Staff Paper

OPPORTUNITY COSTS AND PRESENT VALUE MODELS

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OPPORTUNITY COSTS AND PRESENT VALUE MODELS

Economic theory is intended to study the optimal allocation of scarce resources. Principles of marginal analysis, opportunity cost, utility maximization, profit maximization, etc., are typically applied to lend insights into the optimal allocation of resources. In short-run static analysis, the optimization takes place over a timeless horizon and time preferences for consumption and income are not considered. As the decision framework changes to a multiperiod setting, adjustments must be made to account for the rate of time preference possessed by decision making agents.

Present value analysis is a commonly applied technique to adjust returns in one time period to their equivalent value in another time period by accounting for the returns' opportunity costs across periods (Barry, Hopkin, and Baker, Copeland and Weston). Thus, any return stream can be converted to a single value at a given point in time. This technique allows a meaningful economic comparison of alternative return streams provided certain consistency conditions are met.

Unfortunately, a number of difficulties arise in the applications of present value techniques which, if handled incorrectly, can lead to incorrect investment decisions. To further complicate, matters, there is disagreement on how to apply present value models in both theory and practice. For example, Lins and Robison (1989) discuss the assessment of cash flows for restructuring loans and the discount rate to use in present value models. The correct discount rate they argued is determined by the return available on the alternative use of funds. Rao and Pederson argue that the procedures suggested by Lins and Robison for selecting a discount rate are "theoretically more defensible" than an alternative proposed by Duft and Willett but disagree with their choice of a discount rate.[1] The remainder of this paper addresses the correct

application of present value models when the objective is to maximize wealth over some time horizon.

Optimization And Opportunity Cost

Accountants refer to the price of objects purchased when referring to costs. Their estimates of income and expenses include only the prices of outputs sold and inputs purchased. Obviously, this difference between economists and accountants in the meaning of costs may cause confusion. So to make sure the economist's concept of cost is clear, several examples are created in which opportunity costs are discovered by calculating the marginal opportunity cost.

At the heart of present value models is the discount rate. It represents a rate of return given up on the next best alternative investment. Hence, it is called an opportunity cost of capital. If, of course, there is no alternative use of the resource or if the resource is in such abundance it cannot command a price, the resource is free and the opportunity cost of capital is free. Frequently, the opportunity cost of capital is confused with interest rates on different money instruments or rates of return on some other investments. But, these represent an opportunity cost only if they represent the next best use of funds. To help demonstrate and interpret the discount rate in present value models, we develop analogies with a static (timeless) production optimization problem Using this static model, we show the similarities between it and present value models.

When a resource is available in continuously divisible quantities, the allocation rule familiar to economists is: allocate the resource until its marginal value is equivalent to the marginal opportunity cost -- the returns sacrificed by diverting the resource to another use. Consider the standard production problem:

$$\pi = pf(x) - p_x x \tag{1}$$

where π is profit from the production activity; p is the price of the output produced; $f(x)$ is production function conditional on the level of input where $f'>0$, $f''<0$; x is input level; and p_x is price of input. Maximizing π in (1) is accomplished by solving the following first-order condition:

$$pf'(x)-p_x=0 \tag{2}$$

The firm maximizes profit by setting the marginal value product of the input $pf'(x)$ equal to the marginal cost of the input, p_x. In this case, the input price, p_x, is also the opportunity costs, or what the firm gives up to earn the marginal value product $pf'(x)$. Because the firm operates under no constraint in its use of x, it only has to pay for it at the price p_x. Thus, in this single case, the acquisition price p_x is also the opportunity cost of acquiring x.

A slight modification to the problem will yield a result where the opportunity cost is not equal to the purchase price of the input. Consider the same production problem above only now the firm is constrained by budget B. Moreover, the firm has an alternative to investing its resources in production. This alternative is an investment I that earns a rate of return r. The firm's new profit function is:

$$\pi = pf(x) - p_x x + rI \tag{3}$$

$$\text{s.t.} \quad B = I + p_x x$$

$$I, x \geq 0$$

Substituting the budget constraint into (3) yields:

$$\pi = pf(x) - p_x x - r(B - p_x x) \tag{4}$$

The optimal allocation of x to the production process is found by solving the first-order condition:

$$pf'(x) - p_x(1+r) = 0 \tag{5}$$

The input will be allocated up to the point where marginal value $pf'(x)$ is equal to the marginal opportunity cost of production, $p_x(1+r)$. The marginal opportunity cost has changed because the firm not only gives up the purchase price p_x when using the input, but also the return it could have earned if it had invested in I. Thus, the opportunity cost is $p_x(1+r)$.

An alternative interpretation of opportunity cost will prove useful in the study of present value models. In the standard unconstrained production problem in (1), the cash outlays on the last unit of investment, p_x, produced a marginal return $pf'(x)$ resulting in a rate of return

$$\left(\frac{pf'(x)}{p_x} - 1\right) = 0$$

Because the firm faced no opportunity cost of capital beyond the purchase price, it could expand production until the marginal factor return equals the marginal factor cost and earned a zero rate of return.

The constrained production problem in (3) can be interpreted in a similar manner. Using (5) we find that

$$\left(\frac{pf'(x)}{p_x} - 1\right) = r$$

Now the opportunity cost of returns from the production process is the rate of return foregone on the alternative investment I. In contrast to the earlier case, funds are not idle outside the production process described by $f(x)$. They would be invested and earn at rate r. Thus the

optimization requires that the last unit of x acquired earn a rate of return from the production process that is equivalent to the rate of the opportunity cost of capital, r.

The above examples illustrate the opportunity cost principle. Whenever an investment requires giving up something of value, it creates an opportunity cost equal to the value of the foregone resource in the best alternative use. The important point to recognize is that it is the opportunity cost, not the resource price, that is the decision criteria when employing the marginal principles of economics. The same principle holds in applications of present value models.

Ranking, Optimization, And Valuation

A distinction can be made between optimizing and ranking in present value analysis. First consider a simple two period investment optimization problem. Assume an initial investment I is made at the beginning of the period and repaid with interest cost, r, at the end of the period. A single cash inflow, $R(I)$, which is a function of the investment level is received at the end of the period where $R'(I)>0$, $R''(I)<0$. The profit, at the end of the period π_1, can be written as:

$$\pi_1 = R(I) - I(1+r) \tag{6}$$

The optimal investment level is found by solving the following first-order condition:

$$\frac{d\pi_1}{dI} = R'(I) - (1+r) = 0 \tag{7}$$

Since $R'(I)$ is the return on the last unit of I invested, optimality requires that it pays for the last unit of I invested plus the opportunity cost of capital r. Thus, $R'(I) = (1+r)$.

As seen in the above example, standard marginal economic concepts can be applied to determine the optimal level of investment. Investment decisions are often set in a related, but distinctly different framework. Instead of determining the optimal level of investment to make, decisions often require a discrete choice of whether to make a specific investment although the investment level in that project may not be optimal. In other words, the investment decision is whether to make an investment of some fixed level of I in return for some cash flow $R(I)$. The level of I and $R(I)$ may not be optimal, but they are exogenous to the investment decision.

First consider a two-period investment problem where I is the required investment in the project at the beginning of the period and $R(I)$ is the cash flow at the end of the period. Assume r is the opportunity cost of capital. The end-of-period profit, π_1, from the investment is again:

$$\pi_1 = R(I) - I(1+r) \tag{8}$$

The investment would be made as long as $\pi_1 > 0$, i.e., the cash flow at the end of the period is enough to repay the initial investment plus the end-of-period cost of the investment funds.

Equivalently, the investment should be made if:

$$\frac{\pi_1}{(1+r)} = \frac{R(I)}{(1+r)} - I > 0$$

In other words, if the value of end-of-period profit discounted by one plus the opportunity cost of investment funds is positive, the project should be undertaken because in order for the discounted value of end-of-period profit to be positive, end-of-period profit must be positive. The adjustment made to end-of-period profit by discounting by one plus the opportunity cost of investment fund converts the end-of-period profit values to it equivalent value at the beginning of the period. Thus, $\frac{\pi_1}{(1+r)}$ is additional value, or profit, of the investment in beginning of-the-period dollars over and above the initial investment level I.

The implicit assumption in this approach to the problem is that if $\frac{\pi_1}{(1+r)}$ dollars were available at the beginning of the period, they could be invested to earn a return r during the period. This is essentially the approach taken in the standard net present value (NPV) formulation of the investment problem. The cash flows from the investment are discounted by the opportunity cost of capital to convert the cash flows to a profit measure at the beginning of the life of the investment problem.

A third equivalent formulation of the decision model is to make the investment if:

$$\frac{R(I)}{I} - I > r$$

In other words, the investment should be made if the rate of return on the investment is greater than the cost of funds. This decision rule is equivalent to the standard Internal Rate of Return (IRR) technique, where the return on investment, $\frac{R(I)}{I} - 1$, is defined to be the investment's IRR. Note from (8) that if the IRR is greater than r that end-of-period profit must be positive.[2] Thus projects are then undertaken if the IRR of the investment is greater than the opportunity cost of funds required for the investment. Each of the three approaches to the investment decision are equivalent in the single-period model. The opportunity cost in each model is the cost of investment funds, r. This is the return, or cost, that is given up if the investment is undertaken. Note that there was no optimization involved in the level of investment, only a ranking of the investment relative to the alternative use of the investment funds.

Now, suppose there are two mutually exclusive single-period investments with initial costs $I^A = I^B = I$. The end-of-period profit for each investment can be represented as:

$$\pi_1^A = R^A(I) - I(1+r) \tag{9.a}$$

and

$$\pi_1^B = R^B(I) - I(1+r) \tag{9.b}$$

where r is the opportunity cost of the best alternative use of funds exclusive of the two projects under consideration. The preferred investment will be the one with the highest end-of-period profit, given the profit level is positive.[3] The opportunity cost of each individual project now becomes the larger of the opportunity cost funds, r, or the return given up on the alternative mutually exclusive investment. For example, the return given up on investment B if A is undertaken can be found by calculating B's IRR as:

$$r^B = \frac{R^B(I)}{I} - 1$$

Using the NPV approach to evaluate the project would involve discounting A's end-of-period cash flow by the opportunity cost of the funds invested in A which is: $r^* = \max[r^B, r]$ Thus, the NPV of A can be found as:

$$NPV^A = \frac{R^A(I)}{(1+r^*)} - I$$

This is the net value of investment A in today's dollars given the alternative uses of the funds invested in A. If NPV^A is positive then investment A is the preferred investment.

The IRR approach involves comparing the returns generated by each investment alternative. Investment A would be undertaken if the return from investment A

$$r^A = \frac{R^A(I)}{I} - 1$$

is greater than the opportunity cost of investing in A, i.e., $r*$.

In the single period model, both the NPV and IRR ranking methods produce results which are equivalent to ranking investment by end-of-period profit. Thus, comparing the NPV's or IRR's of the investment alternatives is equivalent to comparing end-of-period profit levels of

each investment. Note again that the above decision problems differ in a sense from a standard optimization problem in that the investment problem involves choosing among discrete fixed investment opportunities. It is the value of these foregone opportunities, not the acquisition price, that is the "choice-influencing" cost that optimally allocates resources.

The appropriate discount rate is often argued to be "the rate of return foregone on the next best investment opportunity." The key term here is that it is the foregone rate of return, not the best alternative rate of return that should be considered as the opportunity cost. Suppose a challenging investment is under consideration. To adopt the proposed investment, the firm must liquidate one of two existing investment, D1 or D2. D1 provides a return of 10 percent while D2 produces a return of 8 percent. If the firm is going to liquidate one of its existing investments, it will choose the one that produces the lowest return. The correct opportunity cost of the challenging investment would not be the return on the best alternative investment, i.e. 10 percent for D1, but the return given up on the foregone investment, which would be D2 because it has a lower return than D1. Thus the opportunity cost for the challenging investment would be the foregone rate of return on D2, i.e., 8 percent.

Accounting Cost Versus Opportunity Cost

How are present value models and accounting cash flows related? The accounting cash flow measure represents difference between revenues and expenses at a common point in time. Present value models contain the sum of accounting cash flow measures across different points in time. These intertemporal cash flow measures can not be compared directly because their accounting units of measurement are not consistent across time. Consequently, present value models adjust each cash flow measure by an opportunity cost which accounts for the time value of the cash flow level if it were invested in the best alternative investment. The correct

opportunity cost is the economist's concept representing a lost opportunity, not an accounting concept such as the interest rate on funds borrowed.

Some confusion has occurred regarding this concept partially due to the introduction of the perfect capital market assumption which assumes that everyone can borrow and lend at the same and known market rate of interest and all assets will provide the same rate of return. Thus, the discount rate is the market rate of interest--an accounting concept. In this instance, the opportunity cost and accountant's concept of cost are the same--the cost of income foregone is measured at the rate of interest prevalent in the market. However, whenever the perfect market assumption fails to hold, the discount rate reflecting the opportunity cost of an investment is no longer necessarily equal to the market rate of interest.

In the single-period profit expression all investment opportunities were endogenized into the problem. This allowed us to find the opportunity cost using maximization techniques. In the present value model, a comparison is made between two investments, a defender and a challenger. The difficulty is how to endogenized the opportunity cost associated with the defender while evaluating the challenger? We do this in the same way that we subtract off costs of purchased inputs in static profit functions. For example, we never asked where did p_x come from. We were willing to accept this as exogenous to the model. Something we would have to pay to acquire the input. Only if there were no other constraints or opportunities would it equal the opportunity cost of making the investment. Likewise, the discount rate in a present value model is an exogenous parameter. It turns out to also be the opportunity cost because it is the <u>only</u> opportunity being considered for the stream of cash flows under consideration.

Calculating Opportunity Costs

The opportunity cost of specific investment alternatives is the rate of return foregone from failure to invest in that investment. In the single-period examples discussed earlier, we considered projects with an initial investment I at the beginning of the period and a single cash flow at the end of the period, R_1. The rate of return for the investment is calculated as:

$$r_1 = \frac{R_1}{I} - 1 \tag{10}$$

If a particular project is not taken, then the rate of return r is foregone. The opportunity cost of all single period projects available to a firm can be calculated and the highest rate of return available on a single period investment can be found. Let this value be denoted r_1^*. Thus, the opportunity cost for a single period project under consideration by a firm is r_1^*. Similarly, the opportunity cost for a two period investment can be calculated. The two period investment consists of an initial investment, a cash flow at the end of one period, and a cash flow at the end of two periods. Note that the cash flow at the end of the first period can be reinvested during the second period. The reinvestment rate will be the best single period investment rate r_1^*. Thus, the average rate of return per period of the two period investment can be found by solving for r_2 in the expression:

$$R_2 + R_1(1+r_1^*) - I(1+r_2)^2 = 0 \tag{11}$$

The solution is:

$$r_2 = \sqrt{\frac{R_2 + R_1(1+r_1^*)}{I}} - 1 \tag{12}$$

Considering all two period projects available to the firm, let r_2^* represent the highest two period return. Proceeding in an iterative fashion, the opportunity cost of a specific n-period project can be found to be:

$$r_n = \left[\frac{\sum_{t=0}^{n-1} R_{n-t}(1+r_t^*)^t}{I}\right]^{(1/n)} - 1 \tag{13}$$

Moreover, the opportunity cost of any n-period project being considered is then found to be $r_n^* = \max\left[r_n^1, r_n^2, \ldots\right]$. The above formulation assumes that all investments are available each time period, i.e., all projects are independent of time. This assumption is easily relaxed by eliminating or adding projects in the future time periods. Also, note that $r_{t+n}^* \geq r_t^*$ for all t by definition.[4]

Consistency

It is important to recognize that present value analysis involves comparing two investments, the investment under consideration which is represented by the cash flow stream and the alternative investment which is represented by the discount rate, or opportunity cost. Call these two investments the challenger and defender, respectively. Robison and Burghardt argue that a principle to be followed when constructing present value models is to measure returns of the challenges and defender in consistent units of measures. Consistency in the measurements of taxes, inflation, risk, size of investment, and timing of defender and challenger must be maintained in the construction of present value models. Detail of how consistency is

achieved is beyond the scope of this paper. The following discussion assumes that the necessary consistency conditions have been satisfied.

Multiperiod Analysis

In the single period investment problem, selection based on end-of-period profits, NPV, and IRR criteria were all consistent. Complications can arise in multiperiod investment problems. As it is usually calculated, the IRR is the compound interest rate that satisfies:

$$\sum_{t=1}^{n} \frac{R_t}{(1+r)^t} - I = 0 \tag{14}$$

The IRR is then interpreted to be the average rate of return that the investment earns over its life. The IRR is then compared to a foregone rate of return, usually the cost of investment funds. If the IRR is greater than the opportunity cost of the investment funds, the project is undertaken. The IRR model suffers a technical difficulty in the fact that (14) is a polynomial equation of the nth degree. Consequently, the solution to (14) has up to n possible roots (values of IRR) which satisfy the equation. Thus, some series of cash flows may have more than one IRR which satisfy (1) and, in some cases, the IRR's may be negative. Interpretation of the IRR becomes difficult in these cases (see Dorfman).

A well-known, but often underemphasized assumption of (14) is that the cash flows received each period can be reinvested at the calculated IRR for that investment. This is easily seen by noting that:

$$\sum_{t=0}^{n} \frac{R_t}{(1+r)^t} = \frac{\sum_{t=1}^{n} R_t(1+r)^{n-t}}{(1+r)^n}$$

Marty and Strung have recognized the implications of this assumption and recommend that the actual reinvestment rate of the intermediate cash flows be accounted for directly in the calculation of the return from a given investment. Using our earlier assumptions, this yields:

$$r_n = \left[\frac{\sum_{t=0}^{n-1} R_{n-t}(1+r_t^*)^t}{I} \right]^{(1/n)} - 1 \tag{15}$$

which is the same as the n-period opportunity cost calculated in Equation (13). The adjusted IRR (AIRR) is really just the investments opportunity cost. Thus, projects can be selected by ranking their correctly calculated opportunity cost. Also note that the AIRR involves only one solution so the difficulties with multiple IRR's is avoided.

Meanwhile, the NPV model is typically calculated as:

$$NPV = \sum_{t=1}^{n} \frac{R_t}{(1+r_n^*)^t} - I \tag{16}$$

As discussed earlier, the NPV is interpreted to be the net value of the investment in today's dollars relative to the next best alternative use of the investment funds. For example, if the NPV of an investment is $100, then undertaking of the investment will provide the same value in today's dollars as the alternative investment plus an additional $100. Thus, an investment with a positive NPV is preferred to the alternative it is being compared to. It is easily seen from (14) and (16) that the standard IRR is the discount rate that equates the NPV to zero. Thus, if $IRR \gtrless r^*$ then $NPV \gtrless 0$ and the two models will provide consistent rankings of investments if

there are no constraints among the investment opportunities. Similar to the IRR model, the NPV model implicitly assumes that all intermediate cash flows are reinvested at that n-period opportunity cost discount rate r_n^*. As argued with the IRR model, the intermediate cash flows are likely to be invested rates which generally differ depending on the size of the cash flow length of time to be invested. These reinvestment rates will not generally be equal to the n-period opportunity cost. The intermediate cash flows should be reinvested at the best return available which results in an adjusted NPV model (ANPV):

$$ANPV = \frac{\sum_{t=0}^{n-1} R_{n-t}(1+r_t^*)^t}{(1+r_n^*)^n} - I \tag{17}$$

The ANPV or AIRR can be used to rank projects as the two will always provide consistent rankings with the end-of-period profit ranking. To see this, suppose two mutually exclusive n-period projects A and B are compared. The end-of-period profit from the two investments can be written as:

$$\pi_n^A = \sum_{t=0}^{n-1} R_{n-1}^A(1+r_t^*)^t - I(1+r_n^*)^n \tag{18.a}$$

and

$$\pi_n^B = \sum_{t=0}^{n-1} R_{n-1}^B(1+r_t^*)^t - I(1+r_n^*)^n \tag{18.b}$$

If $\pi_n^A > \pi_n^B$, the from (17) we see that $ANPV^A > ANPV^B$.

Furthermore, from (15), it is seen that:

$$\sum_{t=0}^{n-1} \frac{R_{n-t}^{A}(1+r_t^*)^t}{I} > \sum_{t=0}^{n-1} \frac{R_{n-t}^{B}(1+r_t^*)^t}{I}$$

Thus, it is clear that $AIRR^A > AIRR^B$. The rankings will always be consistent between the two methods and the end-of-period profit ranking. The ANPV becomes important when questions of investment valuation need to be considered. If the investor is concerned with how much an investment is worth, then ANPV must be used with the opportunity cost of the next best alternative investment used as the discount rate.

A common method of employing present value models to rank mutual exclusive investments is to compare the NPV's of each project using the opportunity cost of some base investment, not necessarily the best alternative investment. For example, a firm may be considering two mutually exclusive investments A and B. The comparison of two projects is made by calculating the NPV using the opportunity cost of a third investment, say $\hat{r}$, which might be the cost of debt funds. <u>This method of comparison will produce a ranking consistent with end-of-period profit maximization</u>. To demonstrate this result, note that

$$\pi^A - \pi^B = \sum_{t=0}^{n-1} R_{n-t}^{A}(1+r_t^*)^t - \sum_{t=0}^{n-1} R_{n-t}^{B}(1+r_t^*)$$

and is independent of $\hat{r}$, the opportunity cost of capital. Thus

$$\frac{\pi^A}{(1+\hat{r})^n} \gtreqless \frac{\pi^B}{(1+\hat{r})^n} \; as \; \pi^A \gtreqless \pi^B$$

and choosing the project with the highest ANPV evaluated using $\hat{r}$ as the discount will yield the profit maximizing project.

But, of course, the value of π^A and π^B do depend on $\hat{r}$. Thus, the procedure will result in the incorrect valuation of each investment. Consider the end-of-period profit functions for the two projects using the opportunity cost for $\hat{r}$:

$$\pi^A = \sum_{t=0}^{n-1} R^A_{n-t}(1+r_t^*)^t - I(1+\hat{r})^n \tag{19.a}$$

and

$$\pi^B_n = \sum_{t=0}^{n-1} R^B_{n-t}(1+r_t^*)^t - I(1+\hat{r})^n \tag{19.b}$$

Suppose $\pi^A > \pi^B$, then using (17) we see that $ANPV^A > ANPV^B$. Thus, project A would be preferred to project B. Now suppose that $AIRR^B > \hat{r}$ thus the opportunity cost of A is $AIRR^B$. Using (15), (19.b) and (17), we see that $ANPV^A$, discounted by the $AIRR^B$ is positive which again indicates that investment A is preferred to B. Thus, even if the incorrect opportunity costs of the original investment funds are used in the ANPV analysis, the projects will be ranked correctly; although, the valuation of the projects will be wrong. The correct value of investment in todays dollars will be $I + \frac{\pi_A}{(1+AIRR^B)^n}$ which is less than the value $I + \frac{\pi_A}{(1+\hat{r})^n}$ found using $\hat{r}$ as the discount rate. It should be clear, at this point, that if project ranking is the purpose of the analysis, one need proceed no further than a comparison of end-of-period profits because the rankings provided by the ANPV and AIRR techniques are always consistent with end-of-period profits but require additional calculations. The ANPV and AIRR are, however, essential to questions of valuation.

Return To Assets Versus Return To Equity

Some confusion exists about whether the discount rate reflects a return to equity (RTE) or a return to total investment or assets (RTA).[5] The RTE approach uses only the portion of the projects cost supplied by equity as the investment level and incorporates the financing costs directly into the projects cash flow. The RTA approach uses the total equity and debt levels needed for the project as the investment level and accounts for the financing of the project in

the choice of the discount rate. The RTA approach implicitly assumes a constant mix of debt and equity financing over the life of the investment. There is considerable debate as to which model to use since the two can produce different results.

Fiske supports the use of the RTA approach arguing it better represents the implicit credit capacities of a given project. Barry, Hopkin, and Baker suggest the RTA approach may be desirable for large firms that have a stable relationship between debt and equity while the RTE approach may be better for small firms whose capital structure fluctuates significantly over time. The correct approach can be found by relating each model back to the profit function problem.

Consider a single period investment consisting of a single outflow I at the beginning of the period and a single inflow R_1 at the end of the period. The end-of-period profit when both debt and equity are used to finance the investment can be represented as:

$$\pi_1^E = R_1 - D(1+r_d) - E(1+r_e) \tag{20}$$

where π_1^E are end-of-period returns to equity; r_d is the opportunity cost of debt funds; r_e is the opportunity cost of equity funds; D is the amount of debt financing; and, E is the amount of equity financing. This is also the formulation of the profit function implicit in the RTA approach. The formulation of the profit function for the RTA approach is:

$$\pi_1^A = R_1 - I(1+r_w) \tag{21}$$

where r_w is the weighted average cost of funds equal to $r_w = \frac{r_d D + r_e E}{I}$ and D + E = I. Note that π^A and π^E in equations (20) and (21) are equal.

The RTE and RTA measures of NPV are found by dividing (20) and (21) by r_e and r_w, respectively. This yields:

and

$$RTE = \frac{\pi_1^E}{(1+r_e)} = \frac{R_1 - D(1+r_d)}{(1+r_e)} - I \tag{22.a}$$

$$RTA = \frac{\pi_1^A}{(1+r_w)} = \frac{R_1}{(1+r_w)} - I \tag{22.b}$$

In this simple form, it is easy to see that RTA and RTE are simply alternative formulations of the NPV model which is intended to indicate the incremental value in today's dollars of the investment under consideration given some alternative use of the investment funds. However, if π_1^A equals π_1^E then $RTA = \frac{\pi_1^A}{(1+r_w)}$ cannot equal $RTE = \frac{\pi_1^E}{(1+r_e)}$ unless r_e is equal to r_w.[6]

The reason the two valuation models produce different results is that they assume a different opportunity cost for the end-of-period profit. The RTA approach values the end-of-period profit as though its value today could be invested to earn r_w during the period. The RTE approach values the end-of-period profit as though its value today could be invested to earnr_e during the period. Which reinvestment rate actually describes the reinvestment opportunities of the firm determines which, if either, model is correct.

Clearly, the end-of-period profits which both models calculate represent additions to equity. Moreover, it is this equity which is being discounted when converting from π_1^A or π_1^E to their equivalent in the current period. Thus we conclude that the correct reinvestment rate to assume is the average rate of return earned on the equity amount of π_1^E. This rate, however, under most conditions, will be neither r_e or r_A. It might equal r_e if π_E equalled $I(Hr_e)$ and the investment costs had not changed. But this is unlikely. The correct rate will only be found by consulting the firm's investment opportunity schedule for investments whose size is equivalent to π_1^E -- in which case the reinvestment rate is r_1^*. Even if the firm maintained a relatively constant

balance of debt and equity funds, it would still be incorrect to assume that the equity portion of the funds π_1^E earns at the rate of r_w unless, of course, r_w were equal to r_e.

Another difficulty with the RTA approach is encountered when multiperiod problems are considered. Consider a two-period problem where an initial investment I is made and cash flows R_1 and R_2 are received at the end of one and two years, respectively. Furthermore, assume that all data and equity are repaid at the end of the second year. The profit at the end of the second period can be written as:[7]

$$\pi_2^E = R_2 + R_1(1+r_1^*) - D(1+r_d)^2 - E(1+r_e)^2 \tag{23}$$

Note that the cash flow at the end of the first period, R_1, is assumed to be reinvested in a asset providing the best one period return. Now, consider formulating the problem in the RTA framework.

$$\pi_2^A = R_2 + R_1(1+r_1^*) - I(1+r_w) \tag{24}$$

It is clear from (23) and (24) that π_2^E equals π_2^A only if

$$r_w = \left[D(1+r_d)^2 + E(1+r_e)^2\right]^{1/2} - 1$$

. Thus, the standard definition of r_w does not produce the correct end-of-period profit level and results in an erroneous valuation of the investment.

The definition of r_w which produces the correct value of end-of-period profit for a given investment is a function of the life of the investment and the type of financing employed. However, even if r_w is defined to produce the correct end-of-period income, use of the RTA approach will produce the wrong valuation of investment in today's dollars. Still, if r_w is defined to produce end-of-period profits, consistent with the RTE approach, both the RTA and RTE approach will produce the correct ranking of investments; since dividing π_2^A and π_2^E either by $(1+r_e)^2$ or $(1+r_w)^2$ will not reverse their relative values.

In Sum, differences between the RTA and RTE model should be based on the argument: which is the correct reinvestment rate, r_e or r_w. The most likely answer is neither.

Conclusion

Present values models are an important tool in economic investment analysis. However, in many cases, difficulties related to alternative specifications of cash flows and discount rates result in incorrect applications of present value models. This paper explores the correct specification of cash flows, reinvestment rates, and discount rates in present value models when the objective is to maximize the profit level at the end of given time period. The notion of opportunity cost is explored and the correct procedure for calculating the opportunity cost of investment is discussed. The necessary adjustments to the standard net present value and internal rate ranking and valuation methods are developed so that the methods produce results project rankings that are consistent with a ending period profit maximization goal. The adjusted net present value model is also shown to be the correct model to use for valuing a given investment alternative. The debate between formulating the present value model in terms of equity investment or total investment is considered and neither method is found to be satisfactory if the objective is to maximize end of period profits.

The models all assume the certain consistency conditions are maintained in the investments being compared and that all cash flows are known with certainty. Future research will focus on the necessary adjustments to ensure that consistency conditions are met and incorporating risk into the analysis.

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1. See also Lin's & Robison's reply (1990).

2. Note that if the NPV model were being used to evaluate a loan, the discount rate becomes the internal rate of cost (IRC) and increasing the IRC decreases the net present cost (NPC) of the loan.

3. If the profit level for both projects was negative, neither project would be selected because a larger end-of-period level could be obtained by investing funds in the alternative investment which yields a return r.

4. It is also important to recognize that the best reinvestment rate for each periods cash flow depends on the size of investment. For example, R_1^* may be quite different when R_1^* then it will be when R_1 = \$1,000.000.

5. This approach is often called the Weighted Average Cost of Capital Approach (WACC) in the corporate finance literature.

6. The RTA and RTE approaches would also be identical in the trivial case of $\pi^A = \pi^E = 0$..

7. For simplicity the debt is assumed to be entirely repaid at the end of the second year.