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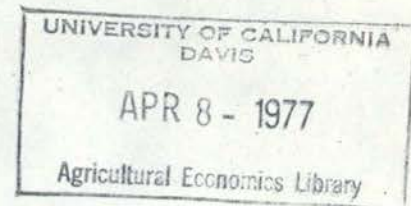
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OPTIMAL INTERNATIONAL BORROWING AND  
CREDIT-WORTHINESS CONTROL

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CREDIT-WORTHINESS CONTROL†

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The paper develops a two-sector model incorporating the relation between debt-servicing capacity and the terms of credit facing the borrowing country. Debt-servicing capacity is reflected via economic variables which are endogenous to the system and thus can be affected by related economic policies. It is shown that optimal growth requires subsidizing of the export sector while simultaneously taxing private borrowing of foreign funds.

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OPTIMAL INTERNATIONAL BORROWING AND  
CREDIT-WORTHINESS CONTROL

The determination of appropriate international borrowing policy has attracted the attention of many economists because economic growth, particularly in developing countries, depends heavily on the inflow of foreign funds. Work on this topic dates back to Adam Smith, but more recent contributions have been made by K. Hamada (1966, 1969), P. K. Bardhan (1967), Ngo Van Long (1974), and R. Manning (1972). The most common approach in related economic analyses has involved a "small country" assumption that has been taken to imply an infinitely elastic supply of loans at a given interest rate. It has been recently argued by J. A. Hanson, however, that the supply of loans may not be completely elastic even for the small country because, as debts get large, the probability of a debt crisis for the individual country increases. That is, as debts get large relative to the country (not the market), it becomes more likely that the borrower will not be able to make repayments on schedule and, hence, lenders will become more reluctant to lend as the expected, discounted returns decline. As indicated by A. F. Mohammed and F. Saccomanni (1973), there is no doubt that this risk plays a major role in the lending practices of commercial banks. Nonprofit-oriented lending institutions are also likely to be concerned with the probability that reschedulings will be required; hence, their lending decisions may also be influenced to some degree by their borrowers' credit status. It thus follows, as Hanson rightly points out, that even small countries do not face an infinitely elastic supply of foreign funds; rather, the terms of credit are endogenous depending on credit worthiness. Credit worthiness, in turn, is determined by the countries' economic performance (usually measured by a few accepted economic indicators).

Since economic-performance indicators are affected by economic policies, it is important to explore borrowing policies for optimal growth, taking into account the relationship between economic performance and terms of credit. Although empirical results suggest that several economic variables are related to debt-servicing capacity, one may accept the use of only one indicator as a useful simplification for theoretical analysis [Feder and Just (1976, 1977)]. Hanson has suggested the use of the debt-equity ratio for this purpose. However, in the one-sector model developed by Hanson, this approach implicitly leads to the assumption that "resources can always be costlessly allocated to obtain the necessary foreign exchange for debt servicing" [(1974), p. 619, footnote 10]. Hence, the only factor limiting availability of foreign exchange is the overall productive capacity of the country which may be represented by the capital stock.

Once installed, however, capital is not a malleable factor that can be equally productive in any sector. Although some types of capital are more flexible than others (transportation equipment, power plants, etc.), it is more likely with many exports, particularly in developing countries where exports are composed mainly of nonmanufactured goods, that investments are highly specialized (mines, dams, etc.). Even when reallocation of capital is possible, the time lag involved may be too long for lenders to consider the productive capacity of other sectors as relevant for the evaluation of export potential in the case of debt payment difficulty. Since the export sector is the main source of foreign exchange earnings in many countries and capital reallocation may not be possible, it thus seems that the most appropriate case for developing countries may be where the terms of credit depend specifically on capital stock in the export



sector (as well as the debt burden). This is consistent with the findings of a recent study regarding lending behavior in the Eurodollar market where two variables related to export performance (the ratio of debt service to export earnings and a measure of export variability) appear to be considered as relevant by lenders, while the ratio of debt to GNP (which is a proxy for the debt-equity ratio) was found insignificant [Feder and Just (1976)].

The purpose of this paper is to consider a two-sector model where an export sector is defined explicitly. The crucial assumption is that capital, once allocated to the export or nonexport sectors, cannot be reallocated.<sup>1</sup> Accordingly, the indicator of credit worthiness is not the ratio of debt to total capital but, rather, the ratio of debt to the capital stock of the export sector. Although other indicators involving export capacity and debt burden could be suggested, the use of an indicator similar in form to Hanson's is preferred so that conclusions are comparable. However, the optimization model presented herein is dynamic (unlike Hanson's model which describes only the steady state); hence, the conclusions hold along an optimal time path rather than in a static sense. Only loans are considered since direct investment may be affected by other risk indicators.

#### 1. A model with capital inflexibility

Consider an economy composed of two productive sectors, say F and G, where good F can serve for both consumption and investment while good G is exported. Domestic use of good G is assumed negligible and is ignored. A small country assumption is adopted so that the economy faces a given international price of both commodities. For simplicity, only one (limiting) factor of production—capital—is considered, and population growth is ignored. The economy may borrow in the international capital market,

but the rate of interest will depend positively on its credit status as reflected by the ratio of outstanding debt to export sector capital stock, namely (the time index omitted for simplicity)

$$r = r \left( \frac{B}{K_g} \right); \quad r' > 0 \quad (1)$$

where  $r$  is the average rate of interest,  $B$  is outstanding external debt, and  $K_g$  is the stock of capital in the export sector. Production is assumed to have declining but positive marginal productivity:

$$F = F(K_f); \quad F' > 0; \quad F'' < 0 \quad (2)$$

$$G = G(K_g); \quad G' > 0; \quad G'' < 0 \quad (3)$$

where  $F$  and  $G$  are the quantities produced of the two respective goods and  $K_f$  is the stock of capital in the  $F$  sector.

Define  $\alpha_t$  and  $\beta_t$  as the proportions of the domestic output of the  $F$  sector allocated for investment in the  $F$  and  $G$  sectors, respectively, at time  $t$ . The quantity of  $F$  output that is not invested is consumed. Consumption can be augmented by imports (denoted by  $M$ ) of the same good. Consumption is thus defined by

$$C_t = (1 - \alpha_t - \beta_t) \cdot F_t + M_t. \quad (4)$$

The balance-of-payments constraint is

$$M_t + (r_t + \theta)B_t = PG_t + L_t \quad (5)$$

where  $\theta$  is the rate of debt amortization (assumed constant),  $L_t$  is the level of new loans taken at time  $t$ , and  $P$  is the international export



price for good G. Import price (the price of F) is assumed to be unity for simplicity (or, alternatively, to serve as numeraire).

The dynamic equations for capital stocks and debt are given by

$$\dot{K}_f = \alpha F(K_f) - \delta K_f \quad (6)$$

$$\dot{K}_g = \beta F(K_g) - \delta K_g \quad (7)$$

$$\dot{B} = L - \theta B \quad (8)$$

where  $\delta$  specifies the depreciation rate for capital in both sectors. To complete the model, it is assumed that the planner perceives a community welfare function of the form

$$W = \int_0^{\infty} e^{-\rho t} U(C_t) dt; \quad U' > 0; \quad U'' < 0 \quad (9)$$

where  $\rho$  is a time discount factor.

The objective is to maximize  $W$  subject to equations (1)-(8) and initial conditions by choosing an optimal time path for  $\alpha$ ,  $\beta$ ,  $L$ , and  $M$ . Using equation (5) to replace  $M$  in equation (4) and inserting (1), (2), and (3), the problem can be treated in an optimal control framework where the current value of the Hamiltonian ( $H$ ) is

$$e^{\rho t} H = U \left\{ (1 - \alpha - \beta) F(K_f) + P G(K_g) + L - \left[ r \left( \frac{B}{K_g} \right) + \theta \right] \cdot B \right\} \quad (10)$$

$$+ \lambda \cdot [\alpha F(K_f) - \delta K_f] + \mu \cdot [\beta F(K_g) - \delta K_g] + \eta \cdot [L - \theta B]$$

and  $\lambda$ ,  $\mu$ , and  $\eta$  are dynamic shadow prices of  $K_f$ ,  $K_g$ , and  $B$ , respectively. Following the Pontryagin Maximum Principle, the following conditions characterize the optimal paths in addition to appropriate transversality conditions:<sup>2</sup>

$$\frac{\partial H}{\partial \alpha} = 0 \Rightarrow U' = \lambda \quad (11)$$

$$\frac{\partial H}{\partial \beta} = 0 \Rightarrow U' = \mu \quad (12)$$

$$\frac{\partial H}{\partial L} = 0 \Rightarrow U' = -\eta \quad (13)$$

$$\dot{\lambda} - \rho\lambda = -\frac{\partial H}{\partial K_f} \Rightarrow \dot{\lambda} = \lambda \cdot [\delta + \rho - F'] \quad (14)$$

$$\dot{\mu} - \rho\mu = -\frac{\partial H}{\partial K_g} \Rightarrow \dot{\mu} = \mu \cdot \left[ \delta + \rho - \left( PG' + r' \cdot \frac{B^2}{K_g^2} \right) \right] \quad (15)$$

$$\dot{\eta} - \rho\eta = -\frac{\partial H}{\partial B} \Rightarrow \dot{\eta} = \eta \cdot \left[ \rho - r - r' \cdot \frac{B}{K_g} \right]. \quad (16)$$

Note that, in the formulation of equations (14)-(16), the results of (11)-(13) were used, which imply

$$U' = \lambda = \mu = -\eta > 0. \quad (17)$$

Equation (17), in turn, yields

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu} = \frac{\dot{\eta}}{\eta}. \quad (18)$$



One can thus derive from equations (14)-(16) the following equalities which hold along the optimal path:

$$F' = PG' + r' \cdot \frac{B^2}{K_g^2} \quad (19)$$

$$F' - \delta = r + r' \cdot \frac{B}{K_g} \quad (20)$$

$$PG' - \delta = r + r' \cdot \frac{B}{K_g} \cdot \left(1 - \frac{B}{K_g}\right). \quad (21)$$

## 2. Implications of the model

Several interesting results are evident from the equations which must hold along the optimal time path. For example, equation (19) suggests that optimal investment policy should lead to a lower direct marginal rate of return (or marginal product of capital) for investment in the export sector than in the other sector. This is a fundamental departure from the common optimization result which suggests that, in the multisector case, the marginal rate of return to investment should be equalized across sectors. Obviously, if the rate of interest on foreign loans were given exogenously, the standard result follows since in that case  $r' \equiv 0$  in equation (19). But, with the rate of interest endogenous as in the present case, the indirect as well as direct marginal benefits of investments in exports must be considered. The direct effects are reflected by the marginal rate of return on capital in the export sector. The indirect benefits due to investing in the export sector are the improved terms of credit which result from improved credit worthiness (brought about by a higher export capacity); these benefits are represented by the term,

$$\left( r, \frac{B^2}{K_f^2} \right).$$

Equation (20) implies that the net marginal rate of return to investment in the other (nonexport) sector should be higher than the interest rate on foreign debt. By comparison, a standard small country growth model suggests an equality between the two. The rationale for the rule implied by (20) is that the marginal cost of borrowing is higher than the average cost since additional indebtedness worsens the credit worthiness of the economy. The marginal benefit generated by investing in the F sector should thus equal the full marginal cost which is higher than the rate of interest. Hanson obtains a similar result for the steady-state solution. However, since he deals with a one-sector economy, the marginal return on investment is not defined for any particular sector. The results in this paper demonstrate that optimal policies differ for different sectors. A related practical implication of this result, in the case where borrowing and investment are performed by private enterprises, is that the government should impose taxes or credit limits on external loans to the nonexport sector so that the private cost of borrowing is equal to the social cost. Otherwise, the cost of borrowing for private producers is only the rate of interest  $r$ , and excessive investment in the nonexport sector will result. Since most institutions lending to private borrowers in less-developed countries require a government guarantee on the repayment of loans, the regulation of the inflow of funds by various sectors is rather easy to perform.

The conclusions relating to equation (21) suggest that the relationship between interest rate on foreign debt and the net marginal rate of return on investments in the export sector may not be the same in all cases.



When the debt/export-capital ratio is low (smaller than one), the rate of return should be higher than the rate of interest on foreign loans; however, the converse relation holds when the debt/export-capital ratio is high (larger than one). It appears, however, that this result depends arbitrarily on the specification of the credit-worthiness indicator. For example, if the indicator is specified as the difference between debt and export capital [i.e.,  $r = r(B - K_g)$ ], then one obtains  $PG' - \delta = r$  along the optimal path while  $F' = PG' + r' \cdot B$ ; hence, the implications are qualitatively the same as suggested by (19) and (20), but the marginal rate of return on export capital is always equal to the interest rate.

In general, the relationship between the marginal rate of return on investment in exports and the rate of interest on foreign debt depends on the relative weight of debt and export capital in the credit-worthiness analysis of the lender. If the former is more (less) influential, the interest rate is likely to be lower (higher) than the marginal rate of return on investment in exports.

In any case where the economy operates via market forces, however, social marginal benefits and marginal costs could be equalized by subsidizing investment in the export sector while taxing all foreign loans. For example, if private decision-makers operate so as to equate the marginal rate of return on capital with its marginal cost and if, because of their insignificant size relative to national debt, they do not perceive their effect on external terms of credit, then free-market optimization obtains

$$F' - \delta = r$$

$$PG' - \delta = r.$$

If taxes are then levied at a rate of  $t_f$  per unit of investment in the F sector and  $t_g$  per unit of investment in the G sector ( $t_g < 0$  would correspond to a subsidy), then private optimization conditions become

$$F' - \delta = r + t_f$$

$$PG' - \delta = r + t_g.$$

A social optimum is attained if  $t_f$  and  $t_g$  vary such that

$$t_f = r' \frac{B}{K_g}$$

$$t_g = r' \frac{B}{K_g} \cdot \left(1 - \frac{B}{K_g}\right) = t_f - r' \frac{B^2}{K_g^2}.$$

It is also interesting to examine the ultimate extent to which investment would be carried in the respective sectors (after sufficient passage of time). These results are found by examining the steady-state solution of the control problem. Imposing the steady-state condition,  $\dot{\lambda} = \dot{\mu} = \dot{\eta} = 0$ , equations (14)-(16) yield

$$F' = \delta + \rho \tag{22}$$

$$PG' = \delta + \rho - r' \cdot \frac{B^2}{K_g^2} \tag{23}$$

$$r = \rho - r' \cdot \frac{B}{K_g}. \tag{24}$$

Equation (22) implies a "golden rule" for capital accumulation in the non-export sector similar to the familiar result of standard optimal growth models (except that in the present case population growth is ignored). That



is, investment should be carried to the point where the marginal rate of return on capital is just equal to the social discount rate  $\rho$ . However, equation (23) implies that accumulation of capital in the export sector should be carried beyond the golden rule point. This result follows because of extra benefits derived from the export-capital stock in terms of interest rate reductions in foreign financial markets. But, obviously, if the rate of interest is exogenous for all sectors, the optimal capital stock follows the golden rule in all cases since  $r' = 0$ . The interesting rule suggested by equation (24) is that the rate of interest in the steady state is lower than the social rate of time discount. This result follows from the fact that the marginal cost of borrowing is higher than the rate of interest because of the effect of debt on interest rates.

### 3. Summary

This paper is based on the observation that the terms of credit for economies engaged in international borrowing are affected by their credit status. The paper improves upon previous works both by adopting a more detailed structure of the economy, which yields sector-specific policies, and by investigating the entire time path rather than simply the steady-state solution. Based on empirical results and plausible economic arguments, there is substantial evidence that the economy's export capacity (relative to its debt obligations) is an important factor in credit-worthiness analysis--more so than the overall productive capacity of the country. This is especially true for less-developed countries where the ability to shift resources between export and nonexport sectors is limited. Adopting the assumption of capital being nontransferable between sectors, a growth model is constructed where international borrowing and investment allocation between the export and the nonexport sectors are control variables.



Maximizing the discounted utility of consumption, it is shown that the introduction of an endogenous rate of interest (which depends on the debt/export-capital ratio) has important policy implications, namely, that in noncentralized economies investments in export industries should be increased beyond free-market levels while investments in nonexport sectors should be limited. This is justified by the fact that the social benefits from investment in the export sector include both their direct marginal productivity and an improvement in the economy's credit status and thus in external terms of credit. The latter is not included in the private rate of return on investment in exports, but a subsidy can be used to bridge the gap and thus encourage optimal investment in the export sector.

As for foreign loans, attention is directed to the difference between the marginal cost of borrowing and the rate of interest. A difference occurs because of the deterioration in credit worthiness related to additional borrowing. Appropriate modifications can be made in the domestic economy by taxing foreign borrowing or possibly by limiting governmental guarantees on foreign loans to the nonexport sector so as to avoid excessive borrowing.

These policies are valid all along the optimal time path. But in the limit or as the economy approaches a steady state, it is also shown that capital accumulation in the export sector should be carried beyond the usual golden rule point and that the rate of interest should be lower than the social rate of time discount.



## Footnotes

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<sup>1</sup>This assumption has been adopted in several other optimal growth models where terms of credit and optimal borrowing are not the major interest; see works by S. Bose (1968), S. Chakravarty (1969), P. S. Dasgupta (1969), L. Johansen (1967), and H. E. Ryder (1969).

<sup>2</sup>It is assumed that the constraint,  $\alpha + \beta \leq 1$ , is not binding. That is, it is assumed that the social optimum is such that domestic production is not used completely for investment but is partially consumed (domestically).

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