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The consequences of using increasing block tariffs to price urban water*

Hugh Sibly and Richard Tooth†

Increasing block tariffs (IBTs) are currently used to price urban water in many Australian mainland capitals and a great many cities worldwide. This paper provides a systematic analysis of the impact of the adoption of IBTs to price urban water under the common constraints of scarce supply and cost recovery. The key tools available to policymakers using IBTs are the volumetric rate in the low tier and the threshold level of that tier. This paper shows how variations in these tools influence (i) the fixed charge set by the firm, (ii) the deadweight loss from the IBT and (iii) the bill paid by customers for particular levels of demand. Our analysis suggests that IBTs are neither fair nor efficient. We propose a modification to IBTs that, while retaining their perception of fairness, results in the efficient allocation of urban water.

Key words: economic efficiency, increasing block tariffs, urban water pricing.

1. Introduction

Efficient provision of urban water is achieved by setting its volumetric rate equal to the marginal cost of supply (Elnaboulsi 2001; Sibly 2006b). A significant departure[1] from this efficient pricing rule commonly applied in urban water markets (and other settings) involves the use of increasing block tariffs (IBTs) – a tariff structure whereby higher tariff rates are charged once consumption increases above a threshold level. IBTs are applied to price urban water in most mainland Australian urban areas and a great many urban centres worldwide. By diverging from marginal cost pricing, IBTs send an inefficient price signal that inevitably results in economic misallocation (Boland and Whittington 2000; Sibly 2006a). In spite of their widespread use, and presumably popularity, there has not been a systematic study of the impact of IBTs.[2]

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† Hugh Sibly (e-mail: hsibly@postoffice.utas.edu.au) is at School of Economics and Finance, University of Tasmania, Hobart, Tasmania, Australia. Richard Tooth is at Sapere Research Group Limited, Sydney, New South Wales, Australia and the School of Business (Sydney Campus), University of Notre Dame, Sydney, New South Wales, Australia.

[1] Another significant departure is the use of rigid prices, prices that do not respond to variations in supply and demand.

[2] Brennan (2006) and Edwards (2006) analyse the impact of IBTs on the water market in Perth and Melbourne respectively. The aim in these papers is to understand the impact in the particular markets analysed rather than identify the general properties of IBTs.

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This paper aims to provide such an analysis of the impact of IBTs. In contrast to the existing literature, the model presented in this paper considers the impact of the use of IBTs in the presence of two common constraints that bind in most Australian water markets and a number of international markets. The first constraint is a restriction on the available supply of water. Water supply may be restricted if the water corporation depletes dams or other water sources at an optimal rate (see Grafton *et al.* 2011). The optimal rate of depletion balances current and future urban and environmental use, given the current and future available supply. Alternatively, in practice, water utilities typically aim to supply a particular level of demand.[3] A second constraint is one imposed by the requirements of competitive neutrality[4]: the water corporations must price to recover the pecuniary cost of their operations.[5]

IBTs are a widely adopted method of pricing urban water in both Australia (where most large water utilities and over 70 per cent of all water utilities reported applying IBTs, see 2010–2011 National Water Reports: Urban Water Utilities, April 2012) and the rest of the world (see Worthington and Hoffmann 2006; and Roseta-Palma and Monteiro 2008). Given the widespread use of IBTs, it is important to understand how changes to their design affect economic outcomes, particularly when policymakers are constrained by water scarcity and the requirement for competitive neutrality. The key design choices available to a policymaker setting an IBT are the volumetric rate in the low tier and the threshold level of that tier. An important contribution of this paper is to show how variations in these choices influence (i) the fixed charge set by the firm, (ii) the deadweight loss from the IBT and (iii) the bill paid by customers with particular demand levels.

There are two common arguments made to account for widespread adoption of IBTs. First IBTs strike many as a fair method of pricing and thereby a fair method by which a target level of urban use may be achieved. IBTs are believed to provide the first tier of consumption to all consumers at a low cost. Thus, each consumer, it is argued, is given a 'fair' allocation at an 'affordable' price. Only those who use an 'excessive' amount face a high volumetric rate. This property of IBTs is also seen by some as an equity measure. For instance, when advocating the introduction of IBTs for water pricing in Sydney, IPART (2004 p.24) recommended an IBT instead of a two-part tariff because vulnerable customers face a higher bill under two-part tariffs.[6] On the other hand, some view IBTs as a rent seeking mechanism on

[3] Mainland Australian urban water supplies are characterised by (i) a high degree of politicisation of pricing and infrastructure expansion and (ii) drought, during which time infrastructure cannot be used to its capacity. In this environment, the constraint on water corporations is the availability of water (particularly ground water).

[4] Competitive neutrality has been a requirement for government business enterprises in Australia following the implementation of National Competition Policy in the late 1990s and early 2000s.

[5] In Australia, this constraint arises from the national competition reforms introduced in the 1990s to ensure that state-owned corporations satisfied competitively neutrality.

[6] More recently, Cole (2011) has recommended time-dependent IBTs to price water. Anglicare and TASCOSS (2010) recommend IBTs to price electricity.

the part of the average voter. In the context of some *developing* countries, Boland and Whittington (2000 pp. 234) argue (without formally demonstrating) that an IBT allows water utilities 'to deliver cheap water to the middle — and upper-income groups while appearing to serve the poor'. In these developing countries, groups of low-income households may share a single connection to access the water network. These low-income households are disadvantaged by the presence of IBTs.

The model presented in this paper is used to assess these arguments. This paper shows that, under water scarcity and cost recovery, the deadweight loss associated with IBTs increases with a decrease in the tier 1 volumetric rate and/or an increase in the threshold. However, consumers with an 'average level' of demand prefer an IBT with a low volumetric rate and a threshold level equal to their consumption level. Furthermore, an increase in the threshold increases the bill facing low demand customers. Low demand customers may have a higher bill under IBTs than under an efficient two-part tariff, in particular when the threshold is set near to the average level of household consumption.

The closest analysis to that presented in this paper is by Boland and Whittington (2000), who provide a broad ranging and insightful discussion of the impact of IBTs in water markets in developing countries. They point out that IBTs are inconsistent with marginal cost (efficient) pricing. They also use a numerical model, calibrated to urban water markets in developing countries, to identify some of the equity issues associated with IBTs; notably that the subsidy that is incorporated in the first block is generally small and that the first block is regressive in nature so that the 'subsidy' is smaller the less water that is used. They also provided an example of how an alternative 'revenue neutral' rate structure could provide a seemingly more equitable outcome. An important limitation of their approach, however, is that it is not apparent whether their results are dependent on the particular set of parameter values they choose or whether it is a general property of IBTs. In contrast, this paper provides a set of general results on the impact of IBTs.

In addition, Boland and Whittington (2000) do not consider IBTs in the presence of both the revenue recovery and water supply constraints. The full equity and efficiency implications of IBTs depend on their impact on higher tier charges and the fixed charges, which means the revenue recovery and water supply constraints must be jointly considered. For instance, it is not immediately clear whether (or under what circumstances) an IBT will provide a more progressive outcome to a simple two-part tariff. For example, advocates of IBTs may argue that large users pay more due to high top tier rates. This is not straightforward as the extent to which this can be the case depends on the nature of demand and the rates applied, which in turn are constrained by water supply. By formally including the revenue recovery and water supply constraints in the model, the analysis in this paper can account for all such effects.

The theoretical literature dealing with utility pricing is concerned almost exclusively with the determination of the optimal pricing schedules. In contrast, we look at the consequences of adopting a very common, yet inefficient, policy. The general theory of utility pricing (in which charges can be related to volume consumed) in the presence of cost recovery was developed by Goldman *et al.* (1984) and Brown and Sibley (1986). There have been applications of this theory to urban water pricing by authors such as Collinge (1992), Kim (1995), Griffin (2001) and Schuck and Green (2002). This literature does not support the use of IBTs. Recently Roseta-Palma and Monteiro (2008) argued that 'increasing marginal prices may come about as a combined result of scarcity and customer heterogeneity under specific conditions'. Monteiro (2010) has attempted to determine whether these conditions hold in Portugal. In all this literature, deviations from a two-part tariff (TPT) come about because a TPT may exclude low demand customers. Having a schedule with increasing marginal prices may reduce the number of low demand customers excluded. (Indeed, we show below that an IBT with a sufficiently low threshold can reduce the bill paid by low demand customers.) However, in Australia, connection to a water network is not a household choice. Regulations require all urban households, particularly new housing developments, to be connected. In any event, water bills are not sufficiently high for households to avoid connection. Hence, arguments in favour of variable marginal prices (either increasing or decreasing) based on exclusion are not applicable, particularly in the capital cities of Australia.[7]

In the next section, the model is described, and the efficient TPT is specified. In section 3, increasing block tariffs are analysed. In section 4, the special case of IBTs in which all households are allocated a free allowance is analysed. Our proposal for a modified IBT is presented and analysed in section 5. Section 6 concludes the paper.

2. Water Industry and efficient two-part tariff

We begin the analysis of this paper by developing the economic model used and demonstrating how both efficiency and cost recovery can be achieved using a TPT. We assume consumers' demand, $X(\alpha, p)$, is given by

$$X(\alpha, p) = \alpha x(p) \tag{1}$$

where p is the volumetric rate faced by the consumer at the margin, and the parameter $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ measures the strength of the customer's demand, and $x(p)$ is a continuous function of p. The specification (1) ensures that demand

[7] To the extent households do not have the financial resources to pay water bills (and are hence disconnected), these households would be best served by a rebate or social security payment. The rebate could be paid from increases in the fixed charge to users. However, given the rebate is a social welfare measure, it could be argued that such a rebate is more properly paid by state or federal governments out of general revenue.

curves of different customer types do not cross, so there is an unambiguous ordering of the strength of consumer demand. Let $N(\alpha)$ be the cumulative distribution function of α (representing the *proportion* of customers with demand lower or equal to α) and let $n(\alpha) = N'(\alpha)$ be the density function of α. The parameters $\underline{\alpha}$ and $\bar{\alpha}$ are the lower and upper support of $n(\alpha)$ respectively.

Total demand for water is $X^T = L^T\hat{\alpha}x(p)$ where $\hat{\alpha} = \int_{\underline{\alpha}}^{\bar{\alpha}} \xi n(\xi)\mathrm{d}\xi$ and L^T is the total number of customers. We restrict our attention to those pricing schemes for which supply equals demand.[8]

Typically water corporations are characterised by large fixed (infrastructure) costs and relatively constant marginal cost (arising from pumping and treatment). Therefore, the water corporation's total cost, C^T, is modelled as

$$C^T = F + cX^T \tag{2}$$

where F is the fixed cost and c is marginal cost of water (which is the marginal social cost of water when the corporation is required to pay royalties to equal to the cost of environmental externalities).

We assume the water authority either produces or releases from storage, the efficient level of water in the period under consideration.[9] In this event, X^T is also the efficient provision of water supply. The efficient linear volumetric rate, τ^*, clears the market, that is, $X^T = L^T\hat{\alpha}x(\tau^*)$. The efficient rate is greater than or equal to marginal cost, that is, $\tau^* \geq c$.[10]

The efficient TPT consists of a fixed charge T and the efficient linear volumetric rate. Total revenue, R^T, under the efficient TPT is

$$R^T = L^T[T + \tau^*\hat{\alpha}x(\tau^*)] \tag{3}$$

If, as is common practice, the fixed charge per household is set to achieve cost recovery once the volumetric charge has been set then

$$T = \frac{F}{L^T} - (\tau^* - c)\hat{\alpha}x(\tau^*) \tag{4}$$

Observe that the fixed charge is lower than the household's share of fixed cost when the efficient price is more than the marginal cost ($\tau^* > c$). Furthermore, if demand is inelastic, an increase in τ^* reduces the fixed charge.

[8] In the event that supply is less than demand, some type of rationing would need to occur (typically water restrictions). See Sibly (2006a) and Grafton and Ward (2008), for a discussion of the impact of water restrictions.

[9] See Grafton *et al.* (2011), for an analysis of the determination of the efficient use level. Note that the efficient use levels ensure that the optimal provision of water has been made for environmental flows.

[10] The efficient rate will be above marginal cost when water availability is limited, either due to environmental conditions (eg drought) or infrastructure capacity limitations.

3. Increasing block tariffs

In this section, the key properties of IBTs are developed. The IBT is modelled in the following subsection. Then, the restrictions imposed by limited water availability and cost recovery are consecutively considered. The impact on customer bills of IBTs and their inefficiency are then detailed.

3.1. The tariff structure

For simplicity, in this paper, we restrict consideration of IBTs to one with two tiers. The IBT $\{\tau_1, \tau_2, \tilde{X}\}$ specifies that consumers pay a 'tier 1' volumetric rate (τ_1) up to a threshold, $\tilde{X}$, and a 'tier 2' volumetric rate (τ_2) beyond the threshold. Specifically, the volumetric rate, p, for the Xth unit is

$$p = \begin{cases} \tau_1, & 0 \leq X \leq \tilde{X} \\ \tau_2, & X > \tilde{X} \end{cases} \tag{5}$$

The IBT $\{\tau_1, \tau_2, \tilde{X}\}$ is depicted in Figure 1. A consumer chooses the level of consumption at the point where their demand curve crosses the IBT. From Figure 1, the demand from type α under the IBT $\{\tau_1, \tau_2, \tilde{X}\}$ is

$$X(\alpha) = \begin{cases} \alpha x(\tau_1), & \underline{\alpha} \leq \alpha \leq \tilde{\alpha}_1 \\ \tilde{X}, & \tilde{\alpha}_1 \leq \alpha \leq \tilde{\alpha}_2 \\ \alpha x(\tau_2), & \tilde{\alpha}_2 \leq \alpha \leq \bar{\alpha}_1 \end{cases}$$

where $\tilde{\alpha}_1 = \tilde{X}/x(\tau_1)$ and $\tilde{\alpha}_2 = \tilde{X}/x(\tau_2)$.

It is observed from Figure 1 that $\tilde{\alpha}_1$ is the customer type with lowest demand who purchases threshold level ($\tilde{X}$), while $\tilde{\alpha}_2$ is the customer type with highest demand who purchases $\tilde{X}$. Unless otherwise stated, it is assumed that the

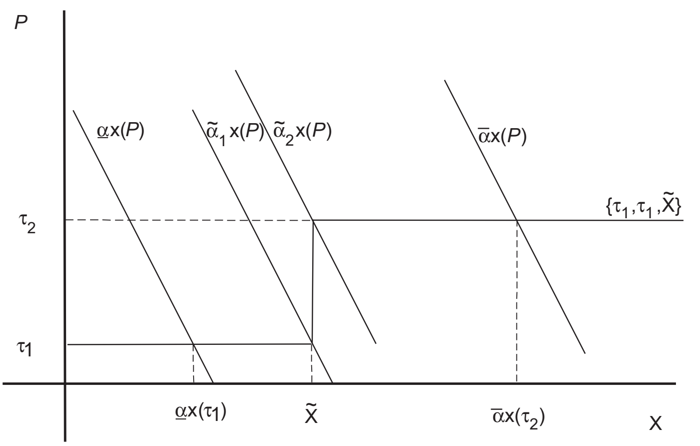

Figure 1 An increasing block tariff.

magnitude of $\tilde{X}$ is such that $\underline{\alpha} < \tilde{\alpha}_1 < \tilde{\alpha}_2 < \bar{\alpha}$. This implies that the proportion of customers purchasing only in tier 1 is $N(\tilde{\alpha}_1)$ purchasing at the threshold is $N(\tilde{\alpha}_2) - N(\tilde{\alpha}_1)$, and purchasing above the threshold level of output is $1 - N(\tilde{\alpha}_2)$.

3.2 Water supply constraint

Water authorities are restricted to supplying X^T. In this subsection, we consider the implications of this restriction.

By integrating consumer demand (1), across customer types, the IBT $\{\tau_1, \tau_2, \tilde{X}\}$ generates the total demand for water, X^T, if

$$X^T = L^T\{\Gamma(\tilde{\alpha}_1)x(\tau_1) + \tilde{N}\tilde{X} + [\hat{\alpha} - \Gamma(\tilde{\alpha}_2)]x(\tau_2)\} \tag{6}$$

where $\Gamma(\alpha) = \int_{\underline{\alpha}}^{\alpha} \xi n(\xi)\xi \mathrm{d}\xi$ and $\tilde{N} = N(\tilde{\alpha}_2) - N(\tilde{\alpha}_1)$. Note that $\Gamma(\alpha)$ represents the average demand of customers who have demand less than or equal to α and $\hat{\alpha} = \Gamma(\bar{\alpha})$ represents the average demand of all customers. Define $\tilde{\alpha}^* = \tilde{X}/x(\tau^*)$, as the customer type who purchases the threshold level of water when facing the efficient volumetric rate. The following result summarises the implications of the water supply constraint.

Result 1: If total water supply (X^T)is fixed, then under an IBT rate structure:

(i) The top tier rate must be above the efficient price level and the low tier rate must be below the efficient price, that is, $\tau_1 < \tau^* < \tau_2$ and consequently $\tilde{\alpha}_1 < \tilde{\alpha}^* < \tilde{\alpha}_2$,
(ii) A decrease in lower tier rate while holding the threshold fixed requires an increase in the top tier rate, and
(iii) An increase in the threshold requires an increase in one or more of the tier rates.

The proofs to all results in this paper are provided in the mathematical Appendix. By lowering the volumetric rate for tier 1, consumption is increased for low demand (i.e. for $\alpha < \tilde{\alpha}^*$) customers. To keep total demand constant, the volumetric rate in tier 2 must be increased to reduce consumption from high demand (i.e $\alpha > \tilde{\alpha}^*$) customers. When the threshold is increased, the number of customers facing the lower tier rate increases, which increases consumption. To compensate for this, the consumption of either low or high demand customers must be reduced by raising either the tier 1 or tier 2 rates.

3.3. Cost recovery

In this subsection, we consider the implications of requiring the water authority to recover costs. To determine the revenue generated by the IBT $\{\tau_1, \tau_2, \tilde{X}\}$, first observe that the water corporation's revenue from a type α customer is

$$R(\alpha) = \begin{cases} T + \tau_1 \alpha x(\tau_1), & \underline{\alpha} \le \alpha \le \tilde{\alpha}_1 \\ T + \tau_1 \tilde{X}, & \tilde{\alpha}_1 \le \alpha \le \tilde{\alpha}_2 \\ T + \tau_1 \tilde{X} + \tau_2[\alpha x(\tau_2) - \tilde{X}], & \tilde{\alpha}_2 \le \alpha \le \bar{\alpha} \end{cases} \tag{7}$$

where T is the fixed charge. Integrating over customer types shows the water corporation's average revenue per household, $\hat{R} \equiv R^T/L^T$, is

$$\begin{aligned} \hat{R} &= T + \tau_1 \Gamma(\tilde{\alpha}_1)x(\tau_1) + \tau_1 \tilde{N}\tilde{X} + \tau_2[\hat{\alpha} - \Gamma(\tilde{\alpha}_2)]x(\tau_2) - (\tau_2 - \tau_1)\tilde{X}[1 - N(\tilde{\alpha}_2)] \\ &= T + \tau_1\{\Gamma(\tilde{\alpha}_1)x(\tau_1) + \tilde{N}\tilde{X} + [\hat{\alpha} - \Gamma(\tilde{\alpha}_2)]x(\tau_2)\} \\ &\quad + (\tau_2 - \tau_1)\{[\hat{\alpha} - \Gamma(\tilde{\alpha}_2)]x(\tau_2) - \tilde{X}[1 - N(\tilde{\alpha}_2)]\} \\ &= T + \tau_1 \hat{X} + (\tau_2 - \tau_1)\hat{E} \end{aligned}$$

where $\hat{X} \equiv X^T/L^T$ is the average household's water use, and $\hat{E}$ is the average use (per household) in excess of the threshold, which is given by

$$\hat{E} = [\hat{\alpha} - \Gamma(\tilde{\alpha}_2)]x(\tau_2) - \tilde{X}[1 - N(\tilde{\alpha}_2)] \tag{8}$$

Cost recovery requires that

$$T = F/L^T - (\tau_1 - c)\hat{X} - (\tau_2 - \tau_1)\hat{E}$$

The above equation is used to analyse the impact of changes to the rate structure under cost recovery.[11] Consider an increase in the threshold. Result 1 shows that this change in the threshold must be accompanied by a simultaneous increase in the top tier rate to maintain the level of total demand. The following result shows how the fixed charge must also simultaneously be changed in order to maintain cost recovery.

Result 2: Assume the water corporation is subject to cost recovery and total supply is fixed. Then, if number of consumers purchasing at the threshold level ($\tilde{N}$) is relatively small and $\tilde{\alpha}_1$ is sufficiently greater than α, an increase in threshold level while holding the tier 1 rate fixed necessitates an increase in the fixed charge, which is approximated by

$$\frac{\mathrm{d}T}{\mathrm{d}\tilde{X}} \approx [1 - N(\tilde{\alpha}_2)](\tau_2 - \tau_1) > 0$$

[11] A case study where a water authority varied the fixed charge to ensure revenue neutrality while adjusting the volumetric charge to satisfy the water supply constraint is given by Loaiciga and Renehan (1997). They report the policy of the City of Santa Barbara during an extended drought in the early 1990s; although they maintained an IBT structure, they offset increases in usage charges with a simultaneous drop in fixed charges (which is, in effect, a rebate).

The impact of an increase in the threshold on the fixed charge is ambiguous because although an increase in threshold reduces payments made up to threshold, it is necessary to increase the tier 2 rate to keep overall demand fixed. When demand is sufficiently inelastic, an increase in the tier 2 rate could raise sufficient revenue that offsets the direct effect of raising the threshold. However, when $\tilde{N}$ is small, an increase in threshold has little impact on demand and thus has a relatively small impact on the tier 2 rate. As a result, the loss of revenue from the increase in the tier 1 range will exceed any gain in revenue in the tier 2 range, and the fixed charge must be increased to maintain revenue neutrality.

3.4. Customer bills and preferences

The size of the bill facing each customer type depends on the level of the threshold. Given water pricing mechanisms are often designed within political environments, it is natural to ask what level of threshold and tier 1 rate is preferred by each household.

Result 3: Suppose $\tilde{N}$ is relatively small. Then, a customer of type α, whose demand is sufficiently larger than α, prefers

(i) A threshold that matches their consumption (i.e $\alpha \in [\tilde{\alpha}_1, \tilde{\alpha}_1,])$, and
(ii) A zero tier 1 volumetric rate.

The requirement in Result 3 that α is sufficiently larger than α means that the result applies to customers in the middle of the demand distribution. Result 3 shows that such an 'average demand' household prefers a threshold which allows them to consume at the threshold. This allows them to receive a low volumetric rate for all the units they consume, thus ensuring the maximum benefit from the lower tier 1 rate. The reduced revenue arising from a low tier 1 rate requires a higher fixed charge, but this increased revenue requirement is spread across all customers. Further, the lower is the tier 1 rate, the higher must be the tier 2 rate to keep total demand constant. Increased revenue from the high demand customers ameliorates the increase in the fixed charge arising from a low tier one rate.

One of the motivations cited for the introduction of an IBT is to guarantee low demand customers a low bill. We now assess this claim. Under an efficient TPT, the revenue from a customer with demand α is given by

$$R(\alpha) = \frac{F}{L^T} - (\tau^* - c)\hat{X} - \tau^* \alpha x(\tau^*) \tag{9}$$

Subtracting (9) from (7) gives the difference between the bill under the IBT and the efficient TPT, $\Delta R(\alpha)$,as

$$\Delta R(\alpha)=\begin{cases}(\tau_1^*-\tau_1)\hat{X}-(\tau_2-\tau_1)\hat{E}+\alpha\tau_1 x(\tau_1)-\alpha\tau^* x(\tau^*), & \underline{\alpha}\le\alpha\le\tilde{\alpha}_1\\ (\tau_1^*-\tau_1)\hat{X}-(\tau_2-\tau_1)\hat{E}+\tau_1\tilde{X}-\alpha\tau^* x(\tau^*), & \tilde{\alpha}_1\le\alpha\le\tilde{\alpha}_2\\ (\tau_1^*-\tau_1)\hat{X}-(\tau_2-\tau_1)\hat{E}-(\tau_2-\tau_1)\tilde{X}+\alpha\tau_2 x(\tau_2)-\alpha\tau^* x(\tau^*), & \tilde{\alpha}_2\le\alpha\le\bar{\alpha}\end{cases} \tag{10}$$

Observe that for small users, where $\alpha \in [\underline{\alpha}, \tilde{\alpha}_1)$, $\Delta R(\alpha)$ is not necessarily positive. In particular, if both α and $\hat{E}$ and are small (which coincides with a low demand customer and a high threshold), $\Delta R(\alpha)$ will be negative. Intuitively, if the threshold is high, the water corporation must increase the fixed charge so high as to overcome the saving on the volumetric charge paid by low valuation customers. Furthermore, an increase in the threshold, accompanied by a simultaneous increase in the top tier rate to keep total demand constant, will increase $\Delta R(\alpha)$ for all customers where $\alpha \in [\underline{\alpha}, \tilde{\alpha}_1)$ if $\frac{\mathrm{d}}{\mathrm{d}\tilde{X}}\left[(\tau_2-\tau_1)\hat{E}\right]<0$. Thus, raising the threshold beyond a low value increases the bill paid by low demand consumers.

To illustrate this argument, the results of a numerical simulation are presented in Table 1 below. In the numerical simulation, it is assumed that α is distributed uniformly between 1 and 10 and that total water availability is sufficient such that that the average demand $(\hat{X})$ is 5.5. Customer demand is given by $X = \alpha p^{-0.2}$. Under these condition, the efficient volumetric rate is $\tau^* = 1$. It is assumed that the tier 1 rate is $\tau_1 = 0.5$.

Table 1 shows that the lowest demand customer (α=1) pays a higher bill under IBTs with any threshold than under the efficient TPT. In addition, the bill facing the lowest demand customer increases as the threshold increases. On the other hand, the bill facing the average demand customer (α = 5.5) is less under IBTs than under the efficient TPT. The bill paid by the average customer falls as the threshold increases to 5. A customer with relatively high demand (eg α = 9) pays a higher bill under IBTs with a low threshold than under the efficient TPT. However, as the threshold increases the bill falls, so that for higher threshold levels the high demand customer has a lower bill under IBTs than the efficient TPT.

Table 1 Revenue difference between the IBT and efficient TPT

α	Threshold level $(\tilde{X})$				
	2	3	4	5	6
1	0.49	0.84	1.14	1.43	1.71
5.5	−0.04	−0.16	−0.39	−0.81	−0.37
9	0.02	3.5E−03	−0.05	−0.16	−0.44

Notes: Results shown the difference in revenue received from customers between the IBT and the efficient TPT. α is distributed uniformly between 1 and 10, $X = \alpha p^{-0.2}$, $\hat{X} = 5.5, \tau_1 = 0.5$

3.5. The deadweight loss associated with an IBT

As shown by Result 1, the IBT imposes a volumetric rate that differs from the efficient volumetric rate. Consequently, customers' consumption decisions are distorted, creating a deadweight loss.

The two likely impacts of political action on the design of the IBT are a lowering of the first tier rate and an increase in the threshold. We show both these actions will increase the total deadweight loss.

Result 4: If total demand is held constant by variations in the top tier rate, the deadweight loss is increased by

(i) A decrease in the bottom tier rate, or
(ii) An increase in the threshold.

Result 4(i) demonstrates that the deadweight loss increases the more the rates diverge away from the efficient level. Recall Result 1 shows that, to ensure the water market clears, $\tau_1 < \tau^*$ and $\tau_2 < \tau^*$. Consequently, the IBTs cause inefficient consumption levels for all customers (with the one exception where $\alpha = \tilde{\alpha}^*$). A reduction in τ_1 must be accompanied by an increase in τ_2 to maintain constant demand. In this case, the volumetric rate in both tiers is distorted away from the efficient level. This distorts the consumption decision of all customers, apart from those who consume at the threshold level (those for whom $\alpha \in (\tilde{\alpha}_1, \tilde{\alpha}_2)$). Thus, total deadweight loss is increased by a reduction in τ_1.

Result 4(ii) is illustrated in Figure 2. The threshold is increased from $\tilde{X}$ to $\tilde{X}^{\uparrow}$. Recall Result 1 shows an increase in the threshold is accompanied by an increase in the tier 2 volumetric rate to maintain total consumption. This is shown as τ_2 increasing to $\tau_2^{\uparrow}$. The increase in the threshold also increases $\tilde{\alpha}_1$ to

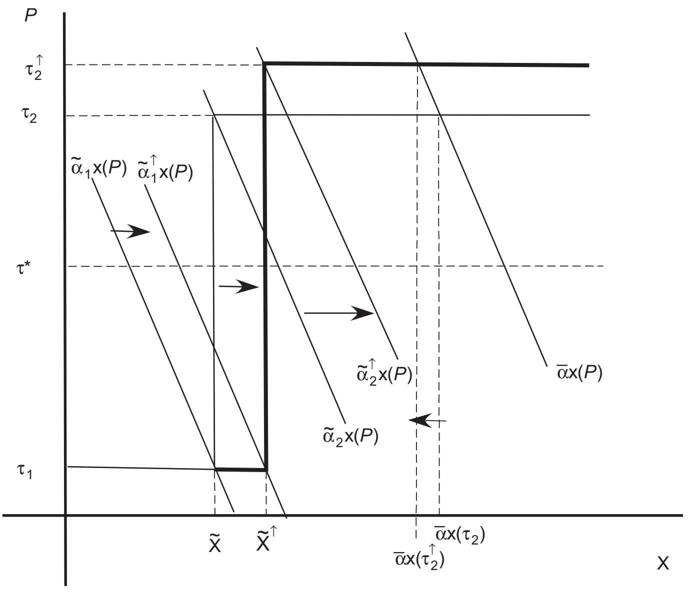

Figure 2 Impact of changing the threshold.

$\tilde{\alpha}_1^{\uparrow}$and $\tilde{\alpha}_2$ to $\tilde{\alpha}_2^{\uparrow}$ as shown in Figure 2. There are the resultant following four impacts on the deadweight loss.

1 The increase in the threshold *does not change* the consumption levels of those customers who consume below the threshold, that is, those with $\alpha \in [\underline{\alpha}, \tilde{\alpha}_1)$.
2 Customers in the set $\alpha \in [\tilde{\alpha}_1, \tilde{\alpha}^*)$ are initially consuming an inefficiently large amount of water. The increase in the threshold causes them to increase consumption, *increasing* the deadweight loss.
3 Those customers in the set $\alpha \in [\tilde{\alpha}^*, \tilde{\alpha}_2)$ are initially consuming an inefficiently low amount of water. The increase in the threshold increases their consumption, thereby *reducing* the deadweight loss.
4 The customers in the set $\alpha \in [\tilde{\alpha}_2, \bar{\alpha}_2]$ face an increase in the tier 2 volumetric rate. These customers, who initially have an inefficiently low level of consumption, further reduce their consumption, thereby *increasing* the deadweight loss.

There are therefore countervailing effects of an increase in the threshold on deadweight loss. Nonetheless, Result 4 shows that the increase in total deadweight loss arising from the customers in the set $\alpha \in [\tilde{\alpha}_2, \bar{\alpha}_2]$ is larger than the fall in deadweight loss arising from the customers in the set $\alpha \in [\tilde{\alpha}^*, \tilde{\alpha}_2)$. Thus, overall total deadweight loss is increased by the increase in the threshold level of consumption.

4. Special case: free allowance for 'essential' water consumption

It often argued that social equity should guarantee households a free allowance of water to cover 'essential' use. For the purposes of analysis,[12] we consider 'essential' use to be a level, X_e such that be that for all reasonable circumstances household demand exceeds this level, that is $X > X_e$ for all households for all $\tau_1 \leq \tau^*$.[13]

This assumption can be formalised in the above model by defining $\alpha_e = X_e/x(\tau^*)$, and then requiring $\alpha_e < \alpha$. Under this assumption, the IBT $\{0, \tau^*, X_e\}$ causes consumer type α to consume $\alpha x\,(\tau^*)$. Thus, the water market is cleared. Furthermore, because customers face the efficient volumetric rate at the margin, water consumption is efficient.

The revenue from type α customers is

$$R(\alpha) = T + \tau^*[\alpha x(\tau^*) - X_e]$$

[12] What is essential is not clear and there is no common accepted definition of what is essential and what is an essential level of use.

[13] In practice, there will always be some circumstances when water usage is very low, for example, when a house becomes vacant for all or part of the metering reading period.

Average revenue is therefore

$$\hat{R} = (T - \tau^* X_e) + \tau^* \hat{\alpha} x(\tau^*) = (T - \tau^* X_e) + \tau^* X^T$$

Cost recovery requires

$$T - \tau^* X_e = F/N^T - (\tau^* - c)\hat{R}$$

Consequently, under cost recovery, the type α customer's water bill is identical to their bill under the efficient tariff, that is,

$$R(\alpha) = F/N^T - (\tau^* - c)\hat{X} + \tau^* \alpha x(\tau^*)$$

Note that $R(\alpha)$ is independent of X_e. In particular $R(\alpha)$ is identical for positive X_e as zero X_e. Household bills are thus not affected by the presence of the free allowance that is sufficienty small.[14] The revenue required to cover the free allowance is simply reallocated to the fixed charge. While the free allowance has no material impact on revenue, it may be perceived to be fairer by redistributing the financial burden.

5. An efficient IBT

The previous section demonstrated that a free allowance IBT, $\{0, \tau^*, X_e\}$, provides an efficient allocation provided the threshold level was below what $\alpha_e < \alpha$. However, there would appear to be a political risk of implementing such an IBT. Political forces may result in the threshold being set, not at an 'essential' level, but at a level that appealed to the average voter. While this may be in the interest of (and thus appear fair to) the average voter, it results in the economic inefficiencies described above.

Given this possibility, it is of interest how the IBT might be restructured so as to be (i) perceived as fair, but (ii) be efficient irrespective of the choice of tariff or threshold level. A simple approach that meets this criterion is to provide consumers with a rebate for water consumed below the threshold level.[15] Under this structure, the household, in effect, receives a water allowance irrespective of use, which given the lump-sum nature of the allowance does not distort consumption incentives. Such an approach may be more likely perceived as fair. As the rebate is larger the lower is the household's consumption below, their 'fair allocation' the rebate

[14] Similarly household bills are not affected by a discounted small allowance.

[15] The idea to use a fixed rebate on water charges, to achieve equity goals, is not new. Some form of this type of rebate has been proposed by a number of economists (Boland and Whittington 2000; Sibly 2006a; Quiggin 2007; Sibly and Tooth, 2008). However, the rebate proposed here differs from those previously proposed as (i) it depends on the level of water 'saved' by the household and (ii) is proposed for efficiency, rather than equity grounds.

could be interpreted as a 'reward' for the household's conservation efforts.[16]

The approach can be formally modelled as follows. If the household consumes water below the threshold, they would receive a rebate of $(\tau^* - \tau_1)(\tilde{X} - X)$. With the rebate introduced, the customer faces the bill, $b(X)$, for consuming X units, where

$$b(X) = \begin{cases} T + \tau_1 X - (\tau^* - \tau_1)(\tilde{X} - X), & 0 \leq X < \tilde{X} \\ T + \tau_1 X - (\tau^* - \tau_1)(\tilde{X} - X), & \tilde{X} < X \end{cases}$$

which can be simply rearranged for all X to give[17]

$$b(X) = T - (\tau^* - \tau_1)\tilde{X} + \tau^* X$$

Given the household faces this bill, type α customers choose to consume the efficient level of water, that is, $X = \alpha x(\tau^*)$ and thus, the average bill (or revenue per customer) is simply

$$\hat{R} = T - (\tau^* - \tau)\tilde{X} + \tau^* \hat{\alpha} x(\tau^*)$$

which under cost recovery must be equivalent to that of an efficient two-part tariff.

To summarise we have

Result 5: Under the modified IBT whereby consumers receive a rebate for water not consumed, the financial incentives and customer bills are identical to that of the efficient TPT.

In its presentation, the modified IBT retains the features that many find fair: low demand users of water are presented with a low volumetric rate. In addition, these low demand users are rewarded with a rebate for using less.[18] High users have to pay the full cost of the 'excess' use.

The modified IBT, by mimicking an efficient two-part tariff, is efficient. Such a modification is also likely to be practical. In practice due to the large fixed costs associated with distribution, treatment and reticulation, the rebate is unlikely to ever exceed the fixed charge.

[16] We understand that during the recent drought in Australia some customers subject to IBTs complained that they were not getting sufficient financial reward for their water conservation efforts.

[17] Rather than modifying an IBT with the addition of a rebate, Griffin (2001) simply proposes a billing system of the form $b(X) = T - \tau^* - (X - \tilde{X})$. Griffen's billing system could be interpreted as a special case of (2) when $\tau_1 = 0$. For Griffen, $\tilde{X}$ is set to ensure cost recovery given optimal network expansion plans. (Griffen does not consider water scarcity issues, so in his analysis the volumetric rate is equal to long run marginal cost.) A key aspect of this paper is analysing the impact of a change in $\tilde{X}$ and τ_1 on market outcomes in the presence of scarcity.

[18] The combined effect of the tier 1 rate and the rebate mean that in effect the user's marginal cost of water use is the efficient price.

6. Conclusions

This paper highlights some important negative economic consequences of the adoption IBTs to price urban water that occur when water corporations face both a constraint on water availability and revenue neutrality. It is demonstrated that IBTs are inferior in many ways to an efficient TPT. In addition to causing efficiency losses relative to the efficient IBT, IBTs have adverse financial impacts on the small users.[19] We also show how undertaking changes that move away from the efficient IBT, such as lowering the tier 1 rate or raising the threshold, exacerbates these problems. Our analysis thus shows how IBTs are a poor method of pricing. Although often motivated as a method to help the poor, IBTs reduce the ability of households, including poor households, to reduce their water bill by reducing consumption. In this way, they do not achieve what their proponents envisage and, in addition, impose an economic inefficiency.

Nonetheless, the (incorrectly) perceived fairness of IBTs makes them politically appealing. We propose a modification to IBTs that, while retaining the perception of fairness, results in households consuming the efficient level of water. The modification involves providing a rebate to customers for water not used below the threshold.

The analysis in this paper makes the dual assumptions that there is full cost recovery and that fixed costs are equally distributed among households. While cost recovery is a widely accepted principle,[20] in practice, water utilities often receive additional public funding. It is also common for water utilities to undertake programs that in effect provide targeted financial support. Such programs may help to mitigate the perverse distributional effects of the IBT, but they do not nullify the results of the analysis.

The paper presents analysis using a two-block IBT. The introduction of additional blocks complicates the analysis but does not change the core findings regarding the distributional effects of an IBT.

The results of the analysis in this paper suggest that equity is not improved by the introduction of IBTs. However, the results suggest some support for rent seeking as the motivation for the introduction of IBTs. One often sees thresholds that are around or above average level of consumption.[21] We show that those who benefit most from IBTs have a level of consumption at the threshold level. If electors vote on the basis of self interest, it would be expected that political forces would drive the threshold to be at the typical level of consumption of voters in marginal electorates. Testing for the presence of this link in practice is the subject of future work.

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[19] See, for example, analysis, in Productivity Commission 2011, Chapter 5.

[20] For example, in Australia, cost recovery of urban water costs through water charges is a pricing principle that has been accepted under the National Water Initiative Pricing Principles (available at http://www.environment.gov.au/water/publications/action/nwi-pricing-principles.html).

[21] For example, a two tier IBT was introduced in Sydney from June 2005 to June 2009. The threshold was set at 400 kl/year, well above average consumption levels per household.

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Appendix: Proofs

Proof of result 1

Part (i) is shown by contradiction.

Assume $\tau_1 \geq \tau^*$ then $\tau_2 > \tau^*$ and $x(\tau_2) < x(\tau_1) \leq x(\tau^*)$. Then, for all $\alpha, X(\alpha, \tau^*) \geq X(\alpha, \tau_1) > X(\alpha, \tau_2))$ which cannot occur for a fixed X^T. Thus, it must be $\tau_1 < \tau^*$ and similarly $\tau_2 > \tau^*$

Part (ii) and (iii):

Taking the total derivative of X^T and simplifying yields:

$$\mathrm{d}X^T/L^T = \Gamma(\tilde{\alpha_1})x'(\tau_1)\mathrm{d}\tau_1 + \tilde{N}\mathrm{d}\tilde{X} + [\tilde{\alpha} - \Gamma(\tilde{\alpha}_2)]x'(\tau_2)\mathrm{d}\tau_2$$

Assuming $\mathrm{d}X^T = 0$

$$\tilde{N}\mathrm{d}\tilde{X} = -\Gamma(\tilde{\alpha}_1)x'(\tau_1)\mathrm{d}\tau_1 - [\hat{\alpha} - \Gamma(\tilde{\alpha}_2)]x'(\tau_2)\mathrm{d}\tau_2 \tag{A1}$$

Result 1 follows. ||

Proof of result 2

Taking the total differential of $\hat{E}$ gives[22]

$$\mathrm{d}\hat{E} = [1 - N(\tilde{\alpha}_2)]\mathrm{d}\tilde{X} + \lceil\hat{\alpha} - \Gamma(\tilde{\alpha}_2)\rceil x'(\tau_2)\mathrm{d}\tau_2 \tag{A2}$$

If X^T and τ_1 are fixed then from (A1) above

$$\tilde{N}\mathrm{d}\tilde{X} = -\lceil\hat{\alpha} - \Gamma(\tilde{\alpha}_2)\rceil x'(\tau_2)\mathrm{d}\tau_2 \tag{A3}$$

And consequently combining (A2) and (A3) gives:

$$\frac{\mathrm{d}E}{\mathrm{d}\tilde{X}} = N(\tilde{\alpha}_1) - 1 < 0$$

[22] Note that $dE/\mathrm{d}\tilde{\alpha}_2 = -n(\tilde{\alpha}_2)\tilde{\alpha}_2 x(\tau_2) + \tilde{X}n(\tilde{\alpha}_2) = 0$

Now differentiating (8):

$$\mathrm{d}T = -(\hat{E} + (\tau_2 - \tau_1)\frac{\partial \hat{E}}{\partial \tau_2})d\tau_2 - (\tau_2 - \tau_1)E\frac{\partial \hat{E}}{\partial \tilde{X}}d\tilde{X}$$

and:

$$\begin{aligned}\frac{\mathrm{d}(\tau_2 - \tau_1)\hat{E}}{\mathrm{d}\tilde{X}} &= (\tau_2 - \tau_1)\frac{\mathrm{d}\hat{E}}{\mathrm{d}\tilde{X}} + \hat{E}\frac{\mathrm{d}\tau_2}{\mathrm{d}\tilde{X}} \\ &= -(\tau_2 - \tau_1)[1 - N(\tilde{\alpha}_1)] - \frac{\hat{E}\tilde{N}}{\lceil \hat{\alpha} - \Gamma(\tilde{\alpha}_2)\rceil x'(\tau_2)}\end{aligned}$$

Consequently:

$$\frac{\mathrm{d}T}{\mathrm{d}\tilde{X}} = \tilde{N}\left(\tau_2(1 - \frac{1}{\varepsilon(\tau_2)} - \tau_1\right) - [1 - N(\tilde{\alpha_2})]\left[\frac{\tilde{N}\tilde{X}}{\lceil \hat{\alpha} - \Gamma(\tilde{\alpha}_2)\rceil x'(\tau_2)} - (\tau_2 - \tau_1)\right]$$

$$\frac{\mathrm{d}T}{\mathrm{d}\tilde{X}} = [1 - N(\tilde{\alpha}_2)](\tau_2 - \tau_2) + \tilde{N}(\tau_2(1 - \frac{1}{\varepsilon(\tau_2)}) - \tau_1 - \frac{[1 - N(\tilde{\alpha}_2)]\tilde{X}}{\lceil \hat{\alpha} - \Gamma(\tilde{\alpha}_2)\rceil x'(\tau_2)})$$

where $\varepsilon(\tau_2) = \tau_2 x'(\tau_2)$, and so:

$$\frac{dT}{\mathrm{d}\tilde{X}} \approx [1 - N(\tilde{\alpha}_2)](\tau_2 - \tau_1) \text{ for small } \tilde{N}$$

Proof of result 3

(i) Differentiating (7) gives

$$\frac{\mathrm{d}R(\alpha \mid \tau_1)}{\mathrm{d}\tilde{X}} = \begin{cases} \frac{\mathrm{d}T}{\mathrm{d}\tilde{X}}, & \underline{\alpha} \le \alpha \le \tilde{\alpha}_1 \\ \frac{\mathrm{d}T}{\mathrm{d}\tilde{X}} + \tau_1, & \tilde{\alpha}_1 \le \alpha \le \tilde{\alpha}_2 \\ \frac{\mathrm{d}T}{\mathrm{d}\tilde{X}} - (\tau_2 - \tau_1) + \alpha x(\tau_2)(1 + \varepsilon(\tau_2))\frac{\mathrm{d}\tau_2}{\mathrm{d}\tilde{X}}, & \tilde{\alpha}_2 \le \alpha \le \bar{\alpha} \end{cases}$$

Observe, using Result 2, for small $\tilde{N}$, $\mathrm{d}T/\mathrm{d}\tilde{X} > 0$ and so $dR(\alpha \mid \tau_1)/\mathrm{d}\tilde{X} > 0$ for $\alpha \in [\underline{\alpha}, \tilde{\alpha}_2]$.

For $\alpha \in [\tilde{\alpha}_2, \bar{\alpha}]$, $\mathrm{d}R(\alpha \mid \tau_1)/\mathrm{d}\tilde{X}$ can be expanded to:

$$\frac{\mathrm{d}R(\alpha \mid \tau_1)}{\mathrm{d}\tilde{X}} = \{-\hat{E} + \alpha x(\tau_2)(1 + \varepsilon(\tau_2))\}\frac{\mathrm{d}\tau_2}{\mathrm{d}\tilde{X}} - (\tau_2 - \tau_1)N(\alpha_1), \qquad \tilde{\alpha}_2 < \alpha \le \bar{\alpha}$$

If $\tilde{N}$ is sufficiently small, then $\mathrm{d}\tau_2/\mathrm{d}\tilde{X} \approx 0$ and hence $\mathrm{d}R(\alpha \mid \tau_1)/\mathrm{d}\tilde{X} < 0$ for $\alpha \in [\tilde{\alpha}_2, \bar{\alpha}]$

Thus, if there are few people consuming at the threshold level an increase in threshold will result in a lower bill for high demand users and a higher bill for small and medium users.

Consider a consumer such that $\alpha \in [\tilde{\alpha}_2, \bar{\alpha}]$ Raising the threshold results in an increase in the tier 2 rate to the consumers detriment, but if $\tilde{N}$ is small, then the rate incease is also small. As raising the threshold reduces the consumers bill, their overall consumer surplus is increased. Now consider a consumer such that $\alpha \in [\underline{\alpha}, \tilde{\alpha}_2]$. Raising the threshold does not result in any change in consumption but results in the consumer's bill being increased, thereby reducing their consumer surplus. It is thus concluded that the preferred threshold is such that $\alpha \in [\tilde{\alpha}_1, \tilde{\alpha}_2]$.

(ii) For

$$\alpha \in [\tilde{\alpha}_1, \tilde{\alpha}_2] \qquad \frac{\mathrm{d}R(\alpha \mid \tilde{X})}{\mathrm{d}\tau_1} = -\hat{E}\frac{\mathrm{d}\tau_2}{\mathrm{d}\tau_1} < 0. \quad ||$$

Deadweight loss: Proof of Result 4

The deadweight loss from the consumption decisions of type α, DWL(α), customers is given by

$$\mathrm{DWL}(\alpha) = \begin{cases} (\tau^* - \tau_1)\alpha x(\tau_1) - \alpha y(\tau_1, \tau^*), & \underline{\alpha} \le \alpha < \tilde{\alpha}_1 \\ (\tau^* - \tau_\alpha)\tilde{X} - \alpha y(\tau_\alpha, \tau^*), & \tilde{\alpha}_1 \le \alpha < \tilde{\alpha}^* \\ 0, & \alpha = \tilde{\alpha}^* \\ -(\tau_\alpha - \tau^*)\tilde{X} + \alpha y(\tau^*, \tau_\alpha) & \tilde{\alpha}^* \le \alpha < \tilde{\alpha}_2, \\ -(\tau_2 - \tau^*)\alpha x(\tau_2) - \alpha y(\tau^*, \tau_2), & \tilde{\alpha}_2 \le \alpha < \bar{\alpha} \end{cases}$$

where $y(a, b) = \int_b^a x(\xi)\mathrm{d}\xi$ and τ_α is such that $\tilde{X} = \alpha x(\tau_\alpha)$.

Hence, $\frac{\mathrm{d}DEL^T}{\mathrm{d}\tilde{X}} > 0$

Integrating across customer types shows the total deadweight loss, DWL^T, is given by

$$\begin{aligned} \frac{\mathrm{DWL}^T}{L^T} &= \Gamma(\tilde{\alpha}_1)[(\tau^* - \tau_1)x(\tau) - y(\tau_1, \tau^*)] + \tilde{D}_1(\tau_1, \tilde{X}) + \tilde{D}_2(\tau_2, \tilde{X}) \\ &\quad + (\hat{\alpha} - \Gamma(\tilde{\alpha}_2))[y(\tau^*, \tau_2) - (\tau_2 - \tau^*)x(\tau_2)] \end{aligned} \tag{A4}$$

where:

$$\tilde{D}_1(\tau_1, \tilde{X}) = \int_{\tilde{\alpha}_1}^{\tilde{\alpha}^*} [(\tau^* - \tau_\alpha)\tilde{X} - \alpha y(\tau_\alpha, \tau^*)]n(\alpha)\mathrm{d}\alpha$$

$$\tilde{D}_2(\tau_1, \tilde{X}) = \int_{\tilde{\alpha}^*}^{\tilde{\alpha}_2} [\alpha y(\tau^*, \tau_\alpha) - (\tau_\alpha - \tau^*)\tilde{X}]n(\alpha)\mathrm{d}\alpha$$

(i) Taking the total derivative of the total deadweight loss with respect to rates, gives

$$\mathrm{dDWL}^T = \Gamma(\tilde{\alpha}_1)(\tau^* - \tau_1)x'(\tau_1)\mathrm{d}\tau_1 - (\hat{\alpha} - \Gamma(\tilde{\alpha}_2))(\tau_2 - \tau^*)x'(\tau_2)\mathrm{d}\tau_2$$

But if total demand is constant then, $\mathrm{d}X^T$=0

$$(\hat{\alpha} - \Gamma(\tilde{\alpha}_2))x'(\tau_2)\mathrm{d}\tau_2 = -\Gamma(\tilde{\alpha}_1)x'(\tau_1)\mathrm{d}(\tau_1)$$

And hence

$$\mathrm{DWL}^T = \Gamma(\tilde{\alpha}_1)x'(\tau_1)\mathrm{d}\tau_1[(\tau^* - \tau_1) + (\tau_2 - \tau^*)]$$

$$\frac{dDWL^T}{\mathrm{d}\tau_1} = \Gamma(\tilde{\alpha}_1)(\tau_2 - \tau_1)x'(\tau_1) < 0$$

(ii) Differentiating each component of (A4) gives

$$\frac{\mathrm{d}\Gamma(\tilde{\alpha}_1)[(\tau^* - \tau_1)x(\tau_1) - y(\tau_1, \tau^*)]}{\mathrm{d}\tilde{X}} = [(\tau^* - \tau_1)\tilde{X} - \tilde{\alpha}_1 y(\tau_1, \tau^*)]\frac{n(\tilde{\alpha}_1)}{x(\tau_1)}$$

$$\frac{\mathrm{d}\tilde{D}_1(\tau_1, \tilde{X})}{\mathrm{d}\tilde{X}} = -[(\tau^* - \tau_1)\tilde{X} - \tilde{\alpha}_1 y(\tau_1, \tau^*)]\frac{n(\tilde{\alpha}_1)}{x(\tau_1)} + \int_{\tilde{\alpha}_1}^{\tilde{\alpha}^*}(\tau^* - \tau_\alpha)n(\alpha)\mathrm{d}\alpha$$

$$\frac{\mathrm{d}\tilde{D}_2(\tau_2, \tilde{X})}{\mathrm{d}\tilde{X}} = [(\tilde{\alpha}_2 y(\tau^*, \tau_2) - (\tau_2 - \tau^*)\tilde{X}]\left(\frac{n(\tilde{\alpha}_2)}{x(\tau_2)} + \frac{\tilde{N}\tilde{X}}{(\hat{\alpha} - \Gamma(\tilde{\alpha}_2))x(\tau_2^2)}\right) + \int_{\tilde{\alpha}^*}^{\tilde{\alpha}_2}(\tau^* - \tau_\alpha)n(\alpha)\mathrm{d}\alpha$$

$$\frac{\mathrm{d}\left(\hat{\alpha} - \Gamma(\tilde{\alpha}_2)\right)[y(\tau^*, \tau_2) - (\tau_2 - \tau^*)x(\tau_2)]}{\mathrm{d}\tilde{X}} = -[\tilde{\alpha}_2 y(\tau^*, \tau_2) - (\tau_2 - \tau^*)\tilde{X}]\left(\frac{n(\tilde{\alpha}_2)}{x(\tau_2)} + \frac{\tilde{N}\tilde{X}}{(\hat{\alpha} - \Gamma(\tilde{\alpha}_2))x(\tau_2)^2}\right) + (\tau_2 - \tau^*)\tilde{N}$$

Adding these components together gives

$$\frac{\mathrm{dDW}L^T}{L^T\mathrm{d}\tilde{X}} = \int_{\tilde{\alpha}_1}^{\tilde{\alpha}^*}(\tau^* - \tau_\alpha)n(\alpha)\mathrm{d}\alpha + \int_{\tilde{\alpha}^*}^{\tilde{\alpha}_2}(\tau^* - \tau_\alpha)n(\alpha)\mathrm{d}\alpha + (\tau_2 - \tau^*)\tilde{N}$$

Note that

$$(\tau_2 - \tau^*)\tilde{N} > -\int_{\tilde{\alpha}^*}^{\tilde{\alpha}_2} (\tau^* - \tau_\alpha) n(\alpha) \mathrm{d}\alpha > 0$$

And

$$\int_{\tilde{\alpha}^*}^{\tilde{\alpha}_2} (\tau^* - \tau_\alpha) n(\alpha) \mathrm{d}\alpha > 0$$

Hence,

$$\frac{\mathrm{d}DEL^T}{\mathrm{d}\tilde{X}} > 0. \quad ||$$