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DEBT SERVICE RESERVE FUND (DSRF) AS A RESPONSE TO REPAYMENT RISK[1]

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Abstract

Debt Service Reserve Fund (DSRF) plan, proposed by Baker (1976), has appeal to both the borrower and the lender as a response to repayment risk. This plan proposes that the borrower and lender establish a pool of liquidity for the exclusive purpose of debt service. For implementation of this plan, DSRF size and other design specifications must be acceptable to both parties. Theoretical models are developed to determine optimal DSRF sizes for the lender and the borrower. The application potential of the model is demonstrated in real lending situations. The sensitivity of the optimal DSRF sizes is also investigated with regard to distributional assumptions made on the stochastic returns generated from loan usage.

DEBT SERVICE RESERVE FUND (DSRF) AS A RESPONSE TO REPAYMENT RISK

Introduction

Any loan contract between a borrower and a lender involves repayment risk to both parties. Even though the return from the project being financed is expected to exceed the contractual repayment amount, some risk of loan default still remains. Loan default or delinquency has an adverse effect on both the lender and the borrower. It reduces the creditworthiness of the borrower and may result in forfeiting the property that has been pledged as security against the loan. In addition, the borrower must pay any penalties the lending agency imposes as a result of loan delinquency. The lender, on the other hand, is directly affected through reduced liquidity and potential loan losses if the secured assets do not completely recover the loan.

Repayment risk and its serious consequences for borrowers and lenders were demonstrated during the recent stress in the farm sector of the United States. Repayment problems due to declining farm incomes caused many farm borrowers to lose their properties held as security against loans. In addition, lending agencies experienced loan security and collateral problems, as well as mounting loan losses and significant deterioration of credit quality.

Possible ways of reducing the lender's risk of loan losses include diversifying the loan portfolio, widening the geographic area of operation, seeking additional collateral, and reducing the availability of loan funds. Similarly, borrowers can reduce default risk by combining different enterprises in their production plan, participating in government programs, utilizing insurance, hedging, improved marketing strategies and other types of risk management practices. However, these measures are not practical to all the lenders and borrowers, especially to specialized lending institutions (e.g. Farm Credit System) and small family farms specializing in one or two enterprises. Hence, new measures to reduce repayment risk must be explored.

Lenders' responses to default risk on real estate and non real estate loans differ. Since lenders tend to monitor non real estate loans more closely, mortgage loans often play a dominant role in default risk. Real estate loans are generally large, and more importantly, the repayment obligations are totally invariant to changes in the borrower's cash flow under traditional amortization plans. Numerous plans have been proposed to reduce repayment risk on mortgage loans. These include a flexible payment plan (Lee, 1979) and a variable amortization plan (Baker, 1976). Prior research has demonstrated the potential effectiveness of these repayment plans (Stone; Aukes). One version of the variable amortization plan that has appeal to both the borrower and lender is a Debt Service Reserve Fund (DSRF). This plan, however, is still subject to many design specifications such as the initial size of the DSRF, how it is established, how it should be used, and its final disposition.

The objective of this paper is to show how the size of a DSRF that is optimal to lender and borrower can be determined. The paper is organized as follows: literature about different repayment plans is first reviewed, followed by operational details of a DSRF plan. Theoretical models to determine the optimal sizes of a DSRF are then developed for the lender and borrower and their comparative statics results presented. The optimal size of a DSRF for the lender and borrower may differ. However, the size of a DSRF must be acceptable to both the lender and borrower before it can be implemented. The paper concludes by discussing how the difference in the optimal sizes of a DSRF for the two parties can be reconciled.

REVIEW OF LITERATURE

Numerous proposals have been made to design loan repayment plans that are flexible enough to accomodate uncertainty in the farmer's cash flow. A flexible payment plan (FPP) proposed by Lee (1979) allows the borrower to reschedule payments by reamortizing the loan balance whenever repayment problems occur. This privilege, however, is limited to only a fixed number of years depending upon the maximum length of loan. The limit allows lenders to take timely action on loans with serious default problems. FPP benefits both the borrower and lender in terms of reducing administrative costs for minor default problems. However, a major disadvantage of this plan is that borrowers have an incentive to default on loans whenever market interest rates on nonreal estate loans exceed the loan rate. Moreover, lenders still face liquidity risk in their availability of loanable funds through uncertainties in loan repayment.

A variable amortization plan (VAP), proposed by Baker (1976), calls for a flexible loan payment plan that is tied to an index of changes in factors affecting cash flow. Crop and/or livestock yields, prices received or paid and other variables affecting cash flow may be used in the index. This plan is appealing to borrowers. However, because of fluctuating periodic payments, it is less acceptable to the lenders.

One version of a variable amortization plan that has appeal to both borrowers and lenders is called the Debt Service Reserve Fund (DSRF) plan (Stone; Aukes; Kent and Lloyd). It is a version of VAP with a debt reserve to stabilize repayment flows to lenders. This plan proposes that the borrower and lender establish a pool of liquidity exclusively for the purpose of debt service. The operational details of the DSRF plan are in Baker (1986). This plan theoretically works as follows:

(1) The DSRF establishes an initial reserve fund as a part of the loan disbursement. Hence, the total loan approved also includes funds for the DSRF. However, the additional cost to the borrower for the increased amount of the loan is offset by the lender by paying the same rate of return on the DSRF as is charged for the loan. Hence the borrower experiences no direct costs in

maintaining this fund.

(2) In periods when realized returns are less than the contractual repayment, the DSRF can be used to meet the obligations. In this case, however, the borrower must pay interest on the amount of the DSRF used. Similarly, the reserve must be replenished during periods of higher returns.

(3) Terms of the loan contract require the borrower to restore the DSRF to its original level with interest by the end of repayment period.

Few studies have evaluated the potential of a DSRF plan. Stone (1976) compared programmed outcomes for a fixed amortized payment (FAP) plan and a DSRF plan for cash grain farmers in Illinois who financed land purchases with Federal Land Bank loans in 1975. The study found the DSRF plan to reduce the probability of default as well as increase the average level of the borrower's cash flow. In addition, it also increased the borrower's non-real estate credit reserves.

Aukes (1980) compared financial outcomes for a sample of Farmers Home Administration Farm Ownership (FO) borrowers who are not qualified for loans from commercial lenders. A modelled farm was stochastically simulated for ten years under both a FAP and a DSRF plan for a wide range of scenarios including price conditions and beginning liquidity. Crop yields and prices were chosen randomly from specified distributions. The study found that the DSRF plan contributed to a higher current ratio in all scenarios and hence reduced the amount of default. In addition, it was also found to result in higher equity growth rates than FAP.

Similarly, Kent and Lloyd (1983) evaluated the effectiveness of a version of the DSRF plan in reducing the probability of default. The study was based on the outcomes of a simulated experiment in which a borrower was required to deposit three-fourths of his disposable cash surplus in a reserve until it reached three times the annual repayment amount. In addition, the borrower had the option of using additional loans to meet annual amortization payments on the farm mortgage loans in each of the first three years. The simulated outcomes suggested that the DSRF plan could reduce the probability of default by one-third.

The DSRF plan seems appealing to both the borrower and lender and has the potential to absorb some repayment risks, as evidenced by the empirical studies reported above. The borrower benefits because it provides an additional source of liquidity to meet repayment obligations, and thereby protects him/her from lender sanctions in response to loan default. This plan also has the potential to stabilize the repayment flows, and in turn reduce the liquidity risk of the lender.

The size of the DSRF is expected to be positively related to the level of risk (perhaps measured by variance of returns from the project financed), and the amount of contractual repayment. The attitudes of the borrower and lender towards default risk could have important influences on the size of an acceptable DSRF because of their differing risk positions in the loan contract. The lender will emphasize loan repayment and safety more than the borrower's expected profitability because the lender does not share directly in the borrower's profits. The borrower, on the other hand, may emphasize expected profitability which generally is the main objective for undertaking business.

LENDER'S OPTIMAL SIZE OF DSRF

Assume a lending agency is considering making a loan, L, to a borrower for a proposed production plan. The return, Y, from the production plan is stochastic and has some distribution. For simplicity, the theoretical model for finding the optimal size of the DSRF is first developed assuming Y is normally distributed. The same methodology is later extended to develop theoretical models under other distribution assumptions.

Assume Y is normally distributed with mean, M, and variance, V. For simplicity, let the loan mature in two periods, and be repaid in two constant amortized repayments, R_1 and R_2 in the first and second periods, respectively. These repayments can be regarded as revenue to the lending agency.

In general, the amount R_1 is much smaller than M. However, because of the probability that realized returns are less than R_1, the contingent repayment in the first period, $(R_1(Y))$, as a function of the stochastic return, Y, is given as (1):

$$R_1(Y) = \begin{cases} R_1 & \text{if } Y > R_1 \\ Y & \text{if } 0 < Y < R_1 \\ 0 & \text{if } Y < 0 \end{cases} \qquad (1)$$

Hence there is a risk that the lender will not receive repayment equal to R_1. The proposed DSRF plan helps reduce this risk by increasing the contingent repayment amount. Under the DSRF, the contingent repayment amount in the first period, ($R_1(Y,D)$), as a function of Y and size of DSRF, D, is given as (2):

$$R_1(Y,D) = \begin{cases} R_1 & \text{if } Y > R_1 - D \\ Y + D & \text{if } 0 < Y < R_1 - D \\ D & \text{if } Y < 0 \end{cases} \qquad (2)$$

The terms of the loan contract do not allow use of the DSRF to meet the loan obligation in the second period. On the contrary, the borrower must replenish any amount drawn in the first period, together with interest, in the second period. As a result, the DSRF plan provides a source of liquidity only in the first period. The lender, on the other hand, can invest DSRF funds in relatively liquid, interest bearing assets until the borrower needs to draw upon the reserve. The rates of return on these assets, however, will be less than the interest rate the lender would charge the borrower if the DSRF is drawn.

In the event of a loan default or delinquency, the lender may impose a penalty on the borrower as discussed later in the borrower's optimal size of DSRF section. The penalty, therefore, should also constitute revenue to the lender. However, the loan default also contributes to lender liquidity problem. In this paper, the lender is assumed to be more concerned with reducing the liquidity risk than the revenue from penalties on loan default. Hence, the expected discounted revenue to the lending institution with the DSRF, ER(Y,D), can be written as (3):

$$ER(Y,D) = \int_{R_1-D}^{\infty} R_1\ f(Y)\ dY + \int_{0}^{R_1-D} f(Y)\ dY + \int_{-\infty}^{0} D\ f(Y)\ dY + D.b + \qquad (3)$$

$$r^{-1} \int_{R_1-D}^{R_1} (R_1-Y)\ (i-b)\ f(Y)\ dY + r^{-1} \int_{-\infty}^{R_1-D} D(i-b)\ f(Y)\ dY$$

Where, f(Y) is the probability density function of the stochastic returns ,Y; b is the rate of return on the DSRF invested in liquid assets; r is the factor used by the lender in discounting expected revenue in the second period to the first period; i is the rate of interest on the amount of the DSRF drawn by the borrower, and other variables are as defined above.

The first part of (3) represents the revenue in the first period when the returns together with the DSRF are sufficient to pay R_1. The second and third parts represent the revenue when the returns together with the DSRF are not enough to pay R_1. The fourth term represents net revenue from investment of the DSRF in liquid assets. Finally, the fifth and sixth terms represent additional interest income in the second period from the borrower's use of the DSRF in the first period conditional on Y being less than R_1 in first period.

In the literature, some researchers (Stiglitz and Weiss; Boyes, Hoffman and Low) assume lenders are risk neutral while others assume they are risk averse (Arvan and Bruekner; Robison and Barry; Barnard and Barry). In this paper, the theoretical model for the optimal DSRF is developed under the assumption of risk neutral lenders.

The utility function of a risk neutral lender is linear. The expected utility from an uncertain prospect is, therefore, equal to the expected profit. Hence, first order conditions for determining the optimal size of the DSRF for the lending agency requires that the expected marginal revenue be equal to the marginal cost. Expected marginal revenue is obtained by differentiating (3) with respect to D.

$$\delta ER(Y,D)/\delta D = \delta/\delta D \int_{R_1-D}^{\infty} R_1 \ f(Y)dY + \delta/\delta D \int_{0}^{R_1-D} (Y+D) \ f(Y) \ dY + \qquad (4)$$

$$\delta/\delta D \int_{-\infty}^{0} D \ f(Y)dY + D.b + \delta/\delta D \ r^{-1} \int_{R_1-D}^{R_1} (R_1-Y)(i-b) \ f(Y)dY$$

$$+ \ \delta/\delta D \ r^{-1} \int_{-\infty}^{R_1-D} D(i-b) \ f(Y)dY$$

Using Leibniz's rule to write out the derivatives of the intergrals, this becomes:

$$\delta ER(Y,D)/\delta D = R_1 \ f(R_1-D) - R_1 \ f(R_1-D) + \int_{0}^{R_1-D} f(Y)dY + \int_{-\infty}^{0} f(Y)dY \qquad (5)$$

$$+ \ b + r^{-1} \ (i-b) \int_{-\infty}^{R_1-D} f(Y)dY$$

Finally, simplifying terms, the expected marginal revenue is:

$$\delta ER(Y,D)/\delta D = [1+r^{-1} \ (i-b)] \int_{-\infty}^{R_1-D} f(Y)dY + b \qquad (6)$$

The right hand side of equation (6) expresses the probability of the stochastic variable, Y, being less than (R_1-D) {i.e. Probability[$Y<(R_1-D)$]}. This is just the distribution function, $F(R_1-D)$. Since Y is assumed to be normal, it can be transformed into a standard normal random variable by subtracting the mean, M, and dividing by the standard deviation, s, of Y to obtain $z =((Y-M)/s)$. Then $P(Y<(R_1-D)) = P(z<(R_1-D-M)/s)$ is the probability that z will be less than $((R_1 - D - M)/s)$ and is given by $\Phi((R_1\text{-}D\text{-})/s)$, the area under the standard normal curve up to $(R_1\text{-}D\text{-}M)/s)$.

Substituting $F(R_1\text{-}D) = \Phi((R_1\text{-}D\text{-}M)/s)$ into eq. (6) results in (7):

$$\delta ER(Y,D)/\delta D = [1 + r^{-1} (i\text{-}b)] \Phi(R_1 - D - M)/s] + b \tag{7}$$

The costs to the lending agency of implementing the DSRF plan come mainly from two sources - (1) The rate of return on the DSRF invested in liquid assets is lower than the interest rate a lender could earn from other investment alternatives. Hence, the lender has to forego an interest rate differential with the DSRF plan; (2) Administrative costs (AC) involved in establishing, maintaining and operating this plan. Assuming a lender allocates administrative costs in proportion to loan volume, which is typical in agricultural lending (Barry and Calvert), the total cost to the lender of the DSRF plan is given by (8):

$$\text{Total Cost} = D(i - b) + AC \tag{8}$$

The interest differential, $(i - b)$, constitutes the marginal cost of the DSRF plan to the lending agency. So for a given interest rate differential, the optimal size of the DSRF for the lending agency can be determined by equating expected marginal revenue to marginal cost:

$$[1+r^{-1} (i\text{-}b)] \Phi((R_1 - D - M)/s) + b = (i\text{-}b) \tag{9}$$

After some algebraic manipulation, equation (10) expresses a lender's optimal size of DSRF as a function of R1, M, s, r and $(i - b)$.

$$D = R_1 - M - s \Phi^{-1} [((i\text{-}b) -b)/(1 + r^{-1} (i\text{-}b))] \tag{10}$$

Some insight into the DSRF size implied by this formula can be gained from the following simple example. Assume a borrower is applying for a loan of \$40,000 for investing in a production plan that yields an expected rate of return of 20 percent. This implies a mean return, M, of \$48,000 from the production plan. Similarly, if the lender charges interest rate, i, of 12 percent, the contractual repayment amount, R_1, equals \$44,800 [40,000(1+0.12)]. For a set of additional values like b = 0.04, r = 1.10, and s = 4800, equation (10) results in the lender optimal size of DSRF of \$5344.00. Thus, the optimal debt reserve of \$5344.00 is large enough to cover a shortfall in the borrower's mean cash flow of upto 1.11 standard deviations which has a 36.65 percent chance of occuring. Moreover, if the realized return is higher than the mean, the borrower can meet his loan obligation. Since, this occurs with the probability of 0.5, the provision of \$5,344.00 as a DSRF ensures that the lender has 86.65 percent chance of receiving the contractual repayment.

The comparative statics properties of equation (10) are straightforward and are presented in (11), (12), (13), (14) and (15).

$$\delta D/\delta R_1 = 1 > 0 \tag{11}$$

$$\delta D/\delta s = -\Phi^{-1}[((i-b)-b)/(1+r^{-1}(i-b))] > 0 \text{ because} \tag{12}$$

$$\Phi^{-1}[((i-b)-b)/(1+r^{-1}(i-b))] < 0 \text{ for } [.] < 1/2$$

$$\delta D/\delta M = -1 < 0 \tag{13}$$

$$\delta D/\delta(i-b) = [-s\Phi^{-1'}(.)][\{1+r^{-1}(b)\}/\{1+r^{-1}(i-b)\}^2] < 0 \tag{14}$$

$$\delta D/\delta r = [-s\Phi^{-1'}(.)][\{r^{-2}(i-b)(i-2b)\}/\{1+r^{-1}(i-b)\}^{2]} < 0 \tag{15}$$

The comparative statics results indicate that the size of DSRF should increase with an increase in the contractual repayment amount and riskiness (variance) of returns of the project. Similarly, the size of the DSRF should decrease with increases in the interest rate differential, the discount rate and the expected value of the returns.

A higher interest rate differential makes it more costly for the lending agency to maintain the DSRF because of increased opportunity costs of the DSRF amount. In addition, a lender with a higher discount factor (i.e. who attaches more importance to present income than future income) would decrease the size of the DSRF. These results conform with apriori expectations.

BORROWER'S OPTIMAL SIZE OF DSRF

Suppose a borrower has decided to invest in a production plan that yields an expected rate of return x. Of the total investment of W dollars needed for investment, he/she can personally finance H dollars out of equity. The remaining amount, L(=W-H), can be borrowed from a financial institution at a rate of interest of i. For simplicity, assume the production plan extends for two periods and the loan amount, L, must be repaid with interest in two constant amortized repayments, R_1 and R_2.

The rate of return from the production plan at each period depends upon the level of output produced and the price received for output. Both of these variables are stochastic at the time the decision is made. However, for a given expected rate of return of x (higher than i), the total revenue, T, at each period is given as:

$$T = H(1+x) + L(1+x) \tag{16}$$

where, H = Amount of equity financing

L = Amount of debt financing

Out of total revenue, T, the amount equivalent to R1(=R2) is needed for loan repayment each period and the balance for meeting other obligations including the opportunity cost on the equity capital.

Because the rate of return is stochastic, realized total revenue, Y, may assume any value in the domain of the distribution. Consequently, loan repayment, R(Y), as a function of Y is given as (17)

$$R(Y) = \begin{cases} R_1 & \text{if } Y > R_1 \\ Y & \text{if } 0 < Y < R_1 \\ 0 & \text{if } Y < 0 \end{cases} \tag{17}$$

If the realized total revenue is greater than or equal to the expected revenue in (16), the borrower will meet the loan obligation. However, if Y is less than its expected level, the borrower faces liquidity problems. If Y is less than the expected level but higher than R_1, he/she can still meet debt obligation and, hence, retain access to his unused line of credit for meeting other obligations. If Y is less than the repayment obligation, the borrower defaults on the loan. Therefore, the probability of loan default is given by the cumulative distribution function, $F(Y<R_1)$.

Parallel to the lender case, assuming Y is normally distributed, it can be transformed into a standard normal variable, z $(=[(R_1 - M)/s]$. The probability that z will be less than $[(R_1 - M)/s]$ is then given by the area under the standard normal curve up to $[(R_1 - M)/s]$.

In the event of a loan default or delinquency, a borrower is subject to lender sanctioned penalties. The nature and size of the actual penalty is an empirical question. Nonetheless, a penalty may include one or more of the following:

(a) A flat fee for defaulting on the loan;

(b) A higher interest rate on the delinquent amount;

(c) Stricter loan terms including higher interest rates on new loans, because of increased credit risk;

(d) A higher interest rate on existing loans if a differential loan pricing program is in effect; and

(e) In an extreme case, the borrower may forfeit the collateral pledged against the loan.

Here, the penalty is modeled as the net present value, P, of the cost of the penalty provisions. The expected penalty on a defaulted loan, EP(Y), is given as the product of the penalty, P, and the probability of loan default (18).

$$EP(Y) = P * \Phi\ [(R_1 - M)/s] \tag{18}$$

The expected penalty can be reduced by raising (lowering) the probability of loan repayment (default). Because a DSRF supplements realized revenue in meeting debt-service requirements, it helps reduce the probability of loan default and is given by cdf, $F(Y<(R_1 - D)$. The borrower, however, must pay interest on the use of the DSRF. As a result, following the standard normal transformation, the expected penalty with a DSRF plan, EP(Y,D), is given as (19).

$$EP(Y,D) = P * \Phi[(R_1 - D - M)/s] - r^{-1}iD\, \Phi[(R_1-M)/s] \qquad (19)$$

The reduction in the amount of the expected penalty constitutes the benefit of the DSRF plan, B, to the borrower and is given as (20)

$$\begin{aligned} B &= EP(Y) - EP(Y,D) \qquad (20) \\ &= P* \Phi[(R_1-M)/s] - \{P* \Phi[(R_1-D-M)/s] \\ &\quad - r^{-1}\, iD\Phi[(R_1-M)/s]\} \end{aligned}$$

The expected marginal benefit of DSRF is obtained by differentiating (20) with respect to D and is presented in (21):

$$\delta B/\delta D = P/s *\phi[\{(R_1 - D - M)/s\} - r^{-1}\, i\Phi[(R_1 - M)/s] \qquad (21)$$

The DSRF plan allows use of DSRF exclusively for debt service. As a result, the borrower has to forego the potential earnings on the amount set aside for the DSRF plan. The foregone rate of return on the DSRF net of the interest rate on use of the DSRF constitutes the cost of the DSRF to the borrower. Assuming the foregone rate of return on the DSRF is x, the expected rate of return from the production plan, and the rate of interest on use of the DSRF is i, the total cost of the DSRF, TC, to the borrower is given by (22). The rate of return net of interest rate i.e. (x - i), therefore, constitutes the marginal cost of the DSRF to the borrower.

$$TC = D * x - D * i = D(x - i) \qquad (22)$$

A borrower's optimal size of DSRF is affected by his/her risk attitude. Like the lender, the borrower is also assumed to be risk neutral. The borrower optimal size of DSRF is then obtained by equating expected marginal benefit to expected marginal cost as in (23).

$$(P/s)*\phi[(R_1-D-M)/s] - r^{-1}\ i\ \Phi[(R_1 - M)/s] = (x - i) \qquad (23)$$

After some algebraic manipulation, equation (24) expresses a borrower's optimal size of DSRF as a function of R1, M, s, r, P and (x-i).

$$D = R_1 - M - s\phi^{-1}\ \{(x-i) + r^{-1}i\ \Phi[(R_1 - M)/s]\ s/p\} \qquad (24)$$

To gain some insight into its usefulness in real lending situations, equation (24) is used to compute borrower optimal size of the DSRF for a set of reasonable assumptions. Since, the provision of DSRF reduces the amount of expected penalty (equation 20), the borrower optimal size of DSRF should be directly related to the size of the penalty in case of loan default. As a result, for the same example used in the lender case, equation (24) is used to compute borrower optimal sizes of DSRF for three levels of penalty. This results in borrower optimal sizes of DSRF of $2,252.80, $3616.00 and $4,432.00 respectively for the penalties of $1,000.00, $2,000.00 and $4,000.00. These results confirm that the higher the penalty in the case of loan default, the higher the incentive for the borrower to set up larger DSRFs.

A borrower optimal size of DSRF of $3,616.00 for a penalty of $2,000.00 is large enough to cover a shortfall in the borrower's expected cash flow of up to 0.75 standard deviations which has a 27.34 percent of occuring. Moreover, there is a 0.5 probability of realizing cash flow higher than the mean. As a result, provision of $3,616.00 as a DSRF ensures that the borrower has 77.34 percent chance of meeting his/her loan obligation.

The comparative statics properties of equation (24) are presented in (25), (26), (27), (28), (29), (30) and (31).

$$\delta D/\delta R_1 = 1 > 0 \tag{25}$$

$$\delta D/\delta M = -1 < 0 \tag{26}$$

$$\delta D/\delta x = [-s\phi^{-1\prime}(.)][1] < 0 \tag{27}$$

$$\delta D/\delta r = [-s\phi^{-1\prime}(.)][-r^{-2}i\Phi\{(R_1-M)/s\}s/p] \tag{28}$$

$$\delta D/\delta s = -\phi^{-1}(.) - s\{\phi^{-1\prime}(.)(r^{-1}i\Phi(.)/p + r^{-1}is/p\ \phi[(R_1-M)/s][(M-R_1)/s^2]\} \tag{29}$$

$$\delta D/\delta p = [-s\phi^{-1\prime}(.)][-r^{-1}i\Phi\{(R_1-M)/s\}s/p^2] > 0 \tag{30}$$

$$\delta D/\delta i = [-s\phi^{-1\prime}(.)\{-1 + r^{-1}\Phi\{(R_1-M)/s\}s/p\}] \tag{31}$$

The comparative statics results of equation (24) indicate that the borrower optimal size of DSRF should increase with the amount of loan obligation and penalty in the case of loan default. Similarly, it should decrease with the increase in the rate of return and mean cash flow from investment. The comparative statics results with respect to increase in discount rate, variance of the returns and risk free interest rate depend upon the actual values of terms in (28), (29) and (31) respectively.

THEORETICAL MODELS UNDER OTHER DISTRIBUTIONS

Theoretical models for lender and borrower optimal DSRF sizes developed in the preceding sections are based on the assumption that stochastic return, Y, is normally distributed. The true distribution, however, may be non-normal. Hence, to assess the robustness of these models to distributional assumptions, theoretical models for optimal sizes of DSRF are also developed for another type of distribution. The specific distribution assumed in this paper is a Beta distribution with pdf presented in (32).

$$f(Y) = [\Gamma(\alpha+\beta)/\Gamma(\alpha)\Gamma(\beta)][(Y-a)^{\alpha-1}(b-Y)^{\beta-1}/(b-a)^{\alpha+\beta-1}] \quad (32)$$

Where, $a < Y < b$

Following the methodology used in the normal case, the theoretical models for lender and borrower optimal DSRF size are developed. For ease of computation, the parameters α and β in beta distribution are assumed to be equal to 2. The pdf and cdf for this assumption are presented in (33) and (34) respectively.

$$f(Y) = [6/(b-a)^3][-Y^2 + (a+b)Y - ab] \quad (33)$$

$$F(Y) = [6/(b-a)^3][(-Y^3/3) + \{(a+b)Y^2/2\} - abY] \quad (34)$$

Using the same principle as in the normal case, the optimal size of DSRF is developed by equating the expected marginal benefit to expected marginal cost. Following equations (9) and (21), the expected marginal benefit and cost of the DSRF to the lender and borrower for any distribution are presented in (35) and (36) respectively.

$$(1 + r^{-1}(i-b)F(R_1 - D) + b = (i-b) \quad (35)$$

$$P * f(R_1 - D) - r^{-1} (i-b) F(R_1) = (Y-i) \quad (36)$$

Substituting the beta distribution cdf into (35) yields the theoretical model for the lender optimal size of DSRF (37) as a cubic equation in D.

$$D^3/3 + R_1{}^2D - R_1D^2 - (a+b)R_1D + ((a+b)D^2/2) - abR_1D = R_1{}^3/3$$
$$- ((a+b)R_1{}^2)/2) - a^3/3+((a+b)a^2/2) - a^2b \quad (37)$$
$$+ ((b-a)^3/6)((i-b-b)/1+r^{-1}(i-b)$$

Similarly, substituting the beta pdf and cdf into (36) results in the theoretical model for borrower optimal size of DSRF (38) as a quadratic equation in D.

$$-D^2 + 2R_1D - (a+b)D = R_1{}^2 - (a+b)R_1 +ab + [(x-i) +$$
$$(6/(b-a)^3)\{(-R_1{}^3/3) + ((a+b) R_1{}^2/2) - abR_1 \quad (38)$$
$$+ a^3/3 - (a+b)a^2/2) - a^2b\}]$$

To assess the sensitivity of optimal sizes of DSRF to the distributional assumptions, lender and borrower optimal sizes of DSRF for same set of assumptions as for the normal case are computed for the beta distribution. Furthermore, a lower and upper bound of the support of the distribution of returns is assumed to be equal to two standard deviations ($4,800.00) away from the expected return of $48,000.00. This results in a lower bound of $38,400.00 compared to the upper bound of $57,600.00. Substituting these values in equations (37) and (38) results in lender and borrower optimal sizes of DSRF of $4,181.00 and $2,151.00 (assuming a penalty of $2,000.00 in case of loan default) respectively. Compared to normal distribution case, both the lender and borrower optimal DSRF sizes are lower under beta distribution.

The assumption that α and β parameters in beta distribution are equal to 2 implies that the distribution is symmetric. As a result, the comparative statics properties of equations (37) and (38) hold as under the normal distribution case described earlier.

RESOLVING THE DIFFERENCES IN OPTIMAL SIZE OF DSRF

The size of the DSRF that is optimal for the lender may not be optimal for the borrower. Hence, the parties will have to resolve the difference before the DSRF can be made a part of the loan contract. The bargaining power of each party in the negotiation depends upon the benefits and costs each party perceives from the DSRF.

A borrower's resources in the event of a loan default include his/her future cash income and current capital assets. If default results in forfeiting collateral (a capital asset), the production structure and capacity of the business are adversely affected. Similarly, if the borrower wants to repay his current obligation from his future cash income, then the present value of the future payment should exceed the current obligation. Otherwise, every borrower will prefer to default even when he/she is able to repay. Hence, the lender has to provide an incentive to repay by offering a refinancing contract with the present value of future payments higher than the current obligation. The difference is the penalty the borrower has to pay for loan default. As a result, the borrower will not be able to keep as much of his future income. The provision of the DSRF reduces the repayment risk and therefore protects the borrower from such lender imposed penalties.

Similarly, the contribution of the DSRF in reducing the probability of default on a long term loan may have many favorable consequences to the borrower in his/her relation to the lender. These include - (1) a reduction in credit risk and more favorable loan contract terms including the interest rate on the loan. (2) a lower probability of default on a long term loan may also increase a borrower's nonreal estate credit capacity. (3) a borrower not qualified for a loan under other repayment terms may qualify with a DSRF plan.

The major benefit of the DSRF to the lender, on the other hand, comes through the stabilization of repayment flows and reduction in liquidity risk. Other benefits include - (1) It may contribute to increased contractual revenue from borrowers who otherwise would have been ineligible for loans; (2) It can help reduce lending risks of current loan portfolios; and (3) It could enable lenders to finance higher risk loan applicants without increasing portfolio risk.

The DSRF plan allows the borrower to draw from the reserve to supplement his cash flow to meet loan obligations. Hence, the lender can not invest this fund in high yielding but relatively illiquid alternatives. The fund, however, may be invested in low yielding liquid assets. Hence, the interest earning differential on the DSRF and administrative costs constitute the cost of the DSRF plan to the lender. On the other hand, the borrower is not allowed to use the DSRF for profitable production plans and therefore has to forego the expected rate of return on the DSRF. Since, the expected rate of return constitutes the major reason for undertaking production plans, the foregone expected rate of return constitutes the cost of the DSRF plan to the borrower. The higher the expected rate of return from production plans, the higher the degree of disincentive to the plan.

The lender's responses to loan delinquency depend upon past credit records of the borrower. The lenders generally do not impose penalties on borrowers with good credit histories. As a result, these borrowers may not have enough incentive to participate in the DSRF plan. Hence, the DSRF proposal may be attractive only to borrowers who would have to pay lender sanctioned penalties in case of loan default. These generally include borrowers with poor credit records or new borrowers whose repayment records are not known to the lenders.

Optimal sizes of DSRF for a loan request of $40,000.00 to invest in a production plan with expected rate of return of 0.20 percent and other reasonable assumptions (discussed earlier) were calculated using the theoretical models developed in this paper. The models resulted in a lender optimal size of DSRF of $5,344.00 compared to $3,616.00 for the borrower under normality assumption of the rate of return from production plan. Similarly, under the beta distribution, the lender optimal size of DSRF of is $4,181.00 compared to borrower optimal size of $2,151.00. As a result, the amounts of differences that must be negotiated between the lender and borrower are $1,728.00 and $2,030.00 respectively under normality and beta distribution assumptions.

The borrower optimal size of DSRF is directly related to the size of lender imposed penalty in case of loan default (equation 30). Hence, the lender can institute a penalty structure such that the borrower and lender optimal sizes of DSRF are equal. This approach, however, would imply that the penalty structure is endogenous to the optimal DSRF problem. In real lending situations, the penalty structure in the case of loan default is already established before the optimal DSRF is computed. Hence, the lender and borrower have to negotiate to resolve the differences in their optimal sizes of DSRF.

The subjective evaluation of benefits and costs associated with the DSRF is instrumental in determining the degree of preference for the plan and hence the bargaining power of the lender and the borrower. It is apparent that the lender seems to be in a strong bargaining position because the cost associated with the DSRF plan is relatively smaller compared to that of the borrower. Moreover, the lender has the option of not extending loans to high credit risk borrowers without the DSRF plan. Therefore, the negotiated size of DSRF is expected to be near the lender's optimal size.

SUMMARY AND CONCLUSIONS

Repayment risk is generally involved in the loan contract between a lender and a borrower. The usual measures of reducing this risk by diversifying the portfolio and widening the geographic area have limited usefulness to family farms (with one or two enterprises) and to lending institutions specialized in one area like the Farm Credit System in the United States. So, the proposed DSRF plan as a measure of reducing repayment risk of long term loans should be appealing especially to these types of lenders and borrowers.

The DSRF plan does not eliminate repayment risk. However, its provisions help buffer the effects of the repayment risk by creating an additional source of liquidity for debt service. The design specifications, including size of initial DSRF, must be acceptable to both the parties. This paper developed a theoretical model for finding the size of the DSRF that is optimal to the lending agency and the borrower. In addition, the empirical applicability of these models was also assessed for a set of reasonable assumptions. Since the lender optimal DSRF size may not be optimal to the borrower, the paper also discussed how this difference can be resolved through negotiation.

The theoretical models in this paper were developed under two distributional assumptions of the stochastic returns from production plans and risk neutral attitudes of the lender and borrower. The model, therefore, may differ under other distributional assumptions of the returns from production plans and also risk averse attitudes. Further studies are needed to explore the sensitivity of the model developed in this study to other distributional assumptions as well as other risk attitudes.

The model developed in this paper determines the optimal size of the DSRF for a one time default possibility. The DSRF plan also envisages that the reserve must be replenished during periods of higher returns. However, in case of mortage loans, it is possible that the borrower's realized return is less than the contractual repayment in succeding years. Under such an event, the borrower will have used the DSRF in meeting first year loan obligation and therefore faces repayment problems in succeding years. The probability of such event is, however, very low. Neverthless, the actual amount of the DSRF to be set aside should depend upon the number of succeding default possibilities agreed to between the two parties throughout the life of the loan. In addition, other design specifications of the DSRF plan such as how it should be used and its final disposition should be acceptable to both the parties if it is to be implemented as a measure for reducing repayment risk.

REFERENCES

Arvan, Lanny and Jan K. Brueckner. "Efficient Contracts in Credit Markets Subject to Interest Rate Risk: An Application of Raviv's Insurance Model". The American Economic Review 76(1986):259-263.

Aukes, R.G. Effects of Variable Amortization Plans on Borrower and Lender Risk: A Simulation Study of Low Equity Illinois Cash Grain Farms. Ph.D. Dissertation, University of Illinois 1980.

Baker, C.B. "Structural Issues in U.S. Agriculture and Farm Debt Perspectives". The University of Kansas Law Review 34(1986):457-467.

Baker, C.B. "A Variable Amortization Plan to Manage Farm Mortgage Risk". Agricultural Finance Review 36(1976):1-6.

Barnard, F.L. and P.J. Barry. "Interest Rate Deregulation and Agricultural Banking: A Risk Programming Analysis". Agricultural Finance Review 45(1985):100-112.

Barry, P.J., Chester B. Baker, and Luis R. Sanint. "Farmers' Credit Risks and Liquidity Management". American Journal of Agricultural Economics 63(1981):216-227.

Barry P.J. and J.D. Calvert. "Loan Pricing and Customer Profitability Analysis by Agricultural Banks". Agricultural Finance Review 43(1983):21-29.

Hogg, R.V. and Elliot A. Tannis. Probability and Statistical Inference, Second Edition, Macmillan Publishing Co. Inc., New York, 1983.

Kent, M.A. and A.G. Lloyd. "Coping with Rural Income Instability: Some Estimates of Probabilty of Default Under Various Loan Repayment Arrangements". Summer Paper Presented at Annual Conference of Australian Agricultural Economics Society, Brisbane, Queensland, Australia, 4 pp.

Lam, C.H. and Andrew H. Chen. "A Note on Optimal Credit and Pricing Policy Under Uncertainty: A Contingent-Claims Approach". The Journal of Finance 31(1986):1141-1147.

Lee, W.F. "Some Alternatives to Conventional Farm Mortgage Loan Repayment Plans". Canadian Farm Economics 14(1979):12-20.

Robison, L.J. and P.J. Barry. The Competitive Firm's Response to Risk Macmillan Publishing Co. 1987.

Sonka, S.T., B.L. Dixon, and B.L. Jones. "Impact of Farm Financial Structure on the Credit Reserve of the Farm Business". American Journal of Agricultural Economics 62(1980):565-570.

Stiglitz, J.E. and Andrew weiss. "Credit Rationing in Markets with Imperfect Information". American economic Review 71(1981):393-410.

Stone, K.E. Effects of a Variable Amortization Payment Plan on the Financial Organization of Illinois Cash Grain Farms. Ph.D. Dissertation. University of Illinois, 1976.