Agricultural Risk, Insurance, and the Land-Productivity Inverse Relationship

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Abstract: s

China is attempting to improve upon existing land entitling policy to stimulate rural land aggregation trend for absorbing rural labors to support urban development. However, the potential negative impact of land size increase on farm productivity, i.e. the Inverse Relationship (IR), is triggering an intensive debate. Given the fact that China's agricultural insurance market has grown rapidly in recent years, the purpose of this study is to investigate whether the insurance boosts large farms' productivity more than smaller farms', so that the establishment of an insurance market can complement the development of land markets, and mitigate the concern of IR. To answer this question, the first part of this study analyzes the role of risk in affecting productivity. A general farm profit model is developed and the result brings an additional layer to the conventional conclusion in literature about the risk and the IR: a constant relative risk averse (CRRA) farmer still suffers from IR problem. The second part of the model shows that insurance can indeed boost productivity, and the impact of the insurance for large farms is bigger than the small farms. In the last theory section, three explanations of IR are decomposed and compared to see how their different roles in affecting productivity. The theoretical findings are econometrically tested using a large-scale filed survey data from 6 provinces in northern China. Insurance policy is employed as IV for dealing with the endogeneity of farmers' insurance purchase behavior. Results show that insurance can significantly boost productivity by 25%, and substantially mitigate the IR. Policy implications for land and insurance market developments are discussed.

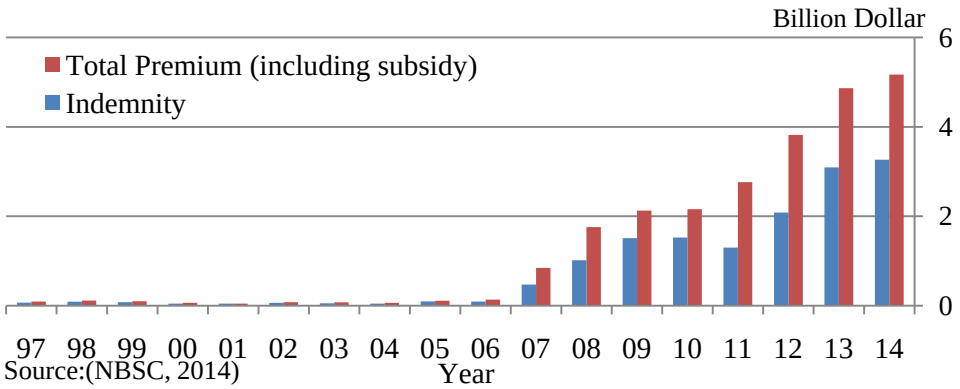
Key Words: Risk, Insurance, Land-Productivity Inverse Relationship, IV

1. Introduction

The Chinese government has consistently been trying to relocate agricultural labors from rural area to urban area for supporting industrial development (Meng, 2012), especially after the increase of urban labor cost was criticized as a main source of the slowdown of its rapid GDP growth rate in recent years (CPG, 2013, 2014; Eggleston et al., 2013; Li et al., 2012). Among the package of policies released to encourage migration, land (use-right) entitlement is widely considered a fundamental policy that secures the development of rural land rental markets, resulting in a promised emergence of land aggregation trends and the increase of farmers' non-farm works (Deininger et al., 2014; Kung, 2002; NDRC, 2009).

However, there is growing concern on the potential negative impacts of the land aggregation trend on agricultural productivity, given that the land-productivity inverse relationship (IR) has commonly been found in the literature (Fan and Chan‐Kang, 2005; Jin and Deininger, 2009). Rada (2015) finds direct evidence showing that grain yields likely will decline as current farm-scale expansion progresses. Wang (2016) and Deininger (2014) also confirm the existence of the IR in China using different data sets. To ensure food security in the world's most populous country, a stream of new subsidies and incentive programs to support farm production have been implemented (MOA, 2013, 2014).

The agricultural insurance subsidy program is relatively new to Chinese farmers and it is also one of the most costly government programs. Before 2007, China's agricultural insurance market was negligible (Wenli, 2004) (Figure.1). After 2007, owing largely to the government's strong support, China's agricultural insurance market began to boom, and it is now the world second largest conventional agricultural insurance market (following the U.S.) (Wang et al., 2011). However, there is little literature exploring the impacts of this rapidly growing insurance market in such a small farms oriented country.



Source:(NBSC, 2014)

Figure.1 Agricultural Insurance Market Booming in China

The purpose of this study is to use risk to explain the Inverse Relationship (IR) and further examine the impact of agricultural insurance on protecting the productivity. In what follows, the section 2 reviews the literature on this topic. Section 3 provides a theoretical framework: we first specify a basic agricultural household model with no risk in it to see whether IR exist. Then, we incorporate production uncertainty to examine whether the risk can cause the IR. Finally, the insurance scheme is introduced to the model and solved to produce testable hypotheses. Section 4 presents the empirical test for the theoretical findings. The last section concludes and points

out the policy implication from this study.

2. Literature Review

Literature has pointed out there are many potential candidates for explaining the IR phenomenon, such as imperfect labor market (Mazumdar, 1965; Sen, 1962), existence of supervision cost on hired labors (Feder, 1985), scale economies (Townsend et al., 1998), land quality (Benjamin, 1995; Bhalla and Roy, 1988), and even methodological shortcomings in studying this topic (sample selection bias based on illiteracy, ignorance of village effect in econometric approach, and measurement error on farm size) (Carletto et al., 2013; Carter, 1984). It's important to note that, sometimes, we may not be able to observe the IR in our data, as yields are endogenously determined through changes in technology and mechanization (Bardhan, 1973; Gorton and Davidova, 2004; Sheng et al., 2015). Whether the IR can be observed in field works depends on the particular external circumstances of the region under study.

Only a handful of papers in the literature point out that risk can be another candidate for explaining the IR.[1] Three papers point out that production risk can be a reason for explaining IR, but two of them specifically require farmers to be increasing relative risk averse (IRRA) to obtain IR result (Feder,1980; Just and Zilberman, 1983).

[1] Surprisingly, they are all among the most cited papers in the research filed of explaining the IR According to Google Scholar search, there are approximately 100 papers focusing on the IR with more than 10 citations by July 10, 2016. Only five papers use risk to explain the IR. One of them has more than 1000 citations, three of them have approximately 300 citations, and the least one has 73 citations.

Srinivasan (1972) shows a boarder conclusion that non-decreasing RRA can lead to IR, but such a finding strictly relies on an assumption on the two technologies specified in the paper. Moreover, actually, all of the three papers model the farmers have two types of risky technologies to choose when making planting decision. This study is going to show that there is a more general and straightforward conclusion between production risk and IR, not relying on the IRRA assumption on farmers' risk altitude or complicated production technologies. Even farmers are constant relative risk averse (CRRA) and only one technology is available for them, the IR still exists.

It is worth to mention that Rosenzweig and Binswanger (1993) and Barrett (1996) study risk and the IR from different points of views. The former study assumes large/wealth farmers have better income smoothing ability than small/poor farmers, giving the large/wealth farmers more risk tolerance, therefore, their expected productivity is higher than small/poor farmers. In other words, Rosenzweig and Binswanger (1993) disagree with the argument that risk causes the IR. But whether large farmers in developing countries have better financial situation than small farmers is questionable. For example, nonfarm departments China are relatively well developed, and small farmers usually have substantial higher non-farm incomes than large farmers. Another thing worth noting is that the large farmers are new emerged ones, they may not have accumulated enough wealth to provide buffers for agricultural shock. The latter study (Barrett, 1996) uses price uncertainty to explain why small farms tend to over invest in crops, while the large ones tend to under invest.

Since small farms are more likely to be net buyers and large farms are more likely to be net sellers in product markets, the price uncertainty generates a positive and negative risk premium for the small and large farms' own productions, respectively, resulting in different input intensity levels.

3. Theoretical Model

3.1 Model Introduction

In this study, we focus on production uncertainty. Three possible explanations for IR will be considered. Except the risk, the other two explanations are the agency cost and the property of return to scale of agricultural technology. Although we include these two explanations in our model setting, for focusing on the role of risk, the results presented here are assuming there is not agency cost and the technology is constant return to scale (CRS), except in the end of this section, we reassume the two factors exist in order to decompose the effects of the three explanations for comparing their different contribution to IR and figuring out is there joint effect of different explanations. Farmers will be assumed to be constant relative risk averse, CRRA, for eliminating the impact of relative risk preference.

Based on the degree of complexity, the modeling process is divided into three phases: Phase-1: Basic Model, in which there is no risk, illustrates the basic relationship between land, labor endowments and farmer productivity. Phase-2: Risk Model includes uncertainty in the production function, so that the effect of risk can be found.

Phase-3: Insurance Model assumes farmers can buy insurance against risk, so the results can be used to answer a practically meaningful question whether insurance can recover the lost productivity due to risk avoiding action, and mitigate the IR.

3.2 Basic Model

3.2.1 Model Setting

- Total Farm Income (Π):

$$\Pi = \pi + N = (Y - C) + N$$

where Π is total farm income, π is farm (net) income, N is non-farm income, Y is total farm output value (price of output is normalized to be 1), and C is total farm cost.

- Farm Product (Y):

Modern agricultural production should at least have three types of inputs: labor, land, and intermediate material inputs (seeds, fertilizers, pesticides, irrigation water, et al.). However, for focusing on the effect of land scale and labor, we use a simplified production function by assuming that the intermediate material inputs per area is fixed, i.e. we are imposing a Fixed-Input Assumption for this study. The production function is denoted as $g(.)$, assuming it has a Cobb-Douglas function form, and $g_i^{'} > 0$, $g_{ii}^{''} < 0$, $g_{ij}^{''} > 0$, i and $j = 1$ and 2:

$$g(E, T) \equiv \alpha E^{\beta} T^{\gamma}$$

where E is the Effective Labor Input, T is the land input, β is the output elasticity

of effective labor; γ is the generalized output elasticity of land; and a is the generalized total factor productivity (TFP).[2] We capture the property of return to scale by simply assigning $\beta + \gamma = (< or >)1$ for constant (decreasing or increasing) return to scale technology assumption.

- Effective Labor Input (E):

$$E = H * [S\left(\frac{L}{T}\right) + \underline{M}\,] + L$$

where H is the number of hired labors; L is the family labors, and denote $\bar{L}$ as family labor endowment, $\bar{L} > 0$. To capture the effect of agency cost in our model, assume the contribution of hired labors on effective labor input need to be discounted by multiplying an agency cost multiplier, $[S\left(\frac{L}{T}\right) + \underline{M}]$, where $S(.)$ measures supervision degree imposed by family labors on hired labors, and $\underline{M} > 0$ is the minimum degree of effort that hired labors will guarantee when there is no supervision activity, driven by their moral standard.[3] Assume $S' \geq 0$, $S'' \leq 0$, which means as family labor per area increases, hired labors will work harder, but this supervision effect marginally decreases. Moreover, we explicitly assume:

$$S\left(\frac{L}{T}\right) = \bar{S} * \left(1 - exp(-d\frac{L}{T})\right)$$

where $\bar{S} > 0$ is the maximum degree of effort of a hired labor that can be affected by

[2] More details of transforming a production function with three inputs to a production function with two inputs using the Fixed-Input Assumption and the meaning of the word "generalized" in describing a and γ can be found in Appendix.I.

[3] The reason to add moral standard constraint, $\underline{M}$, is that when land size goes very large, supervision cost becomes extremely high, and $S(\frac{L}{T})$ is too close to zero, resulting no one want to hire labors. But, in real world, large farms still hire labors. There must be something driving the hired labors to work effectively even there is almost not family labor to supervise them all the time. I assume it is the moral standard or the "spirit of agreement" in the society playing the role to ensure hired labors to work at some least levels.

supervision activity; the $exp(-d\frac{L}{T})$ measures the degree of supervision cost (exp is the exponential operator) (Figure.2). So either less family labor input or larger land size can increase the degree of supervision cost, and one minus the cost is the supervision effect. The d is a parameter for controlling the sensitivity of S(.) (i.e. controlling the curvature of supervision degree function. Figure.1 shows the shape of $S(.)$.

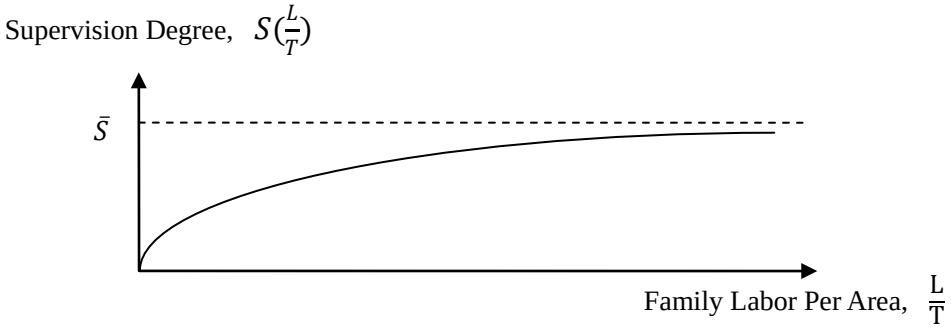


Figure.2 Supervision Degree Function $S(\frac{L}{T})$

If we assume there is not agency cost, we set $S\left(\frac{L}{T}\right) + \underline{M} = \bar{S} + \underline{M} = 1$, so that $E = H + L$. But for avoiding infinite optimal solutions, we assume family labors are always prefered than hired labors, which means if only if optimal $L^* = \bar{L}$, H^* can be positive value, otherwise, $H^* = 0$.[4]

- Land Input (T):

$$T \leq \bar{T}$$

where $\bar{T}$ is land endowment, $\bar{T} > 0$. We assume there is no land market in our model for three reasons. First of all, literature has shown that land market can mitigate IR.

[4] Without this requirement, there will be infinite optimal solutions, for example, if $L^* = 1$and $H^* = 1$ is one optimal solution, then $L^* = 0.9$ and $H^* = 1.1$ or $L^* = 1.11$ and $H^* = 0.89$ will both be optimal solutions as long as $E^* = 2$.

For find other explanations rather than just missing land market, we shut down land market in our model. Secondly, the land markets in developing countries are not well developed in most of cases; even in developed countries, land as an production asset cannot be easily traded due to transaction cost, information unavailability, and geographic inaccessibility. Lastly, land contracts usually last more than one year, so farmers cannot arbitrarily change the land size in one growing season. More accurately speaking, we assume our model is a short-run model, i.e. single production period model, so land endowment is fixed.

- Total Farming Cost (C):

$$C = W\,H + C_x T$$

where W is the wage of hired labors, C_x is the cost of the fixed inputs per unit land. As we discussed before, for simplicity, we assume intermediate material input is fixed. But, in real world, this fixed input cost can be a very small number, such as just seeding cost and harvesting cost.

- Non-Farm Income (N):

$$N = WL_N = W(\bar{L} - L)$$

where L_N is farm household's non-farm works, so $\bar{L} = L_N + L$. One implicit but important assumption here is that the labor hiring department has not risk, so cash flow from doing nonfarm or hiring workers is risk free.

3.2.2 Profit Maximization Problem of Basic Model

Since in the Basic Model, there is no risk, the farmer's utility can be directly optimized by maximizing the total farm income Π. The farmer's profit maximization problem (PMP) is:

$$\max_{L,H,T} \Pi = Y - C + N$$

$$= \alpha E^{\beta} T^{\gamma} - (W\ H + C_x T) + W(\bar{L} - L)$$

$$\begin{aligned} s.t. \qquad & L \leq \bar{L} && \dots\ \rho_1 \\ & T \leq \bar{T} && \dots\ \rho_2 \\ & L \geq 0 && \dots\ \rho_3 \\ & T \geq 0 && \dots\ \rho_4 \\ & H \geq 0 && \dots\ \rho_5 \\ & H(\bar{L} - L) = 0 && \dots\ \rho_6 \end{aligned}$$

where $E = H[\bar{S}\left(1 - exp(-d\frac{L}{T})\right) + \underline{M}\ ] + L$ if we assume there is agency cost, otherwise, $E = H + L$. Denote $\rho_i,\ i = 1,2,\dots,6$ as the Lagrange Multipliers for the corresponding constraints. The mathematical process of solving the PMP can be found in Appendix.II.

Since our primal interesting is in the role of risk, for the Basic Model, we briefly discuss the result without agency cost assumption in Figure.3, which depicts four patterns of optimal solutions (marked by the arrows) in four regions of the land and labor endowment space. First of all, the four regions are divided by three endowment thresholds ($\bar{T}_{\rho_1=0,\rho_2=0}$, $\bar{L}_{\rho_1=0,\rho_2=0}$, and $\bar{L}_{\rho_1=0,\rho_2>0}$) and the solid line displays the final optimal solutions.[5] Region A and B represent the farmer has too many labor

[5] The formulas of the cutoffs will be in the Appendix.II, which will be updated in next version....

endowment, so the farmer will do some nonfarm works, making sure $L^* = \bar{L}_{\rho_1=0,\rho_2=0} < \bar{L}$ in Region A or $L^* = \bar{L}_{\rho_1=0,\rho_2>0} < \bar{L}$ in Region B, i.e. arrows pointing down means selling labors, and the distances measure the number of nonfarm works. In contrast, Region C and D show that when the farmer has limited labor endowment, he/she will hire labors to do farm works, i.e. arrows point up and the distances measure the size of H^*. Between the dash threshold line $\bar{T}_{\rho_1=0,\rho_2=0}$, Region A and D means the farmer has too large land endowment. Since we assume there is fixed cost for cultivating land, so optimal choice is to left some of lands uncultivated. In Region B and C, the farmer has not enough lands, so he/she will cultivate all of available lands, therefore the arrows in Region B and C are strictly vertical, i.e. $T^* = \bar{T} < \bar{T}_{\rho_1=0,\rho_2=0}$.

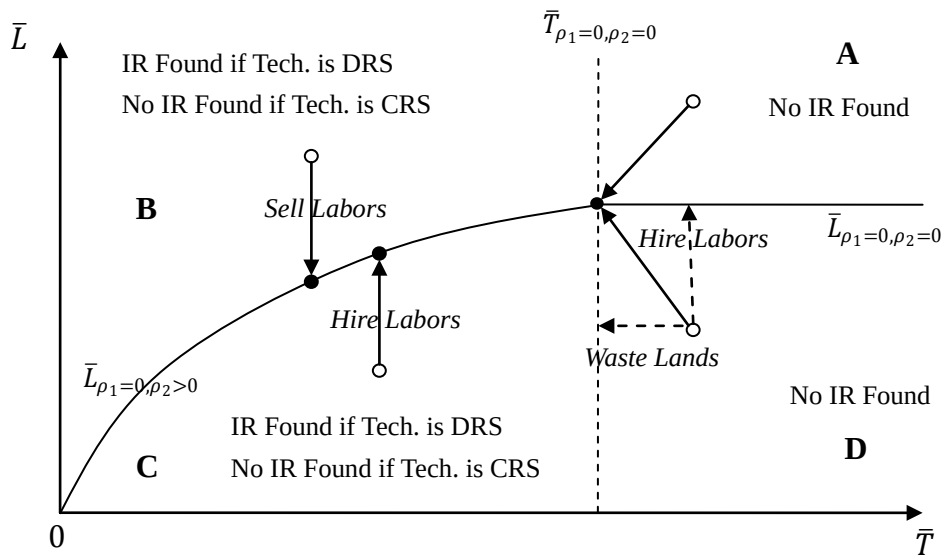


Figure.3 Four Regions of the Optimal Choice Pattern

Note that, if we assume the technology is CRS, then, the Region A and D disappear, since $\bar{T}_{\rho_1=0,\rho_2=0} = \infty$ and $\bar{L}_{\rho_1=0,\rho_2=0} = \infty$ (formulas in Appendix.II), and in Region B and C no IR will be found, so productivity is constant cross all of endowment

combinantions.

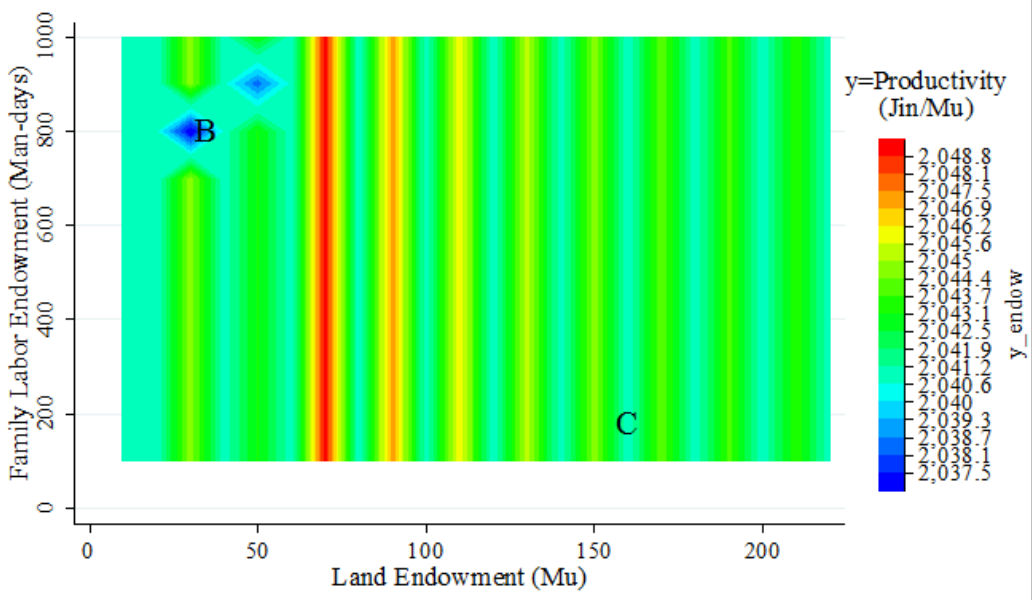
3.2.3 Numerical Simulation of Basic Model

Though the Basic Model is still mathematically traceable, after introducing risk, the model becomes unsolveable, we have to rely on numberical simulation to see the effect of risk and insurance. For comparing with the risk and insurance results, we first run simulation on the Basic Model. Different calibration in this study will change the scale of results presented, but will not fundamentally change the conclusion.

Here is how the simulation is calibrated: first of all, the social parameters are calibrated according to survey data in 6 provinces in the north China: price of output is 1 Yuan/Jin, non-farm wage $W = 120$ Yuan/Man-Day, and input cost per mu $C_x = 500$ Yuan/Mu. Secondly, the technology parameters: $\alpha = 700$ is set by gurranting the yield level (after introcing risk) is arround survey data level. Output elasticities are arbitrily calibrated. Assume $\beta = 0.5$ and $\gamma = 0.5$ for CRS technology, i.e. $\beta + \gamma = 1$. Lastly, the land size is allowed to vary between 0 to 220 Mu, and the labor endowment is allowed to vary between 0 man-day to 1000 Man-Days. Figure.3 shows the result, in which we find that the productivity does not change, i.e. no IR found.[6]

[6] Small fluctuations of productivity is due to lack of simulation accuracy. Better smooth result can be obtained if I run it using more time.

Figure.4 Basic Model: No IR under No Risk, No Agency Cost and CRS



3.3 Risk Model

3.3.1 Introducing Risk and Utility Function

We modify the Basic Model to consider agricultural risks. Assume the farm has possibility p to realize production with α_H as TFP in production function, and has possibility $\bar{p} = 1 - p$ to realize production with α_L as TFP.

The Natural Logarithmic Utility (NLU) is chosen to continue the following analysis, since it represents a decreasing absolute risk averse (DARA) and constant relative risk averse (CRRA) utility. The DARA is the most acceptable assumption for the ARA in the literature, and for avoiding distraction of IRRA and DRRA in finding the IR, we assume the farmer is CRRA. Appendix.III summaries most common used utility functions in microeconomics risk related research, and gives more justification for

why we choose NLU. The NLU takes form:

$$U(\Pi) = ln(\Pi)$$

Where Π is the total farm income.

According to the Expected Utility Theorem, the farmer's utility maximization problem (UMP) is such that:

$$\max_{L,H,T} \begin{aligned} U &= pU\left[\alpha_H E^{\beta} T^{\gamma} - (W\,H + C_x T) + W(\bar{L} - L)\right] \\ &+ pU\left[\alpha_L E^{\beta} T^{\gamma} - (W\,H + C_x T) + W(\bar{L} - L)\,\right] \end{aligned}$$

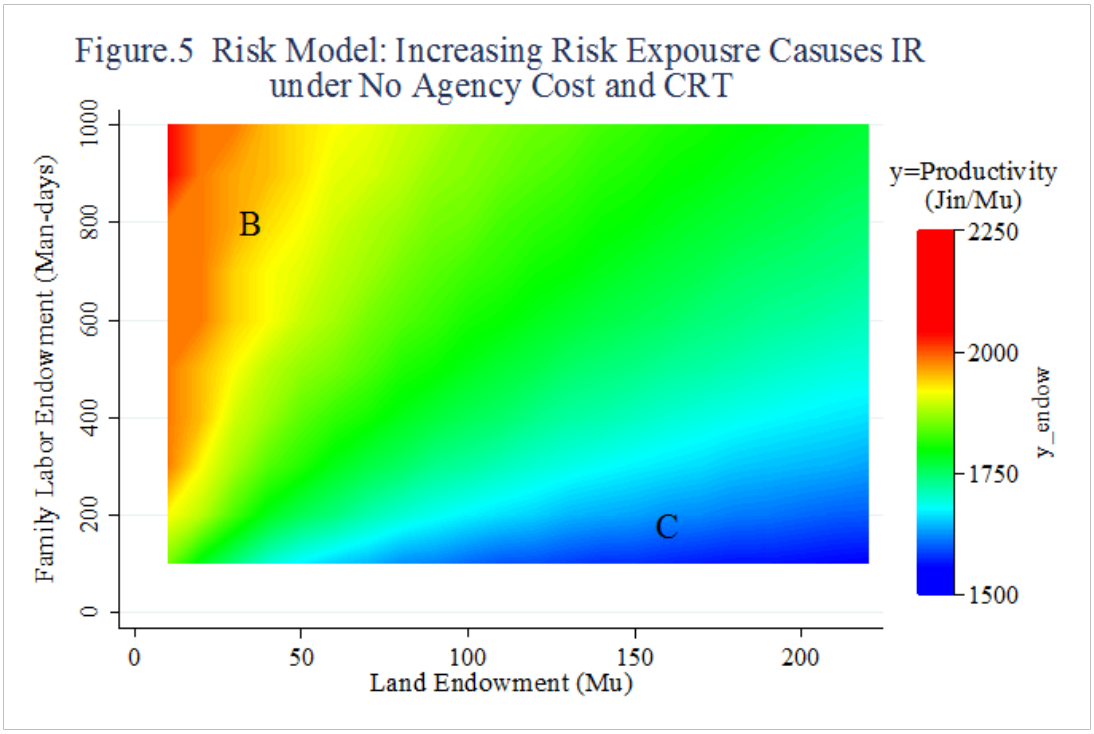
$$\begin{aligned} s.t \quad & L \leq \bar{L} \\ & T \leq \bar{T} \\ & L \geq 0 \\ & T \geq 0 \\ & H \geq 0 \\ & H(\bar{L} - L) = 0 \end{aligned}$$

where $E = H[\bar{S}\left(1 - exp(-d\frac{L}{T})\right) + \underline{M}\,] + L$ if we assume there is agency cost, otherwise $E = H + L$.

3.3.2 Numerical Simulation of Risk Model

Although Basic Model has detailed mathematically analyzable solution forms, after introducing risks, the model becomes mathematically untraceable. Thus we have to employ numerical simulation methods to find the result of the model. As we mentioned in the beginning of this modeling section, for focusing on the effect of risk, the result presented in this subsection assume no agency cost and CRS. Calibrating risk parameters: $\alpha_H = 900,\ \alpha_L = 500,\ p = 0.5,\ \bar{p} = 0.5$. Hence, the expected TFP is 700, which equals the TFP in Basic Model.

Figure.5 shows the simulation result of the Risk Model. As we can see, as land size increases, the farmer is exposed to more agricultural risk, resulting in the labor input intensity (E^*/T^*) decreases, which eventually brings down the expected productivity.[7]



Part of mathematical proof of risk model results are traceable, and they are in the Appendix.IV. To better understand the result of Figure.5, there are more points worth discussing. First of all, how to define that the farmer is becoming more conservative in this model? The answer is focusing on the input intensity, i.e. lower E^*/T^* indicates the farmer is becoming more conservative. Note that, we have two

[7] Note that in Figure.4, all 220 Mu lands are cultivated. If the fixed cost per mu,C_x, increases or the maximum simulated land endowment keeps increasing, some of land eventually will be wasted, since productivity is constantly decreasing as land size increases. But since wasted land is not our research focus, so we don't show this extreme case. Moreover, as we discussed in the model setting about the Fixed Cost Assumption, in real world, the C_x could be very small, such as just seed cost, so no land will be wasted.

departments in this model: agricultural department (farm works) is risky one, and labor department (doing nonfarm works or hiring labors) is risk free one. So the cost of increasing one more effective labor is certain, but the benefit of receiving output is uncertain, the higher labor intensity is chosen the more aggressive action is conducted.

Secondly, why CRRA farmer still has IR as land size increases? The definition of CRRA first appears in Arrow (1965), which basically says that suppose an agent divides his/her asset into two games, one is risky game, the other one is risk free game. As the asset increases, the ratio of dividing the asset between the two games does not change if the agent is CRRA. In such a framework, the environmental risk exposure is an endogenous choice variable -- the agent decides how large the risk game he/she wants to involve. But in our farm Risk Model, the risk exposure is exogenous: as land size increases, the farmer is facing more uncertainty inevitably, as long as no land is wasted, i.e. marginally investing agricultural is still profitable. As the scale of the farmer's risk "game" is increasing, to balance the total risk the he/she can bear as a CRRA investor, he/she has to reduce risky investment intensity, i.e. E^*/T^*. In a word, the risk affects farmers' behavior by two channels: one is their objective risk preference, the other one is the environmental risk exposure they are involved.

Third point, why does the productivity increase as the labor endowment increases? A general answer is that the larger labor endowment means higher initial endowment

in risk free department, which gives the farmer more degree of freedom when balancing the investment between the two departments. In other word, higher labor endowment allows the farmer to use less cost to maintain same investment in labor market, so the farmer can bear more E^*/T^* in risky farm activity.

Last but not the least, note that the IR found here does not rely on existence of agency cost and CRS technology, and the only thing changes between the Basic Model and the Risk Model is just the production uncertainty. Therefore, we don't really need a complicated model setting, such as two production technologies in the current literature, to obtain the result that risk can cause IR.

3.4 Insurance Model

3.4.1 Introducing Insurance

This section introduces an insurance mechanism to examine whether an insurance instrument can recover the sacrificed productivity due to risk aversion, not just smoothing farmers' income flow. For illustration purpose, we assume the insurance market is perfect, i.e. assuming all information is symmetric to both sides and there is no transaction cost, thus insurers know the possibility distribution and they can monitor the outcome as well. For simplicity, I assume insurers set the price at the actuarially fair level.

Using q to denote the premium rate. By assuming actuarially fair pricing, we

have $q \equiv 1 - p$. Denote I as the coverage that the farmer purchases. Hence, the total premium the farmer pays is qI, no matter what the outcome is. The insurer indemnifies the farmer δ if the α_L is realized, and does not indemnify the farmer if α_H is realized.

3.4.2 Utility Maximization Problem with Insurance

The farmer chooses optimal coverage level at given coverage price q. So, the farmer's Utility Maximization Problem with Insurance (UMP-I) is that:

$$\max_{L,H,T,I} \begin{aligned} U = {} & pU\left[\alpha_H E^{\beta} T^{\gamma} - (W\,H + C_x T) + W(\bar{L} - L) - qI\right] \\ & + pU\left[\alpha_L E^{\beta} T^{\gamma} - (W\,H + C_x T) + W(\bar{L} - L) - qI + I\right] \end{aligned}$$

$$\begin{aligned} s.t \quad & L \le \bar{L} \\ & T \le \bar{T} \\ & L \ge 0 \\ & T \ge 0 \\ & H \ge 0 \\ & H(\bar{L} - L) = 0 \\ & I \ge 0 \end{aligned}$$

where $E = H[\bar{S}\left(1 - exp(-d\frac{L}{T})\right) + \underline{M}\,] + L$ if we assume there is agency cost, otherwise $E = H + L$.

Without completely solving the UMP-I, just taking FOC with respect to I, it is easy to show that the optimal coverage level must satisfy the following conditions:

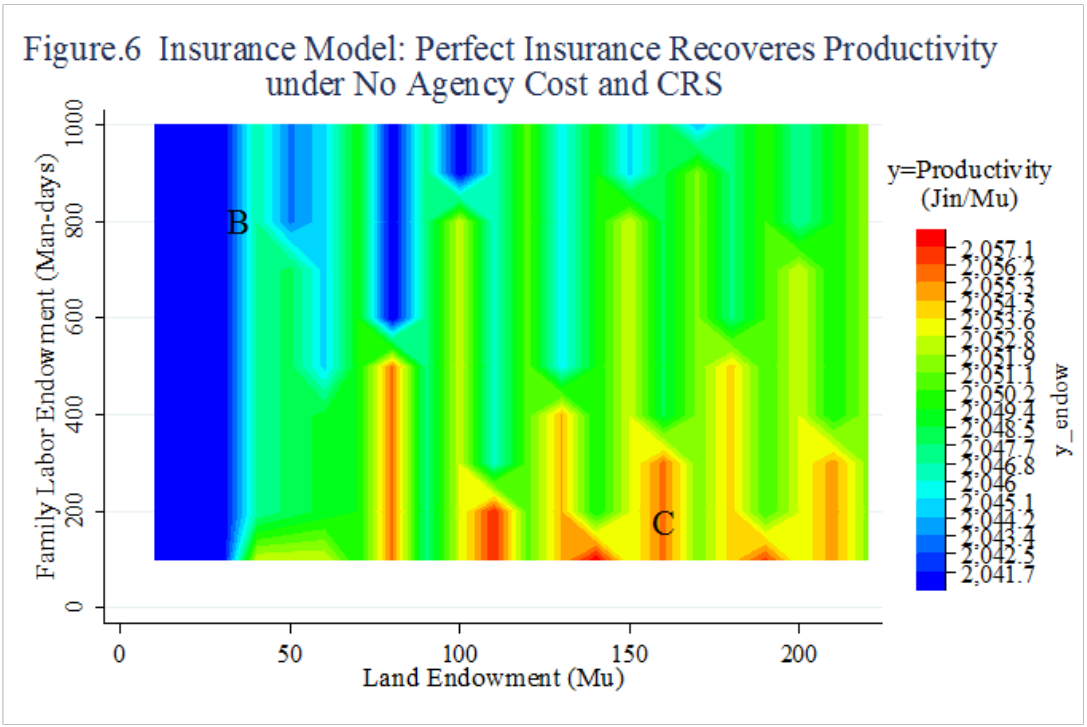
$$I^* = \alpha_H E^{*\beta} T^{*\gamma} - \alpha_L E^{*\beta} T^{*\gamma}$$

In other words, full coverage is the optimal choice of I^* under perfect insurance situation, which is consistent with the conclusion from classic insurance literature (Mas-Colell et al., 1995).

3.4.3 Recovering Productivity under Perfect Insurance

There are two approaches in running numerical simulations to check the effect of insurance on productivity: the first one is directly using the full coverage conclusion to simplify the UMP-I. So the risk is eliminated from the model, in other words, the insurance equalize the two outcomes. Second method, we can also directly run simulation on the original UMP-I, allowing the I to vary, so the I^* is chosen by simulation process itself, and we can test whether our full coverage conclusion is correct by comparing with the result from the first method.

We run simulations using both and obtain exactly the same results from the two approaches. So the full coverage conclusion is justified. Figure.6 shows the effect of insurance on recovering productivity. Compared with Figure.5, the IR disappears, i.e. the productivity is recovered. Moreover, since the insurance market is assumed to be perfect, the productivity actually is recovered to no risk level (comparing Figure.6 with Figure.4), meaning that perfect insurance eliminate the effect of risk on expected productivity.



3.5 Decomposition of the Three IR Factors

The above analysis shows that, due to the risk can cause the IR even if the farmer is CRRA, and a perfect insurance scheme can fully recover the lost productivity, eliminating the effect of risk on the IR. However, as we discussed in literature review, there are many explanations for IR. The specification of the model in this study allows us to decompose the three effects from each other. Figure.7 shows the simulated results.[8]

For focusing on the relationship between productivity and land size, we fix the labor endowment $\bar{L}$ at 500. The simulations are based on Risk Model in which we switch on and off the assumptions about risk, agency cost and return to scale. The $\bar{S}$ and $\underline{M}$

[8] When we introduce agency cost and DRS, the farm activity become less attractive, result the threshold of land being wasted move left. For the green line in Figure.6, the threshold is 230 mu, and for the purple line, the threshold is 170 mu. As we are interested more on comparing risk with agency cost, we cut the results in Figure.6 at 220 mu, so all the top four lines are comparable. The productivity calculated in this study is production divided by total land endowment, so the purple line keeps decreasing after 170 mu.

are set at 0.7 and 0.3, respectively. The d in the $S(.)$ is set at 0.6, and the rest calibrations are the same as we used before. The red line directly uses the result from Figure.4 (no-risk results) and the blue line directly uses the result from Figure.5 (risk results). For green line, we assume there is agency cost, so $S\left(\frac{L}{T}\right) \equiv \bar{S} * \left(1 - exp\left(-d\frac{L}{T}\right)\right)$. For purple line, we further assume technology is DRS by changing $\beta = \gamma = 0.5$ to $\beta = 0.49$ and $\gamma = 0.49$. But for being comparable with other line, we increase the FTP of purple line by 50 both for good and bad outcome, so the initial points of all lines are approximately same. The dash line represents if there is no risk but just agency cost, how does the productivity change. So the top four lines can be used to analyze is there joint effect of risk and agency cost.

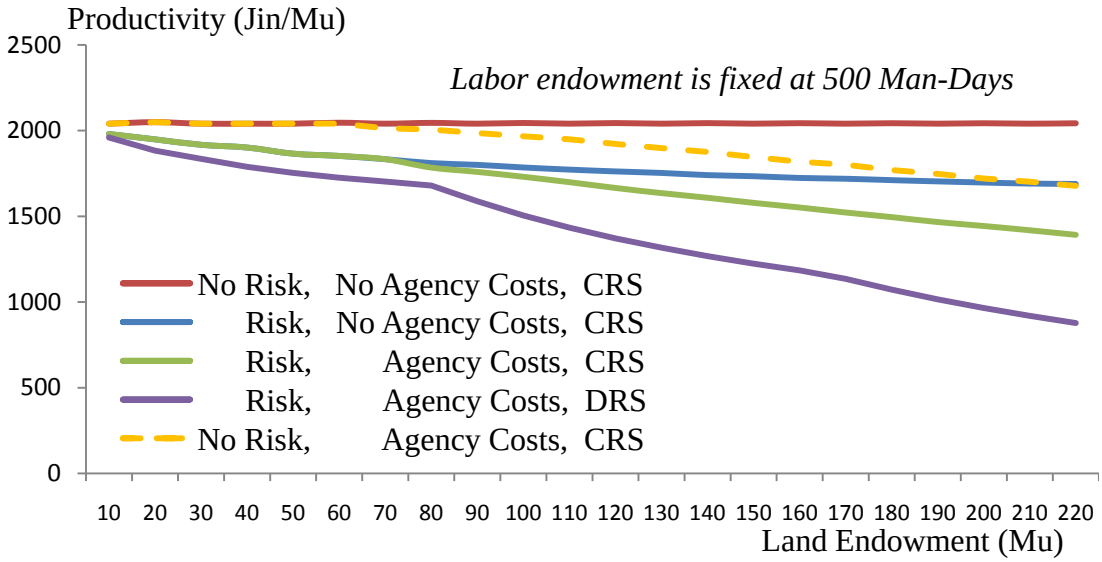


Figure.7 Decomposition of the Three Explanations for IR

We found that the risk and agency cost play different roles in causing the IR. Although the risk affects productivity for all different land sizes, the productivity decreases much faster for small farmers, who have nonfarm incomes, than larger

farmers according to simulation result. In contrast, the agency cost reduces the productivity only when farmers have extra lands, i.e. they have to hire labors to work on their extra lands. Lastly, the DRS can certainly explain IR for all types of farms, the effect of return to scale purely depends on the parameters, i.e. the nature the technology.

The orange dash line starts to deviate from red line at $\bar{T} = 60$, while the green line starts to deviate from blue line at $\bar{T} = 70$. So the start point of hiring is pushed to right when the agency cost and risk both exist. The reason can still be explained by farmer's willingness to balance the investment intensity between risky department and risk free department. As risk exists, the farm activity becomes less attractive, so the farmer inclines to reallocate resource (labor input) from risky farm department to risk free labor department, resulting nonfarm work demand increases, so the starting point of hiring labor is pushed from 60 mu to 70 mu.

When combining the effects of risk and agency cost, the two factors mitigate each other's effect in causing IR (such mitigate effect is small, like 20-60 Yuan/mu), not amplifying each other's effect. The reason could be that the farmer' force or willingness to balance the risky investment and risk free investment is marginally decreasing. So when risk already causes the IR, the effect of agency cost on IR decreases as the farmer becomes less sensitive to one more IR explanation.

4. Empirical Testing

The theoretical section shows the how risks causes the IR, and illustrates that the agricultural insurance can boost farmers' productivity. The following two hypotheses corresponding to the theoretical findings will be tested using field survey data.

Hypothesis 1: Risk can explain the IR even we control farmers for CRRA. Hypothesis 2: Well-functional insurance market can mitigate the degree of IR.

4.1 Data Description

Right now, three data sets from large scale cross-section household agricultural production surveys are available. But right now, the following estimation result just uses the 2013 survey data.

- 2010, 7 provinces in China, 1353 households, 8% of the sample are insured.
- 2012, 9 provinces in China, 3332 households, 38% of the sample are insured.
- 2013, 6 provinces in the northeast and north China, 786 households, 4 agricultural enterprises, and 55 cooperatives. Among total 845 observations, 47% of the sample are insured.

4.2 Identification Strategy

To find the joint effect of insurance and land size on farmers' productivity, the following econometric model will be estimation:

$$Y_{ij} = \alpha + \beta T_{ij} + \gamma I_{ij} + \delta T_{ij} I_{ij} + \lambda X + \epsilon_{ij}$$

where Y_{ij} is the agricultural productivity of individual i in village j, measured by

revenue per mu (Yuan/Mu); T_{ij} represents the land size (Mu) in corresponding growth season (for some province, there may be two growth seasons); I_{ij} indicates the treatment state, if the farmer purchased insurance, $I_{ij} = 1$, otherwise, $I_{ij} = 0$; and $T_{ij} I_{ij}$ is the interaction term of the land size and the insurance treatment state. The above three regressors are variables interested.

The rests, X, are controls, including the local average production loss rate (%) in 2013 for controlling local agricultural risk level; local crop average price (Yuan/Jin), weighted by total crop revenues, for controlling output market disturbance; the individual non-farm income (1000Yuan) in 2013; the farmers' wealth level (1000Yuan); regional dummies (at province level); age (year) and age squared; Plot Distant to Home (Km); Agricultural Experience (Year); family labor per mu (Person/mu); Plain Terrain Rate (%); Number of glowing season (#); 2012 household loss rate (100%); 2012 nonfarm income (1000 Yuan); Gender (1=mal;0=female). . The Greek letters in the model are the coefficients to be estimated, except the last one ϵ_{ij}, which is the error term.

As projected in the conceptual section, large farmers could have lower productivity than small farmers, therefore, the expected sign of β is negative. Moreover, as after being insured, farmers will choose more risky portfolios, which supposedly can bring them higher expected outcomes, hence, in a well-functioning insurance market, the expected sign of γ should be positive. Furthermore, one of the most important contributions of this study comes from the theoretical finding of the joint effect of

insurance and land size on productivity, large farmers will receive more benefits from being insured. This theoretical finding should be able to be justified by empirically testing whether the δ is positive.

Because higher risk farmers may be more likely to purchase insurance than lower risk farmers, resulting endogeneity of the insurance purchase variable, I_{ij}. To estimate the model, an instrumental variable has to be used for correcting the endogeneity (using 2SLS). We use insurance policy accessibility, denote as A_j, as an instrumental variable. $A_j = 1$ means the village j has insurance policy accessibility, otherwise $A_j = 0$. Note that, the data do not come from an RCT project, which means A_j may not be a perfect IV, since it is not randomly assigned to farmers, however, if we can accept an assumption that the degree of endogeneity from insurance policy is less than the degree of endogeneity from farms, then, the estimated results are still meaningful according to Nevo and Rosen (2012) -- the real coefficients γ and δ can be bound by using imperfect IV. Particularly in this study, the real value of γ and δ should be larger than or equal to the maximum values of my estimated coefficients, respectively, i.e. $\gamma \geq \max\{\hat{\gamma}^{OLS}, \hat{\gamma}^{2SLS}\}$ and $\delta \geq \max\{\hat{\delta}^{OLS}, \hat{\delta}^{2SLS}\}$.

As just being mentioned, 2SLS will be employed to estimate the model. Since the endogenous variable I_{ij} is interacted with T_{ij}, the equation actually has two endogenous terms. To fix this problem, we interact the IV, A_j, with L_{ij} for creating a second IV. Then, there are two ways to perform 2SLS. One standard way to run 2SLS with two endogenous terms is to have two regressions in the first stage for

getting fitted $\widehat{I_{ij}}$ and $\widehat{T_{ij}I_{ij}}$, respectively, then use them to run the second stage. However, in this particular model setting, although there are two endogenous terms in the model, there is essentially only one source of the endogeneity (I_{ij}). Using another regression in the first stage to predict the $\widehat{T_{ij}I_{ij}}$ will lead to a variance reduction in the fitted regressor in the second stage, resulting in efficiency loss. Therefore, we use an alternative way to increase efficiency, which only requires one regression in the first stage to obtain $\widehat{I_{ij}}$, then directly use $\widehat{I_{ij}}$ to interact with T_{ij}, hence, we have $T_{ij}\widehat{I_{ij}}$ to act as the second IV for endogenous interaction term $T_{ij}I_{ij}$. This manually perform 2SLS requires correcting the standard deviation.

4.3 Estimation Results

Table.1 provides the estimation results using improved 2SLS (manually multiply predicted insurance purchase variable with real land size variable in the 1st stage). First of all, the general result of model fitting is acceptable. The R-squared ranges from 0.686 to 0.707, which is an acceptable level considering just using cross-sectional data. Secondly, all the signs of the estimated coefficients are consistent with expectations. Furthermore, most estimated coefficients are statistically significant, especially the key variables.

Three key empirical findings will be discussed based on the estimation (4). First of all, temporally assume a farmer does not purchase insurance, i.e. $T_{ij}I_{ij} = 0$, his/her marginal increase of land size will reduce productivity by 0.7 yuan/mu, so IR exist. Based on simple algebra using the survey data, if average land size increases by 25%,

i.e. roughly 1 hectare, the total agricultural production decreases 1%; if average land size is doubled, the total production decreases 3.8%. Note that, in 2013, the rate of China's national agricultural production increase is only 2.6% according to government reports.

Secondly, the estimation of $\hat{\gamma}^{2SLS}$ justifies our conceptual finding, that farmers with insurance are more likely to take more risky but profitable decisions, causing expected productivity to increase. Given the land size at sample mean level, purchasing insurance can increase productivity more than 276 yuan/mu, accounting for about 15% of current productivity mean, which is considerable number in magnitude. If looking at this number from another angle, this 15% productivity can be considered as an average loss due to farmers' risk aversion. This finding provides evidence that developing a well-functioning agricultural insurance market is not only good for smoothing farmers' income but also good for increasing agricultural productivity.

Lastly, the estimated coefficient of the interaction term, $\hat{\delta}^{2SLS}$, is positive, which is consistent with the expectation, that insurance has heterogeneous impact on farmers (large farmers receive more benefits from insurance). More importantly, $\frac{\hat{\delta}^{2SLS}}{\hat{\beta}^{2SLS}} \approx$ 94%, i.e. 94% of IR is explained. Therefore, this empirical finding provides a strong support for urging government to develop a well-functioning agricultural insurance market in order to secure the national food supply and promote the land market transformation.

Table.1 Improved 2SLS Estimation Results: Effect of Insurance

Dependent Variable: Productivity (Yuan/Mu)	(1) SE	(2) Robust-SE	(3) SE	(4) Robust-SE
Key Variables				
Cultivated Land Size(Mu)	-0.701	-0.701	-0.753	-0.753
	(0.239)***	(0.327)**	(0.259)***	(0.402)*
Insurance Purchase (1=Yes; 0=No)	285.8	285.8	276.3	276.3
	(103.8)***	(99.49)***	(101.9)***	(98.63)***
Insurance Purchase * Land Size	0.580	0.580	0.705	0.705
	(0.218)***	(0.288)**	(0.257)***	(0.389)*
Controls				
Non-Farm Income (1000Yuan)	-0.118	-0.118	0.516	0.516
	(0.292)	(0.335)	(1.029)	(1.067)
Local Loss Rate (%)	-15.96	-15.96	-14.73	-14.73
	(1.379)***	(1.610)***	(1.355)***	(1.595)***
Local Crop Price (Yuan/Jin)	771.3	771.3	756.2	756.2
	(42.22)***	(41.89)***	(41.42)***	(42.24)***
Wealth (1000Yuan)	0.0834	0.0834	0.0838	0.0838
	(0.0567)	(0.0445)*	(0.0555)	(0.0446)*
pcode_dummy2	16.81	16.81	24.44	24.44
	(75.32)	(70.28)	(73.46)	(68.68)
pcode_dummy3	223.9	223.9	217.6	217.6
	(54.00)***	(48.04)***	(53.26)***	(48.80)***
pcode_dummy4	831.1	831.1	369.0	369.0
	(59.30)***	(56.91)***	(114.3)***	(123.6)***
pcode_dummy5	815.0	815.0	352.0	352.0
	(83.26)***	(86.11)***	(131.5)***	(147.7)**
pcode_dummy6	894.4	894.4	441.4	441.4
	(59.32)***	(65.29)***	(111.9)***	(125.2)***
Loan Time since 2010(#)			-15.02	-15.02
			(7.083)**	(7.957)*
Age (year)			-36.34	-36.34
			(10.50)***	(12.24)***
Age Squared			0.379	0.379
			(0.0998)***	(0.115)***
Education (Year)			-5.557	-5.557
			(5.075)	(4.923)
Plot Distance to Home (km)			-3.585	-3.585
			(6.255)	(5.152)
Agricultural Experience (Year)			-1.767	-1.767
			(2.001)	(1.894)
Family Labor Per Mu			17.02	17.02

(Person/Mu)				
			(21.07)	(16.30)
Terrain (Plain Terrain Rate) (%)			170.3	170.3
			(66.65)**	(68.74)**
Number of Growing Season(#)			438.1	438.1
			(101.1)***	(119.2)***
2012 HH Yield Loss Rate				
(100%)			-2.479	-2.479
			(1.123)**	(0.997)**
2012 Nonfarm				
Income(1000Yuan)			-0.587	-0.587
			(1.065)	(1.115)
Gender (1=male;0=female)			230.2	230.2
			(102.0)**	(94.13)**
Constant	338.6	338.6	484.2	484.2
	(86.48)***	(78.62)***	(311.3)	(351.0)
Observations	844	844	844	844
R-squared	0.686	0.686	0.707	0.707

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Appendix

Appendix.I A General Production Function and the Fixed-Input Assumption

Firstly, write down a general form of C-D production function with three input elements, $Y = f(E, X, T)$:

$$Y = \alpha_0 E^{\beta} \left(\prod_{i=1}^{Z} X_i^{\gamma_i} \right) T^{\gamma_T}$$

where X_i is total amount of ith material input, such as seeds, fertilizers, pesticides, etc; Z is total number of types of material inputs; a_0 is a total factor productivity (TPF); γ_i is output elasticity of ith material input; and γ_T is output elasticity of land.

One of the important explanations of the IR, rather than the land quality and labor issues, is that small farms tend to have higher input intensity than large farms (Barrett, 1996; Sen, 1962). In other words, the input per area, denoted as x_i, can be a function of T (possibly negatively correlated with T), i.e. $X_i = x_i(T)T$. Therefore:

$$Y = \alpha_0 E^{\beta} \left(\prod_{i=1}^{Z} x_i(T)^{\gamma_i} T^{\gamma_i} \right) T^{\gamma_T} = [\alpha_0 \prod_{i=1}^{Z} x_i(T)^{\gamma_i}] E^{\beta} T^{\gamma_T + \sum_i^Z \gamma_i}$$

Although, by simulation method, using the general production function form still can yield results for analysis (function form of $x_i(T)$ need to be specified explicitly). However, for focusing on risk, it does no harm to simplify the production function by applying a Fixed Input-Assumption, which means $x_i(T) \equiv \bar{x}_i$, therefore:

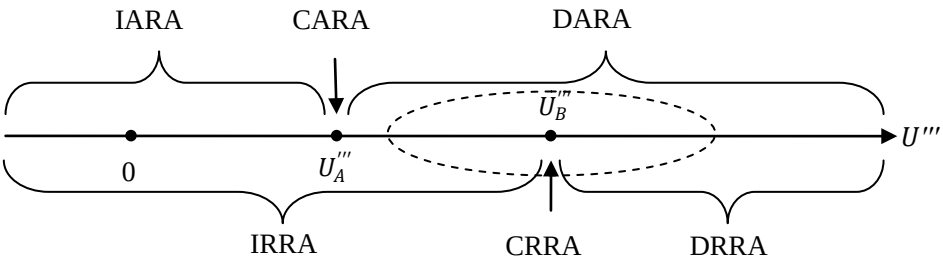
$$Y = [\alpha_0 \prod_{i=1}^{Z} \bar{x}_i^{\gamma_i}] E^{\beta} T^{\gamma_T + \sum_i^Z \gamma_i}$$

Denote a generalized $\alpha = \alpha_0 \prod_{i=1}^{Z} \bar{x}_i^{\gamma_i}$, and a generalized $\gamma = \gamma_T + \sum_i^Z \gamma_i$, so simplified production function is: $Y = \alpha E^{\beta} T^{\gamma} \equiv g(E, T)$

Appendix.II Mathematical Results of the Basic Model (will be updated in next version)

Appendix.III Risk-Averse Utility Function

To choose a utility function form for building the farmer's utility maximization problem (UMP), it is worth to reveal some "secrets" of the risk-averse utility functions. First of all, Figure.IV1 helps us to better understand the relationship between the absolute risk aversion (ARA) and relative risk aversion (RRA). It is easily to show that any smooth risk-averse utility function should have a position in Figure.IV1, according to the size of the third derivative relative to the two cut-off values, U_A''' and U_B''', marked in the figure. The "c" is the final consumption in the utility function.



where $U_A''' = \frac{(U'')^2}{U'}$ and $U_B''' = \frac{(U'')^2}{U'} - \frac{U''}{c} = U_A''' - \frac{U''}{c}$

Figure.IV1 Classification of ARA and RRA for Risk-Averse Utility

The basic findings of Figure.IV1 are that all IARA and CARA utility functions must be IRRA, but DARA is not necessary to be DRRA. Moreover, all CRRA and DRRA utility functions must be DARA, but IRRA is not necessary to be IARA. Drawing Figure.IV1 is useful, since it explains why most of studies agree with that people are DARA, but there are many diversified conclusions on people' RRA (Arrow, 1971; Levy, 1994; Szpiro, 1986)[9]. Currently, empirical studies find that most of people's ARA and RRA locate in the circled area in Figure.IV1. Hence, to choose a utility

[9] Many papers try to measure the RRA and obtain various results. Here I only cite three of them to represent the three directions. Arrow (1971) supports IRRA; Szpiro (1986) finds CRRA, and Levy (1994) stands for DRRA.

function for this study, I adopt a DARA utility function.

Secondly, recall that the former studies build a “one to one relationship" between the types of RRA and the IR, for example, only IRRA results in IR. However, their conclusion is not complete. If we consider the income share effect (ISE), it can be shown that both IRRA and CRRA will lead to the IR, and part of DRRA can also result in the IR. To test whether there is real ISE existing, I assume the farmer’s utility function is CRRA.

In a word, the utility function I use in this analysis should be DARA and CRRA, i.e. the function should locate at point U_B''' in Figure.IV1. The Table.IV1 summarizes most commonly used utility functions in risk related research. The Isoelastic Utility (IU) is chosen to continue the following analysis, but note that, the result does not change if I use the Natural Logarithmic Utility (NLU), and the general form of the Power Utility (PU) can be used to prove that part of DRRA utility function can also lead to the IR.

Table.IV1 Common Used Utility Functions in Risk Related Research

Formulation	ARA	RRA	Locations in Figure.IV1
Quadratic Utility (QU)			
$U(c) = \psi c - \phi c^2, \quad \frac{\psi}{2\delta} \geq \bar{c}, \phi > 0$	*IARA*	*IRRA*	*0-Point*
Exponential Utility (EU)			
$U(c) = \frac{1 - e^{-\psi c}}{\psi}, \psi \neq 0$	*CARA*	*IRRA*	*Point* U_A'''
Power Utility (PU)			

$U(c) = \begin{cases} \dfrac{(c+\eta)^{1-\psi} - \phi}{1-\psi} & ,\psi \neq 1 \\ l\,n(c+\eta) & ,\psi = 1, \phi = 1 \end{cases}$	*DARA*	*IRRA if* η*>0;* *CRRA if* η*=0;* *DRRA if* η*<0*	$U_A^{'''}$ -$U_B^{'''}$ *if* η*>0;* $U_B^{'''}$ *if* η*=0;* $U_B^{'''}$ -∞ *if* η*<0*
Isoelastic Utility (IU) – A Special form of PU			
$U(c) = \dfrac{c^{1-\psi}}{1-\psi}$ $,\psi \neq 1, \eta = 0, \phi = 0$	*DARA*	*CRRA*	*Point* $U_B^{'''}$
Natural Logarithmic Utility (NLU) – A Special form of PU			
$U(c) = l\,n(c)$ $,\psi = 1, \eta = 0, \phi = 1$	*DARA*	*CRRA*	*Point* $U_B^{'''}$

Note: c is final consumption; ψ, η, and ϕ are parameters.

Appendix.IV. Proof of Risk Model Results

First of all, we know that if C_x is zero, then all of $\bar{T}$ will be invested in production. Moreover, if C_x is positive, then there will a threshold in $\bar{T}$ when the $\bar{T}$ passes this threshold, the farmer start to waste extra lands, since the marginal benefit starts to be less than the marginal cost, which is the C_x after that point. We can reasonable assume that C_x is relatively smell so that no land is wasted, since this is more realistic case. Also note that, in our simulation, all of lands are used, as $\bar{T} \leq 220$ mu. In this appendix, I assume $T = \bar{T}$, so only E is left as a choice variable.

Note that the objective function is that:

$$\max_{E} \; E[U(E)] = p\, ln\left(\alpha_H E^{\beta} T^{1-\beta} + W(\bar{L} - E) - C_x T\right) +$$

$$(1-p)\, ln\left(\alpha_L E^{\beta} T^{1-\beta} + W(\bar{L} - E) - C_x T\right)$$

where E is effective labor input. $\bar{L}$ is labor endowment, so if $E^* > \bar{L}$, then the farmer need to hire labors; if $E^* < \bar{L}$, the farmer sell labors.

Labor-Land Elasticity and Inverse Relationship:

Before taking the FOC to find equations for analyzing, it is worth to think about how to prove the IR. Note that the (expected) productivity per unit land is calculated by:

$$E[y] = \left[p\alpha_H E^{*\beta} T^{1-\beta} + (1-p)\alpha_L E^{*\beta} T^{1-\beta}\right]/T$$

$$= [\theta\alpha_H + (1-\theta)\alpha_L] E^{*\beta} T^{-\beta} \equiv \alpha_E E^{*\beta} T^{-\beta}$$

where α_E is expected TFP.

Take derivative of $E[y]$ with respect to T, we have:

$$\frac{dE[y]}{dT} = \alpha_E[\beta E^{*\beta-1}\frac{dE^*}{dT}T^{-\beta} + (-\beta)E^{\beta}T^{-\beta-1}]$$

$$= \alpha_E\beta E^{*\beta}T^{-\beta}[E^{*-1}\frac{dE^*}{dT} - T^{-1}]$$

Therefore, $E^{*-1}\frac{dE^*}{dT} - T^{-1}$ determines the sign of $\frac{dE[y]}{dT}$. Note that, if $E^{*-1}\frac{dE^*}{dT} - T^{-1} < 0$, which means $\frac{dE^*}{dT}\frac{T}{E} < 1$, i.e. if Labor-Land Elasticity, denote as $e \equiv \frac{dE^*}{dT}\frac{T}{E}$, is less than one, then IR can be found.

First Order Derivative:

Since we assume that no land will be wasted ($T^* = \bar{T}$), labor market is available (E^* can be larger than $\bar{L}$), and production function takes C-D form ($E^* > 0$), so we don't need to explicitly apply endowment and non-negative constraints when deriving the results.

Take the FOC of expected utility function with respect to E, we have:

$$\frac{dE[U(E)]}{dE} = p\frac{\beta\alpha_H E^{\beta-1}T^{1-\beta} - W}{\alpha_H E^{\beta}T^{1-\beta} + W(\bar{L} - E) - C_x T}$$

$$+ (1-p)\frac{\beta\alpha_L E^{\beta-1}T^{1-\beta} - W}{\alpha_L E^{\beta}T^{1-\beta} + W(\bar{L} - E) - C_x T} = 0$$

Rearrange the FOC, we get:

$$E^{2\beta-1}T^{2-2\beta}\beta\alpha_H\alpha_L - E^{\beta}T^{1-\beta}W\overline{\alpha_E} + E^{\beta-1}T^{1-\beta}W\bar{L}\beta\alpha_E - E^{\beta}T^{1-\beta}W\beta\alpha_E$$

$$-E^{\beta-1}T^{2-\beta}C_x\beta\alpha_E + W^2E - W^2\bar{L} + WC_xT = 0$$

(1)

where $\overline{\alpha_E} \equiv [(1-\theta)\alpha_H + \theta\alpha_L]$

Total Differentiation of FOC with Respect to T:

Take total differentiation of FOC with respect to T at optimal solution level, we obtain equation (2). Note that E^* is a function of T. Denote $E' \equiv \frac{dE^*}{dT}$, and omit the star mark representing optimal solution in following formulas. the equation (2) contains 12 terms:

$$(2\beta-1)E^{2\beta-2}E'^{T^{2-2\beta}}\beta\alpha_H\alpha_L + (2-2\beta)E^{2\beta-1}T^{1-2\beta}\beta\alpha_H\alpha_L$$

$$-\beta E^{\beta-1}E'^{T^{1-\beta}}W\overline{\alpha_E} - (1-\beta)E^{\beta}T^{-\beta}W\overline{\alpha_E}$$

$$+(\beta-1)E^{\beta-2}E'^{T^{1-\beta}}W\bar{L}\beta\alpha_E + (1-\beta)E^{\beta-1}T^{-\beta}W\bar{L}\beta\alpha_E$$

$$-\beta E^{\beta-1}E'T^{1-\beta}W\beta\alpha_E - (1-\beta)E^{\beta}T^{-\beta}W\beta\alpha_E$$

$$-(\beta-1)E^{\beta-2}E'T^{2-\beta}C_x\beta\alpha_E - (2-\beta)E^{\beta-1}T^{1-\beta}C_x\beta\alpha_E$$

$$+W^2E' + WC_x = 0 \qquad (2)$$

For every term contains E', we can rearrange to obtain the labor-land elasticity $e = E'\frac{T}{E}$, then (2) can be reduced to (3):

$$E^{2\beta-1}T^{1-2\beta}\beta\alpha_H\alpha_L[(2\beta-1)e + (2-2\beta)]$$

$$-E^{\beta}T^{-\beta}W\overline{\alpha_E}[\beta e + (1-\beta)]$$

$$+E^{\beta-1}T^{-\beta}W\bar{L}\beta\alpha_E(1-\beta)(1-e)$$

$$-E^{\beta}T^{-\beta}W\beta\alpha_E[\beta e+(1-\beta)]$$

$$-E^{\beta-1}T^{1-\beta}C_x\beta\alpha_E[e(\beta-1)+(2-\beta)]$$

$$+W^2ET^{-1}e+WC_x=0 \tag{3}$$

Combining FOC and Total Differentiation Results:

Note that, by FOC equation (1), we have:

$$E^{\beta}T^{1-\beta}W\overline{\alpha_E}=E^{2\beta-1}T^{2-2\beta}\beta\alpha_H\alpha_L+E^{\beta-1}T^{1-\beta}W\bar{L}\beta\alpha_E-E^{\beta}T^{1-\beta}W\beta\alpha_E$$

$$-E^{\beta-1}T^{2-\beta}C_x\beta\alpha_E+W^2E-W^2\bar{L}+WC_xT \tag{4}$$

Substitute (4) into (3), cancelling out $\overline{\alpha_E}$ and rearranging, we obtain (5), which is so far the most reduced and analyzable equation we can get.

$$E^{2\beta-1}T^{1-2\beta}\beta\alpha_H\alpha_L(1-\beta)(1-e)-E^{\beta-1}T^{-\beta}W\bar{L}\beta\alpha_E e$$

$$-W^2ET^{-1}(1-\beta)(1-e)+W^2\bar{L}T^{-1}(\beta e+1-\beta)$$

$$-E^{\beta-1}T^{1-\beta}C_x\beta\alpha_E(1-e)+WC_x\beta(1-e)=0 \tag{5}$$

Analysis:

First, we prove that **when there is no risk, $e=1$, i.e. no IR will be found**.

Proof:

Assuming there is no risk by setting $\alpha_r=\overline{\alpha_r}=\alpha_E$.

Then the FOC will imply that:

$$\beta\alpha_E E^{\beta-1}T^{1-\beta} = W \tag{6}$$

Substitute (6) into (5), and multiply the whole obtained equation by $\frac{T}{W}$, we get:

$$[\alpha_r E^{\beta}T^{1-\beta} + W(\bar{L} - E) - C_x T](1-\beta)(1-e) = 0 \tag{7}$$

Note that, the formula inside the square bracket is just the total farm profit, and it cannot be zero, since the farmer can always give up the agricultural production and just do non-farm works to maintain a positive consumption. Therefore the first term at optimal solution level must be positive. β is assumed to be less than one, as technology is marginally decreasing and CRS. Finally, the only way to satisfy (7) is $e^* = 1$. In other words, if there is no risk, land size increases will proportionally increase labor input, so the productivity is constant.

■

Second, we prove that **if there is no labor endowment, then, no IR can be found**

Proof:

Assume $\bar{L} = 0$, the equation (5) becomes:

$$\{[E^{2\beta-1}T^{1-2\beta}\beta\alpha_r\overline{\alpha_r} - W^2ET^{-1}](1-\beta) - E^{\beta-1}T^{1-\beta}C_x\beta\alpha_E + WC_x\beta\}(1-e) = 0 \tag{7}$$

Temporally assume $C_x = 0$, then we have:

$$\left\{\left[E^{2\beta-1}T^{1-2\beta}\beta\alpha_H\alpha_L - W^2ET^{-1}\right](1-\beta)\right\}(1-e) = 0 \tag{8}$$

Therefore, if the first term equal to zero, then:

$$E^{2\beta-1}T^{1-2\beta}\beta\alpha_H\alpha_L = W^2ET^{-1} \tag{9}$$

Rearrange, we get:

$$E = \left[\frac{\beta\alpha_H\alpha_L}{W^2}\right]^{\frac{1}{2-2\beta}} T \tag{10}$$

Hence,

$$e = E'\frac{T}{E} = \frac{\left[\frac{\beta\alpha_H\alpha_L}{W^2}\right]^{\frac{1}{2-2\beta}} * T}{E} = 1 \tag{11}$$

If the first term does not equal to zero, the third term has to be zero, which also means $e = 1$. So, no IR can be found.

Before we relax the assumption on $C_x = 0$, it is worth to note that the equation (10) is not the solution of E^*. $\forall\ constant,\ E = constant * T$ will imply $e = 1$, and satisfies the equation (8). Therefore, the first term in (8) does not determines the solution of E^*. Using simulation method, it can be shown that when $\bar{L} = 0,\ C_x = 0$:

$$E^* \approx \left[\frac{\beta^2\alpha_H\alpha_L}{W^2}\right]^{\frac{1}{2-2\beta}} T \tag{12}$$

Then, we assume $C_x > 0$. Similar idea as we just explained about (10) and (12), the first term in (7) does not necessarily determine the value of E^*. So, we don't know the value of the first term, but we do know that $e = 1$ in the second term in (7) still can

solve the equation. Our simulation results also support this conclusion, i.e. $\bar{L} = 0$ leads to $e = 0$.

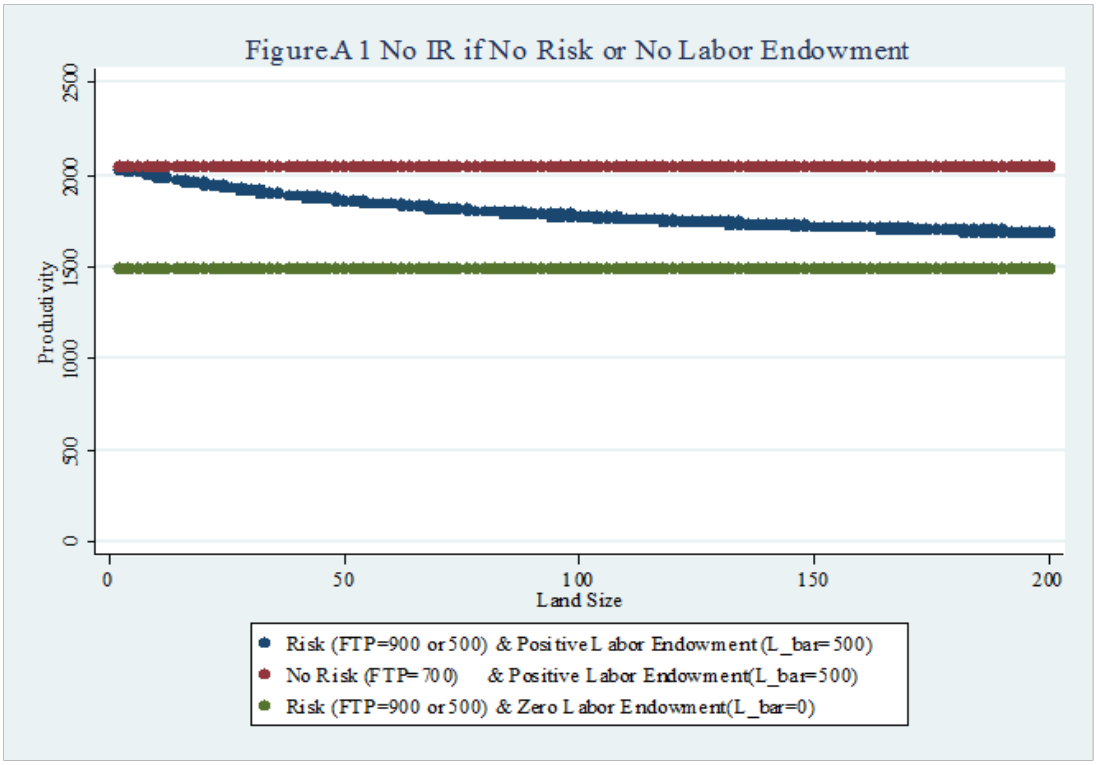
Therefore, we formally proved that when there is no labor endowment and no fixed cost, there is no IR, and informally proved that if fixed cost is positive, $e = 1$ is still a solution of the model, which indicates no IR can be found.

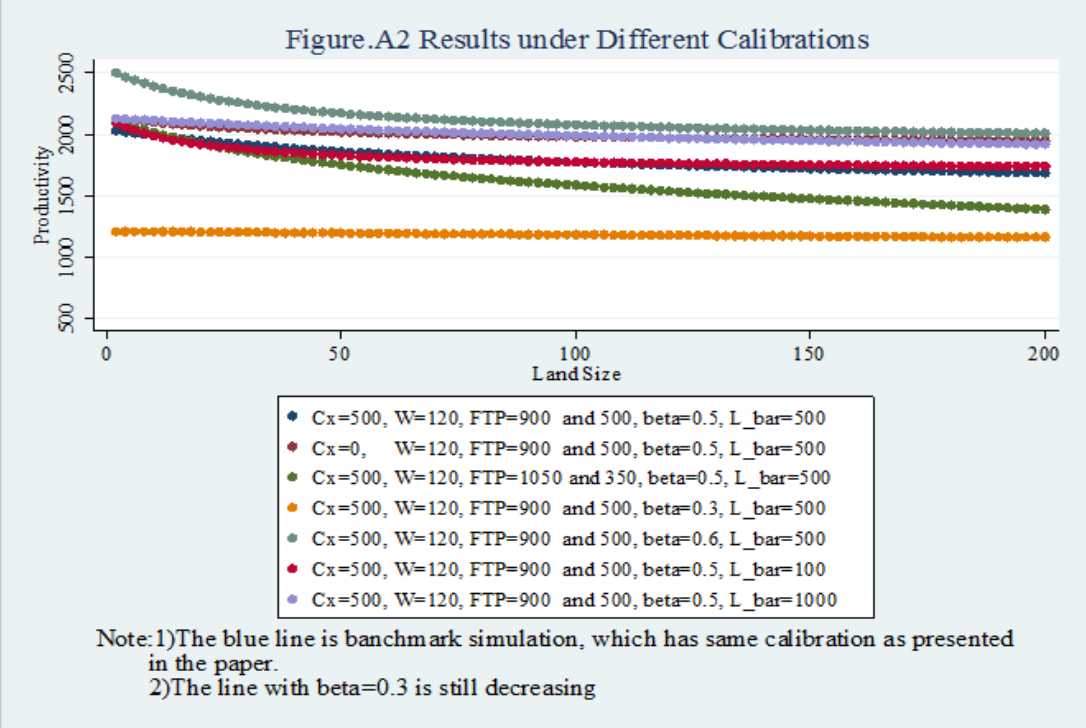
■

The Figure.A1 shows simulation results corresponding to the last two mathematically proved findings.

Unfortunately, so far, we failed in mathematically proving that when labor endowment is positive, and there is risk, the labor-land elasticity in (5) is less than 1. Simulation details show that, at optimal solution levels, the sums of the second to sixth term are always negative, which indicate that the first term must be positive, i.e. $e < 1$. For strengthening our findings, we provide Figure.A2, in which there are a bunch of simulation results with different calibrations. We actually run substantially

more simulations than we present here, all of them show IR results.





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