nash equilibrium of this contracting game reveals a positive sorting where higher ability producers sort themselves into contracts to grow larger chickens and lower ability types sort themselves into contracts to grow smaller birds. We also show that eliciting this type of sorting behavior is profit maximizing for the principal. In the empirical part of the paper, we first estimate growers’ abilities using a two-way fixed effects model and subsequently use these estimated abilities to estimate a random utility model of contract choice. Our results show that higher ability growers are more likely to self-select themselves into contracts with larger expected outputs (larger chickens) and the opposite is true for growers with lower abilities. The empirical results are strongly supportive of the developed theory.

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1 Introduction

Pay for performance ties an employee’s pay to their performance on the job. The idea is that pay-
for-performance compensation schemes will not only offer incentives to motivate and reward
improved performance but will also attract and retain better employees. The two objectives
(incentives and hiring) are tightly related because different people pursue different goals and
respond to incentives differently. In a recent survey, Oyer and Schaefer (2011) pointed out that
personnel economics has made more progress in the area of understanding of how incentives
work than on the subject of matching employees and firms. In particular, relatively little is
known about the mechanisms through which firms strategically design compensation packages
to hire and retain appropriate workers. A key obstacle to advancement in this area, they claim,
has been the paucity of integrated evidence due to the lack of usable real markets data.

The first empirical work in this area is Lazear (2000) who, relying on the Safelite Glass
Corp. data, investigated the effect of changing compensation schemes on the productivity of
windshield installers. He found that worker’s average productivity increased by 44% after
switching from fixed salaries to piece rates and half of the resulting productivity increase was
attributable to attracting and retaining more able workforce. Barro and Beaulieu (2003) studied
the effects of transferring physicians from a salary based compensation to a profit-sharing sys-
tem. They found that the change has a large and significant effect on the quantity of services
provided. In addition, they also detected a sorting/selection effect, where the least productive
doctors left the hospital and more productive doctors joined. More recently, Bandiera et. al
(2015) used administrative sources and survey data to study the match between firms and man-
gers who are different in risk-aversion and talent. They found that policies with tighter link
between performance and reward attract managers who are more talented and less risk-averse
and also that managers respond to incentives by exerting more effort if offered steeper contracts.

In this paper we use a unique and detailed data set which documents the settlements of
contracts for the production of broiler chickens between a company and its contract growers.

1Numerous empirical studies test for the existence of incentives effect and try to quantify its magnitude by
examining the difference in performances under various compensation schemes: see Bull, Schotter and Weigelt

2Poultry companies, frequently called integrators, such as Tyson Foods, Sanderson Farms or Perdue Farms
The common characteristic of virtually all modern broiler production contracts is that contract
growers’ compensation is determined in a cardinal tournament type setting where the individual
grower’s piece rate compensation is determined by comparing her performance against the tour-
nament group average performance (see: Knoeber and Thurman, 1994 and Levy and Vukina,
2004). In our data set we observe heterogeneous ability growers who participate in five differ-
ent broiler production contracts offered by different complexes of a large broiler company in
one geographical area. All contracts are the same when it comes to division of responsibilities
for providing inputs: growers provide housing facilities, utilities and labor and the company
provides birds and feed. Contracts are different with respect to two principal features: the size
of chickens that need to be grown and the payment schedule offered in return. All contract
are written on a take-it-or-leave-it basis and are explicitly short-term (flock-by-flock) so we can
observe multiple contract realizations for each individual grower. The main research objective
is to investigate whether contract growers choose among available contract alternatives based
on some systematic self-selection/sorting patterns that can be uncovered in the data.

In addition to the above mentioned small but growing literature dealing with selection into
alternative remuneration schemes and organizations, our paper is also connected to the theo-
retical literature on selection into contests in general. Leuven, Oosterbeek and van der Klaauw
(2011) investigate how heterogeneous agents choose among contests with different prizes. Their
main finding is that perfect sorting (high-ability agents compete for high prize and low-ability
agents for low prize) does not necessarily obtain. Mixed strategies and reverse sorting are also
possible. Azmat and Möller (2009) study how competing contests should be structured to max-
imize participation. Their model with identical abilities contestants predicts that an increase in
sensitivity with which contest outcomes depend on efforts makes flatter prize structures more
attractive. In equilibrium, contests that focus on maximizing the number of participants will
award multiple prizes if and only if this sensitivity is sufficiently high. Azmat and Möller
(2013), in a model with binary abilities, show tat the distribution of abilities plays a crucial
role in determining the contest choice. Sorting exists only when the proportion of high-ability
contestants is sufficiently small. Morgan, Sisak and Vardy (2014) study how large, heterogeneous population of risk-neutral agents self-select across two mutually exclusive contests. They show that entry into richer contests is non-monotone in ability. This seems to be the only paper which characterizes selection when contests differ in multiple dimensions (entry fees, number of prizes, value of prizes and discriminatoriness/meritocracy) simultaneously.

One of the most difficult obstacles to overcome in empirical studies with real life data is the fact that the choice of compensation schemes in a firm is correlated with observable and unobservable characteristics of the firm and can rarely be considered truly exogenous. The use of controllable laboratory experiments is in this respect attractive because the exogeneity of the change in the compensation scheme is guaranteed by design. Cadsby, Song and Tapon (2007) and Eriksson and Villeval (2008) examined the differences between pay for performance versus fixed salary in experimental settings and found evidence of positive sorting reflected in more productive workers choosing performance pay over fixed salary. Similarly, Dohmen and Falk (2011) comparing output of workers in fixed and variable payment schemes (piece rate, tournament and revenue-sharing) found that variable payment schemes attract more capable workers. They also found that change in the compensation schemes has multidimensional sorting effect with respect to other workers’ characteristics such as risk aversion, relative self-assessment and even gender. Leuven et al. (2011) analyzed the sorting effects within the framework of rank-order tournaments. In their experiment, introductory microeconomics students self-selected themselves to different tournaments with low, medium and high prizes. Their results show that the positive relationship between student’s productivity and prizes are entirely attributable to the sorting effect where participants with higher ability are more likely to sort themselves into a higher reward tournament.

Outside the experimental literature, the contest theory and its applications has been empirically tested exclusively with the sports data. For example, earlier mentioned Azmat and Möller (2009) found empirical support for their findings with professional road running data and Azmat and Möller (2013) used entry data into marathon races for testing their theory. Finally, Lynch and Zax (2000) also relied on professional road racing data and found that races with large prizes record faster times because they attract faster runners and not because they encour-
age all runners to run faster. To the best of our knowledge, ours is the first attempt to test the self-selection into contests based on the real market transactions data.

We start the paper by presenting a theoretical model geared towards deriving testable predictions. We show that the sub-game perfect Nash equilibrium of this contracting game reveals a positive sorting where higher ability growers sort themselves into contracts to grow large chickens and lower ability types sort themselves into contracts to grow smaller birds. We also show that, from the perspective of the principal (integrator), the strategy of offering a menu of contracts that elicits positive sorting generates larger profits than offering one uniform contract to all growers. In the empirical part of the paper, we first estimate growers’ abilities in a two-way fixed effects model and subsequently use these estimated abilities to estimate a random utility model of contract choice. The signs of the estimated model coefficients offer a direct empirical test of the developed theory. We showed that higher ability chicken growers are more likely to self-select themselves into contracts with larger expected outputs (larger chickens) and the opposite is true for chicken growers with lower abilities. The results are strongly supportive of the developed theory.

2 Theoretical Model

The presented theoretical model describes a contractual relationship between a single principal and a number of heterogeneous ability agents. The principal simultaneously offers a menu of contracts to a group of agents and each agent needs to decide which, if any, among the available contracts to sign. The optimal decisions are characterized by relying on the concept of sub-game perfect Nash equilibrium. In the first stage, each agent chooses a contract that gives her the highest expected utility among all available contracts. In the second stage agents exert optimal levels of efforts subject to parameters of the contract being chosen. The equilibrium is solved for by using backward induction. The model is designed to mimic the contracting process prevalent in the poultry industry where companies (integrators) contract the production of live chickens (broilers) with independent farmers (growers).


2.1 Stylized Facts and Preliminary Assumptions

Since all contracts are the same when it comes to defining the exact responsibilities of the contracting parties and other legal provisions, we assume that each contract can be uniquely identified by the payment schedule. In all modern broiler production contracts, the total payment $R_{ik}$ to grower $i$ who participate in contract $k$ is calculated as a variable piece rate times the live pounds of chickens harvested from the farm. The variable piece rate consists of two parts – the base rate and the bonus/penalty rate. The base rate $b_k$ depends on the size of the chickens grown. The contract for growing heavier birds takes longer to complete and therefore has higher base payment rate. The bonus/malus rate is determined as a percentage $\beta_k$ of the difference between the average performance of the entire group of growers whose chickens were harvested in the same period and the individual performance of grower $i$.

$$R_{ik} = [b_k + \beta_k(\frac{1}{N_k} \sum_{j=1}^{N_k} \frac{C_{jk}}{Q_{jk}} - \frac{C_{ik}}{Q_{ik}})]Q_{ik}. \quad (1)$$

As seen from (1), the growers’ performance is measured by the settlement cost $C_{ik}$ per pound of output (live chickens weight) $Q_{ik}$ where the settlement costs includes expenditures for all production inputs supplied by the integrator (such as baby chicks and feed). The above described relative performance scheme, frequently referred to as a two-part piece rate tournament, is a double-margin contest in which growers compete in producing as much output as possible with as little inputs (mainly feed) as possible (Tsoulouhas and Vukina, 1999). The exact modeling of this tournament game is rather difficult because grower’s effort to some degree stochastically influences both feed utilization and final output.\(^3\) To simplify, we assume that effort only influences grower’s performance as measured by the negative of the settlement cost per pound of output and that the mortality is determined by sheer luck.\(^4\) Hence, the performance

\(^3\)A commonly used simplification is to fix one of the two margins. For example, Knoeber and Thurman (1994), Tsoulouhas and Vukina (1999) and Levy and Vukina (2004) fix the output margin by assuming common target weight of finished broilers and constant mortality rate. The alternative is to fix the cost margin and model the tournament as a contest of who can produce more output with given amount of inputs, see Vukina and Zheng (2011). Neither of the two simplifications are suitable to our problem. This is because fixing one margin only works when modeling grower behavior within one contract but not for the purpose of modeling how growers choose among different contracts.

\(^4\)The assumption about random mortality is quite reasonable because broiler mortality largely depends on
function has the following form:

\[ q_{ik} = -\frac{C_{ik}}{Q_{ik}} = e_{ik} + a_i + u_k + w_{ik}. \]  

(2)

Growers are assumed to be heterogeneous in their abilities \( a_i \), they exert efforts \( e_i \) and their performance is influenced by the common shock \( u_k \) and idiosyncratic shock \( w_{ik} \). Each grower only knows her own ability and believes all other growers abilities are randomly drawn from distribution \( G_k(\cdot) \) with density \( g_k(\cdot) \). The two production shocks are independent of each other and normally distributed with means zero, variances \( \sigma_u^2 \) and \( \sigma_w^2 \) and probability density functions \( f_u(\cdot) \) and \( f_w(\cdot) \). Total output \( Q_{ik} \) is randomly drawn from distribution \( H_k(\cdot) \) with probability density function \( h_k(\cdot) \).

Growers are assumed to be risk-neutral and their utility functions are given by \( U_{ik} = R_{ik} - c(e_{ik}) \), where the cost of effort is a strictly convex function of effort. In particular, we assume a quadratic cost of effort \( c(e_{ik}) = \frac{1}{2} e_{ik}^2 \) with \( \gamma > 0 \) and constant for all growers.

When choosing a contract from the menu of available contracts, growers observe the base payment rate \( b_k \) and the bonus/malus coefficient \( \beta_k \). An interesting characteristics of all five contracts in our data set is that they all have the same bonus/malus coefficient equal to 1. Effectively, this feature simplifies the contract choice decision from the need to simultaneously choose the base and the bonus coefficients to choosing only the base rate. Because of the one-to-one correspondence between the base rates and target weights of broilers, by choosing the base rate growers de facto choose the type/size of chickens that they will grow.

Notice that the actual output (realized total live weight), the exact number of players in the settlement group and the final payment are not known to growers at the time of signing the contract. Production is stochastic and depends on things like weather (temperature and humidity), quality of feed and baby chicks (supplied by the integrator) and growers’ idiosyncratic shocks. The production cycle takes 6-8 weeks to complete. Typically, tournament groups consist of all

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the quality of baby chicks and not very much on grower’s effort. Also, the assertion that companies engage in strategic allocation of varying quality inputs (chicks and feed) among heterogeneous ability growers to control cost of production has been empirically refuted by Leegomonchai and Vukina (2005). Therefore, it seems reasonable to assume that the mortality rate is exogenous and common to all growers.

\(^5\)This is directly derived from the assumption of exogenous mortality rate since the initial number of chicks are the same across growers in one contract.
growers whose birds were harvested within the same calendar week, whereas the delivery of new batches is determined by scheduling and logistics of the production process.  
Therefore, in any particular contest (tournament), the competition and final payment will vary in line with the actual size and the composition of the tournament group.

2.2 Self-selection Mechanisms

Each grower maximizes her expected utility/profit by choosing the optimal level of effort:

\[
\max_{e_{ik}} \mathbb{E}(R_{ik} - c(e_{ik})) = \max_{e_{ik}} \int \cdots \int \left[ b_k + \beta_k \frac{N_k - 1}{N_k} (e_{ik} + a_i + u_k + w_{ik}) - \frac{1}{N_k - 1} \sum_{j=1}^{N_k} (e_{jk} + a_j + u_k + w_{jk})) \right] Q_{ik} \prod_{j=1, j \neq i}^{N_k} g_k(a_j) \prod_{j=1}^{N_k} f_{w_k}(w_{jk}) f_{u_k}(u_k) h_k(Q_{ik}) \prod_{j=1, j \neq i}^{N_k} da_j \prod_{j=1}^{N_k} dw_{jk} du_k dQ_{ik} - \frac{1}{2} \gamma e_{ik}^2. \tag{3}
\]

The first-order condition of the above maximization problem gives the closed-form solution for the optimal effort:

\[
e_{ik}^* = \frac{\beta_k (N_k - 1) \bar{Q}_k}{\gamma N_k}
\]

where \(\bar{Q}_k\) denotes the mean of total output of contract \(k\). As we can see, heterogeneous ability growers participating in the same tournament are expected to exert the same level of optimal effort, that is, \(e_{ik}^* = e_{jk}^*, \forall i \neq j\). This is because marginal utility of effort is independent of ability when additive performance function is assumed.

Growers will sign the contract which provides them with the highest expected utility/profit among all available contract alternatives assuming that whatever contract they choose they would always exert the optimal effort. Analytically, the expected utility/profit of grower \(i\) choos-

\[\text{The birds are harvested from a given farm when the integrator’s production manager estimates that birds have reached the target weight and are ready to slaughter. The new cycle will start when the integrator delivers a new batch of birds to the farm. The acceptance of the new batch by the farmer constitutes a tacit renewal of the existing contract.}\]
ing contract \( k \), after inserting the optimal effort \(( \text{E}^*_k ) \) into \(( \text{E} \) \), becomes:

\[
\text{E}U_{ik} = \mathbb{E}(R(e^*_ik) - c(e^*_ik)) \\
= \int \cdots \int [b_k + \beta_k N_k - 1 (e^*_ik + a_i + u_k + w_{ik}) - \frac{1}{N_k} \sum_{j=1}^{N_k} (e^*_jk + a_j + u_k + w_{jk})]Q_{ik} \\
\prod_{j=1}^{N_k} g_k(a_j) \prod_{j=1}^{N_k} f_{wk}(w_{jk}) f_{uk}(u_k) h_k(Q_{ik}) \prod_{j=1}^{N_k} da_j \prod_{j=1}^{N_k} d\omega_{jk} d\omega_k d\omega_{ik} - \frac{\gamma}{2} (e^*_ik)^2.
\]

Because, in equilibrium, growers in the same tournament exert the same level of effort, the above integral can be simplified to obtain a rather tractable representation of grower’s expected utility:

\[
\text{E}U_{ik} = [b_k + \beta_k N_k - 1 (a_i - a_k)]Q_k - \frac{1}{N_k} \beta_k (N_k - 1)Q_k^2.
\]

Among \( K \) alternatives, grower \( i \) would choose to participate in contract \( k \) if \( \text{E}U_{ik} \geq \text{E}U_{il} \), \( \forall l = 1, 2, \cdots, K; l \neq k \). We use expression \(( \text{E} \) \) to obtain several interesting comparative statics sorting results. First, it is easy to see that if it were possible to choose the base payment \( b_k \) independently of targeted output level \( Q_k \), regardless of their ability, all utility maximization growers would pool themselves into the production contract with larger base payment because \(( \text{E} \) \) is strictly increasing in \( b_k \). Second, growers’ utilities also depend on the slope \( \beta_k \). However, as mentioned earlier, all five contracts in the menu have the same bonus/malus coefficients, hence, there is no sorting based on the slope (intensity) of the scheme (\( \beta_k = \beta, \forall k \)).

Next, growers with higher (lower) abilities should be more inclined to sort themselves into contracts with larger (smaller) expected total output. To prove sorting based on expected output, assume that growers hold fixed uniform expectations about the size of tournament groups in all contracts, \( \mathbb{E}(N_k) = N, \forall k \), and compute the partial derivative of grower’s expected utility with respect to expected output:

\[
\frac{\partial \text{E}U_{ik}}{\partial Q_k} = b_k + \beta \frac{N - 1}{N} (a_i - a_k) - \frac{\beta^2}{\gamma} (\frac{N - 1}{N})^2 \bar{Q}_k.
\]

The proof is based on realizing that there exists a threshold ability \( a_0 = \bar{a} - \frac{b_k N}{\beta(N-1)} + \frac{\beta}{2} (\frac{N-1}{N}) \bar{Q}_k \).
such that for growers with abilities $a_i > a_0$, the expected utilities are increasing with output at $\tilde{Q}_k$, that is, $\frac{\partial EU_{ik}}{\partial Q_k} > 0$, which means that they will sort themselves into contracts with expected outputs greater than $\tilde{Q}_k$. On the contrary, growers with abilities $a_i < a_0$ will sort themselves into contracts with expected outputs lower than $\tilde{Q}_k$ because their expected utilities are decreasing with output at $\tilde{Q}_k$.

The presented sorting result is illustrated in Figure 1 which graphs grower’s expected utility against the expected output levels for three growers with heterogeneous abilities, $a_H > a_0 > a_L$. Grower’s expected utility is a polynomial of degree 2 in $Q_k$ represented by an inverted U-shape parabola. The expected utility curve for a high ability grower always lies strictly above that for a low ability grower because at each output level expected utility is always larger for the high ability type. Now let’s analyze the contract choice problems for growers $a_H$, $a_0$ and $a_L$ when facing the contract with expected output $\tilde{Q}_k$. We can see that expected utility reaches its maximum for grower $a_0$ since $\frac{\partial EU_{ik}}{\partial Q_k} |_{a_i=a_0} = 0$ and, hence, this grower should choose the contract with expected output $\tilde{Q}_k$. For high ability grower $a_H$, her expected utility is increasing with output at $\tilde{Q}_k$ and choosing contracts with output larger than $\tilde{Q}_k$ generates higher utility. Therefore, grower $a_H$ should choose contract with expected output $\tilde{Q}_k^*$ to attain the highest possible utility $EU_{H}^*$. On the other hand, the expected utility is decreasing with output at $\tilde{Q}_k$ for low ability grower $a_L$ which means that, in order to achieve highest possible utility, she should choose contract with output lower than $\tilde{Q}_k$, namely $\tilde{Q}_L^*$.

In fact, one can calculate the optimal expected output for each grower $i$ by solving $\frac{\partial EU_{ik}}{\partial Q_k} = 0$ to obtain:

$$\tilde{Q}_i^* = \frac{b_k + \beta_k N_k^{-1}(a_i - \bar{a}_k)}{\frac{\beta^2_k}{\gamma} (N_k-1)^2}.$$  \hspace{1cm} (7)

Expression (7) indicates that optimal output is an increasing function of grower’s ability. If production contracts only vary by output, we expect to observe a positive sorting effect such that high ability growers self-select themselves into contracts with larger expected outputs while low ability growers choose to participate in contracts with lower expected outputs. The underlying rationale behind this result is straightforward. Higher ability growers have larger probability of winning the tournament competition. Therefore, choosing contracts with larger expected output can increase the potential bonus that they can earn. On the contrary, lower ability growers
are more likely to lose the tournament and receive a penalty. Hence, it is wise for them to participate in contracts with smaller expected output to minimize the potential loss.

Similarly, it is easy to show that growers with higher (lower) abilities are more inclined to sort themselves into contracts with larger (smaller) number of players. Assuming that growers hold fixed uniform expectations about the average output in all contracts, \( \mathbb{E}(\bar{Q})_k = \bar{Q}, \forall k \), and taking partial derivative of grower’s expected utility function (5) with respect to \( N_k \) yields:

\[
\frac{\partial \mathbb{E} U_{ik}}{\partial N_k} = \beta (a_i - \bar{a}) \bar{Q} - \frac{\beta^2 \bar{Q}^2 (N_k - 1)}{\gamma N_k^3}.
\]

If \( a_0 = \bar{a} + \frac{\beta \bar{Q}(N_k - 1)}{\gamma N_k} \), then for all growers with ability \( a_i > a_0 \), \( \frac{\partial \mathbb{E} U_{ik}}{\partial N_k} > 0 \), and these growers sort themselves into contracts with number of growers larger than \( N_k \). For growers with ability \( a_i < a_0 \), \( \frac{\partial \mathbb{E} U_{ik}}{\partial N_k} < 0 \), and these growers would choose contracts with number of growers smaller than \( N_k \). This result can be explained by the influence of one player’s performance on the tournament average. Higher ability players would prefer to compete with a large group of contestants because they don’t want their good performances to significantly increase the average performance of the entire group which is used as a benchmark for comparison. Conversely,
lower ability players prefer contracts with smaller number of contestants because, in this case, their poor performance can easily drag down the average performance making their penalty less severe.

2.3 Nash Equilibrium

The self-selection results discussed so far are all based on the assumption that all contracts have the same exogenous average ability, not influenced by growers’ optimal contract choices. However, the description of the self-selection equilibrium is more complicated because the equilibrium average abilities in available contracts are determined endogenously and vary with the change in contract attributes. Clearly, based on the previously obtained comparative statics results, the average ability in contracts with larger output is, ceteris paribus, expected to be higher than in contract with lower output. Hence, a rational high ability grower should anticipate that contracts with larger expected outputs should attract, beside herself, other high ability growers and, as a result, she may end up competing against a very strong pool of contestants. An alternative, perhaps a more profitable strategy, could be to sort herself into a contract with lower expected output but with potentially lower average ability pool of contestants which could guarantee an easy victory.

Therefore, as seen from equation (5), the decision about which contract to choose involves a trade-off between the piece-rate determined by the grower’s ability relative to the average ability in the respective tournament and the total expected output. Choosing a contract with large expected output would earn smaller expected piece-rate, whereas choosing a contract with smaller expected output would earn larger piece-rate. The optimal contract choice for each grower will depend on the multiplication of these two negatively correlated variables. The Nash equilibrium of this game is summarized in the following proposition.

**Proposition 1.** Suppose there are $K$ contracts with increasing output levels $\bar{Q}_1 < \bar{Q}_2 < \cdots < \bar{Q}_K$, each with positive number of participating growers. Growers abilities follow the distribution $G(\cdot)$ with density $g(\cdot)$, minimum and maximum abilities are $a_{\min}$ and $a_{\max}$. There exist $K - 1$ threshold abilities $a_{12} < a_{23} < \cdots < a_{K-1,K}$ such that growers with abilities in $(a_{\min}, a_{12})$ choose contract with the smallest output $\bar{Q}_1$, growers with abilities in $(a_{12}, a_{23})$
choose contract with output $\bar{Q}_2$, and so on, and growers with abilities in $(a_{K-1}, a_{\text{max}})$ choose contract with the largest output $\bar{Q}_K$. The contract average abilities are increasing in output levels, i.e. $\bar{a}_1 < \bar{a}_2 < \cdots < \bar{a}_K$.

Proof. We first prove the proposition under the simplest scenario with only two contracts $\bar{Q}_1 < \bar{Q}_2$. Based on equation (9) and maintaining the assumption that $E_i (N_1) = E_i (N_2) = N$, for every grower $i$'s expected utilities from participating in contract $\bar{Q}_1$ and contract $\bar{Q}_2$ can be written as:

$$\begin{align*}
    E_{U_{i1}} &= \left[ b_1 + \beta \frac{N-1}{N} (a_i - \bar{a}_1) \right] \bar{Q}_1 - \frac{1}{2\gamma} \left[ \frac{\beta (N-1) \bar{Q}_1^2}{N} \right]^2 \\
    E_{U_{i2}} &= \left[ b_2 + \beta \frac{N-1}{N} (a_i - \bar{a}_2) \right] \bar{Q}_2 - \frac{1}{2\gamma} \left[ \frac{\beta (N-1) \bar{Q}_2^2}{N} \right]^2.
\end{align*}$$

Since both expected utility functions are linear in $a_i$, there exists a threshold ability $a_{12}$:

$$a_{12} = -\frac{b_2 \bar{Q}_2 - b_1 \bar{Q}_1}{\beta (N-1) (Q_2 - Q_1)} + \frac{\bar{a}_2 \bar{Q}_2 - \bar{a}_1 \bar{Q}_1}{Q_2 - Q_1} + \frac{\beta (N-1) (\bar{Q}_2 + \bar{Q}_1)}{2\gamma N}$$

obtained by setting $E_{U_{i1}} = E_{U_{i2}}$ such that grower $i$ with ability $a_{12}$ is indifferent between choosing $\bar{Q}_1$ or $\bar{Q}_2$. The threshold ability $a_{12}$ must be between $a_{\text{min}}$ and $a_{\text{max}}$ because both contracts have a positive number of participating growers. It is easy to see that $E_{U_{i1}} > E_{U_{i2}}$ when $a_{\text{min}} < a_i < a_{12}$ and $E_{U_{i1}} < E_{U_{i2}}$ when $a_{12} < a_i < a_{\text{max}}$. Therefore, grower $i$ will choose contract $\bar{Q}_1$ if her ability is between $a_{\text{min}}$ and $a_{12}$ and choose contract $\bar{Q}_2$ if her ability is between $a_{12}$ and $a_{\text{max}}$. This sorting result is clearly illustrated in Figure 2a.

The Nash equilibrium in average abilities is determined by the following two equations:

$$\begin{align*}
    \bar{a}_1 &= \frac{\int_{a_{\text{min}}}^{a_{12}} a_i g(a_i) da_i}{\int_{a_{\text{min}}}^{a_{12}} g(a_i) da_i}, \\
    \bar{a}_2 &= \frac{\int_{a_{12}}^{a_{\text{max}}} a_i g(a_i) da_i}{\int_{a_{12}}^{a_{\text{max}}} g(a_i) da_i}.
\end{align*}$$

Inserting $a_{12}$ from (9) into (10) gives a system of two nonlinear equations with two unknowns $\bar{a}_1$ and $\bar{a}_2$. The equilibrium average ability in contract $\bar{Q}_1$ is smaller than in contract $\bar{Q}_2$ because...
Figure 2: Sorting Effect In Nash Equilibrium

\[ \bar{a}_1 < a_{12} < \bar{a}_2, \text{ i.e.:} \]

\[ \bar{a}_1 = \frac{\int_{a_{\min}}^{a_{12}} a_i g(a_i) da_i}{\int_{a_{\min}}^{a_{12}} g(a_i) da_i} < \frac{\int_{a_{\min}}^{a_{12}} a_{12} g(a_i) da_i}{\int_{a_{\min}}^{a_{12}} g(a_i) da_i} = a_{12} \]

\[ \bar{a}_2 = \frac{\int_{a_{12}}^{a_{\max}} a_i g(a_i) da_i}{\int_{a_{12}}^{a_{\max}} g(a_i) da_i} > \frac{\int_{a_{12}}^{a_{\max}} a_{12} g(a_i) da_i}{\int_{a_{12}}^{a_{\max}} g(a_i) da_i} = a_{12}. \]

Notice that the base payment \( b_k \) is not having any impact on growers’ monotonic self-selection strategies. It only changes the values of the cut-off abilities.

Next, suppose a third contract is added to the pool of contract alternatives with \( Q_1 < Q_2 < Q_3 \). The threshold ability which equates the expected utility from contract \( Q_2 \) and contract \( Q_3 \),

\[ a_{23} = -\frac{b_3 \bar{Q}_3 - b_2 \bar{Q}_2}{\beta(N-1)(Q_3 - Q_2)} + \frac{\bar{a}_3 \bar{Q}_3 - \bar{a}_2 \bar{Q}_2}{Q_3 - Q_2} + \frac{\beta(N-1)(\bar{Q}_3 + \bar{Q}_2)}{2\gamma N} \]

must be greater than \( a_{12} \) and smaller than \( a_{\max} \). This is because when \( a_{23} < a_{12} \), contract \( Q_2 \) would be dominated by either contract \( Q_1 \) or contract \( Q_3 \) and would not be chosen by any grower, and when \( a_{23} > a_{\max} \), even the best grower would pick contract \( Q_2 \) and no grower would choose contract \( Q_3 \). Hence, it must be that \( a_{\min} < a_{12} < a_{23} < a_{\max} \). With this condition, growers with abilities in \((a_{\min}, a_{12})\) would choose contract \( \bar{Q}_1 \), growers with abilities in \((a_{12}, a_{23})\) would choose contract \( \bar{Q}_2 \) and growers with abilities in \((a_{23}, a_{\max})\) would choose contract \( \bar{Q}_3 \). This sorting result for three contracts is illustrated in Figure 2b. Similarly to two
equations in (10), the Nash equilibrium in average abilities for 3 contracts case is determined by solving a system of 3 nonlinear equations with 3 unknowns \( \bar{a}_1, \bar{a}_2, \bar{a}_3 \), with equations (9) and (11) used as the thresholds. Here too, the equilibrium average abilities are positively related to the expected contract outputs because \( \bar{a}_1 < a_{12} < \bar{a}_2 < a_{23} < \bar{a}_3 \).

The same reasoning can be extended to \( K \) contract alternatives. Threshold abilities are expressed as

\[
a_{k-1,k} = -\frac{b_k Q_k - b_{k-1} Q_{k-1}}{\beta \left( \frac{N-1}{N} \right) (Q_k - Q_{k-1})} + \frac{\bar{a}_k Q_k - \bar{a}_{k-1} Q_{k-1}}{Q_k - Q_{k-1}} + \frac{\beta (N - 1)(Q_k + Q_{k-1})}{2 \gamma N}
\]

\( \forall k = 2, 3, \ldots, K \)

and growers with abilities between \( a_{k-1,k} \) and \( a_{k,k+1} \) would choose contract \( k \). Following the same argument as before, the contracts’ equilibrium average abilities are increasing with the expected contracted output.

Q.E.D.

The result in Proposition 1 shows that the menu of contracts offered by the integrator company generates an equilibrium with monotonic positive self-selection of growers with heterogeneous abilities into contracts with differentiated expected output size. For high ability types the attractiveness of larger output outweighs the negative impact of higher average ability in the chosen tournament and potentially lower piece rate. Hence, it is optimal for these types to choose contracts with large output levels. On the other hand, for low ability growers the incentive to mix themselves with high ability types and enter the contract with large contracted output is too weak to overturn the chance of earning higher piece rate by competing against low average ability pool, leading them to self-select themselves into contracts with smaller expected output.

A parallel equilibrium where contract tournaments are separately varied by the number of growers can be easily obtained. In this equilibrium high ability types would choose tournaments with more players and low ability types would choose tournaments with fewer players.\(^7\) The

\(^7\)The proof of this equilibrium directly follows the proof of Proposition 1 and is available from authors upon request.
equilibrium selection of heterogeneous ability agents into contests with vector-valued characteristics (expected output and the number of players) would be much more difficult, perhaps even impossible, to characterize; for an example see Morgan, Sisak and Vardy (2014). However, as seen from (3), this jointly determined equilibrium would depend on the product of \( \frac{N_k - 1}{N_k} \) and \( Q_k \) and since \( \frac{N_k - 1}{N_k} \) is close to unity for any meaningful number of players, the equilibrium should decisively depend on the sorting based on the expected output and not so much on the number of players.

2.4 Principal’s Problem

Keeping the hypotheses testing objective in mind, the usefulness of the result showing positive selection of growers with different abilities into contracts for growing different sizes of chickens hinges on the ability to prove that the whole scheme is meaningful from the integrator’s perspective. This means that offering a menu of contracts designed to motivate high ability growers to pick contracts for growing heavier birds and low ability types to pick contracts for growing smaller birds has to maximize principal’s profit. There are two alternative possibilities that could potentially maximize profits.

First, instead of offering a menu of contracts which elicits positive sorting of growers into contracts, an integrator could instead offer a menu that would elicit negative sorting. This possibility is ruled out by the technological (nutritional) fact that feed conversion deteriorates with the size of animals grown. Because feed is the most significant production input, as a result, the cost of production per pound of live weight is always higher in contracts for heavier birds. Therefore placing better growers in contracts for larger broilers makes perfect sense because in this situation they can better manage feed and minimize the production cost.

Second, we still need to show that offering a menu of contracts is welfare superior (in the expected profit sense) to offering one uniform contract to all contract growers. Let’s start by calculating the principal’s expected profit from one tournament in contract \( k \) as the difference
between the expected total revenue and the total payment to growers and the cost of production:

\[
\mathbb{E}[N_k \bar{Q}_k] = \mathbb{E}[p N_k \bar{Q}_k] - \sum_{i=1}^{N_k} (b_k \beta_k \frac{N_k - 1}{N_k} \bar{a}_k) \bar{Q}_k - \sum_{i=1}^{N_k} C_{ik}(e_{ik}^\ast)].
\] (13)

Referring to the grower’s performance function from equation (2) and given that two production shocks are assumed to have zero means, expected cost of production becomes:

\[
\mathbb{E}[C_{ik}(e_{ik}^\ast)] = -(e_{ik}^\ast + a_i) \bar{Q}_k.
\] (14)

Substituting in the closed-form solution for optimal effort (4), the expected profit of the principal becomes:

\[
\mathbb{E}[\Pi] = (p - b + \bar{a}_k) \bar{Q}_k N_k + \beta_k (N_k - 1) \bar{Q}_k^2 \gamma.
\] (15)

Notice that the aggregate bonus-malus payment has dropped out from principal’s expected profit. In a cardinal tournament scenario, positive bonus payments and negative bonus (malus) payments cancel each other out precisely by construction. The total payments to contract growers that the principal has to make simply equals the base piece rate multiplied by the total quantity produced (number of pounds).

To prove that a positive selection menu generates higher profits than a uniform contract we assume that the menu consists only of two contracts: contract \(H\) with larger expected output \(\bar{Q}_H\) and contract \(L\) with smaller expected output \(\bar{Q}_L\) and that higher ability growers would choose contract \(H\) and lower ability growers would choose contract \(L\). The resulting total expected profit of the principal is:

\[
\mathbb{E}[\Pi] = (p - b + \bar{a}_H) \bar{Q}_H N_k + \frac{\beta_k (N_k - 1) \bar{Q}_H^2 \gamma}{\gamma} + (p - b + \bar{a}_L) \bar{Q}_L N_k + \frac{\beta_k (N_k - 1) \bar{Q}_L^2 \gamma}{\gamma}.
\] (16)

In the alternative scenario, the principal offers each grower only one contract for both sizes of birds. We assume that half of growers will be tasked with the production of heavier birds (contract \(H\)) and the other half with the production of lighter birds (contract \(L\)). Since growers abilities are private information and there is no revelation mechanism in place, in expectation,
the average ability in a uniform contract is $\frac{a_H + a_L}{2}$. The principal’s expected profit from this contractual arrangement is:

$$E\Pi_3 = (p - b + \frac{a_H + a_L}{2})Q_H N + \frac{\beta(N - 1)Q_H^2}{\gamma} + (p - b + \frac{a_H + a_L}{2})Q_L N + \frac{\beta(N - 1)Q_L^2}{\gamma}. \quad (17)$$

The difference

$$E\Pi_1 - E\Pi_3 = \frac{(\bar{a}_H - \bar{a}_L)(\bar{Q}_H - \bar{Q}_L)N}{2}, \quad (18)$$

is strictly positive which proves the claim. The generalization of the result for the menu of three or more contracts is straightforward.

### 3 Estimation and Results

The objective of the empirical part of this paper is to test Proposition II. For this purpose we use contract settlement data from a large broiler company in the United States. The contract data set contains contract settlements information for five different broiler production contracts during a two-year period. These five contracts are differentiated by the size of the birds produced. The contracts for growing heavier birds have higher base payment rates. For the target weights varying between 5 and 6.2 pounds of live weight per bird, the base rate varies in the interval between 3.55 cents and 4.53 cents per pound. The slope parameter in all five contracts is equal to one. There are total of 7,421 observations and each observation provides one contract settlement information between the integrator company and an individual grower. Each observation includes starting and settling date, heads started and sold, weight sold, and total amounts and values of various production inputs provided by the integrator. Table II presents summary statistics for several key variables. The data shows that it usually takes 48 to 57 days for one-day old baby chicks to reach the target weight (5-6 pounds). For all five contracts, the feed conversion ratios are close to 2, which means that two pounds of feed is required for a bird to gain one pound of weight. The mortality rate fluctuates around 5% per flock per growing cycle.

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8Everything said about sorting based on the expected output is true for sorting based on the number of players. Using the same line of reasoning one can easily show that the integrator benefits from offering a menu of contracts where high ability types pick contracts with more players and low ability types contracts with fewer players.
All growers whose birds were harvested during the same calendar week will settle their contracts at the end of that week and will form a tournament. Table 2 provides the summary of tournament statistics and contains entries on the total number of observations (flocks grown) $K$, number of tournaments $T$, number of growers $N$ in each contract, average number of growers in each tournament $n$, total output produced by each grower expressed in pounds of live weight and their corresponding total settlement costs. Obviously, the settlement costs are directly related to weight: the heavier the birds, the larger the costs of producing them.

The actual testing of the sorting result involves several steps. In the first step, we obtain growers’ abilities by estimating a two-way fixed effect model based on the grower’s additive performance function as specified in equation (19):

$$
q_{it} - e^*_it = a_N + \sum_{j=1}^{N-1} (a_j - a_N)d_{jt} + \sum_{k=1}^{T-1} u_k g_{kt} + w_{it}
$$

where $q_{it} = -C_{it}/Q_{it}$ is the performance measure equal to the negative of the adjusted prime cost (APC) for grower $i$ in tournament $t$ and $e^*_it$ is optimal effort of grower $i$ calculated using equation (4). APC measures the average cost accrued to the integrator of producing each pound of live broilers. It is computed as total settlement cost, which is the sum of cost of chicks, feed, fuel, medications, vaccinations and other customary flock costs charged to grower $i$, divided by the total pounds of live weight moved from the grower’s farm. We assume that ability $a_j$ is a tournament-invariant variable specific to grower $j$ and common production shock $u_k$ is a grower-invariant variable specific to tournament $k$. $d_{jt}$ and $g_{kt}$ are grower and tournament dummy variables with $d_{jt} = 1$ if $j = i$ and 0 otherwise and $g_{kt} = 1$ if $k = t$ and 0 otherwise. To avoid singularity, only $N - 1$ grower dummies and $T - 1$ tournament dummies are included in the regression. That way, $a_N$ is the ability of grower N and it is estimated as the constant term of the regression. Growers’ fixed effects are the differences between each grower’s ability and grower N’s ability. Assuming the common shock in tournament T is zero, common shocks for all other tournaments are estimated as tournament specific fixed effects. Idiosyncratic shock of grower $i$ in tournament $t$, $w_{it}$, is estimated as the regression error.

With estimated parameters of equation (19), the sample variance of common production
shocks $\hat{\sigma}_u^2$ can be computed as $\hat{\sigma}_u^2 = \frac{1}{T-1} \sum_{t=1}^{T} (u_t - \bar{u})^2$ where $\bar{u} = \frac{1}{T} \sum_{t=1}^{T} u_t$. Similarly, the sample variance of idiosyncratic shocks can be computed as $\hat{\sigma}_w^2 = \frac{1}{K-1} \sum_{k=1}^{K} \sum_{t=1}^{n_k} (w_{it} - \bar{w})^2$ where $\bar{w} = \frac{1}{K} \sum_{k=1}^{K} \sum_{t=1}^{n_k} w_{it}$ and $K = \sum_{t=1}^{T} n_t$.

Next, we conduct a pairwise t-tests for each pair of contracts with the null hypotheses stating that the average growers’ abilities in each pair of contracts are equal. In addition, we also conduct the Kolmogorov-Smirnov tests for each pair of contracts to examine whether the distribution of growers abilities are the same for each pair of contracts.

The test of Proposition 1 is based on the estimation of a random utility model. We assume that at the beginning of the data, a grower is presented with a menu of contracts specifying which type/weight of broiler chickens will be grown under each contract and the parameters of the tournament payment scheme. At this point, based on her idiosyncracies (in this case ability) she needs to decide which contract to sign. Specifically, we assume that the utility that grower $i$ expects to derive from participating in contract $k$ is given by:

$$
\mathbb{E}U_{ik} = \alpha_k + \lambda_Q (a_i - \bar{a}_k) \bar{Q}_{ik} + \lambda_N (a_i - \bar{a}_k) \bar{N}_{ik} + \epsilon_{ik}
$$

(20)

where $\alpha_k$ is the contract-specific constant, $a_i$ is the ability of grower $i$, $\bar{a}_k$ is the average ability in contract $k$, $\bar{Q}_{ik}$ is the expectation held by grower $i$ about the daily output per chicken house in contract $k$ and $\bar{N}_{ik}$ is the expectation held by grower $i$ about the number of growers in her tournament. The iid random error term $\epsilon_{ik}$ is assumed to follow a type I extreme value distribution which implies a simple logit model. Grower $i$ would choose contract $k$ if $\mathbb{E}U_{ik} \geq \mathbb{E}U_{il}$ for all $l \neq k$.

Since growers make their contract choices prior to the actual realization of their efforts and random shocks, the expected outputs instead of actual outputs are used to calculate grower’s

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9Contract specific constant for contract $D$ is assumed to be zero and all other constants are estimated relative to contract $D$.

10Growers’ final total outputs largely depend on the target weight of finished birds (decided by the integrator) and the number of chicken houses they own. Since growing larger birds takes longer time to finish and operating more chicken houses means housing more birds, using pounds of live chickens per house per day is a measure of output which is comparable across growers and contracts.
expected utility. In addition, we also assume that growers choose which contract to participate in based on the expectations about the average number of growers that would participate in those tournaments. Of course, not all growers currently on assignment in contract $k$ compete simultaneously in the same tournament but the expectation about the actual number of players in a particular tournament is informed by the total number of growers in that contract. In estimating (20), $Q_{ikt}$ and $N_{ikt}$ are assumed to be exogenous and the same across all growers. We calculate these two explanatory variables as the average output and the average number of growers in one tournament for the entire contract $k$:

$$
Q_{ik} = \frac{\sum_{t=1}^{N_k} \sum_{i=1}^{T_{ik}} Q_{ikt}}{\sum_{i=1}^{N_k} T_{ik}}
$$

(21)

$$
N_{ik} = \frac{\sum_{t=1}^{T_k} N_{kt}}{T_k}
$$

(22)

where $N_k$ is total number of growers in contract $k$, $T_{ik}$ is number of tournaments grower $i$ participated in for contract $k$, $N_{kt}$ is number of growers in tournament $t$ in contract $k$, and $T_k$ is total number of tournaments in contract $k$.

Finally, notice the absence of the tournament slope parameter as a determinant of contract choice in the random utility model. This is because the slopes in all contracts are the same (equal to 1) so there is no choice to be made with respect to this contract attribute.

The expected signs of the estimated parameters are determined by the presented theory. If the estimated parameter $\lambda_Q$ is positive, it means that growers with higher than average abilities would prefer contracts with larger expected output. The positive sign on $\lambda_N$ would mean that higher ability growers sort themselves into tournaments with higher expected number of growers.

Because the estimation of the logit models require growers’ abilities which were not observed but rather estimated, the inference from models with generated regressors is invalid because the standard errors and test statistics are obtained without taking sampling variation into consideration (see Efron and Tibshirani 1993, Wooldridge 2002). To address the problem, both stages of the model are estimated using 1000 bootstrap samples with replacement and the
The parameter estimates of equation (19) are suppressed for brevity. For all five contracts, the adjusted $R^2$ is around 0.9, which indicates that growers’ heterogeneous abilities and tournament common shocks capture most of the variations in growers’ performances. The statistical tests for the differences in estimated growers’ abilities across contracts are presented in Table 3. The first column is the contract identifier, the second column presents the total number of growers, followed by the estimated mean ability and its standard deviation. In the next five columns we show the pairwise differences in estimated abilities between the column contract and the row contract. The corresponding t-statistics are displayed in the parentheses beneath them. As we can see, the null hypotheses of equal average abilities cannot be rejected for only three contract pairs: AC, AE and CE. These results show prima facie evidence that heterogeneous growers are not randomly selected into contracts with different characteristics, but instead, there could be some systematic rules governing the selection process.

The results of the pairwise Kolmogorov-Smirnov test statistics shown in Table 4 reinforce the previous conclusion. The K-S statistics for a pair of contracts is calculated as

$$D_{ij} = \max_a(|F_i(a) - F_j(a)|)$$

(23)

where $F_i(a)$ and $F_j(a)$ are the empirical cumulative distribution function (cdf) of abilities of contract $i$ and $j$. The results show that the null hypotheses that growers abilities of two production contracts come from the same continuous distribution cannot be rejected for only two pairs of contracts: CE and AE at 10% significance level. For the remaining 8 pairs, the null hypotheses are rejected at 10% significance level (some are rejected at 1%) indicating a possible systematic sorting of heterogeneous abilities agents into contracts with different attributes. These results are also illustrated in Figure 3 which depicts the distributions of estimated growers abilities for all five contracts.

The estimation results of the random utility model are summarized in Table 5. The contract specific constant for contract D is assumed to be zero and the constants for all other four contracts are estimated in relationship to it. There are several important conclusions that can be derived from the obtained results. First, the estimated model coefficients show strong evidence
of positive sorting of growers into contracts with different expected outputs based on their abilities because the parameters $\lambda_Q$ are positive and statistically significant. Controlling for other unobservable contract attributes captured by the contract specific fixed effects $\alpha’s$, higher than average ability growers are more likely to sort themselves into contracts with larger expected output (heavier birds) while lower than average ability growers would prefer contracts with smaller output levels (lighter birds).

The sorting behavior based on the expected number growers in one tournament is also shown to be positive, yet it is not statistically significant. Apparently, the number of contestants has a small impact on the average performance in any given tournament and as such it does not represent a decisive factor in growers’ decisions about which contract to choose. In addition, formulating reliable expectations about the number of players in a given tournament is very difficult because integrators routinely change the size of the tournament groups in response to logistical considerations (scheduling of production) and market conditions (demand for poultry...
meat). This results indirectly confirm previous findings of Levy and Vukina (2004) who found that the organization of contract production leads to a rapid dissipation of tournament groups over time and that the composition of those groups becomes random after about five subsequent tournaments.

4 Conclusion

Pay for performance compensation schemes have been used for quite some time in many sectors of the economy, such as manufacturing, agriculture and sales. Recently they have even penetrated several non-traditional sectors such as health care and education. Labor economics literature has recognized the fact that pay for performance can accomplish two tasks: it can provide incentives for workers to work hard and it can also help recruiting and retaining high ability employees who chose to work in highly competitive job environments. The literature on the provision of incentives has significantly outpaced the literature on hiring and job design. In particular, relatively little is known how firms design job packages to hire and keep workers they want. This paper contributes to the literature in this area in two important ways.

First, we develop a relatively simple theoretical model of sorting into cardinal tournament type of contests. The most interesting aspect of sorting into any type of relative compensation schemes is that high ability types face confusing and counter-balancing incentives. On one hand, they want to self-select themselves into a tournament with steeper incentives which they would surely do in any type of individual scheme such as simple piece rate. On the other hand, they would rather not select themselves into tournaments with steep incentives because they anticipate that other high ability types may also want to join the same tournaments which would make the competition in those tournaments rather stiff and the probability of winning rather remote. Instead, they might disguise themselves as low ability types and join a tournament with lower incentives but also with less severe competition which would earn them lower piece-rate but would surely improve their chances of winning the tournament. The incentives structure for low ability types is straightforward, i.e. they have no incentives to disguise themselves as high ability types and play in a tournament with high ability types. We were able to show that
for high ability agents the first incentive trumps the second and one still obtains a separating equilibrium where high ability types choose tournaments with steeper incentives.

Secondly, we test the theoretical propositions with a unique real market data on the settlements of chicken production contracts. We were able to show that, indeed, the chicken companies have the incentive to offer a menu of contracts which elicit positive sorting of heterogeneous ability contract growers into contract reflective of their types. High ability types need to choose contract for growing heavier birds and low ability types need to choose contracts to grow lighter birds. And they do. This results makes perfect sense from the nutritional and genetic point of view. It is generally true that smaller animals have better (lower) feed conversion ratios than larger animals. So as chickens grow larger, their feed conversion ratio deteriorates. Because the integrator company always pays for feed and because feed is the single largest line item in the cost structure of live broilers production, the cost of production per pound of live weight is significantly higher in contracts for heavier birds. And, therefore, placing better growers in contracts for larger broilers makes sense because this job design places them into a situation where they can better utilize production inputs and minimize the production cost to the greatest extent.

Of course, this work is not without some problems. Strictly speaking, the equilibrium selection of heterogeneous ability agents presented in the theoretical part of this paper pertains to contests differing in one dimension only (expected output or the number of players), yet the estimated random utility model uses both dimensions to specify the contract choice. The theoretical characterization of sorting equilibrium for contracts with a relative performance evaluation function that differ in multiple dimension simultaneously would be considerably more difficult, perhaps even impossible, to obtain. However, as we discussed in the theory section of the paper, this jointly determined equilibrium would depend on the product of \( \frac{N_k - 1}{N_k} \) and \( Q_k \) and since \( \frac{N_k - 1}{N_k} \) is close to unity for any empirically observed tournament size, the equilibrium should decisively depend on the sorting based on the expected output and not so much on the number of players. The obtained empirical results confirmed this conjecture.
References


Table 1: Summary Statistics of Broiler Production Contracts

<table>
<thead>
<tr>
<th>Contract</th>
<th>Days</th>
<th>Days (St. Dev.)</th>
<th>Broiler Weight (Lbs.)</th>
<th>Broiler Weight (St. Dev.)</th>
<th>Feed Conversion</th>
<th>Feed Conversion (St. Dev.)</th>
<th>Mortality Rate</th>
<th>Mortality Rate (St. Dev.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>49</td>
<td>1.48</td>
<td>5.0</td>
<td>0.25</td>
<td>2.08</td>
<td>0.06</td>
<td>5.24%</td>
<td>2.58%</td>
</tr>
<tr>
<td>B</td>
<td>50</td>
<td>1.62</td>
<td>4.8</td>
<td>0.29</td>
<td>2.03</td>
<td>0.08</td>
<td>2.91%</td>
<td>3.31%</td>
</tr>
<tr>
<td>C</td>
<td>56</td>
<td>1.99</td>
<td>5.9</td>
<td>0.25</td>
<td>2.19</td>
<td>0.12</td>
<td>4.67%</td>
<td>2.99%</td>
</tr>
<tr>
<td>D</td>
<td>57</td>
<td>1.55</td>
<td>6.0</td>
<td>0.20</td>
<td>2.17</td>
<td>0.06</td>
<td>4.45%</td>
<td>1.80%</td>
</tr>
<tr>
<td>E</td>
<td>57</td>
<td>1.81</td>
<td>6.2</td>
<td>0.26</td>
<td>2.19</td>
<td>0.11</td>
<td>5.39%</td>
<td>2.98%</td>
</tr>
</tbody>
</table>

Table 2: Summary Statistics of Contract Tournaments

<table>
<thead>
<tr>
<th>Contract</th>
<th>K</th>
<th>T</th>
<th>N in each T</th>
<th>n in each T</th>
<th>Output (Lbs.)</th>
<th>Output (Lbs.)</th>
<th>Cost ($)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>908</td>
<td>48</td>
<td>196</td>
<td>19</td>
<td>232,520</td>
<td>120,510</td>
<td>75,691</td>
<td>39,495</td>
</tr>
<tr>
<td>B</td>
<td>3234</td>
<td>104</td>
<td>344</td>
<td>31</td>
<td>240,260</td>
<td>131,040</td>
<td>75,316</td>
<td>41,955</td>
</tr>
<tr>
<td>C</td>
<td>1361</td>
<td>104</td>
<td>280</td>
<td>13</td>
<td>332,330</td>
<td>138,430</td>
<td>106,270</td>
<td>45,287</td>
</tr>
<tr>
<td>D</td>
<td>958</td>
<td>76</td>
<td>184</td>
<td>13</td>
<td>302,880</td>
<td>169,180</td>
<td>100,890</td>
<td>56,288</td>
</tr>
<tr>
<td>E</td>
<td>959</td>
<td>103</td>
<td>240</td>
<td>9</td>
<td>364,810</td>
<td>153,960</td>
<td>117,560</td>
<td>51,789</td>
</tr>
</tbody>
</table>

a K = number of observations; T = number of tournaments; N = number of growers in a contract; n = number of growers in a tournament.
Table 3: Differences in Estimated Abilities Across Production Contracts

<table>
<thead>
<tr>
<th>contract</th>
<th>$N_k$</th>
<th>mean(a)</th>
<th>std(a)</th>
<th>t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>199</td>
<td>0.03694</td>
<td>0.00614</td>
<td>0.001290** -0.000566 -0.000614</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2.442) (-0.961) (-0.037)</td>
</tr>
<tr>
<td>B</td>
<td>339</td>
<td>0.03565</td>
<td>0.00578</td>
<td>-0.001856*** 0.004225*** -0.001903***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-3.786) (8.281) (-3.853)</td>
</tr>
<tr>
<td>C</td>
<td>307</td>
<td>0.03750</td>
<td>0.00668</td>
<td>0.006081*** -0.000047</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(10.577) (-0.087)</td>
</tr>
<tr>
<td>D</td>
<td>181</td>
<td>0.03142</td>
<td>0.00507</td>
<td>-0.006128***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-10.646)</td>
</tr>
<tr>
<td>E</td>
<td>292</td>
<td>0.03755</td>
<td>0.00663</td>
<td></td>
</tr>
</tbody>
</table>

*a* $H_0$ states that means are equal; t-statistics are in parentheses.

*b* $* p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 4: Kolmogorov-Smirnov Tests of Differences in Estimated Abilities

<table>
<thead>
<tr>
<th>contract</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.1104*</td>
<td>0.1180*</td>
<td>0.4006***</td>
<td>0.1084</td>
</tr>
<tr>
<td></td>
<td>(0.0875)</td>
<td>(0.0641)</td>
<td>(5.35 E-14)</td>
<td>(0.1158)</td>
</tr>
<tr>
<td>B</td>
<td>0.1792***</td>
<td>0.3601***</td>
<td>0.1712***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.17 E-05)</td>
<td>(4.92 E-14)</td>
<td>(0.0002)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>0.5063***</td>
<td>0.0142</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.14 E-26)</td>
<td>(1.000)</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td>0.4965***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(6.00 E-25)</td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\) p-value in parentheses.  
\(^{b}\) * p < 0.10, ** p < 0.05, *** p < 0.01
Table 5: Random Utility Model Estimation Results

<table>
<thead>
<tr>
<th>parameter</th>
<th>estimate</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_A$</td>
<td>0.4525</td>
<td>1.4410</td>
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<tr>
<td></td>
<td>(0.3140)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_B$</td>
<td>1.6564***</td>
<td>6.3994</td>
</tr>
<tr>
<td></td>
<td>(0.2588)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>0.8495***</td>
<td>2.9993</td>
</tr>
<tr>
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<td>(0.2832)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_E$</td>
<td>0.6570**</td>
<td>2.0993</td>
</tr>
<tr>
<td></td>
<td>(0.3129)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_Q$</td>
<td>0.0139**</td>
<td>2.4190</td>
</tr>
<tr>
<td></td>
<td>(0.0057)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_N$</td>
<td>0.4532</td>
<td>0.1807</td>
</tr>
<tr>
<td></td>
<td>(2.5074)</td>
<td></td>
</tr>
</tbody>
</table>

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*a p < 0.10, ** p < 0.05, *** p < 0.01. Standard deviation in parentheses.