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Learning to Learn: A Case for the Heterogeneous Expectations Hypothesis in Industrialized Markets

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ABSTRACT

A cobweb model is developed where the heterogeneous expectation hypothesis is examined. An agent's heterogeneous expectation involves the development of a "higher ordered learning" process in which agents over time develop expectations that are consistent with rational expectations. In addition, as cob web models are production based systems, an agents' heterogeneous expectations are influenced by a specialization of activities. The case of the industrialization of the U.S. hog-pork industry is used to illustrate the influence of these features on the equilibrium and non-equilibrium properties of a modified cob-web model.

Keywords. : prices dynamics, heterogeneous expectations, learning, and industrialized supply chain.

1 Introduction

1.1 Overview

As expectations have played a key role in modelling dynamic phenomena (Hommes, 2011; Sonnemans et al., 2004), cobweb models (Ezekiel, 1938) have had a long history in explaining the non-equilibrium price dynamics of agricultural commodity markets (e.g. Barten and Vanloot, 1996; Harlow, 1960). A unique appeal to such models is the attention placed on the price expectations of the producing agent. In that, while naïve and rational expectations have historically played an important role to understanding the price dynamics of cob web models, more recent developments in heterogeneous expectations research have appealed to an adaptive expectations framework. A benefit of adaptive expectations is that differences in the weighting of an agent's past prices and forecasts supports a heterogeneity of expectations that evolve as new price information becomes available (Muth, 1961; Sonnemans et al., 2004). This heterogeneity of expectations has been used to explain convergent, cyclical, and divergent price movements of cob web models (Branch and McGough, 2008; see also Brock and Hommes, 1997).

Yet, as cobweb models are noted for their convergent properties, adaptive expectations fail to yield a convergence of prices that are consistent with a rational equilibrium outcome (Hommes, 2011; p. 21; Sonnemans et al., 2004). This convergence has been described as the "heterogeneous expectations hypothesis" (Hommes, 2011; see also Brock and Hommes, 1997; Branch and McGough, 2008). It argues that an agent's adaptation to past prices and forecasts will reduce an agent's forecast errors. This yields a convergence that is consistent with a rational expectation equilibrium outcome (Heemeijer et al., 2009; Hommes, 2011; Sonnemans et al., 2004). Yet, a pervasive empirical finding of heterogeneous expectation research is that "deviations from the rational equilibrium fundamental benchmark seem to be the rule rather than the exception" (Hommes, 2011, p. 21). Hence, while adaptive expectations form the basis of heterogeneous expectations research, the research challenge facing an adaptive expectation framework is

that it does not support the equilibrative tendencies of cob web models (e.g. Sonnemans et al., 2004). This failure is attributed to two shortcomings in the adaptive expectations framework.

First, as rational expectations are based on knowledge of an underling structural model (Muth, 1961), an adaptive expectations framework does not account for differences in an agent's ability to incorporate this structural understanding into their price expectations. For instance, agricultural markets have become increasingly "industrialized" where the various demand and supply relationships of the agricultural supply chain have become increasingly coordinated into a vertical system (e.g. Boehlje, 1999, Gray and Boehlje, 2007; Hurt, 1994; Ng, 2008). Yet, since an adaptive expectations framework does not account for the demand and supply relationships of this vertically coordinated system, cob web models that draw on an adaptive expectations framework will yield a heterogeneity of expectations where prices will not converge to a rational expectations outcome.

Second, with the industrialization of agricultural markets, adaptive expectations fail to recognize that an agent's heterogeneous expectations operate within a highly specialized context. In particular, since cob web models have a strong production focus, the "industrialization" of markets has been attributed to an increasing specialization of supply chain activities (Drabenstott, 1995; Ng, 2008). Ng (2008) shows that in an industrialized system, the specialized activities of one stage of the supply chain have a convergent effect on the prices of other vertically coordinated stages. Yet, because an agent's adaptive expectations are influenced by their past prices and forecasts, cob web models that draw on these adaptive expectations will fail to account for the influence of this specialization in attaining a rational equilibrium outcome.

1.2 Problem outline and organization of discussion

In order to understand the price dynamics of agricultural markets, the objective of this study is to develop a "modified" cob web model that addresses these shortcomings in adaptive expectations framework. The case of the industrialization of the U.S. hog-pork supply chain is used to illustrate the price dynamics of this proposed model. Specifically, in this model, this model examines the influence of an agent's heterogeneous expectations and the specialization of supply chain activities on the attainment of a rational equilibrium outcome. To examine these industrialized features, a concept of "learning" expectations is developed where agents are heterogeneous with respect to their understandings of the demand and supply relationships of the U.S. hog-pork supply chain. Furthermore, as an agent's heterogeneous expectations operate within an increasingly specialized supply chain; this specialization is also examined by our modified model. By accounting for these industrialized features, a modified cob web model is developed to not only address the shortcomings of the adaptive expectations framework, but this model also explains how these industrialized features can influence the attainment (or lack of) of a rational equilibrium outcome.

To organize the development of this modified cob web model, this paper is organized into six parts. The first part provides a review of the role of heterogeneous expectations in cob web research. In the second part, a concept of learning expectations is developed that offers an alternative to the shortcomings of heterogeneous expectation research, especially from the standpoint of adaptive expectations. In the third part, this notion of heterogeneous expectations is examined within the specialized context of an industrialized U.S. hog-pork supply chain. The fourth part models these industrialized features within a modified cob web model. A mathematical Lemma is then devised where the role of heterogeneous expectations and the specialization of supply chain activities are modelled within our proposed cob model. Through this Lemma, the fifth part introduces three thought experiments where the influence of these industrialized features on the price dynamics of the modified cob web model is examined. Lastly, this paper concludes with its implications and contributions to heterogeneous expectations research.

2 Review of Heterogeneous Expectations Research in Cob Web Models

2.1 Cobweb Model

Often regarded "as one of the most successful attempts at dynamic economic theories" (Muth 1961, p.33), cobweb models have been the subject of much theoretical and empirical interest in explaining endogenous price movements in hog markets (e.g. Barten and Vanloot, 1996; Branch and McGough, 2008; Chavas, 1999; Chavas and Holt, 1991; Hommes, 2011). Such price movements are attributed to a production lag where production decisions in a given period are based on the price expectations of an earlier period. With this production lag, a key feature of cobwebs models is that the "conditions" surrounding the price dynamics, PD_t, for a given time period, *t*, are dependent on the ratio of the slope coefficient of supply, s_t, to the slope coefficient of demand, d_t (Dean and Heady, 1958; Harlow, 1960).

These price predictions (equation 1) are summarized by the following set of price dynamic, PD_t, relationships and have been confirmed by a variety of empirical studies (Brock and Hommes, 1997; Chavas and Holt, 1991; Dean and Heady, 1958; Harlow, 1960).

(1) $PD_t = s_t / d_t$

PD_t <1(Convergent prices),

PD_t >1(Divergent prices),

PD_t =1(Oscillating prices).

2.2 Heterogeneous Expectations

Yet, since the price dynamics of cobweb models are also influenced by a producer's price expectations, heterogeneous expectations research have traditionally appealed to a producer's naïve and rational expectations. Naïve expectations (Ezekiel, 1938) reflect the simplest or least cognitive demanding form of expectations where the future is an extension of the recent past. In the case of hog price expectations, *e*, Ezekiel's (1938) naïve, *n*, expectations attribute the expected hog price in period t, $P_{t,e-n}^{hog}$, to the previous period's hog supply price, P_{t-1}^{hog} (equation 2) (Chavas, 1999; Chavas and Holt, 1991). An implicit assumption held by naïve expectations is that an agent's expectations are based on a limited or simplistic understanding of the data generation process. That is, agents lack a structural understanding of the system of supply and demand equations that are used in formulating a producer's expectation of prices. In the absence of this structural understanding, a naive agent's forecasted price is based on an extension of past prices.

(2) $P_{t,e-n}^{hog} = P_{t-1}^{hog}$.

In contrast, Muth's (1961) rational expectations are based on a structural understanding of the data generating process. A basic tenet of rational expectations is that agents make efficient use of all available information in which their "anticipated future values of relevant variables are equal to their expectations conditional on all past data and the model itself which describes the behavior based on those expectations" (Nerlove and Fornari 1998, p. 130). In particular and in accordance to Muth's (1961) rational expectations hypothesis, *r*, agents engage in a projection of prices that not only draws on past prices, but this projection of prices also draws on a full understanding of market supply and demand side equations. This yields an expectation of prices that is consistent with realized market prices (Branch and McGough, 2008; Chavas, 1999). In the context of hog prices, rational expectations, $P_{t,e-r}^{hog}$, arise when the realized equilibrium hog price, $P_t^{hog} = P_*^{hog}$, is equated with an agent's rational expected price, $P_{t,e-r}^{hog}$ (equation 3). Yet, while rational expectations offer a structural understanding of the data generating process, it imposes highly unrealistic cognitive demands on the agent. Rational expectations assume that the agent can fully understand the set of supply and demand equations that will result in a rational equilibrium outcome.

(3) $P_{t,e-r}^{hog} = P_t^{hog} = P_*^{hog}$

2.3 Adaptive Expectations

In an effort to overcome the shortcomings of naïve and rational expectations, there has been a long standing interest to broaden this set of heterogeneous expectations to account for adaptive expectations (Hommes, 2011; Nerlove, 1958). Adaptive expectations have been used to examine the non-equilibrium properties of the cob web phenomena (Muth, 1961; Nerlove, 1958) and have formed the basis of heterogeneous expectations research (e.g. Hommes, 2011). Adaptive expectations are defined by the weighted sum of its previous period's forecasts and the previous period's prices (Hommes, 2011). Adaptive expectations have also been alternatively expressed as the sum of its past expectations and the weighted forecast errors of the previous period (Muth, 1961; Nerlove, 1958).

A benefit of adaptive expectations is that differences in the weighting of past prices and forecasts support a heterogeneity of expectations. In particular, while agents with adaptive expectation cannot know the underlying structural model or what Hommes (2011) describes as the "true law of motion of the economy" (p. 2), agents can over time learn about the "parameters of their *perceived law of motion* as more observations become available" (p. 2).

Hommes (2011) argues that due to an agent's lack of understanding of the data generating process, agents initially formulate an adaptive expectations based on naïve expectations. Yet, as an agent's expectations adapts to new price information, this adaptation evolves an agent's naïve expectations to those of a rational expectations framework. Hence, through this adaption, agents formulate an expectation where prices are based on a "perceived understanding" of the structural model. This adaptation underlies Hommes' (2011) heterogeneous expectations hypothesis where he argues that "given the limited market information one cannot expect that all individuals have rational expectations at the outset, but one can hope that in such a simply, stationary environment, individuals would *learn to have rational expectations*" (p. 6).

2.4 Heterogeneous Expectations Hypothesis

To elaborate on this heterogeneous expectations hypothesis, Hommes (2011) drew on an agent's naïve expectations as a starting point. With this starting point, individual producers have no structural knowledge of their system, but nevertheless, the subjects understood that there was a negative feedback relationship between an individual's forecasts and the market clearing or rational expectation equilibrium price. They found that with a stable treatment –involving a scaling parameter, *l*, on their non-linear supply curve-, agents with initially naïve expectations quickly converged to the rational expectation equilibrium outcome. But under the unstable treatment, heterogeneous expectations led to persistent and excessive volatility. When considering these treatment effects as well as other parameter settings, Hommes (2011) concluded that a convergence to a rational expectation equilibrium outcome was an "exception" rather than a rule (Hommes, 2011. p.21). Other studies have similarly shown that 60% of their price fluctuations are chaotic or non-equilibrative, while a convergence to a unique RE steady stay occurs only 10% (see also Brock and Hommes, 1997; Sonnemans et al. 2004).

While there are various factors that can explain for this lack of support for the heterogeneous expectation hypothesis (e.g. Brock and Hommes, 1997), this lack of support stems from a heterogeneity that fails to "directly" account for an agent's understanding of the data generating process. Namely, adaptive expectations offer a variety of different weight assignments that reduce forecast errors. These weight assignments offer a "perceived" understanding of the underlying structural model. Yet, a reduction in forecast errors does not imply that an agent has knowledge of the underlying model structure. This is because the attainment of rational equilibrium outcome can be attained by a variety of different weight assignments and forecasts strategies. Adaptive expectations thereby cannot offer a suitable basis for examining the heterogeneous expectations hypothesis. This is because an expectation based on past prices and forecasts do not reveal knowledge about the system and thus cannot yield an expectation of prices that converge to a rationally equilibrium outcome state.

3 Learning Expectations

As an alternative to adaptive expectations, an agent's heterogeneous expectations are explained by a concept of "learning" expectations. Learning research has found that organizations have the capacity to engage in "single" and "double loop learning" or "learning to learn" behaviors (Argyris, 1976, 2003; Sterman, 1994). According to Argyris (2003), "single loop learning occurs when a mismatch is detected and corrected without changing the underlying values and status quo that govern these behaviors" (p. 1178-9). In its most simplest form, naïve expectations reflects a single loop learning process in which the detection and correction of a producer's price expectations are updated by its previous period's prices. Adaptive expectations offer a more complex form of a single loop learning process where the accumulation of past prices updates the weights used in correcting past forecast errors (e.g. Heemeijer et al., 2009; Hommes, 2011). Yet, a challenge with naïve and adaptive expectations is that while they can be useful in detecting and correcting forecast errors, the single loop learning process cannot detect errors in the equations used in generating the forecast estimate itself. That is, single loop learning cannot detect and correct errors relating to model misspecification errors. Since naïve and adaptive expectations cannot make corrections in the underlying structure of the model, these expectations are bound by a single loop learning process that precludes agents from developing expectations consistent with rational expectations.

In contrast to single loop learning processes, double loop learning appeals to a higher ordered learning activity where it involves an ability to detect and correct past decision errors by changing the prevailing assumptions and information used in that decision (Argyris, 1976). More precisely, "double loop learning occurs when a mismatch is detected and corrected by first changing the underlying values and other features of the status quo" (Argyris, 2003). An agent's learning expectation appeals to this form of double loop learning in which agents detect and correct errors by learning to change over time the underlying set

of equation (s) used in their expectation of prices. Specifically, as naïve and rational expectations are commonly conceived as expressions of different degrees of rationality, learning expectations refer to a producer's capacity to not only "naively" project future prices from past prices, but over time, learn to understand the system of demand and supply equations used in rational expectations. Hence, unlike the single loop learning processes of adaptive expectations, this learning expectation introduces a heterogeneity where individuals over time differ in their ability to learn about those system of equations used in generating a rational equilibrium outcome. This is consistent with expectations research whereby Conlisk's (1996) review finds that individuals with repeated experience tend to move towards more rational expectations (see also Branch and McGough 2008; Colucci and Valori, 2011; Goeree and Hommes, 2000; Ranyard et al., 2008; Sonnemans et al., 2004;).

To model this concept of learning expectations, an individual's learning expectations, *l*, is modeled by a hog producer's learning coefficient, α_t (equation 4). Since hog producers can learn from their past experiences, this learning coefficient, α_t increases with time, t. Namely and consistent with Hommes (2011), hog producers initially formulate hog prices based on naïve expectations where α_t =0. However, over time, hog producers develop expectations consistent with the rational expectations hypothesis where α_t asymptotically converges to 1 (Hommes, 2011). With these learning expectations, heterogeneous expectations are defined by a range of values taken by the learning coefficient, α_t, where the heterogeneity, *h* , in a hog producer's expected prices in period t, $P_{t,e-h}^{hog}$, has the generalized form (equation 4):

(4) $$P_{t,e-h}^{hog} = \alpha_t P_{t,e-r}^{hog} + (1-\alpha_t) P_{t,e-n}^{hog},$$

Where, $0 \le \alpha_t \le 1$, and $d\alpha_t / dt \ge 0$.

As models of heterogeneous expectations are commonly defined at a group level (Branch and McGough, 2008; Chavas, 1999; Hommes, 2011; Pfajfar, 2013), individual level expectations are aggregated in accordance to three heterogeneous groups. Based on the values of the learning coefficient, α_t, individual hog producers are assigned to one of the following groups: naïve ($\alpha_t = 0$), learning ($0 \le \alpha_t \le 1$) and rational expectations ($\alpha_t = 1$) groups . These group level expectations are then defined by a proportion of agents, H_t, who respectively subscribe to naïve, H_t^n, rational, H_t^r, or learning expectations, H_t^l (see also Branch and McGough, 2008). These proportions satisfy the condition where $H_t^n + H_t^r + H_t^l$ =1 in which the expectations of each group are assumed to be independent of the other (see also Muth, 1961). Furthermore, a hog producer's ability to evolve from their naïve expectations to rational expectations would need to satisfy the following inequalities in equation 5 (see also Hommes, 2011):

(5) $$dH_t^r / dt \ge 0, dH_t^n / dt \le 0$$

4 The Industrialization of the Hog-Pork Supply Chain

Since an agent's heterogeneous expectations are based on an agent's understanding of an underlying structural model, this structural understanding is informed by the industrialization of U.S. hog-pork supply chain (Drabenstott, 1995; Gray and Boehlje, 2007). In explaining the industrialized features of this supply chain, the U.S. hog-pork supply chain is distinguished by an: 1) increasing specialization of supply chain activities and 2) increasingly vertically coordinated system of supply and demand relationships (Boehlje, 1999; Ng, 2008). Each of these constituent components is examined as follows:

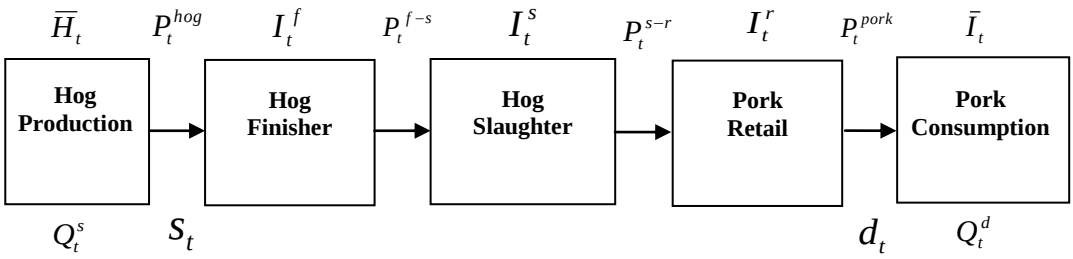

Figure 1. Specialization of the U.S. Hog-pork supply chain

4.1 Specialization of the U.S. Hog-pork supply chain

Figure 1 illustrates the specialization of supply chain activities in the U.S. hog-pork supply chain. Figure 1 consists of the following supply stages: Hog Production that would include farrow to feeder operations, Hog Finisher, involving feeder to finisher operations, Hog Slaughter that include processing and distribution companies, and Pork Retail involving supermarkets, restaurants of food suppliers (Lowe and Gereffi, 2008). With figure 1, the degree of specialization, I_t, that is involved with the conversion of inputs to outputs is denoted by the following specialization variables: hog finisher specialization, I_t^f, hog-slaughter specialization, I_t^s, pork retailing specialization, I_t^r. These specialized variables reflect the extent to which assets are used in transforming the outputs of an upstream stage to inputs of an adjacent downstream stage. These specialization variables have values that range from a value of 0 to 1 where a value of 0 and 1 respectively denote a 0% and 100% utilization of that supply stage's specialized assets. Furthermore, since the specialized tasks of any given supply stage influences the efficiency of subsequent supply stages, the degree of specialization for the entire supply chain is captured by an aggregate supply chain specialization variable, $\bar{I}_t$ (equation 6). It is computed as the product of the specializations of each supply stage where

(6) $$\bar{I}_t = I_t^f \cdot I_t^s \cdot I_t^r \text{, and } 0 < \bar{I}_t < 1.$$

In addition, each supply stage experiences an increasing degree of specialization where this specialization is modeled by the following inequalities:

$$dI_t^f/dt \geq 0,\ dI_t^s/dt \geq 0,\ dI_t^r/dt \geq 0 \text{ and } d\bar{I}_t/dt \geq 0.$$

4.2 Heterogeneous Expectations: Vertical Coordinated System

Another important attribute of the industrialization process is that the specialized activities of the U.S. hog-pork supply chain are coordinated through a series of supply and demand exchange relationships. In particular, as the efficiencies of the various specialized activities of the hog-pork supply chain can only be leveraged through their greater coordination (e.g. Ng, 2008; Freudenburg, 1993), an individual's price expectations play an important role in coordinating the supply and demand relationships of this hog-pork supply chain. In appealing to this study's concept of learning expectations, agents are heterogeneous with respect to their understanding of these supply and demand equations. This heterogeneity yields an expectation of prices that not only reflects difference in an agent's structural understanding but such differences subsequently impacts the coordination of the supply and demand relationships of the hog-pork supply chain

By drawing on this concept of learning expectations, hog producers are assigned to one of the three heterogeneous expectations groups: naïve ($\alpha_t = 0$), learning ($0 \leq \alpha_t \leq 1$) and rational expectations ($\alpha_t = 1$) groups. Each of these heterogeneous groups reflects a different and increasingly sophisticated understanding of the supply and demand relationship of the U.S. hog-pork supply chain. Hence, by drawing on this characterization of heterogeneous expectations, differences in an agent's structural understanding of the demand and supply relationships influences the extent to which the specialized activities of the U.S. hog-pork supply chain can be coordinated.

5 Modelling the Industrialization of the U.S. hog-pork supply chain in a Modified Cob Web Model

To model these two industrialized features of the U.S. hog-pork supply chain, a modified cob web model is developed. This modified cob web model consists of a heterogeneous group of hog producers where each group operates within an increasingly specialized supply chain. In particular, the production of hogs by each group is not only influenced by this increasing specialization, but each group's production is also dependent upon a heterogeneous expectation of prices. By appealing to these industrialized features, the influence of heterogeneous expectations and the specialization of supply chain activities on the price dynamics of the U.S. Hog-pork supply chain are then examined within this study's modified cob web model.

In explaining this modified cob web model, the final pork consumption demand, $Q_t^d = D\left(P_t^{pork}\right)$, is equated with a supply of hogs, $S\left(P_{t,e}^{hog}\right)$ that have been transformed through the various specialized activities of the hog-pork supply chain. This supply of hogs, $S\left(P_{t,e}^{hog}\right)$, is determined by taking the product of the aggregate supply chain specialization variable, $\bar{I}_t$, with the output of hogs, Q_t^s, produced by the entire supply chain (Equation 7). Yet, since each group of hog producers supplies an output, $Q_t^s = S\left(P_{t,e}^{hog}\right)$, that is based on a heterogeneity of price expectations, the price expectations (equation 2, 3 and 4) for each of these heterogeneous groups are then substituted into their respective supply responses. The proportion of agents with rational, naïve and learning price expectations - H_t^r, H_t^n, H_t^l - and their corresponding output or supply - S^R, S^N, S^L - are then aggregated across these heterogeneous groups. Since the specialization of supply chain activities influences each group's productive response, each heterogeneous group's supply response is then made a product of the aggregate supply chain specialization variable, $\bar{I}_t$. The supply responses for each of these heterogeneous groups are then aggregated into single supply response where the specialization of supply chain activities and each group's heterogeneous expectations are jointly used in determining the aggregate supply for the hog-pork supply chain.

To coordinate the supply and demand relationships of this supply chain, this aggregate supply is then equated with the inverse demand function where each heterogeneous group formulates a production response based on their respective understanding of the supply and demand relationships of the hog-pork supply chain. This modified cobweb model thereby not only accounts for the specialized aspects of the industrialized process but it also accounts for a heterogeneity of price expectations that would vertically coordinate the supply and demand relationships of the U.S. hog-supply chain[*].

$$(7)\quad Q_t^d = D\left(P_t^{pork}\right) = \bar{I}_t\, Q_t^s = \bar{I}_t\, S\left(P_{t,e}^{hog}\right) \text{ or}$$
$$D\left(P_t^{pork}\right) = \bar{I}_t\, H_t^r S^r\left(P_{t,e-r}^{hog} = P_t^{hog}\right) + \bar{I}_t\, H_t^n S^n\left(P_{t,e-n}^{hog} = P_{t-1}^{hog}\right) +$$
$$\bar{I}_t\, H_t^l S^L\left(P_{t,e-l}^{hog} = \alpha_t P_t^{hog} + (1-\alpha_t) P_{t-1}^{hog}\right).$$

In explaining the demand side attributes of this modified cobweb model, the demand price of pork is based on the sum of the price of hogs and the per unit supply chain profits, $\bar{\pi}_t$. Supply chain profits, $\bar{\pi}_t$, consists of the aggregate profits accrued from transforming hogs into pork. Yet, as these supply chain profits, $\bar{\pi}_t$, stem from the various specialized tasks of the hog-pork supply chain, the demand price of pork is adjusted by the aggregate specializations of the supply chain, $\bar{I}_t$. This adjustment in shown in equation 8[†].

$$(8)\quad P_t^{pork} = \left(P_t^{hog} + \bar{\pi}_t\right) / \bar{I}_t\ ,$$

5.1 Lemma: Price Dynamics of Modified Cobweb Model

Since hog prices monotonically influence pork consumption prices (equation 8), the price dynamics of our modified cobweb model originate from the price dynamics at the hog production stage. These price

[*] The assumptions of this modified cob web model are listed in appendix 1

[†] Proof of this equation in shown in Appendix ii

dynamics are examined with Lemma 1, L_t. Lemma 1, L_t, underscores the lagged nature of the hog production cycle where hog prices in period t are represented by a series of past prices. Equation 9 shows that Lemma 1, L_t, has the following price dynamic properties[‡]:

Lemma 1.

(9) $$L_t \equiv \frac{dP_t^{hog}}{dP_{t-1}^{hog}} = \frac{P_{t+1}^{hog} - P_t^{hog}}{P_t^{hog} - P_{t-1}^{hog}},\ t = 1,2,\cdots,$$

(i) *If* $|L_t|$*<1, then the prices series {* P_t^{hog} *} is convergent.*

(ii) If $|L_t|$>1, then the prices series { P_t^{hog} } is divergent.

(iii) If $|L_t|$=1, then the prices series { P_t^{hog} } oscillates.

The price dynamics of Lemma 1, L_t, are examined within this study's modified cobweb model (equation 7). Specifically, by substituting the pork mark-up price (equation 8) into the inverse demand $D\left(P_t^{pork}\right)$ function, we can differentiate the modified cob web model (equation 7) with respect to its previous period's hog prices, P_{t-1}^{hog}, to yield a reformulated Lemma 1 shown in equation 10[§]. This reformulated Lemma 1 enables us to examine the price dynamics of our modified cob-web model where the convergent, divergent and oscillating price movements of our cob web model can be explained in terms of the industrialized features of the U.S. hog-pork supply chain. This reformulated Lemma 1 is shown in equation 10.

(10) $$\left|L_t\left(\overline{H}_t, APD_t\right)\right| = \frac{\left(1-\overline{H}_t\right)APD_t}{1+\overline{H}_t APD_t} \text{ or } \frac{1-\overline{H}_t}{\frac{1}{APD_t}+\overline{H}_t}$$

Where, $\overline{H}_t \equiv H_t^r + \alpha_t H_t^l$, $APD_t \equiv \bar{I}_t^{\,2} \cdot \frac{s_t}{|d_t|}$ or $\bar{I}_t^{\,2} \cdot PD_t$

Specifically, through equation 10, the price dynamics of the reformulated Lemma 1, L_t, is examined by two key variables of interest: 1) the heterogeneous learning index, $\overline{H}_t$, and 2) the adjusted price dynamic index, APD_t. The heterogeneous learning index, $\overline{H}_t$, is a measure of the proportion of agents who learn to formulate prices that are consistent with rational expectations. This index not only includes the proportion of agents who have formulated prices based on rational expectations, H_t^r, but also a proportion of those agents, $\alpha_t\ H_t^l$, that have learned to formulate prices consistent with rational expectations. Specifically, the heterogeneous learning index, $\overline{H}_t$, initially takes on a value of 0 because in the absence of experience, all individuals formulate prices that are consistent with naïve expectations where H_t^n =1 or H_t^r = H_t^l =0. However, over the time and consistent with the heterogeneous expectation hypothesis (Hommes, 2011), agents learn to develop expectations consistent with the heterogeneous expectation hypothesis where α_t converges to 1 such that H_t^n = H_t^l =0 and H_t^r =1. This heterogeneous learning index, $\overline{H}_t$, thereby appeals to a heterogeneous expectation where each group learns to develop an understanding of the vertical system of demand and supply relationship of the U.S. hog-pork supply chain. Through the reformulated Lemma 1, this heterogeneity enables us to examine the influence of this heterogeneity on the price dynamics of our modified cob-web model.

With respect to the adjusted price dynamic index, APD_t, the increasing specialization of the U.S. hog-pork supply is captured by the aggregate supply chain efficiency variable, $\bar{I}_t$. Increases in the supply chain efficiency variable, $\bar{I}_t$, results in an increase in the adjusted price dynamic index, APD_t. Through the adjusted price dynamic index, APD_t increases in this supply chain efficiency variable, $\bar{I}_t$, captures the increasing specialization of the hog-pork supply chain. In particular, as the supply chain efficiency variable, $\bar{I}_t$, converges to one, each member of the hog-supply supply chain has maximized the specialized efficiencies of their respective supply stage. Under this setting, members of each stage are specialized to the productive experiences of the other. This yields a vertically coordinated series of specialized supply chain activities where the hog-supply chain can be viewed as a highly integrated system. Hence and not

[‡] The proof and non-equilibrium properties of Lemma 1 are shown in Appendix iii

[§] See appendix iv for proof.

surprisingly, when the supply chain efficiency variable, $\bar{I}_t$, converges to one, the adjusted price dynamic index, APD_t, is identical to the price dynamics, PD_t, of earlier cobweb models. However, unlike these earlier models, the reformulated Lemma 1, enables us to examine the influence of this increasing specialization on the price dynamics on a modified cob-web model that extends beyond the single demand and supply framework of earlier models.

6 Thought Experiments

In order to examine the impact of the heterogeneous learning index, $\overline{H}_t$, and the adjusted price dynamic index, APD_t, on the price dynamics of the reformulated Lemma 1, three thought experiments were introduced. By substituting an increase in the values of each of these variables into the reformulated Lemma 1, we can examine their respective influences on the price dynamics of this study's modified cob web model. With respect to the first thought experiment, this is achieved by first holding the value of the adjusted price dynamic index, APD_t, to a fixed value, and then examining changes in the heterogeneous learning index, $\overline{H}_t$, on the price dynamics of the reformulated Lemma 1. In the second thought experiment, the value of the heterogeneous learning index, $\overline{H}_t$, is held constant, while allowing the supply chain efficiency, $\bar{I}_t$, variable to vary. The resulting change in the adjusted price dynamic index, APD_t, on the price dynamics of the reformulated Lemma 1 is then examined. Lastly in the third thought experiment, the joint impact of the heterogeneous learning index, $\overline{H}_t$ and the adjusted price dynamic index, APD_t, on the price dynamics of the reformulated Lemma 1 are examined.

6.1 Thought Experiment 1: Impact of Heterogeneous Learning Index, $\overline{H}_t$, on Hog Price Dynamics

In order to isolate the effects of the heterogeneous learning index, $\overline{H}_t$, on the price dynamics of the reformulated Lemma, different values of the heterogeneous learning index, $\overline{H}_t$, are examined for given values of the adjusted price index, APD_t. For each of the given values of the adjusted price index, APD_t, the supply chain efficiency variable, $\bar{I}_t$, takes a value of 1. With this assumption, the adjusted price index, APD_t, yields a price dynamic that is identical to earlier cobweb models (equation 1). This enables us to examine the influence of an increase in the heterogeneous learning index on the predictions made by these earlier models (i.e. Hommes, 2011).

6.1.1 Thought Experiment 1: $APD_t = PD_t \leq 1$ for any $\overline{H}_t$

The reformulated Lemma 1 suggests that as long as the adjusted price dynamic index, APD_t, is less than 1, the modified cob web model yields a convergence of prices for any increase in the value of the heterogeneous learning index, $\overline{H}_t$. Hence, unlike the findings of heterogeneous expectation research (Hommes, 2011), the heterogeneous learning index yields an adaptive process where all values of the heterogeneous learning index, $\overline{H}_t$, will result in a convergence to a rational expectations outcome. This outcome can also be shown by differentiating the reformulated Lemma 1 with respect to the heterogeneous learning index, $\overline{H}_t$. As a result, Lemma 1 proposes the following price dynamics:

Proposition 1: When the adjusted price dynamic index, APD_t, is less than 1, hog prices have a convergent effect for increases in all values of the heterogeneous learning index where $0 \leq \overline{H}_t \leq 1$.

6.1.2 Thought Experiment 1: $APD_t = PD_t > 1$, for different values of $\overline{H}_t$

While earlier cobweb models predict that divergent prices arise when the price dynamic variable, PD_t, has a value greater than 1 (equation 1), such divergent behavior is however dependent on the range of values taken by the heterogeneous learning index, $\overline{H}_t$. These range of values are summarized in table 1. In table 1, the heterogeneous learning index, $\overline{H}_t$, falls within one of three interval values. These interval values are determined by respectively equating the reformulated Lemma 1 to each of the following inequalities: convergent ($|L_t| < 1$), divergent ($|L_t| > 1$) and oscillating prices ($|L_t| = 1$). The reformulated Lemma 1 is then solved for the heterogeneous learning index, $\overline{H}_t$ to yield a critical interval value of $\frac{1}{2} - \frac{1}{2APD_t}$. By comparing the values of the heterogeneous learning index, $\overline{H}_t$ that are less than, greater than and equal to this critical value, the price dynamics of the reformulated Lemma 1 can then be examined. These price dynamics are summarized in table 1.

Table 1.

Interval Value of $\overline{H}_t$ for Hog Price Dynamics where APD_t >1

Lemma 1< 1 (Convergent)	Lemma 1 >1 (Divergent)	Lemma 1=1 (Oscillating)
$\frac{1}{2}-\frac{1}{2APD_t}<\overline{H}_t<1$	$0<\overline{H}_t<\frac{1}{2}-\frac{1}{2APD_t}$	$\overline{H}_t=\frac{1}{2}-\frac{1}{2APD_t}$

According to table 1, the reformulated Lemma 1 suggests that despite having an adjusted price dynamic index, APD_t, that is greater than 1 (i.e. divergence), such divergent behavior is contingent on a value of the heterogeneity index, $\overline{H}_t$, that is less than its critical interval value. Table 1 also shows that that the reformulated Lemma 1 yields convergent and oscillating price movements when the heterogeneous learning index, $\overline{H}_t$, is respectively greater than or equal to its critical interval value. Hence in addition to the convergent tendencies of proposition 1, the following price dynamics from Lemma 1 are also proposed:

Proposition 2: Given a APD_t that is greater than 1, a heterogeneous learning index, $\overline{H}_t$ whose value that is greater than, less than or equal to its critical interval value of ($\frac{1}{2}-\frac{1}{2APD_t}$), yields a respective convergence, divergence and oscillation of prices.

6.2 Thought Experiment 2: Impact of adjusted price dynamic index, APD_t, on Hog Price Dynamics

To examine the influence of specialization on the modified cob web model, increases in the adjusted dynamic price index, APD_t, on the price dynamics of the reformulated Lemma 1 are examined. Yet, since the price dynamics of the reformulated Lemma 1 are dependent on values of the heterogeneous index, $\overline{H}_t$, the influence of the supply chain efficiency variable, $\overline{I}_t$, on the reformulated Lemma 1 is examined for a heterogeneous learning index value, $\overline{H}_t$, of 0.5. This threshold value was chosen to reflect the expectations of the "learning expectation" group where the population is neither strictly naïve ($\overline{H}_t$=0) nor strictly rational ($\overline{H}_t$=1). In that, a heterogeneous learning index value, $\overline{H}_t$, of 0.5 not only directly appeals to this study's concept of a "learning expectation", but this "learning expectation" offers a potentially more realistic and plausible alternative to naïve and rational expectations. Yet, since populations can vary in the degree to which they exhibit such learning expectations, increases in the value of the adjusted price dynamic index, APD_t, are evaluated for a heterogeneous learning index, $\overline{H}_t$, that is greater, less and equal to this threshold value of 0.5.

6.2.1 Thought Experiment 2.1: $\overline{H}_t \geq \frac{1}{2}$ for any increase in the value of APD_t

The reformulated Lemma 1 indicates that as long as the heterogeneous learning index, $\overline{H}_t$, is greater or equal than 0.5, price convergence arises for any increase in the value of the adjusted price dynamic index, APD_t. This result suggests that increases in the specialized efficiencies of the U.S. hog-supply chain offer opportunities for producers to exploit these efficiency gains. These efficiency gains yield a downward pressure on hog prices whereby according to the reformulated Lemma 1, the following is proposed:

Proposition 3: *When the heterogeneous learning index, $\overline{H}_t$, is greater than or equal to 0.5, hog prices have a convergent effect for any increases in the value of the adjusted price dynamic index, APD_t.*

6.2.2 Thought Experiment 2.2: $\overline{H}_t < \frac{1}{2}$, for different values of APD_t

On the other hand, when the heterogeneous learning index, $\overline{H}_t$, falls below its threshold value of 0.5, the influence of an increase in the adjusted price dynamic index, APD_t, on the reformulated Lemma 1 is dependent upon a range of critical values. This set of critical values is shown by three interval values shown in table 2. Each of these intervals is calculated by equating the reformulated Lemma 1 to values that respectively correspond to the following inequalities: convergence ($|L_t|$<1), divergence ($|L_t|$>1) and

oscillation ($|L_t|$=1). The reformulated Lemma 1 is then solved for the adjusted price dynamic index, APD_t, to yield a critical interval value of $\frac{1}{1-2\overline{H}_t}$. Relative to this critical interval value, there are three sets of values where changes in the adjusted price dynamic index, APD_t, will result in convergent, divergent and oscillating price dynamics.

Table 2.
Interval Value of APD_t for Hog Price Dynamics where $\overline{H}_t < 1/2$

Lemma 1< 1 (Convergent)	Lemma 1 >1 (Divergent)	Lemma 1=1 (Oscillating)
$APD_t < \frac{1}{1-2\overline{H}_t}$	$APD_t > \frac{1}{1-2\overline{H}_t}$	$APD_t = \frac{1}{1-2\overline{H}_t}$

Table 2 shows that relative to its critical value, an increase in the adjusted price dynamic index, APD_t, will result in non-convergent price movements. For instance, when values of the adjusted price dynamic, APD_t, fall below its critical value, the reformulated Lemma 1 predicts a period of convergent prices. However, as the values of the adjusted price dynamic, APD_t, increases to the point where it is equal to its critical value, the reformulated Lemma 1 predicts a period of oscillating prices. As the values of the adjusted price dynamic, APD_t increases beyond its critical value, the reformulated Lemma 1 shows that prices will diverge. Such changes in the adjusted price dynamic, APD_t, offers a more robust explanation than earlier cobweb models. Cob web models predict non-convergent price movements arise when the value of the price dynamic, PD_t, is greater than or equal to 1 (see equation 1). Yet, according to the reformulated Lemma 1, divergent behaviors are also dependent upon the relationships between the adjusted price dynamic, APD_t, and its critical value. By drawing on the reformulated Lemma 1, the effect of this increasing specialization on the price dynamics of the modified cob web model can be proposed as follows:

Proposition 4: *Given a heterogeneous learning index, $\overline{H}_t$, that is less than its critical value of 0.5, increases in the adjusted price dynamic index, APD_t, relative to that of its critical interval value ($\frac{1}{1-2\overline{H}_t}$), yields non-convergent price dynamics*.

6.3 Though Experiment 3: The Joint Influence of Heterogeneous Expectations, $\overline{H}_t$, and adjusted price dynamic index, APD_t, on Hog Prices

Since an agent's heterogeneous expectations operates within an increasingly specialized supply chain, their joint effects are examined in this third thought experiment. Figure 2 illustrates their joint of effects. Relative to their respective critical values, figure 2 shows that when the value of the heterogeneous learning index, $\overline{H}_t$, is equal or greater than that of its critical value of 0.5, prices converge. In particular, area I in figure 2 shows that while increases in the adjustment dynamic price index, APD_t, favors convergent prices, agents with learning expectations, $\overline{H}_t$, that are equal or greater than its critical value of 0.5 will yield a convergence that is robust to any increases in the value of the adjustment dynamic price index, APD_t. This finding suggests that an agent's learning expectations, $\overline{H}_t$,yields a rational equilibrium outcome that is robust to any increase in the adjusted price dynamic price index, APD_t.

Furthermore, this price convergence can arise even if the heterogeneous learning index, $\overline{H}_t$, is below its critical value of 0.5 (area II). Specifically, area II shows that price convergence occurs in the area below the upward sloping portion of figure 2. This upward sloping portion is a graph of the critical values, $\frac{1}{1-2\overline{H}_t}$, of the adjusted price dynamic index, APD_t. When increases in the adjustment dynamic price index, APD_t, is below this upward sloping portion, prices converge even when agents fail to develop a structured understanding of the demand and supply relationships of the hog-pork supply chain (i.e. $\overline{H}_t$ <0.5). That is, in the absence of learning expectations (i.e. $\overline{H}_t$ <0.5), the specialized activities of the industrialized process can yield a convergence where prices are consistent with a rational equilibrium outcome. However, as the adjusted price dynamic index, APD_t, increases beyond its critical value of

$\frac{1}{1-2\overline{H}_t}$, prices diverge from this rational equilibrium state. This divergence is shown by area III of figure 2 where divergent prices arise in the area above the upward sloping portion of figure 2. This is because in the absence of learning expectations (i.e. $\overline{H}_t$ <0.5), agents cannot develop an expectation of prices that will coordinate the specialized activities of the industrialized process. Prices thereby diverge from a rational equilibrium outcome state. This affirms our earlier findings that indicate the attainment of a rational equilibrium outcome cannot occur in the absence of learning expectations.

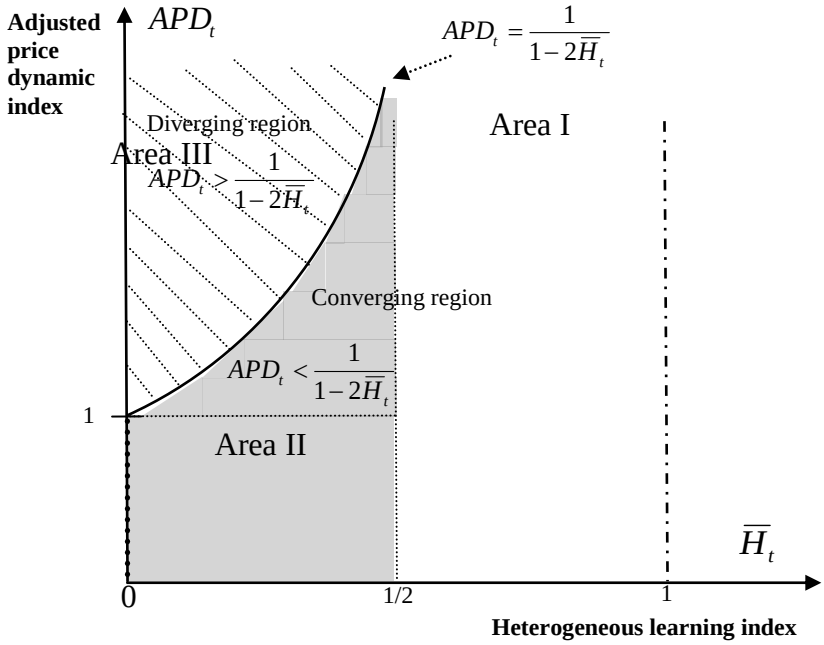

Figure 2. Joint effects of heterogeneous learning index and adjusted price dynamic index to hog price dynamics

7 Discussions and Conclusions

While adaptive expectations have been central to heterogeneous expectations research, the objective of this study was to develop a "modified" cob web model that addresses the shortcomings of the adaptive expectations framework. Specifically, the U.S. hog-pork supply chain was used to illustrate the influence of an agent's heterogeneous expectations and the specialization of supply chain activities on the price dynamics of our modified cob web model. In this model, an agent's heterogeneous expectations are based on a concept of learning expectations. These learning agents have a heterogeneous understanding of the various demand and supply relationships of the U.S. hog-pork supply chain. This heterogeneity offers a structured understanding that coordinated the supply and demand relationships of the hog-pork supply chain. Furthermore, this heterogeneity was also examined within a context of an increasing specialization of supply chain activities. This specialization appeals to an industrialized process where the production efficiencies of the supply chain are also important to coordinating the various production activities of the hog-pork supply chain. By developing this modified cob web model, this study addresses the two shortcomings of adaptive expectations research where an agent's heterogeneous expectations and the specialization of supply chain activities were found to have distinctive influences on the price dynamics of an industrialized system. This model offers three contributions / implications to the study of endogenous price dynamics.

First, while a number of empirical findings have failed to confirm the heterogeneous expectations hypothesis (Hommes, 2011), this study's modified cob web model identifies those conditions where a rational equilibrium outcome can be attained. By increasing an agent's heterogeneous expectations, this modified cob web model shows that an agent's expectation not only coordinates the various demand and supply relationships of a vertical supply chain, but that this model also shows a convergence of prices that is consistent with a rational equilibrium outcome. In that, unlike adaptive expectations, this heterogeneity of expectations is based upon a concept of learning expectations where the equilibrium tendencies of a

cob web model can arise even when agents fail to fully reflect the information and computational demands of rational expectations. This is because unlike adaptive expectations, learning expectations offers a heterogeneity where an agent's forecast is not based upon a "perceived" understanding of a structural model, but rather this forecast is based upon knowledge of the structural model itself. This concept of learning expectations is consistent with various empirical and experimental studies where they have found that it takes subjects repeated experiences or trials to gain a full understanding of the structure of equations ascribed by rational expectations (Branch and McGough 2008; Colucci and Valori, 2011; Conlisk, 1996; Goeree and Hommes, 2000; Sonnemans et al., 2004; Ranyard et al., 2008). This study's modified cob web model offers an approach to analytically examine such learning behaviors.

Second, as heterogeneous expectations operate within a context of a highly specialized supply chain, this study's modified cob web model shows that this specialization can offer additional insights to the heterogeneous expectations hypothesis (Hommes, 2011). In appealing to the productive driven focus of cob web models, heterogeneous expectations research has found that this production focus is subject to negative feedback influences. Such negative feedback can result in a set of price expectations that favor a convergence to a rational expectations equilibrium outcome state (Heemeijer et al., 2009). This study's modified cob web model offers insights to this negative feedback process where it shows that the specialization of production activities has a convergent influence on price. Hence, this specialization not only yields prices that are consistent with a rational equilibrium outcome, but that this specialization may be an important consideration when examining the negative feedback influences of cob web models.

Third, while cobweb models are noted for their convergent properties, this study's modified cob web model also identifies conditions where prices will diverge from a rational equilibrium outcome state. In this study's model, the critical values associated with the heterogeneous expectations and supply chain specialization variables were important to identifying such divergent outcomes. Specifically, a pervasive finding of heterogeneous expectation research is that "deviations from the RE(Rational Expectations) fundamental benchmark seem to be the rule rather than the exception" (Hommes, 2011; p. 21). For instance, Sonnemans et al. (2004) study show that 60% of their price fluctuations are chaotic or non-equilibrative in nature, while a convergence to a unique RE(Rational Expectations) steady stay occurs only 10% (see also Brock and Hommes, 1997). Relative to their critical values, changes in the values of the heterogeneous expectations and supply chain specialization variables can result in such non-equilibrium price outcomes. As a result, one of the contributions of this study is it not only introduces these variables as important considerations to heterogeneous expectations research, but their critical values identifies the "scope" or "boundary" conditions of the rational expectations hypothesis.

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Appendix I

This set of heterogeneous expectations are assumed to be risk neutral where the price expectations for each group of supplying agents respond to the same linear production function $S(\cdot)$. This means that the slope coefficient of supply, s_t, for each heterogeneous group in equation 8b is assumed to be identical for all groups. This yields the following restrictions:

(8b) $$s_t = \frac{\partial S\left(P_{t,e-r}^{hog}\right)}{\partial P_{t,e-r}^{hog}} = \frac{\partial S\left(P_{t,e-n}^{hog}\right)}{\partial P_{t,e-n}^{hog}} = \frac{\partial S\left(P_{t,e-l}^{hog}\right)}{\partial P_{t,e-l}^{hog}} > 0,$$

Where, $s_t = \dfrac{dS\left(P_{t,e}^{hog}\right)}{dP_{t,e}^{hog}}$

In explaining the demand side assumptions of this modified cobweb model, the slope coefficient of demand, d_t, is downward sloping and convex (Greenhut, Hwang and Ohta, 1975) in pork prices such that:

(9) $$d_t \equiv \frac{dD\left(P_t^{pork}\right)}{dP_t^{pork}} < 0,$$

Where, $\dfrac{\partial^2 D\left(P_t^{pork}\right)}{\partial\left(P_t^{pork}\right)^2} \geq 0$

Appendix II

According to Marsh and Brewster (2004), they find that hog and pork prices are co-integrated in which increases or decreases in pork price follow respective increases or decreases in hog prices. However, such co-integrated price movements involve aspects of asymmetric price transmission whereby upward movements in hog prices are followed by larger increases in pork prices. As such asymmetric price transmissions are typically associated with a market power influences (e.g. Boyd and Brorsen, 1988), this article assumes the presence of such market power influences in which successive agents in the hog-pork value chain "pass through" price variability to adjacent members of the hog-pork supply chain. This is consistent with a "mark-up" pricing model commonly found in the U.S. hog industry where an agent's profit is independent of change in hog prices and pork prices.

From this standpoint, π_t^d, π_t^s and π_t^r respectively denotes the unit profit realized by hog distribution, slaughter and pork retail agents at period t. In accordance to figure 1, P_t^{d-s} and P_t^{s-r} denotes the prices found in the adjacent stages of hog distribution-slaughter and pork slaughter-retail segments of Hog-Pork supply chain in period t.The unit profits for each of the downstream stages of this hog-pork supply chain are represented as follows:

(A.1) $$\pi_t^d = \frac{I_t^d Q_t^s \cdot P_t^{d-s} - Q_t^s \cdot P_t^{hog}}{Q_t^s} = I_t^d P_t^{d-s} - P_t^{hog}.$$

(A.2) $$\pi_t^s = \frac{I_t^d I_t^s Q_t^s \cdot P_t^{s-r} - I_t^d Q_t^s \cdot P_t^{d-s}}{I_t^d Q_t^s} = I_t^s P_t^{s-r} - P_t^{d-s}.$$

(A.3) $$\pi_t^r = \frac{I_t^d I_t^s I_t^r Q_t^s \cdot P_t^{pork} - I_t^d I_t^s Q_t^s \cdot P_t^{s-r}}{r_t^h r_t^s Q_t^s} = I_t^r P_t^{pork} - P_t^{s-r}.$$

By substituting equation (A.3) of π_t^r into (A.2) of π_t^s, and then equation (A.2) of π_t^s into (A.1) of π_t^d, the following equation (A.4) or equation 8 is deduced:

(A.4) $\bar{I}_t \cdot P_t^{pork} = P_t^{hog} + \bar{\pi}_t$ or (10) $P_t^{pork} = \left(P_t^{hog} + \bar{\pi}_t\right)/\bar{I}_t$,

Where $\bar{\pi}_t \equiv I_t^d I_t^s \pi_t^r + I_t^d \pi_t^s + \pi_t^d$

Equation (A.4) will be used in the proof of the reformulated Lemma 1 for the derivation of equation (13).

Appendix III: Proof of Lemma 1

Consider that

(A.5) $$L_t = \frac{P_{t+1}^{hog} - P_t^{hog}}{P_t^{hog} - P_{t-1}^{hog}},$$

set $L_t^M \equiv \max_{1 \le i \le t} \{|L_i|\}$, $L_t^m \equiv \min_{1 \le i \le t} \{|L_i|\}$.

Whe $|L_t| < 1, t = 1,2,\cdots$, which means that, $L_t^M < 1$, to yield:

(A.6) $$\left|P_{t+1}^{hog} - P_t^{hog}\right| = \left|\prod_{j=1}^{t} L_j\right| \cdot \left|P_1^{hog} - P_0^{hog}\right| < \left(L_t^M\right)^t \left|P_1^{hog} - P_0^{hog}\right| \to 0, \text{ as } t \to \infty$$

So $\{P_t^{hog}\}$ is converging.

Similarly, when $|L_t| > 1, t = 1,2,\cdots$, which means that, $L_t^m > 1$, to yield:

(A.7) $$\left|P_{t+1}^{hog} - P_t^{hog}\right| = \left|\prod_{j=1}^{t} L_j\right| \cdot \left|P_1^{hog} - P_0^{hog}\right| > \left(L_t^m\right)^t \left|P_1^{hog} - P_0^{hog}\right| \to \infty, \text{ as } t \to \infty$$

So $\{P_t^{hog}\}$ is diverging.

Appendix IV: Proof of the reformulated Lemma 1

At period t, the market equilibrium of the hog-pork supply chain is expressed by the follow equation:

(A.8) $$D\left(P_t^{pork}\right) = \bar{I}_t\ H_t^r S\left(P_t^{hog}\right) + \bar{I}_t\ H_t^n S\left(P_{t-1}^{hog}\right) + \bar{I}_t\ H_t^l S\left(\alpha_t P_t^{hog} + (1-\alpha_t) P_{t-1}^{hog}\right).$$

Substituting P_t^{pork} from equation (A.4) into the equation (A.8) yields:

(A.9) $$D\left(\left(P_t^{hog} + \bar{\pi}_t\right)/\bar{I}_t\right) = \bar{I}_t H_t^r S\left(P_t^{hog}\right) + \bar{I}_t H_t^n S\left(P_{t-1}^{hog}\right) + \bar{I}_t H_t^l S\left(\alpha_t P_t^{hog} + (1-\alpha) P_{t-1}^{hog}\right).$$

Differentiating both sides of this equation (A.9) w. r. t. P_{t-1}^{hog}, yields

(A.10) $$\frac{\partial D\left(P_t^{pork}\right)}{\partial P_t^{pork}} \cdot \frac{1}{\bar{I}_t} \cdot \frac{dP_t^{hog}}{dP_{t-1}^{hog}} = \bar{I}_t H_t^r \frac{\partial S\left(P_t^{hog}\right)}{\partial P_t^{hog}} \cdot \frac{dP_t^{hog}}{dP_{t-1}^{hog}} + \bar{I}_t H_t^n \frac{\partial S\left(P_{t-1}^{hog}\right)}{\partial P_{t-1}^{hog}}$$
$$+ \bar{I}_t H_t^l \frac{\partial S\left(P_{t,e-l}^{hog}\right)}{\partial P_{t,e-l}^{hog}} \cdot \left(\alpha_t \frac{dP_t^{hog}}{dP_{t-1}^{hog}} + (1-\alpha_t)\right).$$

By substituting equations (A.8), (A. 9), (A.5) into (A.10), we have:

(A.11) $$-|d_t| \cdot \frac{1}{\bar{I}_t} \cdot L_t = \bar{I}_t H_t^r \cdot s_t \cdot L_t + \bar{I}_t H_t^n \cdot s_t + \bar{I}_t H_t^l \cdot s_t \cdot \left(\alpha_t L_t + (1-\alpha_t)\right),$$

i.e.

(A.12) $$-\left|d_t\right| \cdot L_t = \bar{I}_t^{\,2} H_t^r s_t L_t + \bar{I}_t^{\,2} H_t^n s_t + \bar{I}_t^{\,2} H_t^l s_t \alpha_t L_t + \bar{I}_t^{\,2} H_t^l s_t \left(1-\alpha_t\right).$$

And then solving for L_t

(A.13) $$L_t = -\frac{\bar{I}_t^{\,2} s_t \left(H_t^n + H_t^l - \alpha_t H_t^l\right)}{\left|d_t\right| + \bar{I}_t^{\,2} s_t \left(H_t^r + \alpha_t H_t^l\right)}.$$

Where

(A.14) $$\overline{H}_t \equiv H_t^r + \alpha_t H_t^l$$

By drawing on equation (A.14), equation (A.13) is rearranged and is instead expressed by equation (A.15):

(A.15) $$L_t = -\frac{\bar{I}_t^{\,2} s_t \left(1-\overline{H}_t\right)}{\left|d_t\right| + \bar{I}_t^{\,2} s_t \overline{H}_t} \equiv -\frac{\left(1-\overline{H}_t\right) APD_t}{1+\overline{H}_t APD_t},$$

Where,

(A.16) $$APD_t = \frac{\bar{I}_t^{\,2} s_t}{\left|d_t\right|}.$$

Since $0 \le \overline{H}_t \le 1$, the reformulated Lemma 1has the following form:

(A.17) $$\left|L_t\right| = \frac{\left(1-\overline{H}_t\right) APD_t}{1+\overline{H}_t APD_t} = \frac{1-\overline{H}_t}{\overline{H}_t + \dfrac{1}{APD_t}}.$$

Where if $\overline{H}_t \ge \frac{1}{2}$, $\left|L_t\right| < 1$.