AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

NC

#53

ECONOMICS RESEARCH REPORT

Measurement of Returns to Fixed Factors on North Carolina Dairy Farms

by

Victor G. Ganoza

Richard A. King

GIANNINI FOUNDATION OF
AGRICULTURAL ECONOMICS
LIBRARY
OCT 20 1988

North Carolina State University
Department of Economics and Business
Raleigh, North Carolina

Economics Research Report No. 53

Measurement of Returns to Fixed Factors on North Carolina Dairy Farms

Victor G. Ganoza

and

Richard A. King

Economics Research Report No. 53

Department of Economics and Business

North Carolina State University at Raleigh

September 1988

ABSTRACT

A common method used to measure the returns to fixed factors on dairy farms is a residual approach in which all but one of the fixed factors are valued at current market prices and any remaining income is taken as the return to that factor. When using this method it often is found that the returns to the residual factor is negative, which implies that some or all of the returns to fixed factors are below current market values.

This report presents the findings of experiments with several statistical models that allow simultaneous estimation of the contribution of each factor to variable profit defined as the difference between cash income and cash expenses adjusted for inventory change. The assumptions of each model are described and the mathematical relationships among the models are summarized. Empirical estimates are calculated for a set of dairy farms in North Carolina for the year 1982.

The translog model is the most elegant of those tested, but the data set was too small to meet requirements for statistical significance of many individual parameters. The estimated values are of interest, however. More restrictive Cobb-Douglas models with fewer variables were estimated and the results compared with the translog estimates.

In each of the models reported, here the return to land was negative, although the coefficients from which the value of land was calculated were not different from zero. In seeking an explanation for this result it was noted that a number of farms reported little or no land used by the dairy operation, suggesting that off-farm sources of feed provided a profitable alternative to farm-produced roughage. However, a high proportion of farms in the data set rented land in addition to that owned. Land rent is included

in the crop variable expense category. A second explanation might be that the marginal contribution of land to variable profit was offset by additional land rent expense and thus the shadow price of land was zero.

Returns to cow numbers ranged from $300 to $400 per head for the translog models to as high as $1200 for Cobb-Douglas models and tended to fall as herd size increased. Returns to investment in buildings and equipment were found to be between 5 and 38 percent per year including depreciation. The value of family labor varied from $1.10 to $2.87 per hour for the translog models, was as high as $4.01 per hour for the Cobb-Douglas models, and increased steadily with size of herd.

Use of the statistical models described here represents an improvement over the usual residual method of measuring returns to fixed factors and offers new insights into the relative contribution of the fixed factors used on North Carolina dairy farms in 1982. According to these models, one would expect that (1) herd size would increase in response to the large variable profit associated with additional cows and (2) operators of small herds would either exit from dairying or increase the size of their herd because of the low returns to operator and family labor on farms with small herds. These expected adjustments in the North Carolina dairy industry are consistent with the 24 percent increase in December daily deliveries per farm and with the 36 percent decrease in number of Grade A herds observed between December 1982 and December 1987.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

This report is based on the Ph.D. dissertation presented by the senior author to the Graduate Faculty of North Carolina State University in 1985. Appreciation is expressed by Dr. Geoffrey A. Benson who graciously provided the data base for the study and to Stephen R. Sutter for assistance in interpreting of individual farm records.

Helpful editorial comments on earlier drafts were received from Drs. Benson, Stephen A. Hatchett, E. C. Pasour, Richard K. Perrin, Ronald A. Schrimper and Walter N. Thurman. The authors take full responsibility for interpretation of the findings reported here.

This study is a contribution to Southern Region Dairy Marketing Research Project S-166 and its successor project S-217.

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

Objective

Milk production in North Carolina is influenced by the income dairy farmers receive from resources used in milk production compared to potential returns from those resources in other uses. Cash receipts and cash expenses associated with milk production are relatively easy to measure, but valuing the contribution of different types of resources owned by farmers is more complicated. This is because resources owned by farmers provide a flow of services over several years, making it difficult to assign an appropriate economic value in any one year.

The objective of this study is to calculate the returns to owners of resources used in milk production when selected statistical procedures are employed. The four resource groups considered are land, dairy cows, buildings and machinery, and unpaid labor supplied by the operator and his family. Financial reports summarized by the North Carolina Farm Business Records Program for the year 1982 provide the data base for the study.

Recent Changes in Milk Production and Income

To provide background for the analysis, recent changes in milk production and income may be of interest. Production of Grade "A" milk in the state increased slowly over the period 1975-85 then declined as a consequence of the federal dairy herd buyout program (Table 1). Class I (fluid milk and cream) sales to North Carolina consumers showed little change from 1975 through 1982 but have drifted upward since then.

Table 1. Grade A milk purchases, Class I sales, farms numbers, average deliveries, and blend prices received by farmers, North Carolina, 1975-1987

Year	Milk purchases[a]	Fluid milk and cream sales[ab]	Grade A producers, December	Average daily deliveries December	Average blend price
	(mil. lbs.)	(mil. lbs.)	(no.)	(pounds)	($/cwt)
1975	1340.0	1104.9	1586	2350	10.51
1976	1419.0	1147.5	1528	2600	10.71
1977	1419.2	1128.2	1437	2703	10.95
1978	1375.4	1116.7	1349	2866	11.47
1979	1397.6	1181.0	1301	3040	12.83
1980	1445.6	1161.0	1280	3196	14.12
1981	1465.3	1138.2	1250	3369	15.08
1982	1511.4	1126.2	1231	3461	14.80
1983	1525.0	1146.1	1222	3546	14.94
1984	1492.8	1164.7	1166	3632	15.18
1985	1583.6	1208.4	1139	3896	14.70
1986	1546.5	1234.3	1012	4031	14.14
1987	1403.3	1238.3	912	4277	14.82

[a]1975-78: Purchases and sales by N.C distributors.
1979-87: Purchases and sales adjusted to state line.

[b]Includes Class IA military sales and milk shake mix and cream items now classified as Class II.

Source: North Carolina Dairy Report, various issues, N. C. Department of Agriculture, Raleigh.

The number of Grade "A" dairy farms in the state decreased dramatically over this period, from 1586 in December 1975 to 912 in December 1987 - a 42 percent drop. Over the same period, daily December milk deliveries per farm increased 82 percent, from 2350 pounds per day in 1975 to 4277 pounds in 1986. Clearly, fewer farmers found milk production to be the best use of their resources, but those who remained in dairy farming expanded milk production from their operations to more than compensate for those who left.

Between 1975 and 1981 the average blend price received for fluid milk rose from $10.51 to $15.08; blend prices stabilized from 1981 to 1984 but declined to $14.14 by 1986. When adjusted for increases in prices paid by farmers, there was a slow but unbroken decline in the real blend price from 1975 through 1986.

Measurement of Returns to Fixed Resources

Two broad classes of resources used on dairy farms can be identified: variable inputs and fixed inputs. The level of use of variable inputs is readily changed. Examples include purchased feed, hired labor, crop and livestock cash expenses and overhead cash expenses. "Fixed" inputs is the term applied to those resources that in the short run can be regarded as given, although the term "quasi-fixed" might be more accurate. Examples include cropland, number of dairy cows, buildings, equipment inventory, and owner and family labor.

Clearly, it is possible to change the level of use of inputs in the fixed category through land purchase or rental, purchase or leasing of milking cows, custom machine use, and off-farm work. However, these resources are less subject to change than inputs in the first group. As noted earlier, the cost of variable inputs is easy to measure. The value to be placed on fixed inputs presents more of a problem. Market prices provide a starting place, but these may not reflect their contribution to current dairy farm profits.

Two methods can be used to calculate the value of fixed inputs using financial records of dairy farm operations. Both make use of variable profits, which are measured as the difference between cash receipts and cash expenses, with appropriate adjustment for change in inventories. The residual method assigns current prices to all but one of the fixed inputs and the value of the remaining input is calculated as a residual claimant on variable profits. The second method uses statistical estimating procedures. The values of all fixed inputs are estimated simultaneously through a function that relates the levels of use of each of the fixed factors to variable profits of the farm as a whole.

Use of the residual method to estimate the contribution of fixed resources to profits on the farms in the N.C. Farm Business Records Program in 1982 is illustrated in Table 2. Interest on net worth is assigned a value of 11 percent. Operator's labor is valued at $5.35 per hour, the average hourly wage paid to textile workers in North Carolina knitting mills. Unpaid family labor is valued at $3.35, the federal minimum wage. Management is assigned a value of 5 percent of cash receipts. When interest on net worth is deducted from net farm income the return to management and to operator and family labor is a negative $8807. On the other hand, when the assigned value of unpaid labor and management is subtracted from net income, the result is a return to net worth of $953 for the year.

Table 2. Average return to fixed assets using residual method, 117 dairy farm records, 1982

Item	Amount	Assumed value	Annual value
Assets:			
Land	$183,132		
Buildings	29,900		
Machinery	62,199		
Dairy livestock	167,170		
Other assets[a]	102,673		
Total	545,074		
Liabilities	146,837		
Interest on net worth	398,237	11%	43,806
Labor and management:			
Operator and partner unpaid labor	3505 hr	$5.35/hr	18,752
Unpaid family labor	613 hr	3.35	2,054
Management[b]	264,818	5%	13,240
Total unpaid labor and management			34,046
Residual valuation of management and unpaid labor:			
Net farm income[c]			34,999
less interest on net worth			43,806
Return to management and unpaid labor			-8,807
Residual valuation of return on net worth:			
Net farm income			34,999
less unpaid labor and management			34,046
Return on net worth			953

[a]Includes feed, crops, supplies (44,138), other livestock (1624), milk base and allotment (28,312) and accounts receivable.

[b]Return to management taken as 5 percent of cash receipts.

[c]See Table 3 for calculation.

Source: Benson, G. A. and S. R. Sutter. 1982 Dairy Farm Business Summary and Business Evaluation Workbook. N. C. Agr. Ext. Service Bulletin AG-39 (Revised), Raleigh, undated, 50 pp.

Comparison of the financial performance of dairy farms participating in the N.C. Farm Business Program for the years 1980-1984 is provided in Table 3. As explained in the table footnotes, the values assigned to each of the fixed inputs vary from year to year, reflecting changes in current market values. Clearly, the return to each of the fixed factors depends on the assumed value placed on other resources.

An alternative to the residual method is statistical estimation of the value of each fixed input using the variability among farms to calculate the effect of level of fixed input use on net farm income. This study presents the results of several statistical estimating procedures.

The study is based on the financial records kept by farmers participating in the N.C. Farm Business Records Program in 1982. This program has been in operation for two decades and is offered to all farmers in the state for a small annual fee. Cooperating farmers receive support from county agents and state specialists to assure accurate and complete reporting methods.

A summary of the records for 1982 is provided in Benson and Sutter. Income, expenditure and performance measures included in that report are found in Appendix Table 1. The records are suitable for farmers to use in filing income tax returns, to locate strengths and weaknesses in their businesses and to assist in evaluating business opportunities. A detailed description of each item and an explanation of how it is obtained can be found in Benson and Sutter.

Characteristics of Farms in Data Set

Farmers participating in the farm records program do not represent a random sample. However, comparison with results of a mail survey of randomly

Table 3. Financial performance of dairy farms participating in N. C. State Farm Business Records Programs, 1980-1984

Item	1980	1981	1982	1983	1984
Number of farms	127	130	117	106	80
Number of cows	105	114	109	100	108
Blend price ($/cwt.)	14.08	15.00	14.71	14.85	15.11
Total farm receipts[a]	253,336	284,788	264,818	246,212	288,576
Operating expenses	194,620	228,475	210,439	203,095	226,332
Net operating income	58,716	56,313	57,582	43,117	62,244
Less depreciation[b]	16,893	20,654	22,583	23,917	27,470
Net farm income	41,823	35,659	34,999	19,200	34,774
Less interest on net investment[c]	45,342	59,912	43,806	34,938	41,003
Return to management and unpaid labor	-3,519	-24,253	-8,807	-15,738	-6,229
Less value of operator and unpaid family labor[d]	20,478	20,456	20,805	23,665	25,270
Return to management	-23,997	-44,709	-29,612	-39,403	-31,500

[a]Adjusted for changes in inventories, excluding land appreciation.

[b]Buildings and machinery.

[c]Assumed to be 12 percent (1980), 14 percent (1981), 11 percent (1982 and 1983), and 10 percent (1984).

[d]Operator's labor valued at $4.70 (1980), $5.10 (1981), $5.35 (1982 and 1983), and $5.80 (1984). Family labor valued at $3.10 (1980), and $3.35 (1981, 1982, 1983, and 1984).

Source: G. A. Benson and Stephen R. Sutter, Dairy Farm Business Summary and Business Evaluation Workbook, N. C. Agricultural Extension Service Bulletin AG-39, Selected issues, Raleigh.

selected dairy farmers in 1983 (King, Benson and Ganoza) indicates a close correspondence between the two, as shown in Table 4 using the size groupings found in the mail survey report. Herds with less than 50 cows are underrepresented and those with 140 or more cows overrepresented in the records program when compared with the survey proportions. The average number of milk cows is 18 percent larger and milk sales 27 percent larger on record-keeping farms. One reason for this difference is that the records include only those farms receiving 70 percent or more of total farm receipts from milk sales.

In summary, farms analyzed in this study tend to be a bit more specialized than those of a random sample and tend to be somewhat more concentrated in the Piedmont region. However, the differences are not so great as to render invalid the results from the analysis that follows. Valuable insights into farmer behavior can be gained, but any inferences drawn must take into account these characteristics.

Three output categories, five variable inputs and four fixed inputs were used. Measured outputs were milk sales, livestock sales and crop sales. The grouping of farm expense categories used in the analysis is shown in Table 5. Milk sales represent total sales less freight costs. No adjustment is made for milk used on the farm. Livestock and crop sales are those reported by farmers. Miscellaneous fixed expenses include all costs not included elsewhere such as interest, taxes, insurance, and general farm expenses. Variable profit is defined as the difference between income and expenses measured in this way.

Table 4. Milk production, sales, and farm acreage, by herd size

Item	Units	Sample Total		Size of Herd: Less than 50 cows		50 - 79 cows		80 - 109 cows		110 - 139 cows		140 cows or more	
		SURVEY	FRP	SURVEY	FRP	SURVEY	FRP	SURVEY	FRP	SURVEY	FRP	SURVEY	FRP
Percent of farms	(%)	100.0	100.0	19.3	8.5	33.8	33.3	20.7	18.8	12.5	15.4	13.7	23.9
Average number of cows	(No.)	92.4	108.9	36.4	38.1	63.0	63.4	91.8	94.5	122.8	124.3	217.9	199.0
Milk production per cow	(lbs.)	14596	14405	13269	12491	14387	14304	14982	14454	15645	15313	15140	14605
Milk sold, 1982	(thous. lbs.)	1253	1588	465	493	820	900	1205	1368	1720	1905	2978	2908
Acreage per farm:													
Cropland	(acres)	208.1	208.5	87.1	100.3	157.1	133.2	217.8	205.5	261.0	202.5	437.8	358.6
Pasture	(acres)	93.1	81.4	79.6	43.0	74.4	62.0	83.1	85.2	131.0	61.1	149.4	132.1
Total	(acres)	301.2	289.9	166.7	143.3	231.5	195.2	300.9	290.7	392.0	263.6	587.2	490.7
Acreage per cow:													
Cropland	(acres)	2.25	1.91	2.39	2.63	2.65	2.10	2.37	2.17	2.13	1.63	2.01	1.80
Pasture	(acres)	1.01	.75	2.19	1.13	1.18	.98	.91	.90	1.07	.49	.69	.66
Total	(acres)	3.26	2.66	4.58	3.76	3.67	3.08	3.28	3.07	3.20	2.12	2.70	2.46

Sources: Survey: 1983 survey of N. C. dairy farms, King, Benson and Ganoza.

FRP: 1982, N. C. Farm Business Records Program, Benson and Sutter.

Table 5. Financial summary for average farm by herd size, N.C. Farm Business Records Program farms, 1982

Item	Less than 50 cows	50-79 cows	80-109 cows	110-139 cows	140 cows or more
		(thousands of dollars)			
INCOME:					
Milk sales	79.9	126.6	195.5	269.7	412.8
Dairy livestock sold	8.3	11.1	20.8	16.4	31.9
Crop sales	4.7	6.4	9.8	5.0	12.4
Total sales	83.9	144.1	226.1	291.1	457.1
EXPENSES:					
Hired labor	3.8	8.7	21.7	22.6	58.2
Purchased feed	21.8	41.5	60.8	94.6	129.3
Crop expenses					
Seed	1.6	1.5	2.7	2.7	4.9
Fertilizer and lime	7.5	10.3	15.7	14.6	32.4
Chemicals	0.7	1.0	2.5	2.4	4.7
Conservation expense	0	0	0	0	0.5
Machine repairs	3.7	5.8	9.4	13.0	19.0
Gas, fuel, and oil	3.2	4.9	8.1	8.2	16.5
Machine hire	1.0	1.1	1.6	2.0	3.3
Land rent	1.5	3.8	5.5	7.6	11.3
Livestock expenses					
Breeding fees	0.6	1.1	2.8	4.0	4.7
Vet and medicine	0.9	2.7	3.6	4.2	9.6
Livestock marketing	1.0	1.3	1.6	1.2	3.8
Miscellaneous livestock	0.1	0.5	1.1	0.7	0.9
Dairy supplies	0.7	1.3	1.3	1.5	2.1
Herd testing	0.5	0.9	1.4	1.7	2.4
Utilities	2.0	2.7	3.9	5.1	8.1
Fixed Expenses					
Interest	5.4	6.7	11.1	18.5	30.4
Taxes	0.7	1.3	2.3	2.7	5.2
Insurance	0.9	1.3	1.8	3.2	5.7
Other repairs	0.3	0.9	1.6	2.4	4.3
Supplies	2.4	3.8	5.1	8.9	15.0
Miscellaneous	0.9	1.9	2.0	2.3	4.4
Total expenditures	61.2	105.2	167.9	224.3	376.6
Variable Profit	22.7	38.9	58.3	66.9	80.5
ASSETS					
Land	142.3	122.8	190.5	153.9	285.9
Livestock	49.7	101.1	142.4	197.3	331.4
Bldgs. & Equip.	21.3	58.0	88.9	107.3	155.7
Price of output ($)	14.12	13.82	14.05	13.73	13.64

Land inputs include both owned and rented cropland and pasture used. Cow numbers represent the average herd size during 1982. The value of buildings and equipment represents the tax basis (undepreciated balance) as reported by the farmer, excluding the residence. Unpaid family and operator labor are expressed in hours as reported by cooperating farmers. A total of 117 records were available. Of these, four farms reporting cash expenses exceeding cash receipts were deleted from the set, since there was no variable profit to allocate.

Organization of the Report

The theoretical foundation for this study and model development are discussed in the following section. The use of shadow prices to value the fixed inputs used in milk production is explained. Selection of the variable profit function form and specific statistical models are discussed.

A comparison of the results obtained from several forms of the translog model is provided in the section that follows. Next, results from the use of linear estimation procedures are provided and compared with the earlier findings. These models differ from the translog models in that a linear expansion path is assumed, which implies that the proportions in which the fixed factors are used are invariant across herd sizes. Model performance and implications are summarized in the final section of the report.

THEORETICAL FOUNDATIONS AND MODEL DEVELOPMENT

The allocation of resources and the evaluation of their opportunity cost in alternative uses is an ongoing process on dairy farms. Decisions to allocate certain resources to milk production occur at very short intervals, while other decisions are made at rather lengthy intervals. Resources for which the allocative decision occurs frequently are commonly referred to as variable inputs. Resources for which allocative decisions are made at rather lengthy intervals are referred to as fixed or durable inputs. Once the allocative decision has been made for durable inputs, there often can be substantial costs involved if reallocation is desired.

Selection of Variable Profit Functions

The choice of inputs that will result in profit maximization is a process that can be evaluated at several different levels, as suggested by Figure 1. The underlying production function (A) summarizes the relationship between inputs used and the output which results. In the short run the selection of inputs may be constrained by certain inputs that are available in fixed quantities (B). Given sufficient time, the levels of these inputs can be varied by the firm manager, but for some questions it is of interest to regard these quantities as fixed. In the short run the manager must choose among the attainable set of outputs (C).

When the prices of the variable inputs (D) are known, a cost function (E) can be constructed from the attainable production set. If the prices of outputs (F) are given, it is possible for the manager to select the desired output level, Y^*.

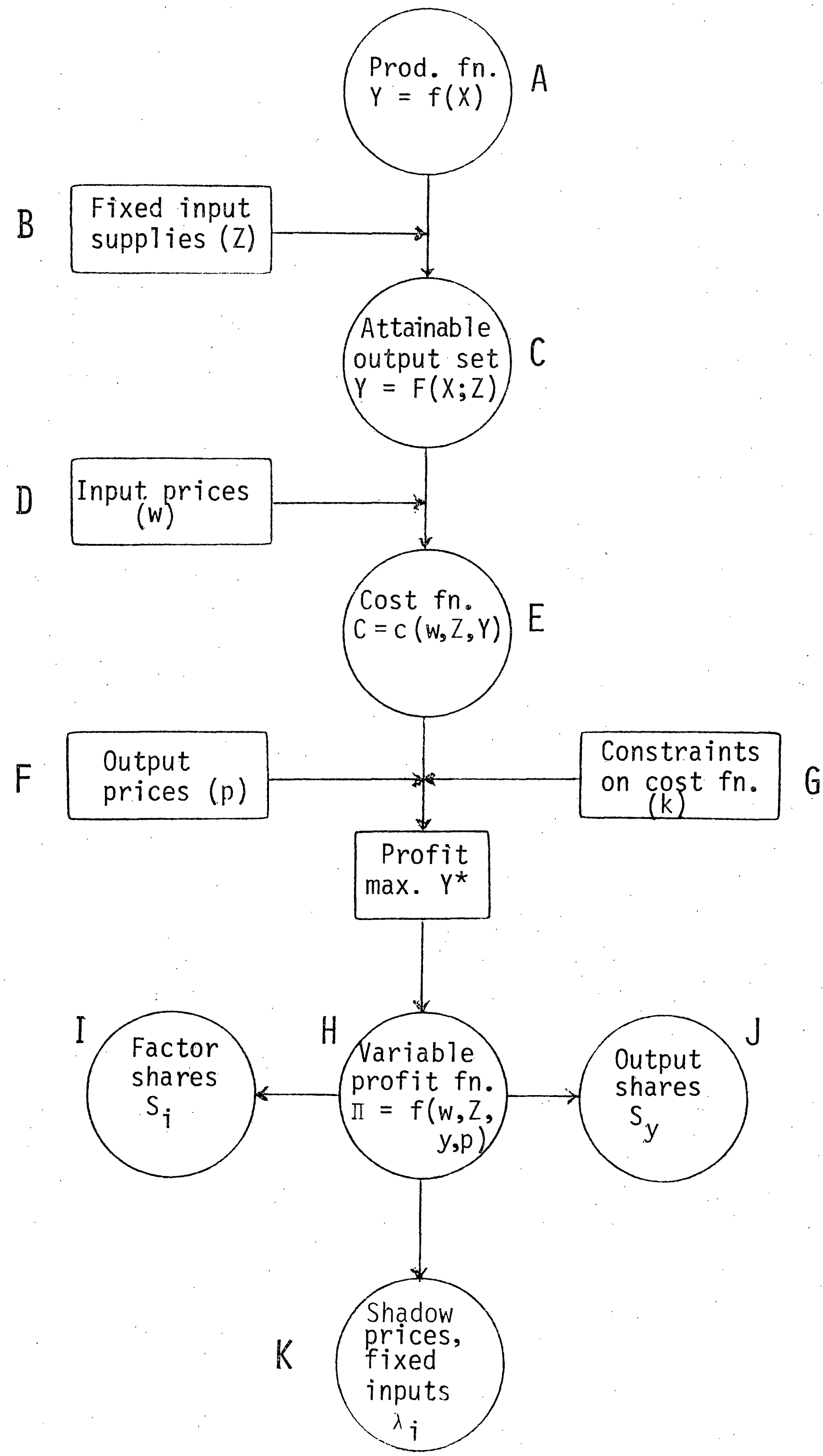

Figure 1. Relation between production, cost and profit function.

The outcome of this output selection process may be viewed as maximization of a variable profit function (H), which represents returns to the set of fixed factors, (B) in Figure 1. The form of the variable profit function reflects the constraints (G) placed on the cost function. Using the variable profit function, it is possible to derive factor shares (I) attributable to each of the variable inputs used in the production process and the profit-maximizing output shares (J).

Quantitative estimates of any of the functions shown in Figure 1 are feasible. In the present study the variable profit (H) function was selected as the relationship to be estimated. Given the variable profit function, it is possible to calculate the shadow prices of each of the fixed inputs (K). These are the measures of particular interest in this study, which investigates the returns to fixed inputs used by North Carolina dairy farmers.

Profit Maximization

Using a profit function (H) to estimate production relations has certain advantages as compared to cost (E) or direct production function (A) approaches. Given the firm's set of feasible outputs and inputs, and assuming price-taking behavior in both product and input markets, the maximization process determines the firm's profit function, π. Under certain regularity conditions on the feasible input and output set, the profit function π may be used to determine the input-output set. Derived input demand and output supply can be obtained simply by differentiating the profit function. Behavioral characteristics such as profit maximization can be empirically investigated and no endogenous variables need be used as explanatory variables as called for in estimating a production function.

In the most general or long-run case, the farm owner seeks to maximize expected profits before any resources have been committed to the productive process. Under this scenario, there is a given production function for the firm: $Y = F(\underline{X})$, where Y is firm output and $\underline{X} = (X_1, \ldots, X_I)$ is a vector of inputs. If output price and input prices are known and equal to p and w_i, respectively, the allocation of resources is dictated by the following maximization problem:

$$\text{Max } \pi = pF(\underline{X}) - \sum_i w_i X_i \qquad (1)$$

where π is profit (revenue minus costs). This unrestricted profit maximization process yields the well known result that each input is used up to the point where its marginal value product equals its price.

A more common situation is that in which the farm owner, at a given point in time, has fixed amounts of m of the I inputs needed in the production process. Maximization under these circumstances is assumed to be carried out over the I - m variable inputs. Thus, the short-run profit maximization problem is given by

$$\text{Max } \pi = pF(\underline{X}, \underline{Z}) - \Sigma w_i X_i \qquad (2)$$

where $\underline{X}$ is an (n x 1) vector of variable inputs, $\underline{Z}$ is an (m x 1) vector of fixed inputs (m + n = I), and π is variable profit defined as revenue minus outlays on variable inputs.

The first order conditions on maximization of equation (2) with respect to the i variable inputs are given by equation (3)

$$p \frac{\partial F(X)}{\partial X_i} = w_i \qquad i = 1, 2, \ldots, n \qquad (3)$$

The demand for the i^{th} variable input of the profit maximizing firm can be solved from the system as

$$X_i^* = f_i(\underline{w};\ \underline{Z}) \qquad i = 1, \ldots, n \tag{4}$$

where X_i^* is the optimal quantity of the i^{th} input and $\underline{w}$ is the n x 1 vector of variable input prices.

Shadow Prices of Fixed Inputs

The implicit cost of a fixed input is referred to as its shadow price. Estimation of shadow prices involves the allocation of variable profit among the fixed inputs. The case of one fixed input presents no problem since all variable profit is allocated to the single fixed input and returns per unit of that input can be easily calculated. When two or more inputs are fixed, one approach is to select one of those inputs for which the shadow price will be calculated while assigning a value (generally the market price of some input considered to be similar to the farm input) to each unit of the other fixed factors. However, the opportunity cost of the fixed factor already in use may be higher or lower than that of the market proxy.

By assigning a specific form to the term $F(\underline{X};\underline{Z})$ in equation (2), Diewert (1974) shows it is possible to estimate the shadow price of each fixed factor by solving the maximization problem first and then assume that the levels of fixed factors in use were chosen so as to minimize the cost of producing the observed level of variable profit. Given a variable profit function $\pi(\underline{p}, \underline{w}; Z)$ that is differentiable at a point $(p^*, w^*; Z^*)$ with respect to the components of Z, and specifying that the expenditure on fixed inputs equals the value of outputs minus the cost of variable inputs, the shadow price or imputed value of a marginal unit of the k^{th} fixed input is defined as $\partial\pi(p^*, w^*; Z^*)/\partial Z_k$.

Selection of Profit Function Forms

There remains the matter of selecting particular functional forms for the profit function. The following criteria have been used in choosing desirable functional forms (Diewert [1973]);

(i) They are linear in unknown parameters so that linear regression techniques can be used to estimate the parameters of interest.

(ii) They provide a second-order approximation to an arbitrary, twice-differentiable profit function satisfying appropriate regularity conditions.

(iii) Only simple restrictions on the unknown parameters are needed to ensure sufficient conditions for satisfying appropriate regularity conditions.

The form of the variable profit function reflects the restrictions placed on the underlying production and cost functions. It is assumed that neither input prices nor output prices are influenced by the individual dairy farmer and so can be regarded as given. To compare the alternative forms that might to be considered it is convenient to use the cost functions shown in Figure 2. The mathematical relationship among the different forms is provided in Appendix B.

The generalized Box-Cox function can be taken as a starting place. By imposing appropriate restrictions, a function can be derived that is homogeneous in factor prices. This implies that if all factor prices are changed by a given proportion, the profit-maximizing output will remain

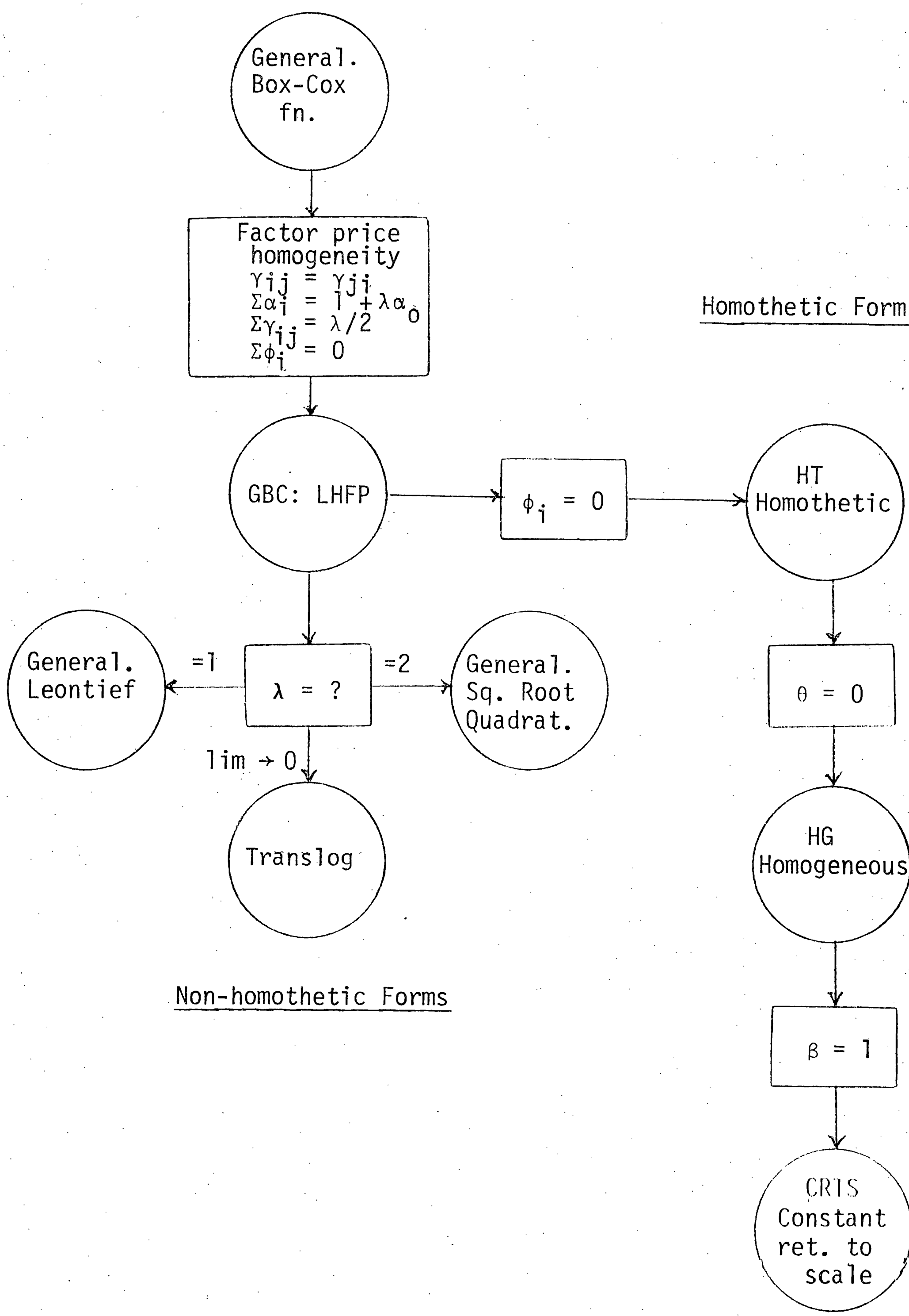

Figure 2. Generalized Box-Cox family of cost functions. (Note: Derivation found in Appendix B.)

unchanged. Selecting specific values for the coefficient λ leads to three common functional forms; the Generalized Leontief form (λ=1), the Generalized Quadratic Square Root form (λ=2) and the Translog function ($\lambda \rightarrow 0$). All are nonhomothetic functions that permit the expansion path to be nonlinear, allowing factor proportions to vary with output. The Translog form was selected as the initial functional form to be used in this study.

An alternative constraint ($\phi_i = 0$) leads to a set of homothetic functional forms, those having linear expansion paths. This constraint implies that level of output has no effect on the relative use of inputs. If, in addition, the coefficient θ is set equal to zero, a homogeneous functional form with scale elasticity of $1/\beta$ is produced. Finally, if β is restricted to 1, the result is a linear homogeneous function of degree 1, reflecting constant returns to scale. The latter two homogeneous forms were also used in estimating returns to the fixed factors employed on North Carolina dairy farms.

As in production cost estimation, there are numerous forms available for approximating the unknown profit function of a firm. For a single-output firm, the normalized restricted profit function developed by Lau (1978) has been widely used. A normalized profit function, π^*, can be obtained from the maximization process to yield

$$\pi^* = G(w^*; Z) \tag{5}$$

where G is a convex function of $w^* = (w_1^*, \ldots, w_n^*)$, a vector of normalized input prices defined as $w_i^* = w_i/p$ where w_i is the i^{th} input price and p is the aggregate output price.

It is not necessary to solve the maximization process to obtain normalized profit functions that yield demand and supply functions consistent with profit maximization subject to a production function. As long as one starts out with a normalized function satisfying certain regularity conditions, the appropriate demand and supply functions are simply derived by Hotelling's Lemma (Lau [1978], p. 139). The normalized profit function is a tool for the estimation of shadow prices, which reflect returns to the fixed inputs used on North Carolina dairy farms.

Empirical Models Selected

Two model types were employed to estimate the value of fixed inputs on farms in the 1982 dairy farm records data set. The first type uses the translog profit function, which allows testing the appropriateness of generalized Cobb-Douglas assumptions.

The second set of models includes three versions of the generalized Cobb-Douglas profit function. The first of these allows estimation of several efficiency measures. The second investigates the importance of farm size and geographic region effects. The final Cobb-Douglas model assumes that all farms in the set are homogeneous. Each of these models is discussed in some detail and the results compared.

ASSET VALUES BASED ON TRANSLOG MODELS

Description of the Translog Model

The normalized translog profit function, $\ln \pi^*$, may be written as:

$$\begin{aligned} \ln\pi^* = \alpha_o &+ \sum_i \alpha_i \ln q_i^* + 1/2 \sum_i \sum_j \alpha_{ij} \ln q_i^* \ln q_j^* + \\ &+ \sum_i \beta_i \ln Z_i + 1/2 \sum_i \sum_j \beta_{ij} \ln Z_i \ln Z_j + \\ &+ \sum_i \sum_j \delta_{ij} \ln q_i^* \ln Z_j \end{aligned} \tag{6}$$

Using * to denote normalized variables, π^* is variable profit (total revenue less variable costs) divided by the price of output, q_i^* is the price of the ith variable input divided by the price of output, and Z_j is the quantity of the jth fixed input.

Equation (6) is estimated subject to restrictions to ensure homogeneity of degree one in prices:

$$\begin{aligned} &(a) \quad \sum_i \alpha_i = 1 \\ &(b) \quad \sum_i \alpha_{ij} = 0, \text{ and} \\ &(c) \quad \sum_i \delta_{ij} = 0 \end{aligned} \tag{7}$$

It is further assumed that the symmetry conditions of Young's theorem hold:

$$\begin{aligned} &(a) \quad \alpha_{ij} = \alpha_{ji}, \text{ and} \\ &(b) \quad \beta_{ij} = \beta_{ji} \end{aligned} \tag{8}$$

These restrictions are illustrated in the flowchart in Figure 3.

Variable input shares of normalized profits, S_i, may be obtained by taking derivatives of the profit function:

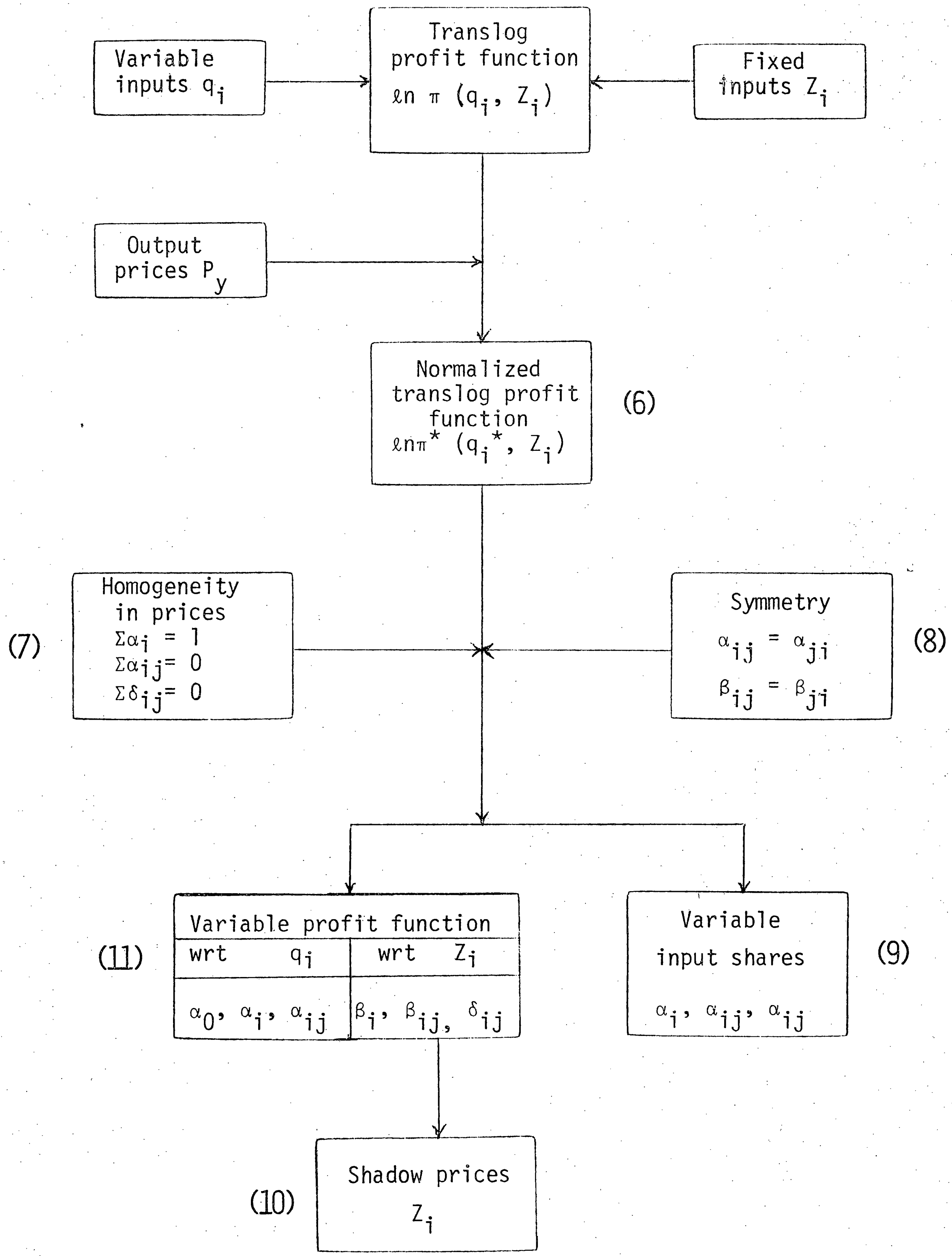

Figure 3. Flowchart for normalized translog profit function. (Note: Equation number in text shown in brackets.)

$$S_i = \frac{\partial \ln \pi^*}{\partial \ln q_i^*} = \frac{\partial \pi^*}{\partial q_i^*} \cdot \frac{q_i^*}{\pi^*} \quad , \; i = 1, \ldots, n$$

$$= \alpha_i + \sum_j^n \alpha_{ij} \ln q_j^* + \sum_j^m \delta_{ij} Z_j \tag{9}$$

If the ratio of output value to variable profit, S_y, is

$S_y = P_y Y/\pi$,

then $\sum_i S_i + S_y = 1$

The profit function (6) and the input equations (9) may be estimated as a system using seemingly unrelated regression (SUR) methods or the profit function may be estimated alone using ordinary least squares (OLS).

Shadow prices for each of the m fixed inputs, Z_i, are obtained by taking the derivative of the normalized translog profit function with respect to each input:

$$\frac{\partial \ln \pi^*}{\partial \ln Z_i} = \frac{\partial \pi}{\partial Z_i} \cdot \frac{Z_i}{\pi^* P_y} = \lambda_i \frac{Z_i}{\pi} \quad , \; i = 1, \ldots, m$$

or $$\lambda_i = \frac{\partial \ln \pi^*}{\partial \ln Z_i} \cdot \frac{\pi}{Z_i}$$

$$= [\beta_i + \sum_j^m \beta_{ij} \ln Z_j + \sum_j^n \delta_{ij} \ln q_j^*] \frac{\pi}{Z_i} \tag{10}$$

It will be noted that three sets of variables are included in the formula for the shadow price of fixed input Z_i. The first is the direct effect, β_i, of the input Z_i. The second set measures the cross-effects of other fixed inputs, β_{ij}. Finally, there are the terms δ_{ij} measuring the effects of variable input prices on the shadow prices of fixed input Z_i.

In the present study based on farm financial records the price of labor was reported, but only total expenditures on other variable inputs were available. As in Garcia et al., expenditures on purchased feed, crop expenses, livestock expenses and overhead expenses were substituted for the price of these input groups and therefore capture both price and quantity effects. The terms δ_{ij} thus will measure the effects of variable input prices on the fixed inputs, Z_i, only if the quantity of the variable input used is constant across farms having any given mix of the fixed inputs.

Estimated Equation Set

Expressed in actual variables, the normalized restricted translog profit function for the set of North Carolina dairy farms can be specified in actual variables as:

$$\begin{aligned} \mathrm{Ln}\pi^{*} = {} & \alpha_0 + \alpha_1 \mathrm{Lnw} + \alpha_2 \mathrm{LnT}_2 + \alpha_3 \mathrm{LnT}_3 + \alpha_4 \mathrm{LnT}_4 + \alpha 5 \mathrm{LnT}_5 \\ & + \tfrac{1}{2}\, \alpha_{11}(\mathrm{Lnw})^2 + \tfrac{1}{2} \sum_{i=2}^{5} \alpha_{1i}(\mathrm{LnT}_i)^2 \\ & + \sum_{i=2}^{5} \alpha_{1i}\, \mathrm{LnwLnT}_i + \sum_{1=3}^{5} \alpha_{2i}\, \mathrm{LnT}_2\mathrm{LnT}_i \\ & + \sum_{i=4}^{5} \alpha_{3i}\, \mathrm{LnT}_3\mathrm{LnT}_i + \alpha_{45}\mathrm{LnT}_4\mathrm{LnT}_5 \\ & + \beta_1 \mathrm{LnZ}_1 + \beta_2 \mathrm{lnZ}_2 + \beta_3 \mathrm{LnZ}_3 + \beta_4 \mathrm{LnZ}_4 \\ & + \tfrac{1}{2} \sum_{i=1}^{4} \beta_{ii}(\mathrm{LnZ}_i)^2 + \beta_{12}\mathrm{LnZ}_1\mathrm{LnZ}_2 + \beta_{13}\mathrm{LnZ}_1\mathrm{LnZ}_3 \\ & + \beta_{23}\mathrm{LnZ}_2\mathrm{LnZ}_3 + \sum_{i=1}^{5}\sum_{j=1}^{4} \delta_{ij}\, \mathrm{LnT}_i\, \mathrm{LnZ}_j + e_1 \end{aligned} \tag{11}$$

where π^* is normalized variable profit per farm;

w, normalized hourly wages;[1]

T_2, expenditures on purchased feed normalized by the price of output;

T_3, expenditures in crop-producing activities normalized by the price of output;

T_4, expenditures on livestock normalized by the price of output; and

T_5, "overhead" or miscellaneous expenditures per farm normalized by the price of output.

The fixed inputs are defined as:

Z_1, the total amount of land used by each farm;

Z_2, average number of cows per farm;

Z_3, book value of buildings and equipment (undepreciated) owned by the farm;

Z_4, hours of unpaid operator and family labor reported.

No interaction was assumed to exist between unpaid hours of operator and family labor reported per farm, Z_4, and the other fixed factors.

Following the development of (9), the S_i equations for wages, purchased feed, crop expenses, livestock expenses, and "overhead" expenses are:

$$-\frac{w_1}{\pi^*} = \alpha_1 + \alpha_{11}\ln w + \alpha_{12}\ln T_2 + \alpha_{13}\ln T_3 + \alpha_{14}\ln T_4 + \alpha_{15}\ln T_5$$

$$+ \delta_{11}\ln Z_1 + \delta_{12}\ln Z_2 + \delta_{13}\ln Z_3 + \delta_{14}\ln Z_4 + e_2 \quad (12)$$

$$-\frac{T_2}{\pi^*} = \alpha_2 + \alpha_{21}\ln w + \alpha_{22}\ln T_2 + \alpha_{23}\ln T_3 + \alpha_{24}\ln T_4 + \alpha_{25}\ln T_5$$

$$+ \delta_{21}\ln Z_1 + \delta_{22}\ln Z_2 + \delta_{23}\ln Z_3 + \delta_{24}\ln Z_4 + e_3 \quad (13)$$

---

[1]Hourly wages were obtained by dividing total wage bill per farm by hours of total paid labor per farm as reported by the farmer. Differences in hourly wages are assumed to represent differences in the opportunity cost of time for laborers rather than quality differences.

$$- \frac{T_3}{\pi^*} = \alpha_3 + \alpha_{31}\ln w + \alpha_{32}\ln T_2 + \alpha_{33}\ln T_3 + \alpha_{34}\ln T_4 + \alpha_{35}\ln T_5$$

$$+ \delta_{31}\ln Z_1 + \delta_{32}\ln Z_2 + \delta_{33}\ln Z_3 + \delta_{34}\ln Z_4 + e_4 \qquad (14)$$

$$- \frac{T_4}{\pi^*} = \alpha_4 + \alpha_{41}\ln w + \alpha_{42}\ln T_2 + \alpha_{43}\ln T_3 + \alpha_{44}\ln T_4 + \alpha_{45}\ln T_5$$

$$+ \delta_{41}\ln Z_1 + \delta_{42}\ln Z_2 + \delta_{43}\ln Z_3 + \delta_{44}\ln Z_4 + e_5 \qquad (15)$$

$$- \frac{T_5}{\pi^*} = \alpha_5 + \alpha_{51}\ln w + \alpha_{52}\ln T_2 + \alpha_{53}\ln T_3 + \alpha_{54}\ln T_4 + \alpha_{55}\ln T_5$$

$$+ \delta_{51}\ln Z_1 + \alpha_{52}\ln Z_2 + \alpha_{53}\ln Z_3 + \alpha_{54}\ln Z_4 + e_6 \qquad (16)$$

Equations (11) to (16) were estimated using the data from the Farm Business Records Program described earlier. For statistical specification, additive errors with zero expectations and finite variance are assumed for each of the six equations of the model. Covariance of errors of any two of the equations for the same farm may be different than zero, but the covariance of errors of any two equations corresponding to different farms is assumed to be identically zero. Under these assumptions, an asymptotically efficient method of estimation, Zellner's Seemingly Unrelated Regression Method (SUR) is used to jointly estimate equations (11) to (16).

For ease of interpretation, the system of equations shown above can be arrayed as in Table 6. The parameters of the system are reported in Appendix Tables 2 and 3.

Table 6. Format for estimated coefficients, translog function

Variable	First order terms	Second order terms $\ln q_i^*$	Z_i
Variable profit function[a]			
Intercept	α_0	-	-
$\ln q_i^*$	α_1	α_{ij}	δ_{ij}
$\ln Z_i$	β_i	-	β_{ij}
Input/profit ratio equations:[b]	Intercept	Purchased inputs	Fixed inputs
Hired labor	α_1	α_{1i}	δ_{1i}
Purchased feed	α_2	α_{2i}	δ_{2i}
Crop expense	α_3	α_{3i}	δ_{3i}
Livestock expense	α_4	α_{4i}	δ_{4i}
Overhead expense	α_5	α_{5i}	δ_{5i}

[a]See text Equation 11.

[b]See text Equations 12 to 16.

Estimated Shadow Prices

Translog system estimates of shadow prices for the fixed inputs are found in Table 7, Column 1. The shadow price of land was $-24, the value per cow was $307, return to investment in buildings and equipment was $.25 per dollar, and operator and family labor was valued at $1.24 per hour.

Size of herd influenced the value of each of the fixed inputs. Land increased slightly with herd size but remained negative; the value of cows was highest in small herds, as was the return to buildings and equipment. Returns to labor on farms with small herds half that of farms with medium and large herds.

In the second model the translog profit function was estimated as a single equation. Equality of the δ_{ij} terms in the profit function and in the variable input share equations seen in Figure 3 no longer holds. The effect was to raise the shadow prices of each of the fixed inputs except land, where larger negative returns were found (Column 2, Table 7). The signs of the herd size effects were not changed from the unrestricted system estimates except for buildings and equipment, which now increased with size.

The third model used the translog profit function only with all variable input cross-terms restricted to zero, producing a generalized Cobb-Douglas function. Estimating shadow prices using the first two sets of terms in Equation 10 had the effect of lowering all shadow prices for the fixed inputs relative to the unrestricted translog model (Column 3, Table 7). The value of cows and returns to buildings and equipment again declined with herd size; land returns changed little, while operator and family labor increased substantially with herd size.

Table 7. Estimated shadow prices for fixed inputs using three alternative translog models

	Herd size	Translog system[a]	Profit function only: Translog[b]	Profit function only: Cobb-Douglas[c]
			($ per acre)	
Land (acres)	All farms	-24.43	-50.92	-5.59
	Small	-26.69	-65.12	-5.76
	Medium	-20.63	-48.62	-6.65
	Large	-17.01	-31.93	-4.66
			($ per cow)	
Cows (number)	All farms	306.90	374.50	248.80
	Small	389.80	441.50	302.40
	Medium	362.20	410.50	289.80
	Large	216.30	298.70	192.30
			($ per $1 invested)	
Buildings & equipment ($)	All farms	.252	.342	.221
	Small	.284	.276	.258
	Medium	.250	.294	.239
	large	.237	.378	.185
			($ per hour)	
Owners' labor (hours)	All farms	1.238	2.869	1.103
	Small	.797	1.846	.679
	Medium	1.689	3.816	1.241
	Large	1.596	3.894	1.733

[a]Simultaneous estimation of profit function (11) and variable input equations (12-16) using SUR. Weighted R^2 for the system is .650.

[b]Equation 11 estimated using OLS. Uncorrected R^2 = .758.

[c]Equation 11 estimated using OLS with all second order terms assumed equal to zero. Uncorrected R^2 = .550.

These three translog models lead to the conclusion that land had little effect on variable profits on this set of dairy farms in 1982. A number of farms reported little or no land used by the dairy operation, suggesting that off-farm sources of feed provided a profitable alternative to farm-produced roughage. However, a high proportion of farms in the data set rented land in addition to that owned. Land rent is included in the crop expense category. A second explanation might be that the marginal contribution of land to variable profit was offset by additional land rent expense and thus, the shadow price of land was zero. If rent paid by farmers is treated as income to land, the shadow prices per acre for the translog system model become $-8.61, $2.40 and $6.11 for small, medium and large herds, respectively.

There was a clear incentive for operators to expand the number of cows in the herd, both because of the positive effect of cow numbers on profits and because of the higher returns to labor with larger herds. The estimated returns to buildings and equipment seem a bit high, but include depreciation allowances as well as return on the inventory value reported for these inputs. Returns to labor as estimated by these models was substantially below minimum wage levels, suggesting that some operators might find off-farm employment an attractive alternative to dairy farming.

Comparison of Statistical Models

The unrestricted translog system model was compared with the Cobb-Douglas model in which all second-order terms were restricted to zero. This test indicated that the restricted model could not be rejected ($F(87,576) = .053$ compared with the critical value of $F(.05)$ of 1.32). This result is not surprising, since the translog profit function requires identification of 51 parameters while the data set consisted of 113 observations, well below the desired level of five observations per parameter.

A comparison of the shadow prices calculated using the system estimates with the estimates based on the profit function alone shows that the two methods provide very similar results. The models presented in the following section use the Cobb-Douglas form for the profit function alone rather than for the complete system of equations derived from the theoretical model.

ASSET VALUES BASED ON LINEAR MODELS

Description of the Linear Model

It was established in the previous section that the translog model with second-order terms constrained to be zero was an acceptable functional form for this data set. In this section the linear profit function model is used to measure the effects of three additional characteristics thought to affect fixed asset values: size of herd group, productivity measures, and region where the farm is located.

The Cobb-Douglas profit function as derived in Appendix C can be written as follows:

$$\begin{aligned} \ln\pi^* = {} & \alpha_0^* + \alpha_1^*\ln w + \alpha_2^*\ln T_2 + \alpha_3^*\ln T_3 + \alpha_4^* T_4 + \alpha_5^* \ln T_5 + \\ & + \beta_1^*\ln Z_1 + \beta_2^*\ln Z_2 + \beta_3^*\ln Z_3 + \beta_4^*\ln Z_4 + \phi_1 S_S + \phi_2 S_L + \\ & + \delta_1 \ln \mathrm{PROD} + \delta_2 \ln \mathrm{MPCT} + Y_1 R_1 + Y_2 R_2 + v_1 \end{aligned} \tag{17}$$

where the parameters ϕ_1 and ϕ_2 measure the effect of small herd and large herd size, respectively, relative to the base medium herd size group.

Two productivity characteristics were added. One measures the effect of milk production per cow as the ratio of individual farm average yield per cow to the average of all herds, PROD. The second measures specialization in milk production using the relative importance of milk sales to total sales, MPCT. It should be noted that only those farms receiving 70 percent or more of cash receipts from milk sales were included in the data set. A total of 100 farm records were used in estimating the Cobb-Douglas profit function models reported in this section.

The effect of farm location on variable profits was investigated by adding a discrete variable for Coastal Plain counties, R_1, and a variable for Mountain locations, R_2.

Estimated System Equations Using SUR

A set of system models was estimated using the Cobb-Douglas framework with both variable profit function and the variable input equations described earlier for the translog models (see equations 6 and 9) Location effects were omitted from these SUR models. The system was estimated using the variable profit function (equation 17) and the following variable input equations, which allow for variations among the three herd sizes:

$$
\begin{aligned}
- \; wX_L/\pi^* &= \alpha_1^{**S} S_S + \alpha_1^{**M} S_M + \alpha_1^{**L} S_L + v_2 \\
- \; T_2/\pi^* &= \alpha_2^{**S} S_S + \alpha_2^{**M} S_M + \alpha_2^{**L} S_L + v_3 \\
- \; T_3/\pi^* &= \alpha_3^{**S} S_S + \alpha_3^{**M} S_M + \alpha_3^{**L} S_L + v_4 \\
- \; T_4/\pi^* &= \alpha_4^{**S} S_S + \alpha_4^{**M} S_M + \alpha_4^{**L} S_L + v_5 \\
- \; T_5/\pi^* &= \alpha_5^{**S} S_S + \alpha_5^{**M} S_M + \alpha_5^{**L} S_L + v_6 \qquad (18)
\end{aligned}
$$

The discrete size variables are $S_S = 1$ for farms with small herds and zero otherwise, $S_M = 1$ for medium size herds and $S_L = 1$ for farms with large herds. Small herds are defined as those with less than 80 cows, medium herds are those with 80 to 139 cows and large herds are those with 140 or more cows.

The parameters of the profit function and five factor equations estimated for the three SUR models are shown in Table 8. In the unrestricted model no constraints were placed on any of the equations. In Column 2 equal technical efficiency was assumed across farm size groups by setting ϕ_1 and ϕ_2 in the profit function equal to zero. The equal technical efficiency

Table 8. Profit function and factor equations for three Cobb-Douglas system models estimated using SUR

Variable	ID	Unrestricted	Equal technical efficiency	Neutral efficiency
		Model characteristics		
	Variable Profit Function:			
Intercept:	$\alpha0^*$	3.1444**	4.5545**	4.2652**
Purchased Inputs:				
Wage	$\alpha1^*$	- .1149	- .1064	- .1101
Purchased feed	$\alpha2^*$	- .3519**	- .3451**	- .3414**
Crop expenses	$\alpha3^*$	- .1699**	- .1586	- .1563
Livestock expenses	$\alpha4^*$	- .0999	.1239	.1235
Overhead expenses	$\alpha5^*$	- .2837**	- .2676**	- .2682**
Fixed Inputs:				
Acres of land	$\beta1^*$	- .0087	- .0313	- .0222
Number of cows	$\beta2^*$	1.8363**	1.4203**	1.4716**
Buildings & equipment	$\beta3^*$	.0590	.0511	.0507
Own labor	$\beta4^*$	.1550*	.1920**	.1875*
Farm Characteristics:				
Productivity index	PROD	1.7287**	1.7413**	1.7125**
Milk sales percent	MPCT	-1.6499**	-1.5615**	-1.5760**
Small herd group	$\phi1$	.2315*	-	-
Large herd group	$\phi2$	- .3448**	-	-
	Factor Equations:			
Wage	S_1	- .2793**	- .3162**	-
	S_2	- .9860**	- .8633**	- .4648**
	S_3	- .3727**	- .4020**	-
Purchased feed	S_1	-1.1013**	-1.1810**	-
	S_2	-2.4279**	-2.1631**	-1.4860**
	S_3	-1.3750**	-1.4381**	-
Crop expenses	S_1	- .8638**	- .9272**	-
	S_2	-1.5951**	-1.3847**	-1.0239**
	S_3	- .8745**	- .9247**	-
Livestock expenses	S_1	- .2756**	- .2922**	-
	S_2	- .4968**	- .4418**	- .3234**
	S_3	- .2773**	- .2903**	-
Overhead expenses	S_1	- .4348*	- .4874**	-
	S_2	-1.2572**	-1.0824**	- .6728**
	S_3	- .6033**	- .6450**	-
Sum of squared error		514.7975	515.0486	513.5273
Degrees of freedom		505	507	517

hypothesis cannot be rejected based on a Chow test of the difference in sum of squared errors for the two models (F(2.505) = .12 vs critical value of F(2,400) of 3.02 at the .05 level).

The assumption of neutral efficiency (Column 3) requires zero values for ϕ_1 and ϕ_2 in the profit function as well as equality of the herd size coefficients α_i in each factor equation. The sum of squared errors is smaller, but the weighted R^2 for the system drops from .52 to .37.

Shadow prices calculated from these Cobb-Douglas system models are shown in Table 9. As in the case of the translog estimates, the contribution of land to variable profits was small but negative. The shadow price of cow numbers was substantially higher than those from the translog models, but again decreased with size of herd. Returns to investment in buildings and equipment were less than 5 percent with little difference among herd sizes. Returns to operator and family labor ranged from \$2.10 to \$2.60 per hour and increased with herd size, reaching \$4.00 per hour for the large herds.

Estimated Profit Functions Using OLS

The normalized variable profit functions for three models estimated using ordinary least squares are shown in Table 10. The location effects for Mountain and Coastal Plain farms found in the first model were negative but not statistically significant, possibly because of the small number of farms outside the Piedmont area. Using a Chow test to compare the first and second OLS models, it was found that the second model in which location effects are constrained to zero could not be rejected. This second model is the OLS counterpart to the SUR system model shown in column 1, Table 8.

The third model, in which both region and herd size parameters are constrained to be zero, when compared with the first model was marginally

Table 9. Estimated shadow prices for fixed inputs using three alternative Cobb Douglas system models

Input	Herd size	Unrestricted	Equal Tech. Efficiency	Neutral Efficiency
			($ per acre)	
Land (acres)	All farms	-1.77	-6.35	-4.51
	Small	-1.81	-6.50	-4.61
	Medium	-2.16	-7.77	-5.51
	Large	-1.40	-5.05	-3.57
			($ per cow)	
Cow (number)	All farms	976	755	782
	Small	1202	930	963
	Medium	1147	877	919
	Large	551	551	511
			($ per $/invested)	
Buildings & equipment ($)	All farms	.038	.033	.033
	Small	.045	.039	.038
	Medium	.041	.036	.036
	Large	.031	.027	.027
			($ per hour)	
Owners' labor (hours)	All farms	2.10	2.60	2.54
	Small	1.36	1.69	1.65
	Medium	2.38	2.95	2.87
	Large	3.28	4.07	3.97

Table 10. Profit function for three Cobb-Douglas models estimated using OLS

Variable	ID	Model Characteristics: With size and region	No region effects	No size or region effects
Intercept:	$\alpha 0$	2.3642*	2.3958*	4.3351**
Purchased Inputs:				
Wage	$\alpha 1^*$	- .1928**	- .1889**	- .1792*
Purchased feed	$\alpha 2^*$	- .5247**	- .5246**	- .5294**
Crop expenses	$\alpha 3^*$	- .3109**	- .3144**	- .3312**
Livestock expenses	$\alpha 4^*$	- .0435	- .0467	- .0004
Overhead expenses	$\alpha 5^*$	- .3108**	- .3113**	- .2907**
Fixed Inputs:				
Acres of land	$\beta 1^*$	- .0254	- .0233	- .0464
Number of cows	$\beta 2^*$	2.4023**	2.4074**	1.9069**
Buildings & equipment	$\beta 3^*$	.1116	.1101	.0981
Own labor	$\beta 4^*$	.3028**	.3027**	.3482**
Farm Characteristics:				
Productivity index	PROD	2.2044**	2.2080**	2.2331**
Milk sales percent	MPCT	-2.1289**	-2.1566**	-2.0979**
Small herd group	$\phi 1$	.3195*	.3199**	-
Large herd group	$\phi 2$	- .3509**	- .3463**	-
Coastal Plain Region	Y_1	- .0756	-	-
Mountain Region	Y_2	- .0058	-	-
Sum of squared error		7.6699	7.6854	8.7277
Degree of freedom		73	75	77
R^2		.7540	.7535	.7200

rejected at the .05 level (F ratio of 2.52 with a critical F(4,70) value of 2.50) but was not rejected at the .01 level. When compared with the second, model three was rejected at the .01 level. This third model is the OLS counterpart to the SUR models found in Columns 2 and 3 of Table 8.

Estimated Shadow Prices

The shadow prices for fixed inputs calculated from the three OLS models are found in Table 11. The first two which include size of herd produce similar estimates with and without the region variables. Returns to land are negative but small, returns to cow numbers are large and decline with size of herd, returns to investment in buildings and equipment range from 5 to 9 percent, while returns to operator and family labor range from $2.46 per hour for small herds to $5.89 for large herds.

When both size of herd and region effects are restricted to zero (column 3), the estimated shadow price per cow drops from over $1200 to $982 per cow in the average size herd, returns to land and investment in buildings and equipment drop slightly and the value of operator and family labor increases to $4.01 per hour. The OLS estimates of the variable profit function provide additional support for the view that returns per cow are greatest on small herds but that the value of family labor rises with herd size.

Table 11. Estimated shadow prices for fixed inputs using three alternative Cobb-Douglas OLS models

Input	Herd size	With size and region	No region effects	No size or region effects
			($ per acre)	
Land (acres)	All farms	-5.19	-4.76	- 9.49
	Small	-5.41	-4.96	- 9.88
	Medium	-6.09	-5.59	-11.13
	Large	-3.80	-3.49	- 6.95
			($ per cow)	
Cow (number)	All farms	1237	1240	982
	Small	1416	1419	1124
	Medium	1350	1353	1072
	Large	844	846	670
			($ per $/invested)	
Buildings & equipment ($)	All farms	.077	.076	.067
	Small	.091	.090	.080
	Medium	.079	.078	.069
	Large	.053	.053	.047
			($ per hour)	
Owners' labor (hours)	All farms	3.49	3.49	4.01
	Small	2.47	4.46	2.83
	Medium	4.26	4.25	4.89
	Large	5.89	5.89	6.77

EVALUATION OF MODELS AND RESULTS

This study investigates the factors that influence the profitability of Grade A milk production in North Carolina. The usual method for evaluating fixed resources used on dairy farms is a residual procedure in which current market values are placed on all but one factor and the remainder of variable profits allocated to the selected input. This method produces negative values when the profitability of milk production is not great enough to cover current market prices for all fixed factors.

In this study a theoretical model of resource allocation was presented. The most general model for estimating the values of fixed factors considered here is the translog variable profit system. This system relates prices of variable inputs and quantities of fixed factors to variable profit, the difference between cash receipts and cash expenses adjusted for inventory change.

When financial records for a set of farms in a particular year are used for estimation, variability in factor prices is not observable. As a result, it is necessary to substitute expenditures on selected variable factors for the prices called for in the theoretical model. While this is a serious weakness of the empirical model when used to estimate certain characteristics of the system, it is possible to obtain estimates of the marginal value of the fixed inputs. These shadow prices are of primary interest in this study.

The first set of shadow prices of fixed inputs on dairy farms was developed using the translog profit function and five factor supply equations estimated as a seemingly unrelated regression (SUR) system. These estimates were compared with those derived from the variable profit function alone using ordinary least squares (OLS). The translog system requires a larger data set than the 117 farm records used in this study to provide desired levels of statistical significance.

It was shown that a more restrictive linear model could not be rejected. This model is the generalized Cobb-Douglas form. Estimates were developed using both SUR estimates of the equation system and OLS estimates of the variable profit function alone. It was found that region of the state in which the farm was located did not have a significant effect on variable profit.

The results from all models reported here suggest that variation in the quantity of cropland and pasture used on these farms had little impact on variable profits. Investment in buildings and equipment also played a small role in explaining variable profit differences among farms, ranging from as much as $.38 per dollar invested for one model to $.05 for other models.

Number of cows in the herd was an important determinant of variable profit. Shadow prices for cows averaged between $300 and $400 per head for translog forms but reached as high as $1200 per cow for some Cobb-Douglas models. In every case the shadow price of cows was highest on small herds and lowest on large herds.

The fourth fixed factor considered, hours of unpaid operator and family labor, was also important in explaining differences in profits among farms. Translog estimates for the average farm varied from $1.10 to $2.87, while Cobb-Douglas estimates were as high as $4.01 per hour. In contrast to the decline in value of cows, operator's labor was valued least on small herds and rose steadily for medium and large herds.

The shadow prices of fixed factors reported in this study should be regarded as marginal values calculated at the mean rather than average values. This implies that multiplying shadow prices by average farm use levels need not exactly equal the average variable profit per farm. The values per cow derived using two of the three Cobb-Douglas models seem unreasonably high, but the translog forms and the last Cobb-Douglas model seem to provide sensible estimates.

Insights can be gained concerning the relative contribution to dairy farm profits of the four types of fixed factors using the statistical procedures described here. They make it possible to derive consistent estimates of the marginal values of the fixed factors and the effects of herd size and productivity that are not possible using the traditional residual method.

Based on the models reported here which analyze the financial performance of a set of dairy farms in 1982, it would be expected (1) that herd size would increase in response to the large contribution to variable profit from additional cows and (2) that operators of small herds would either exit from dairying or increase the size of their herd in response to the low returns to operator and family labor in small herds as compared with large herds. These expected adjustments in the North Carolina dairy industry are consistent with the 24 percent increase in December daily deliveries per farm and with the 36 percent decrease in number of Grade A herds observed between December 1982 and December 1987.

REFERENCES

Benson, G. A., and S. R. Sutter, "1982 Dairy Farm Business Summary and Business Evaluation Workbook." North Carolina Agricultural Extension Service Bulletin AG-39 (Revised), Raleigh, 1983.

Benson, G. A., "The Cost of Producing Milk on N. C. Farms, 1982. A Report to the North Carolina Milk Commission." Department of Economics and Business and N. C. Agricultural Extension Service. North Carolina State University, Raleigh, 1983.

Berndt, E. R. and M. S. Khaled, "Parametric Productivity Measurement and Choice Among Flexible Functional Forms." J. Pol. Econ. 87(1979): 1220-45.

Diewert, W. E., "Functional Forms for Profit and Transformation Functions." Journal of Economic Theory 6(1973): 284-316

Diewert, W. E., "Applications of Duality Theory." In Frontiers of Quantitative Economics II, M. D. Intrilligator and D. A. Kendrick (Eds.), North-Holland Publishing Co., Amsterdam, 1974.

Ganoza, V. G., "Estimating the Value of Fixed Assets Used on North Carolina Dairy Enterprises." Ph.D. dissertation, Department of Economics and Business, North Carolina State University, Raleigh, 1985.

Garcia, P., S. J. Sonka and M. S. Yoo, "Farm Size, Tenure, and Economic Efficiency in a Sample of Illinois Grain Farms." Amer. Jour. Agr. Econ. 64(1982): 119-23.

King, R. A., G. A. Benson, and V. G. Ganoza, North Carolina Dairy Farming, 1983." Economics Information Report No. 71, North Carolina State University, Raleigh, 1984.

Lau, L. J., "Applications of Profit Functions." Production Economics, A Dual Approach to Theory and Applications, Vol. I, M. A. Fuss and D. McFadden (Eds.), North-Holland Publishing Co., Amsterdam (1978), pp. 133-216.

North Carolina Department of Agriculture, Crop and Livestock Reporting Service, "North Carolina Agricultural Statistics 1982," Raleigh.

North Carolina Milk Commission, North Carolina Dairy Report, various issues, Raleigh.

APPENDIX A. APPENDIX TABLES

Appendix Table 1. Income, expenditure, and performance data available for each farm in the Electronic Farm Business Records Program

FARM RECEIPTS

(1) Milk sales
(2) Dairy livestock sold
(3) Other livestock and livestock products
(4) Crop sales
(5) Rent, interest, dividends
(6) Other income
(7) Total cash receipts
(8) Change in feed, crops and supply inventory
(9) Change in livestock inventory
(10) Total farm receipts

OPERATING EXPENSES

(11) Hired labor
(12) Purchased feed

<u>Crop expenses</u>

(13) Seed
(14) Fertilizer and lime
(15) Chemicals
(16) Conservation expense

<u>Livestock expenses</u>

(17) Breeding fees
(18) Vet and medicine
(19) Livestock marketing
(20) Livestock purchases
(21) Miscellaneous livestock expense
(22) Dairy supplies
(23) Herd testing

<u>Power and machinery expenses</u>

(24) Machine repairs
(25) Gas, fuel and oil
(26) Machine hire
(27) Freight--trucking
(28) Utilities

<u>Interest, taxes, insurance expenses</u>

(29) Interest
(30) Taxes
(31) Insurance

<u>General farm expenses</u>

(32) Other repairs
(33) Supplies
(34) Rent
(35) Miscellaneous expenses
(36) Total operating expenses

(37) NET OPERATING INCOME
(38) Less machinery depreciation
(39) Less building depreciation
(40) NET FARM INCOME
(41) Less interest on net worth at 11 percent
(42) RETURN TO MANAGEMENT AND UNPAID LABOR
(43) Less operator and partner unpaid labor
(44) Less unpaid family labor
(45) RETURN TO MANAGEMENT
(46) RETURN TO MANAGEMENT PER COW
CHARGE FOR MANAGEMENT @ 5% CASH RECEIPTS
(47) RATE OF RETURN ON NET WORTH

OTHER FACTORS

(48) Blend price per cwt.
(49) Percent farm receipts used for operating expenses
(50) Percent farm receipts for depreciation
(51) Net operating income per cow
(52) Percent cash receipts from dairy

BALANCE SHEET, ASSETS

(53) Accounts receivable
(54) Land
(55) Buildings
(56) Machinery
(57) Feed, crops and supplies
(58) Dairy livestock
(59) Other livestock
(60) Base and allotment
(61) Total assets
(62) Liabilities
(63) Net worth

MEASURES OF SIZE OF BUSINESS

(64) Number of cows
(65) Pounds of milk sold
(66) Farm receipts
(67) Total acres cropland
(68) Man-years of dairy labor
(69) Man-years of farm labor
(70) Total assets
(71) Total value of assets used

Appendix Table 1 (continued

MEASURES OF RATES OF PRODUCTION

(72) Pounds of milk sold per cow
(73) Milk sales per cow
(74) Tons of corn silage per acre
(75) Tons per hay per acre
(76) Bushels of corn grain per acre

Roughage harvested (tons of silage equivalent)

(77) Hay
(78) Corn silage
(79) Small grain silage
(80) Other silage
(81) Total tons of silage equivalent harvested per cow

MEASURES OF LABOR EFFICIENCY

(83) Number of cows per man-year
(84) Pounds of milk sold per man-year of dairy labor
(85) Crop acres per man-year
(86) Farm receipts per man-year

ITEMS RELATED TO FEED COSTS

(87) Pounds of milk sold per cow
(88) Feed purchased per cwt. of milk sold
(89) Feed purchased as percent of milk sales
(90) Crop expense per cwt.
(91) Total purchased feed and crop expenses per cwt. milk

OTHER FACTORS

(92) Silage equivalent per cow, tons
(93) Acres of cropland cow
(94) Acres of pasture per cow
(95) Fertilizer and lime per crop acre
(96) Seed and chemicals per crop acre
(97) Heifers as percent of cow numbers

POWER AND MACHINERY EXPENSE

(98) Machinery depreciation per cwt. of milk sold
(99) Interest per cwt. of milk sold
(100) Utility cost per cwt. milk sold
(101) Machinery oper. expense per cwt. milk sold
(102) Total P&M expense per cwt. milk sold
(103) Percent of milk sales for P&M expense

LABOR AND MACHINERY EXPENSE

(104) Total P&M expense per cwt. milk sold
(105) Hired labor expense per cwt. milk sold
(106) Value operator and unpaid labor per cwt. milk sold
(107) Total labor and machinery expense per cwt. milk sold
(108) Percent of milk sales for labor and machinery expense

INVESTMENT ANALYSIS

Total investment

(109) Per man-year
(110) Per cow
(111) Mach. & equip. investment/cow
(112) Land and building investment per cow
(113) Capital turnover (investment/ farm receipts)

FARM DEBT ANALYSIS

(114) Percent equity
(115) Farm debt per cow
(116) Available for debt service, living & taxes
(117) Annual debt payment per cow
(118) Debt payment as percent of milk sold

NET COST OF PRODUCING MILK

(119) Farm receipts
(120) Less milk sales
(121) Other receipts
(122) Total operating expenses
(123) Depreciation
(124) Interest
(125) Operator and unpaid labor
(126) Total cost
(127) Less other receipts
(128) Net cost
(129) Pounds of milk sold
(130) Net cost per 100 pounds
(131) Blend price received for milk

Appendix Table 2. Variable definitions and mean values, Cobb-Douglas mdels

Item	Farm Record Identification Name[a]	Source[b]	Mean C-D farms	LN Norm by POUTPUT Cobb-D	Trans.
Farm Location[c]	Region	Piedmont Base	.80		
		CPlain R1=1	.05		
		Mountain R2=1	.15		
Size of herd	Small	<80 cows S1=1	.43		
	Medium	80-139 Base	.34		
	Large	140+cows S2=1	.23		
Dollar values:					
Milk sales		1			
Milk less freight	X1	1-27	221435		
Livestock sales	X2	2+3	17475		
Crop sales	X4	4	5644		
Total sales	Sales	X1+X2+X4	244554		
Total op expenses	Expenses	36-20-27	187019		
Variable profit	Profit	Sales-expenses	57535	8.0729	8.02277
Prices:					
Milk	PMilk	131	14.876		
Livestock	PStock	Geo. mean of:	49.7702		
Calves		55.02**(2446.74/16403.94)*			
Cull cows		40.00**(11059.2/16403.94)*			
Milk cows		105.00**(2898.0/16403.94)			
Crops	PCrop	State mean corn pr.	4.8124		
Output	POutput	PMilk**(X1/Sales)* PStock**(X2/Sales) *PCrop**(X4/Sales)	15.1452		
Variable Inputs($)					
Hired labor:					
Labor expense	T1	11	23744		
Hours hired	HLabor	69*2640+N25	5626		
Wage rate, W	Wage	11/HLabor	3.85	-1.4157	-1.3027
Purchased feed	T2	12	70202	8.2618	8.2852
Crop expenses	T3	13 To 16+24 To 26+34	47134	7.8306	7.8621
Livestock expense	T4	17 To 19+ 21 To 23+28	16630	6.795	6.8296
Overhead expense	T5	29 To 33+35	29308	7.2275	7.2770
Total variable expense	TEXP	T1 To T5	187019		
Milk sold/cow (lb.)	Yield	87	14387		
Rel. productivity	Prody	87/14387		-.0187	
PCT milk sales	MPCT	X1/Sales		-.1095	
Owned inputs ($)					
Land (crop+past)	Z1	54	181892		
Livestock	Z3	58+59+20	169102		

Appendix Table 2 (continued)

Item		Farm Record Name[a]	Identification Source[b]	Mean C-D farms	LN Norm by POUTPUT Cobb-D	Trans.
Building & equip.		Z4	55+56	88807	11.1565	11.1668
Value family labor		Z5	43+44	21399		
Owned inputs (Quant.)						
Cropland	(AC)	NZ1	67	202.67		
Pasture	(AC)	NZ2	64*94	80.83		
Total land	(AC)	NZT	NZ1+NZ2	283.50	5.4616	5.4673
Milk cows	(NO)	NZ3	64	108.21	4.5382	4.5468
Family Labor	(HR)	NC5	43/$5.35+ 44/$3.35	4241.4	8.3383	7.7867

Note: The following farm numbers were deleted: Trans; 40, 50, 58, 98. C-D: Also 1, 14, 15, 22, 38, 47, 56, 65, 85,91, 101, 105, 106.

[a]Ganoza, Appendix B, pp. 160-169.

[b]Ganoza, Table 5, pp. 19, 20.

[c]Ganoza, Figure 1, p. 18.

Appendix Table 3. Estimated profit function, unrestricted translog model

			Second order terms								
			Variable inputs					Fixed inputs			
Variable	ID	1st order terms	Wage rate w	Feed exp. T_2	Crop exp. T	Livestock exp. T_4	Overhead exp. T_5	Land Z_1	Cows Z_2	Build. & equip. Z_3	Own labor Z_4
Intercept		5.7866		-	-	-	-	-	-	-	-
		(16.2934)									
<u>Purchased inputs:</u>											
Wage	w	1.7904	-.2525*	-.3688	.1405	.2738	.1324	.3067	-.5320	-.0400	-.1537
		(2.5953)	(.2622)	(.3181)	(.2750)	(.2248)	(.2119)	(.3018)	(.4637)	(.2225)	(.1481)
Purchased feed	T_2	-.4077	-	-.4991	-.4417	.6764	.1998	-.1901	.5429	-.1695	.2063
		(6.0722)		(1.0361)	(.4870)	(.6175)	(.2804)	(.4951)	(1.1991)	(.4925)	(.1800)
Crop expense	T_3	.7702	-	-	-.0274	.2683	.2454	.0198	.2075	-.0342	-.0998
		(2.8157)			(.4823)	(.3530)	(.2429)	(.2988)	(.6323)	(.2936)	(.1058)
Livestock expense	T_4	-1.6657	-	-	-	-.7425	.0648	-.2038	-.6891	.4238	-.1407
		(3.4660)				(.8492)	(.3204)	(.4895)	(.7682)	(.3849)	(.1224)
Overhead expense	T_5	0.2604	-	-	-	-	-.3535	-.3598	-.2072	-.0674	.0282
		(2.0496)					(.2181)	(.2702)	(.4490)	(.2708)	(.0666)
<u>Fixed inputs:</u>											
Acres of land	Z_1	.7509						.0253	.5994	.1941	-
		(2.6925)						(.2641)	.8968)	(.2223)	
Number of cows	Z_2	-.0543						-	-.0768	.2573	-
		(7.0921)							(2.4736)	(.7086)	
Build. & equip	Z_3	.4264						-	-	-.1532	
		(2.7295)								(-.3637)	
Own labor	Z_4	-2.2511						-	-	-	-.0019
		(.5656)									(.0285)

*Diagonal entries are $PA_{11} = 1/2\ A_{11}$, etc. Standard errors are shown in parentheses.

Appendix Table 4. Estimated input/profit ratio equations, unrestricted translog model

Variable	ID	Factor equation: Hired labor w	Purchased feed T_2	Crop expenses T_3	Livestock expenses T_4	Overhead expenses T_5
Intercept		-1.0227	-6.4960	-1.9795	-1.4634	-.8520
		(1.4373)	(3.8385)	(2.1950)	(.6999)	(2.0750)
Purchased inputs:						
Wage	w	-.1224	-.3993	-.2232	-.0570	-.0340
		(.1376)	(.3674)	(.2101)	(.0670)	(.1986)
Purchased feed	T_2	-.1922	-1.5783**	-.3674	-.0421	-.1784
		(.2351)	(.6279)	(.3590)	(.1145)	(.3394)
Crop expenses	T_3	-.0733	.1901	-.9218**	.0559	.0850
		(.1717)	(.4586)	(.2622)	(.0836)	(.2479)
Livestock expenses	T_4	-.0843	.4884	.2655	-.2439*	.0051
		(.2201)	(.5878)	(.3361)	(.1072)	(.3177)
Overhead expenses	T_5	-.3073*	-.6803	-.4524*	-.1776**	-.9874**
		(.1355)	(.3618)	(.2069)	(.0660)	(.1956)
Fixed inputs:						
Acres of land	Z_1	-.2445	-.4298	-.1182	-.0730	-.2484
		(.1549)	(.4136)	(.2365)	(.0754)	(.2236)
Number of cows	Z_2	.1343	.6083	.8294	.1994	.6999
		(.3426)	(.9148)	(.5231)	(.1668)	(.4945)
Build & equip.	Z_3	.4506**	1.3948*	.7462**	.2672**	.5058*
		(.1524)	(.4071)	(.2328)	(.0742)	(.2200)
Own labor	Z_4	.1238**	.1731	.1041	.0474**	.0635
		(.0383)	(.1022)	(.0584)	(.0186)	(.0552)

*Significant at .05 level.

**Significant at .01 level.

Standard errors are shown in parentheses.

APPENDIX B

GENERALIZED BOX-COX FAMILY OF COST FUNCTIONS

APPENDIX B. GENERALIZED BOX-COX FAMILY OF COST FUNCTIONS

Function class	Characteristics
1. Generalized Box-Cox	Unconstrained
2. Linear homogeneous in factor prices	Nonlinear expansion path
a. Generalized Square Root Quadratic	$\lambda = 2$
b. Generalized Leontief	$\lambda = 1$
c. Translog	$\lambda \to 0$
3. Homethetic production technology	Linear expansion path
a. Nonhomogeneous	$\phi_i = 0$
b. Homogeneous, degree $1/\beta$	$\theta = 0$
c. Constant returns to scale	$\beta = 1$ (Cobb Douglas)

The generalized Box-Cox function, C, can be expressed as

$$C = [1 + \lambda G(P)]^{1/\lambda} \, Y^{\beta(Y,P)}$$

where $$G(P) = \alpha_i + \sum_i P_i(\lambda) + 1/2 \sum_i \sum_j \gamma_{ij} P_i(\lambda) P_j(\lambda)$$

$$\beta(Y,P) = \beta + 1/2\ \theta \ln Y + \sum_i \phi_i \ln P_i, \text{ and}$$

$$P_i(\lambda) = (P_i^{\lambda/2} - 1)/1/2\lambda, \quad \gamma_{ij} = \gamma_{ji}$$

The function, C, can be constrained to be linear homogeneous in factor prices.

If $$\sum_i \alpha_i = 1 + \lambda \alpha_o$$

$$\sum_i \gamma_{ij} = 1/2\ \lambda \alpha_i, \text{ and}$$

$$\sum_i \phi_i = 0$$

then $$C = [2/\lambda \sum_{ij}\sum \gamma_{ij} P_i^{\lambda/2} P_j^{\lambda/2}]^{1/\lambda} Y^{\beta(Y,P)}$$

Three nonhomothetic cases can be identified.

If $\lambda = 2$, then the nonhomothetic generalized square root quadratic form is:

$$C = [\sum_{ij}\sum \gamma_{ij} P_i P_j]^{1/2} Y^{\beta(Y,P)}$$

If $\lambda = 1$, then the nonhomothetic generalized Leontief form is:

$$C = 2 \sum_{ij}\sum \gamma_{ij} P_i^{1/2} P_j^{1/2} Y^{\beta(Y,P)}$$

Rewriting the general form as:

$$G(P) = 1/\lambda\{[C/Y^{\beta(Y,P)}]^{\lambda} - 1\}$$

and taking the limit as $\lambda \to 0$, we obtain the nonhomothetic version of the translog cost function

$$\ell n\, C = \alpha_0 + \sum_i \alpha_i\, \ell n P_i + 1/2 \sum_{ij}\sum \gamma_{ij}\, \ell n\, P_i \ell n P_j +$$

$$+ \beta\, \ell n\, Y + 1/2\theta\, (\ell n Y)^2 + \sum_i \phi_i\, \ell n\, P_i \ell n Y$$

which is the linear homogeneous in prices form with the additional restriction that

$$\lim_{\lambda \to 0} \sum_i \alpha_i = 1$$

Alternatively, three homothetic forms are obtained by placing constraints on ϕ_i, Θ and β. In each case $\phi_i = 0$. If $\Theta = 0$, the function is homogeneous with returns to scale equal to β. Constant returns to scale requires that $\beta = 1$.

Source: Berndt, E.R. and M.S. Khaled (1979) pp. 1222, 1223.

Appendix C

GENERALIZED COBB-DOUGLAS PROFIT FUNCTION

APPENDIX C. GENERALIZED COBB DOUGLAS PROFIT FUNCTION

The generalized Cobb-Douglas form of the translog production function, extended to include fixed inputs Z_j following Lau (1978)and Garcia and Sonka (1984), can be written as:

$$Y = A \prod_{i}^{m} X_i^{\alpha i} \prod_{j}^{n} Z_j^{\beta j} \ . \tag{1}$$

Where Y is homogeneous of degree μ and w_i^* are normalized input prices, the normalized profit function is expressed as:

$$\pi^* = A^{(1-\mu)^{-1}} (1-\sum_{i}^{m} \alpha_i) [\prod_{i}^{m} \alpha_i^{\alpha_i (1-\mu)^{-1}}] \text{ x}$$

$$\text{x } [\prod_{i}^{m} (w_i^*)^{-\alpha_i (1-\mu)^{-1}}] [\prod_{j}^{m} Z_j^{\beta_j (1-\mu)^{-1}}] \tag{2}$$

where

$$\mu = \sum_{i}^{m} \alpha_i < 1. \tag{3}$$

When extended to include efficiency difference parameters for farm group g, the second term in (2) is written

$$[1 - \sum_{i}^{m} (\alpha_i / k_i^g)] [\prod_{i}^{m} (k_i^g)^{-\alpha_i (1-\mu)^{-1}}] \tag{4}$$

To incorporate relation (4) in the normalized profit function define a new constant term

$$A_g^* \equiv A^{(1-\mu)^{-1}} [1 - \sum_{i}^{m} (\alpha_i / k_i^g)] [\prod_{i}^{m} (k_i^g)^{-\alpha_i (1-\mu)^{-1}}] [\prod_{i}^{m} \alpha_i^{\alpha_i (1-\mu)^{-1}}] \tag{5}$$

and define the exponents of w_i^* and Z_j as

$$\alpha_i^* \equiv {}^{-\alpha_i \ (1-\mu)^{-1}} \tag{6}$$

and $\beta_i^* = \beta_j \ (1-\mu)^{-1}$. (7)

We now rewrite equation (2) as

$$\pi_g^* = A_g^* \prod_i^m (w_j^{*g})^{\alpha_j^*} \prod_j^n (Z_j^g)^{\beta_j^*} \tag{8}$$

or, in natural logarithms, as

$$\ell n \pi_g^* = \ell n A_g^* + \sum_i^m \alpha_i^* \ell n w_i^{*g} + \sum_j^n \beta_j^* \ \ell n \ Z_j^g \tag{9}$$

Equation (9) permits both the constant term and the coefficients to vary with farm group. If it is assumed that farms in different groups have identical production functions up to a neutral efficiency parameter, the coefficients in (9) are identical for all groups. In the case of three herd size groups S, M and L, the following three equations

$$\ell n \pi_s^* = \ell n A_m^* + \ell n(A_s^*/A_m^*) + \sum_i^m \alpha_i^* \ \ell n w_i^* + \sum_j^n \beta_j^* \ell n \ Z_j \tag{10}$$

$$\ell n \pi_m^* = \ell n A_m^* \qquad\qquad + \sum_i^m \alpha_i^* \ \ell n w_i^* + \sum_j^n \beta_j^* \ \ell n \ Z_j \tag{11}$$

$$\ell n \pi_L^* = \ell n A_m^* + \ell n(A_L^*/A_m^*) + \sum_i^m \alpha_i^* \ \ell n w_i^* + \sum_j^n \beta_i^* \ \ell n \ Z_j \tag{12}$$

may be combined as:

$$\ell n \pi^* = \ell n A_m^* + \phi_1 S_s + \phi_2 S_L + \sum_i^m \alpha_i^* w_i^* + \sum_j^n \beta_i^* \ \ell n \ Z_j \tag{13}$$

The parameters ϕ_1 and ϕ_2 measure the effect of small herd (S) and large herd (L) size, respectively, relative to the base medium size herd group (M) where S_S is a discrete variable equal to 1 if farm size is small and zero otherwise, and S_L is equal to 1 if size is large and zero otherwise.

Agricultural Research Service

North Carolina State University
Raleigh, North Carolina

Ronald J. Kuhr
Director of Research

Agricultural Research Service

North Carolina State University
Raleigh, North Carolina

Ronald J. Kuhr
Director of Research