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Essay on Wavelet analysis and the European term structure of interest rates

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We analyse the generalized, dynamic Nelson-Siegel approach including five factors to ensures the absence of arbitrage. In contrast to previous empirical analyses we define our risk factors so that they are observable and determine their significance using a series of cross sectional regressions. We then decompose the risk factors and test whether they are significant on each time scale. The results allow us to distinguish expected and unexpected components which are used in out of sample forecasts. We find good forecasting abilities of this approach; the one month forecast remains high even during times of financial market distress.

JEL Classifications: E43, E44, G15

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Introduction

Models to explain the Term Structure of Interest Rates have been of interest to researchers for a long time. The models differ in the purpose they are built for. First, term structures of interest rates are needed to evaluate a great variety of assets and derivatives in financial markets. In this context a number of models have been developed with the specific goal to calculate fair prices for derivatives. General Equilibrium models calculate the term structure of interest rates at any point in time on the basis of a stochastic short term interest rate. The short term interest rate is modeled so that the interest rates become a martingale under a probability measure which cannot be observed directly in financial markets. These models have been extended to include more than one stochastic factor. In these models the price of the risk factor can deviate from its actual market price. Therefore, these models do not provide good forecasts for future term structures. In contrast to this, no-Arbitrage models use observable market prices to estimate coefficients of the models, e.g. Ho and Lee (1986), Hull and White (1990), Black and Karasinski (1991). Although these models have theoretical and mathematical foundations, they demonstrate poor forecasting abilities for the term structure of interest rates.

In our analysis we assume that the data generating process for term structure of interest rates can be expressed as an approximate factor model (see Chamberlain and Rothschild (1983). This approach has been used by a numbers of researchers with the purpose to explain and forecast interest rates. Only recently researchers concentrated on the question how the different types of models can be reconciled so that they might generate better forecast, e.g. Diebold and Li (2006), Christensen et al. (2007), and Christensen et al. (2008). The latter generalize the Nelson-Siegel approach (Nelson and Siegel, 1987)) and derive five factors so that the Nelson-Siegel term structure model can be put in an arbitrage-free, general equilibrium model setting. Their model defines the explanatory variables to represent the level, slope, and curvature of the term structure of interest rates.

In our analysis we collect market data for the risk factors and determine their significance through the Fama and MacBeth (1973) t-statistic. Econometric results are questionable if aggregated over time scales with different data generating processes. In financial markets, market participants have varying objectives with different corresponding investment horizons. Arbitrage dealers search for price discrepancies and exploit them within milliseconds to their advantage. Day traders act according to technical analyses within a day or on a weekly basis. Asset managers focus on the next performance reporting date which is usually at the end of a month or a quarter. Furthermore, buy-and-hold investors and central banks will have a much longer investment horizon, one that can vary from a month to a year or even decades. We therefore do not want to impose the assumption that the data generating process is the same on each constituting time scale. Furthermore, in standard econometric estimation techniques non-stationary features of the data are removed prior to the econometric analysis resulting in the known problem that relationships seem to change in times of financial distress. We therefore apply wavelet analysis to decompose the data into their respective time scales and perform the analysis on a scale-by-scale basis. In doing so, we do not have to remove extreme values prior to our analysis. The model remains applicable in times of financial market distress.

Only recently researchers applied wavelet analysis to macro-economic and financial theories (see Ramsey and Lampart, 1996; Kiermeier (1998), Kim and Haueck In (2003), Raihan at al. (2005), Gallegati et al. (2006), Gencay et al. (2009) among others).

The paper is organized as follows. The following section presents the Nelson-Siegel Term Structure Models and explains how this model is extended to an arbitrage-free setting. The subsequent section motivates the use of the Fama and McBeth approach. Furthermore, wavelet analysis is described, and it is pointed out how to decompose the explanatory variables into their respective components on different time scales. Section with the empirical analysis presents data and the estimation and forecasting results.

Nelson-Siegel term structure models

The Nelson-Siegel model (Nelson and Siegel, 1987) approximates the actual yield curve observed in the market on any specific date t for zero rates y with maturity τ through the following equation:

$$y_t(\tau) = \beta_{0t} + \beta_{1t}\left(\frac{1-e^{-\gamma\tau}}{\gamma\tau}\right) + \beta_{2t}\left(\frac{1-e^{-\gamma\tau}}{\gamma\tau} - e^{-\gamma\tau}\right) \qquad (1)$$

$\beta_{0t}, \beta_{1t}, \beta_{2t}, \gamma$ are model parameters, Nelson and Siegel (1987).

Diebold and Li (2006) are able to interpret the parameters in the Nelson-Siegel model. The respective β_i`s can be viewed as dynamic factors that represent short-, medium-, and long term behaviour. They identify the factors, level (β_0), slope (β_1) and curvature (β_2), γ being the mean reversion rate.

The Dynamic Generalized Nelson-Siegel (DGNS) model is introduced by Christensen et al. (2008) to embed the above approach in an arbitrage free setting. In order to ensure the absence of arbitrage five factors are needed.

$$y_t(\tau) = \beta_{0t} + \beta_{1t}\left(\frac{1-e^{-\gamma_1\tau}}{\gamma_1\tau}\right) + \beta_{2t}\left(\frac{1-e^{-\gamma_2\tau}}{\gamma_2\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\gamma_1\tau}}{\gamma_1\tau} - e^{-\gamma_1\tau}\right) + \ldots$$
$$+ \beta_{4t}\left(\frac{1-e^{-\gamma_2\tau}}{\gamma_2\tau} - e^{-\gamma_2\tau}\right) \qquad (2)$$

As in the previous models, the authors assume that the data generating process for this model is a first-order autoregressive process.
The above models increase the number of explanatory, risk factors according to theoretical considerations. In our analysis, we focus on the question if observable market data supports the idea of the five factors being significant risk factors in the determination of term structures of interest rates.

In the next section we outline the different steps of the empirical analysis.

Identification of significant risk factors for the term structure of interest rates

In our macroeconomic factor model observable, explanatory variables are selected according to the above theoretical foundations. In accordance with Christensen et al. (2008) five variables (one level, two slope, and two curvature variables) are chosen to explain and forecast the term structure.

We use the identified k factors and assume an approximate factor structure to hold (see Chamberlain and Rothschild (1983)). It follows that interest rates can then be expressed in the following way:

$$\underset{(nx1)}{y_t} = \underset{(nxk)}{B} * \underset{(kx1)}{\lambda} + \underset{(nx1)}{\xi(t)} \qquad (3)$$

y_t is a vector that comprises all Zero Rates $y(\tau)$ with maturity τ ($1 \leq \tau \leq n$) at time t. $\xi(t)$ is assumed to be normally distributed with $E(\xi(t)) = 0$. B is the matrix of sensitivities with regards to the k risk factors β_j ($1 \leq j \leq k$), λ`s are the model parameters of interest rates with regards to the k risk factors. Fama and McBeth (1973) show that the t-test for the sample mean of the time series of estimated coefficients for $\lambda_t$ of the cross section regressions can be used to determine factors` significance as explanatory risk factors.
In a following step equation 3 is used to estimate the λ`s in a cross section regression which we run for every point t = 1,…,T. The OLS estimator $\hat{\lambda}(t)$ at time t is then given by equation (4) where the estimated scaled eigenvectors of the factor analysis are $\hat{B}_k$.

$$\hat{\lambda}(t) = (\hat{B}_k^{'} \hat{B}_k)^{-1} \hat{B}_k^{'} y_t \qquad (4)$$

This procedure results in a time series of estimated coefficients $\hat{\lambda}(t)$ which is used to construct a t-distributed test statistic with T-1 degrees of freedom. The statistic allows us to test the hypothesis, if the estimated coefficients are significantly different from zero, see Fama and McBeth (1973).

$$t(\bar{\lambda}_k) = \frac{\bar{\lambda}_k T^{\frac{1}{2}}}{s(\hat{\lambda}_k)} \qquad (5)$$

Where, T - numbers of observations; $\bar{\lambda}_k$ - arithmetic mean of $\hat{\lambda}_{k,t}$; $s(\bar{\lambda}_k)$ - standard deviation of the monthly estimates $\hat{\lambda}_{k,t}$.

This procedure allows us to determine the significance of explanatory variables although model parameters may not be constant over time.

In a next step, we apply wavelet analysis to decompose the identified risk factors in their time-scale components and test whether the significance can be detected on every time scale. Furthermore, it will allow us to distinguish expected and unexpected components of the risk factors.

Wavelet analysis

Wavelets decompose data to different scales which allows analyzing economic relationships at different time horizons (resolutions). With the time-scale analysis we can extract characteristics of the time series that have a limited effect over time and thereby allow for non-stationarities in the data. Wavelet analysis (i.e. time-frequency analysis) exhibits frequency characteristics of time series at a given point in time.

A wavelet is a function that oscillates around zero over a finite support on the time axis. Wavelet analysis analyzes the time series at different frequencies that have varying support on the time line, i.e. varying resolutions. In the multiresolution analysis wavelets are dilated, compressed, and shifted in a systematic way to extract features for all frequencies localized on the time axis. For a thorough comparison of the two methods see Gallegati et al. (2011). A general review of wavelet analysis is given by Chui (1992).

Wavelet filters are a short sequence of values, the number of values represents the width of the wavelet filter. $\{h_l\}$ denotes a wavelet filter, where h_l is called the respective wavelet coefficient. The set of all filter coefficients is the impulse response sequence. The scaling filter $\{g_l\}$ combined with the wavelet filter $\{h_l\}$ map a time series from the time domain to a time-scale domain. This is done by a procedure called the discrete wavelet transform (DWT). Let dj and sj be the set of wavelet and scaling coefficients for a given level j. Then the DWT consists of $[d_1,d_2,\ldots,d_J,s_J]$. A crystal is a set of coefficients at a level j. High level coefficients are capturing information in a time series at high frequencies, whereas the scaling coefficients capture the very lowest frequency information of the time series.

The time series variation can be approximated by a relatively small number of wavelet functions, i.e. wavelets serve to compress the data with respect to specific resolution levels. The theory of approximation by translates relates the accuracy of approximation to the Fourier transforms of the approximating function, i.e. the scaling function. For more details see Strang and Fix (1973).

The discrete wavelet transform (DWT) calculates the coefficients of a time series approximated by wavelets. It maps the vector f onto a set of n wavelet coefficients and uses a fast pyramid algorithm. The pyramid algorithm involves use of low-pass and high-pass filters, along with down-sampling or up-sampling algorithms.

The aim is to decompose the time series $\beta(t)$ into its components at different scales (levels), indexed by j. At each new level the support h is cut in half and the number of wavelet coefficients is doubled. In our analysis we use a non-decimated form of DWT the maximum overlap DWT (MODWT) to allow for any length of time series and to get more robust estimators independent of when we start the observations for our time series.

A wavelet multiplied with its respective coefficients is called an atom (i.e. $d_{j,k}*\psi_{j,k}$ and $s_{J,k}\varphi_{J,k}$ $j=1,\ldots,J$ and $k = 1,\ldots, n$) with $\psi_{j,k}$ and $\varphi_{J,k}$ being the wavelet and scaling functions at level j and J respectively.

The coefficients for a given scale are defined to be a crystal. Let $s_J = \left(s_{J,1} s_{J,2}, s_{J,3}, ..., s_{J,\frac{n}{2^J}} \right)$ and $d_j = \left(d_{j,1}, d_{j,2}, d_{j,3}, ..., d_{j,\frac{n}{2^j}} \right) \forall j = 1,..., J$ then a crystal can be described as (s_J d_J) and d_j $j \neq J$.

Each wavelet and scaling coefficient is localized in time. In order to localize them correctly we use symmlet wavelets which are linear phase filters, so that the coefficients can be shifted to align them with the time of occurrence of the features of the time series.

A signal can be approximated using only parts of the coefficients and their respective wavelets. For this purpose the multiresolution decomposition of a time series β(t) is defined in the following way:

$$\beta_i(t) = S_J(t) + D_J(t) + D_{J-1}(t) + ... + D_1(t) \qquad (6)$$

With

$$S_J(t) = \sum_{k=1}^{\frac{n}{2^J}} s_{J,k} \varphi_{J,k} \qquad (7)$$

$$D_j(t) = \sum_{k=1}^{\frac{n}{2^j}} d_{j,k} \psi_{j,k} \forall j = 1,..., J \qquad (8)$$

We decompose the data of the identified risk premium to analyze the relationship between interest rates and the risk factors on a scale by scale matrix.

Empirical analysis

Data

In our empirical analysis we use European zero coupon curves estimated by ICAP which are available in the Thompson/Reuter Datastream Database. Zero rates with a maturity less than 6 month are interest rates directly observable in the money markets. Forward Rate Agreements are used to derive zero rates with maturities seven to eleven months. From 12 months onwards swap rates serve to calculate the zero rates. A term structure of zero rates at time t (t=1,…,T) consists of monthly zero rates with maturities that range between one month and 600 months (i.e. 50 years). Each term structure of interest rates at any point in time t therefore consists of 600 observations. The term structure of interest rates is calculated as of the beginning of a month for the time period May 2002 to September 2010. The data in the years 2008 to 2010 demonstrate the impacts of the last financial markets' crisis. All in all, we have 101 monthly observations of term structures where each consists of 600 observations representing maturities ranging from one month to 50 years.

As macroeconomic factors we define one level variable, three slope variables and three curvature variables. To capture the level we use the 10-year-interest rate (Level). To calculate slopes we choose variables that account for short term (Slope1), medium term (Slope3), and long term (Slope2) behaviour. The difference in interest rates which accounts for the short term behaviour is calculated from the 10-year interest rate minus the 1-year interest rate. The medium term slope is calculated by subtracting the 10 year interest rate from the 30 year interest rate. The long term slope behaviour is the difference between 50 years and 30 years interest rates. The variables that represent characteristic of the curvature of the term structure are calculated by subtracting the one and ten year

interest rates from the five year rate (Curve1). Reducing the 40 year rate with 30 and 50 year rates (Curve2), and the 20 year rate with 10 and 30 years (Curve3) capture long term and medium term behaviour of the curvature of the term structure respectively.

Estimation results

In order to follow the reasoning put forward by Christensen et al. (2008) we include five variables in the cross section regressions to test for the significance of risk factors. We apply the Fama and McBeth t-test to test for the significance of the risk factors.
The variables Level, Slope1, Slope2, Slope3, Curve1, Curve2, and Curve3 are used as risk factors. The factor sensitivities are estimated for the time period from May 2002 to December 2006. To avoid problems arising from multi-collinearity we include the variables Level, slope1, slope3, curve1, curve2 and a constant in the regressions for the time period May 2002 to December 2006 that result in an estimation of the sensitivities with regards to potential risk factors. The factor sensitivities are then used in cross sectional regressions for each point in time for the time period January 2007 to September 2010. According to Fama and McBeth (1973) we calculate the relevant t-statistics for the means of the estimated coefficients which are summarised in Table 1.

TABLE 1. FAMA AND MCBETH T-STATISTICS FOR THE MEANS OF ESTIMATED COEFFICIENTS (JAN 2007-SEP. 2010)

	Level	Slope1	Slope3	Curve1	Curve2
Mean	1.19	0.05	1.86	1.24	5.16
SE of Mean	1.16	0.23	0.90	1.05	1.23
Fama McBeth t-stat	6.88	1.55	13.78	7.90	28.07
t-prob	0.00	0.13	0.00	0.00	0.00

We find that the variables Level, Slope3, Curve1, and Curve2 are evaluated by the market. Slope1 does not contain information that is viewed to be significant at the usual confidence levels.

We question if risk factors are correctly identified through this procedure because it implicitly assumes that the factors` influence is the same on each time scale. We therefore decompose the data of the significant risk factors for two purposes. First we test whether every time-scale component of the factor can be viewed as being significant. Second, we use the results from the regressions on the different time scales to distinguish expected and unexpected components in the data.

The maximum overlap discrete wavelet transform of the explanatory variables are performed using three levels (j) and the least asymmetric (symmlet) wavelet functions. These filters have the same gain functions as other wavelets and are smoother and more symmetric.

We find that the broadest and finest scales (scaling function at level three) explain close to 98% of the time series variations.

The results from the discrete wavelet transform of the explanatory variables Level, Slope1, Slope3, Curve1, and Curve2 are summarized in Table 2. We indicate the impact of the most important atoms, and the number of atoms that are required to explain a certain level of variation in the time series.

TABLE 2. EXPLANATORY POWER OF DISCRETE WAVELET TRANSFORMS

Explained Variance (in %) of the Explanatory Variables by number of Wavelets for the Time Period May 02 to Dec.06							
Number of wavelets	1	2	3	6	9	12	15
Level	20.5	35.2	49.9	89.1	99.8	99.9	99.9
Slope1	23.3	46.3	62.7	93.1	99.6	99.8	99.9
Slope3	31.0	52.3	69.4	95.8	98.5	99.1	99.4
Curve1	21.8	39.6	54.1	87.3	99.9	99.9	100.0
Curve2	17.5	33.6	49.5	88.8	99.9	99.9	100.0

For the purpose of extracting markets expectations we analyze the percentage of variation that can be explained by wavelets including their respective coefficients at specific levels j, i.e. the crystals at level j (see Table 3).

TABLE 3. EXPLANATORY POWER OF DISCRETE WAVELET TRANSFORMS BY CRYSTALS

Explained Variance (in %) of the Explanatory Variables by Crystals (DWT) for the Time Period May 02 to Dec.06					
	Level	Slope1	Slope3	Curve1	Curve2
Crystal d1	0.05	0.16	0.40	0.03	0.03
Crystal d2	0.08	0.10	0.74	0.05	0.05
Crystal d3	0.10	0.29	0.67	0.05	0.07
Crystal s3	97.74	97.81	97.38	97.70	97.97

To determine whether the risk factors are significant on every time scale and not only on an aggregate level, we test for the factors` significance on every time scale.

The following Table 4 summarizes the significance of the time scales d1 to s3 of the risk factors indicated by the FamaMcBeth t-statistic.

TABLE 4. SIGNIFICANCE OF THE TIME SCALES D1-S3 OF THE RISK FACTORS (2007-2010)

Fama McBeth t-statistic	Level	Slope1	Slope3	Curve1	Curve2
d1					
FMcBeth t-stat	-10.42	-23.28	-22.31	9.59	84.94
t-prob	0.00	0.00	0.00	0.00	0.00
d2					
FMcBeth t-stat	-2.22	-7.98	-8.09	2.22	38.93
t-prob	0.03	0.00	0.00	0.03	0.00
d3					
FMcBeth t-stat	-5.08	7.28	-4.16	0.57	38.93
t-prob	0.00	0.00	0.00	0.57	0.00
s3					
FMcBeth t-stat	-2.27	-1.17	-7.35	-2.21	38.97
t-prob	0.03	0.25	0.00	0.03	0.00

The results show that the significance varies according to different time scales. On scale d1 every factor is significantly evaluated by the market. On scale d2 only the Variables Slope1, and Slope3 remain important. On scale d3 Curve1 cannot be considered a significant risk factor. Last but not least on scale s3 significant factors are Slope3 and

Curve2 only. From this we deduce that averaging over different time scales for the determination of significant risk factors is at best questionable.

By reconstructing the time series using the significant time scales we can concentrate on a relatively small number of wavelet functions. We distinguish the expected and unexpected components. We assume that the unexpected variation cannot be forecasted, i.e. is "news". In order to model market expectations for the explanatory variables we reconstruct the series using the scaling and wavelet functions of level one and three. We extract the part of the explanatory variables that is expected by the market by removing crystals with low explanatory power. We find that with the exception of Slope1 the variables can be considered significantly evaluated by the market.

In a next step we test the model for its ability to forecast the term structure of interest rates. The best forecasting results are achieved for the one to four month forecasts of the term structure of interest rates. During the beginning of the last financial market crisis (September 07 to June 08) the forecast quality for five months and higher become low, the short term forecasts however remain good.

Conclusion

In this analysis we summarise approaches to model and forecast term structure of interest rates. Christensen et al. (2008) generalize the Nelson-Siegel approach and identify five factors. Due to financial market crisis we do not make the assumption of parameter constancy in the underlying model, therefore we apply the Fama and Mc Beth t-statistic to test the explanatory variables` for their significance so that they can be interpreted as risk factors evaluated by the market. The Fama and McBeth t- test for significance of the pre-specified macro-economic variables indicate that a maximum of four variables are evaluated by the market as risk factors. Since econometric analyses can be misleading if the data generating process is different on various time scales, we test whether the significance can be confirmed on every time scale. We find that on various scales different variables are tested to be significant in the determination of the term structure. We therefore decompose the explanatory variables in their expected and unexpected components. By doing so, we assume that market participants form their expectations according to the coarsest and finest time scale of the significant risk factors. To generate out of sample forecasts for one to twelve months, we forecast the explanatory variables with ARIMA models. The estimation and forecasting has been repeated monthly for four years, data of financial market distress is included in the analysis. The best forecast quality is achieved for term structures to manifest in one to four months. The one month forecasts remains of good quality even during the time period of financial market crises.

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