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Price Uncertainty and the Long Run Equilibrium of the Competitive Firm

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Long run equilibrium conditions of the riskless competitive firm have been examined in a series of recent articles (e.g., Ferguson and Saving, Portes, Bassett and Borcherding, Silberberg, 1974a). By long run, it is assumed that unrestricted entry and exit force output price to the minimum average cost of the representative firm.[1/] Thus, each firm chooses optimal levels of output and inputs subject to the condition that profits are zero. In these long run analyses, it was discovered that many of the traditional short run comparative static results are modified.[2/] In particular, long run factor demand curves are not generally downward sloping and factor demand reciprocity (or symmetry) conditions do not hold except under homotheticity of the production function. However, relative factor demands, input quantity per unit of output, are negatively sloped and symmetric.

The purpose of this paper is to examine the implications of a risk averse equilibrium admitting positive expected profits. The model considered assumes output price risk where all input decisions are ex ante. Equilibrium is characterized by minimizing average cost plus an average risk premium. The long run risk averse output is less than the long run risk neutral output. Input price and expected output price changes move in the same direction regardless of risk preferences. Further, if input price changes affect only expected output price via entry and exit of firms, then relative factor demands are symmetric and downward sloping. When other moments of the distribution of output price are changed as supply shifts, then relative factor demands are not generally symmetric nor downward sloping.

I. Representative Firm Equilibrium

The paradigm presented under certainty argues that firms enter and exit the competitive industry in such a way as to drive profits to zero or where output price equals minimum average cost of the representative firm. Hence, it follows that cost curves of every firm are generally assumed identical. Few would argue that this assumption is very descriptive, but it is often felt that the representative firm paradigm is a reasonable postulate in order to enhance an understanding of long run firm behavior.

In a similar spirit, it is assumed here that there is a firm with representative beliefs and preferences. The existence of risk responsive market behavior is empirically well documented (e.g., Behrman, Just, and Jensen); hence, a movement towards the recognition of risk in the characterization of long run equilibrium appears imperative.

A. Short Run Equilibrium

Consider initially the conventional short run representation of an expected utility maximizing firm. Let $U(\pi)$ denote the utility of profit, where U is the preference function and π is profit. Let profit be given by

$$\pi = Pq - C(q, c) \equiv Pq - C(q),$$

where P is the random output price, q is the quantity of output, $C(q, c) \equiv \sum_{j=1}^{N} c_j X_j (q, c) \equiv C(q)$ is the cost function, while X_j and c_j denote quantities of inputs and their prices respectively.[3/] The output price, P, is distributed with finite mean, $\bar{P}$, and variance, σ. It is assumed that the firm maximizes expected utility of profit, $E[U(\pi)]$, postulated to be concave in output.[4/]

The necessary condition for equilibrium output is given by

$$E[U'(\pi)\ (P - C'(q))] = 0$$

or,

$$\bar{P} - C'(q) = -\frac{Cov[U'(\pi),\ P]}{E[U'(\pi)]}, \qquad (1)$$

where $C'(q)$ is marginal cost and Cov denotes covariance. Marginal utility, $U'(\pi)$, is assumed positive; but, Baron, Sandmo and others have shown the covariance to be negative under risk aversion. Hence, expected price exceeds marginal cost under risk aversion.

Following Pratt and Baron, the risk premium, R, is that quantity of money which makes the entrepreneur indifferent between a sure return of $E(\pi) - R$ and the risky prospect, π. That is, $U[E(\pi) - R] = E[U(\pi)]$ and $R = 0$ for a risk neutral firm. In equilibrium, differentiation of this definition with respect to output, q, and using (1) gives

$$U'[E(\pi) - R]\ [\bar{P} - C'(q) - R'(q)] = \bar{P} - C'(q) + \frac{Cov[U'(\pi),\ P]}{E[U'(\pi)]}$$

or,

$$R'(q) = -\frac{Cov[U'(\pi),\ P]}{E[U'(\pi)]} = \bar{P} - C'(q), \qquad (2)$$

where $R'(q) = dR(q)/dq$. From (2), at equilibrium output, the marginal change in the risk premium as q changes is given by $- Cov[U'(\pi),\ P]/E[U'(\pi)]$ which is positive under risk aversion.

The question now occurs as to the existence of any meaningful long run equilibrium output for the risk averse firm. The rationale for such an equilibrium is apparent and is found elsewhere in the literature (see particularly Baron, and Pazner and Razin). Suffice it to say that demands are considered random and induce a probability distribution on market price where

price is determined by the intersection of the market supply and the demand schedules. It is assumed that there is a representative level of risk aversion which is possessed by participants (or potential participants) in production decisions. That is, the level of risk aversion is bounded away from zero such that all active producers have a representative level of risk aversion.

B. Long Run Equilibrium

Let $E[U(\pi|q^*)]$ denote the expected utility given the optimal output, q^*. If q^* is optimal, then $E[U(\pi|q^*)] \geq E[U(\pi|q)] \geq E[U(\pi|q = 0)]$. It is assumed that $(\pi|q = 0) = 0$ and that exit and entry occurs such that the expected utility of producing is equal to the expected utility of not producing or, $E[U(\pi|q^*)] = E[U(\pi|q = 0)]$.[5/]

It follows that $U[E(\pi) - R] = E[U(\pi|q^*)] = 0$, or,

$$R = E(\pi) - U^{-1}[E(U(\pi|q^*))] \tag{3}$$
$$= E(\pi).$$

From (3), it is clear that expected output price is forced to an equilibrium value such that $E(\pi) - R = 0$, or

$$\bar{P} = \frac{C(q)}{q} + \frac{R}{q}, \tag{4}$$

where the first right hand side term in (4) is average cost and the second term is the average risk premium. The sum of the two terms will be referred to as the subjective average cost, SUAC.

Using (2) and (4) the first order conditions for a firm maximizing expected utility subject to the long run equilibrium constraint, (3), is

$$C'(q) - AC(q) = \frac{R(q)}{q} - R'(q) \tag{5}$$

where $AC(q) = C(q)/q$ or average cost. Second order conditions require that $C''(q) + R''(q) > 0$ at optimum. It is noted that (5) corresponds to the minimization of SUAC. It will be assumed that SUAC has a unique minimum; hence, $\bar{P}$ = min SUAC = SUAC* specifies long run equilibrium output of the firm.[6/] Further, the cost curves are classically U-shaped (Hanoch) with $C''(q) > 0$; hence, $R(q)$ may be concave or convex, but $C''(q) > |R''(q)|$ must hold.[7/]

When the firm is risk neutral $R = 0$ and (5) reduces to the familiar conditions: marginal cost equals average cost, or,

$$C'(q) = AC^*(q), \tag{6}$$

where $AC^*(q)$ is the minimum average cost. When $R(q) > 0$, (5) is rewritten as

$$AC(q) + \frac{R(q)}{q} = SUAC^* = C'(q) + R'(q). \tag{7}$$

Here, it is clear that in the long run, the firm equates subjective marginal cost $[C'(q) + R'(q)]$ with subjective average cost $[AC(q) + R(q)/q]$ = SUAC*. Below, the relationship of long run risk averse equilibrium output and that level of output which minimizes average cost is explored.[8/]

1. Equilibrium Output

Since R is positive under risk aversion, it is clear that the risk averse equilibrium expected output price is higher than the risk neutral equilibrium expected price. However, given classically shaped cost curves and risk aversion, it is interesting to examine long run equilibrium output of the risk averse firm vis-a-vis marginal and average cost. For, if at equilibrium, marginal cost exceeds (is less than) average cost, then the risk averse output would exceed (be less than) the risk neutral equilibrium output.

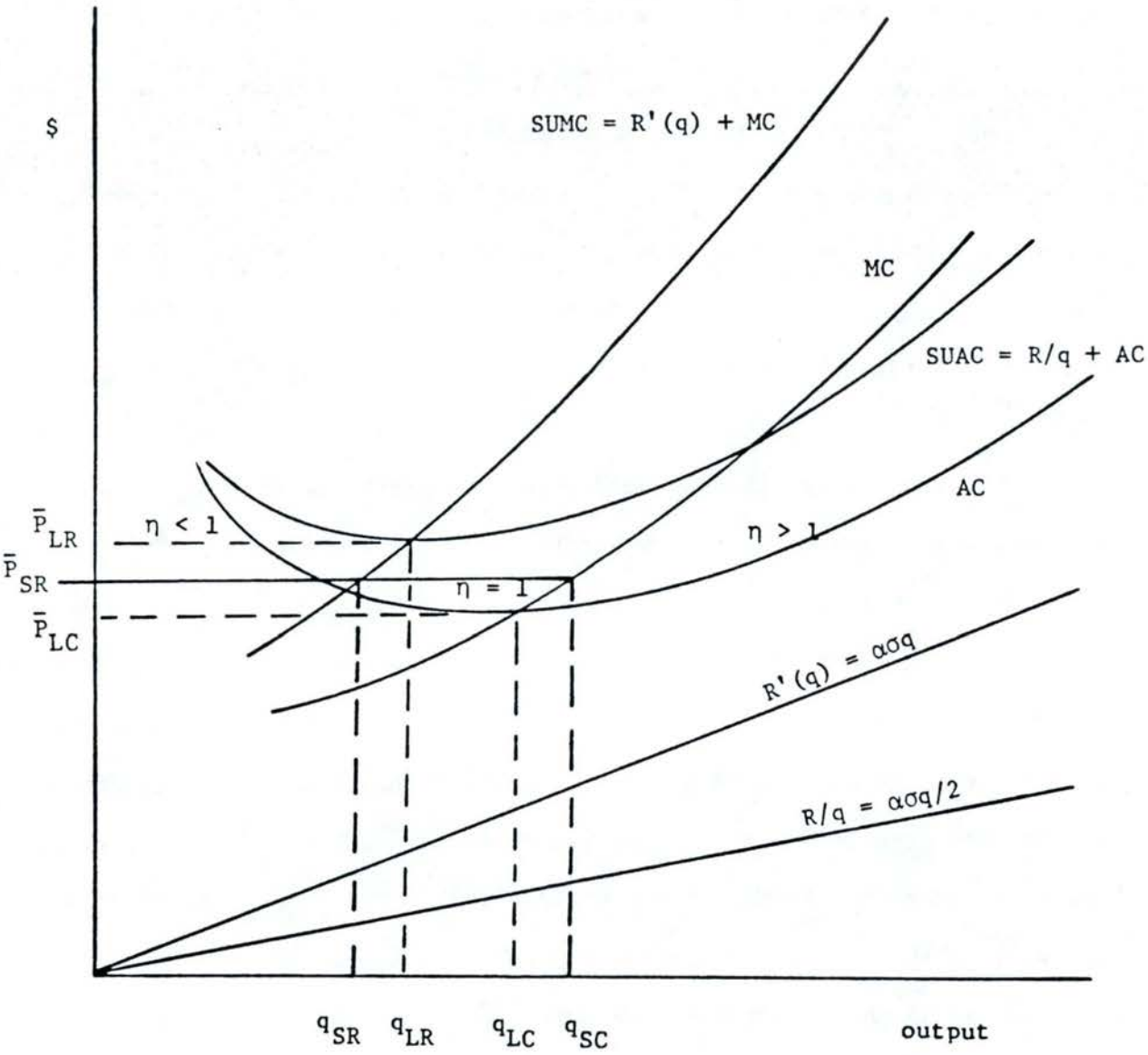
Since $R(0) = 0$, it follows that if $R(q)$ is convex, then the average risk premium exceeds the marginal risk premium. Assuming risk aversion, (2) implies that $R''(q) = - E[U''(\pi)\ (P - C'(q))^2] > 0$. Since R is convex, $R(q)/q < R'(q)$ which from (5) verifies Proposition 1.[9/]

Proposition 1. The long run risk averse equilibrium output occurs at the minimum of average cost plus an average risk premium. The risk averse equilibrium expected output price is higher than the risk neutral equilibrium expected price. Under risk aversion, the average risk premium is increasing. Analogously, the risk premium, R, is convex in output, and the risk averse equilibrium output is smaller than the long run risk neutral equilibrium output.

In summary, Proposition 1 implies that the long run risk averse equilibrium output is less than the long run risk neutral equilibrium output: this is qualitatively analogous to the short run equilibrium output comparisons. The Proposition is illustrated in Figure 1. Because of the widespread empirical and theoretical use of the constant risk averse utility function (Freund, Baron, Connors) under normality, this expected utility function will be employed for illustrative purposes. In such case, $U(\pi) = - e^{-\alpha\pi}$, where $\alpha > 0$ is the Arrow-Pratt risk aversion measure ($\alpha = - U''(\pi)/U'(\pi)$). $E[U(\pi)] = - \exp[- \alpha\{E(\pi) - (\alpha/2)\sigma q^2]$, where σ is the variance of price. In fact, from the definition of R, it is easily shown that $R = (1/2)\alpha\sigma q^2$; hence, $R'(q) = \alpha\sigma q$ and R is clearly convex since $R''(q) = \alpha\sigma > 0$. In the figure, let $\overline{P}_{SR}$ represent the short run expected price. A risk neutral firm would choose output level q_{SC}. However, as Sandmo has shown, a risk averse firm would produce at $q_{SR} < q_{SC}$. The level of output, q_{SR}, is determined by the intersection of expected price, $\overline{P}_{SR}$, and the subjective marginal cost curve, SUMC. The SUMC curve is the

FIGURE 1

Long Run Equilibrium and Risk



sum of the marginal cost curve, MC, and the marginal risk premium, $R'(q) = \alpha\sigma q$. In the case of long run equilibrium for risk neutral firms, expected price would adjust such that expected profits are zero, or equivalently, the long run expected price, $\overline{P}_{LC}$, is equal to the minimum of the average cost curve. The corresponding level of output is q_{LC}. Alternatively suppose all firms are risk averse with average risk premium given by $R/q = \alpha\sigma q/2$. The subjective average cost curve, SUAC, is the sum of the average cost curve, AC, and the average risk premium, R/q. Long run equilibrium is characterized by long run expected price, $\overline{P}_{LR}$, equalling the minimum of SUAC. The corresponding equilibrium output is q_{LR}. Note that since R/q is increasing, $q_{LR} < q_{LC}$.

Consider now the long run adjustment process for risk averse firms in the industry. At the short run price, $\overline{P}_{SR}$ < min SUAC, firms are not receiving a sufficiently high expected price so as to self-insure against risk. Hence, firms will exit the industry driving expected price higher until $\overline{P} = \overline{P}_{LR}$ = min SUAC with a corresponding output of q_{LR} (for this example, it is assumed that adjustments in expected price do not alter higher moments of the distribution. Hence, R/q and $R'(q)$ do not shift as $\overline{P}$ changes). In the next section, comparative static results are analyzed when SUAC is minimized.

2. Comparative Static Analysis

It is necessary here to assume a mechanism relating the effects of entry and exit of firms on the moments of the distribution of price. It is assumed that price, P, is given by $P = h(\overline{P}, \varepsilon)$, where e is a random disturbance. It is also assumed that input price changes affect $\overline{P}$ through entry and

exit of firms but leave the parameters of the distribution of ε unchanged. Functionally, SUAC* is denoted by

$$\begin{aligned} SUAC^* &= AC(q^*) + R(q^*)/q^* \qquad (8) \\ &= \bar{P} - \frac{1}{q^*} U^{-1}[E\{U(P\ f(X^*(q^*, c)) - c'\ X^*(q^*, c)\}] \\ &= \bar{P} - \frac{1}{q^*} U^{-1}[0] \\ &= \bar{P}. \end{aligned}$$

It is important to distinguish the relationship in (8) from the special case of risk neutrality where $R = 0$ and $\bar{P} = AC^*$. In this risk neutral case, note that AC* is a function of only the input price vector. Hence, as an input price, c_j, rises so also must $\bar{P}$ such that $\partial\bar{P}/\partial c_j = \partial AC^*/\partial c_j$.[10/] Therefore, equilibrium values of $\bar{P}$ are determined by c_j, or $\bar{P} = \bar{P}(c)$. In contrast to the risk neutral case, under risk aversion, R is positive and generally $R(\bar{P}, c, \tau)$, where τ represents relevant moments of the distribution of ε. In general, equilibrium $\bar{P}$ and τ are determined by c. We shall consider only the case where a change in an input price changes only $\bar{P}$ and not τ. Therefore, a change in an input price affects the risk premium causing entry and exit of firms shifting industry supply. This in turn leads to a change in $\bar{P}$ which leads to a change in the risk premium. Therefore, $\bar{P} = \bar{P}(c, \tau)$ with $\partial\tau/\partial c_j = 0$ by assumption.

Examination of (8) reveals that equilibrium implies that $\partial E[U(\pi)]/\partial c_j$ must be zero since $E[U(\pi)] = 0$ in long run equilibrium. Hence, $\partial SUAC^*/\partial c_j = \partial\bar{P}/\partial c_j$. These results are derived explicitly below:[11/]

$$\frac{\partial SUAC^*}{\partial c_j} = \frac{\partial\bar{P}}{\partial c_j} - \frac{1}{q}\ \frac{\partial E[U(\pi)]/\partial c_j}{U'[U^{-1}(E\{U(\pi)\})]}$$

$$- \frac{1}{q} \frac{\partial E[U(\pi)]/\partial \bar{P}}{U'[U^{-1}(E\{U(\pi)\})]} \frac{\partial \bar{P}}{\partial c_j}$$

$$+ \frac{\partial SUAC}{\partial q} \frac{\partial q}{\partial c_j} + \frac{\partial SUAC}{\partial q} \frac{\partial q}{\partial \bar{P}} \frac{\partial \bar{P}}{\partial c_j} . \tag{9}$$

In analyzing the above expression, it is important to note that the minimization of SUAC with respect to q is equivalent to the problem Max $E[U(\pi)]$ subject to $E[U(\pi)] = 0$. Therefore expected utility evaluated at optimal output must remain constant, or, $\partial E[U(\pi)]/\partial c_j$ must be zero. This implies that the last four terms in (9) vanish. The latter two terms vanish since $\partial SUAC/\partial q = 0$. The remaining two terms correspond to the derivative of the indirect expected utility function with respect to an input price which must be zero in equilibrium. That is,

$$\frac{\partial E[U(\pi\{\bar{P}, c, \tau\})]}{\partial c_j} = \frac{\partial E[U(\pi)]}{\partial c_j} + \frac{\partial E[U(\pi)]}{\partial \bar{P}} \frac{\partial \bar{P}}{\partial c_j} = 0.$$

Hence,

$$\frac{\partial SUAC^*}{\partial c_j} = \frac{\partial \bar{P}}{\partial c_j} = - \frac{\partial E[U(\pi)]/\partial c_j}{\partial E[U(\pi)]/\partial \bar{P}} . \tag{10}$$

In order to evaluate (10), it is necessary to describe the distribution of price. Let $h(\bar{P}, \varepsilon) = \bar{P} + \varepsilon$, $E(\varepsilon) = 0$; then (10) reduces to

$$\frac{\partial SUAC^*}{\partial c_j} = \frac{X_j}{q} \frac{E[U'(\pi)]}{E[U'(\pi)]} = \frac{X_j}{q} . \tag{11}$$

From (11), it is clear that a rise (reduction) in input price raises (lowers) expected output price.[12/] This result is summarized in Proposition 2.

Proposition 2. Assuming an additive error in the price distribution, a rise (fall) in an input price raises (lowers) the long run risk averse equilibrium expected output price. The corresponding marginal response is given by the equilibrium relative factor demands (input/output quantities).

Thus, the changes in expected output price, (SUAC*) as an input price (c_j) is changed, is given by the corresponding relative factor demand (X_j/q) regardless of the nature of risk preferences. It is remarked here that similar to the certainty case, input price and expected output price changes are in the same direction. In the case of an additive disturbance, $\partial\bar{P}/\partial c_j = X_j/q$ which is the result obtained under certainty by Silberberg. However, in general X_j/q under risk aversion will differ from X_j/q under risk neutrality or certainty. Therefore, the marginal changes, $\partial\bar{P}/\partial c_j$, are not equal under both sets of risk preferences.

In analyzing further comparative static results, it is convenient to use the envelope approach developed by Samuelson and Silberberg (1974b). Denote the primal dual function

$$L = SUAC^* - SUAC, \tag{12}$$

where SUAC* is the minimum SUAC function or SUAC* = SUAC* ($\bar{P}$, τ, c). The matrix

$$L_{ij} = \partial^2 SUAC^*/\partial c_i \partial c_j - \partial^2 SUAC/\partial c_i \partial c_j$$

must be negative definite and symmetric when the primal function (SUAC) is well behaved. This implies that $L_{jj} \leq 0$. From (11),

$$L_{jj} = \frac{\partial^2 SUAC^*}{\partial c_j^{\,2}} - \frac{\partial^2 SUAC}{\partial c_j^{\,2}} \leq 0 \tag{13}$$

or,

$$\frac{\partial(X_j/q)}{\partial c_j} \leq 0. \tag{14}$$

That is, relative factor demands are downward sloping.[13/] This is consistent with the qualitative results under certainty. It is clear that risk neutrality implies (14) directly (see footnote 10, and 13). Next, the symmetry conditions are examined. Since $L_{ij} = L_{ji}$ by Young's theorem, it follows from (11) that

$$\frac{\partial(X_j/q)}{\partial c_i} = \frac{\partial(X_i/q)}{\partial c_j} \tag{15}$$

such that symmetry of relative factor demands is preserved as stated below.

Proposition 3. Assuming additive price uncertainty, relative factor demands are symmetric and downward sloping when SUAC* is concave.

Again, it is commented that the qualitative results under risk aversion are similar to the risk neutral case. Therefore, one cannot distinguish between behaviors on the basis of symmetry and negativity conditions. However, the behaviors could be identified by the testing of the significance of coefficients associated with higher order moments of price.

Finally, as Silberberg has shown, under certainty factor demands and relative factor demands are symmetric and downward sloping if $\partial q/\partial c_j = 0$.[14/] He also showed that $\partial q/\partial c_j = 0$ when production is locally homothetic where the production elasticity, $\eta = (\partial X_j/\partial q)(q/X_j) = C'(q)/AC(q)$, equals unity.

That is, at the risk neutral long run equilibrium, marginal cost equals average cost and $\eta = 1$ which implies $\partial q/\partial c_j = 0$.

However, given the results of Proposition 1, the risk averse equilibrium implies $C'(q) < AC(q)$ or $\eta < 1$ (under homotheticity, all elasticities are equal). Therefore, homotheticity does not guarantee symmetric and negatively sloped factor demands. This is best illustrated with a familiar example. Consider the negative exponential expected utility function under normality, where $E[U(\pi)] = -\exp[-\alpha E(\pi) - (\alpha/2)V(\pi)]$, where V denotes variance and $\alpha > 0$ is the Arrow-Pratt risk aversion coefficient. It is easily verified that $R(q) = (\alpha/2)q^2 V(P)$. Differentiation of (7) with respect to the jth input price using the above risk premium gives[15/]

$$\frac{\partial q}{\partial c_j} = \frac{1}{C''(q) + R''(q)} \frac{X_j}{q} [1 - \eta_{jq}]. \tag{16}$$

Second order conditions require that $R''(q) + C''(q) > 0$. Since risk aversion implies $\eta_{jq} < 1$, (16) yields $\partial q/\partial c_j > 0$ and a rise in an input price raises output as well. In contrast, risk neutrality implies $R''(q) = 0$ and $\eta_{jq} = 1$, and equilibrium output is unaffected by input price changes. It is concluded that homotheticity does not imply downward sloping and symmetric factor demand curves for the long run risk averse equilibrium.

Summarizing this section, many of the above results indicate similar qualitative or quantitative restrictions for both the risk averse and risk neutral cases. Yet, it is important to realize that these results are implied because of the assumed additive price uncertainty. Briefly, the comparative static results in Propositions 2 and 3 are discussed when price is assumed to be multiplicatively distributed as $P = \bar{P}\varepsilon$, $\varepsilon > 0$ and $E(\varepsilon) = 1$. In a mean variance context, an increase in $\bar{P}$ raises the mean and variance of

P. This may be the case if exit from the industry raises expected price but leads to greater variability in the market clearing price distribution.

Evaluation of (10) in the multiplicative case implies that

$$\frac{\partial SUAC^*}{\partial c_j} = \frac{X_j}{q} \frac{E[U''(\pi)\varepsilon]}{E[U'(\pi)]} . \tag{17}$$

In this case the essential result in Proposition 2 still holds assuming that entry and exit does not alter the distribution of ε. That is, expected price, $\bar{P}$, rises as an input price rises. However, (17) does not imply that relative factor demands are negatively sloped and symmetric. It can also be verified that decreasing absolute risk aversion does not guarantee the result. This is so because under multiplicative risk, a rise in $\bar{P}$ (ceteris paribus) increases expected profits but also leads to a rise in other moments of the distribution of profits (e.g., dispersion).

II. Conclusions

The results of Proposition 1 indicate that there exists a very natural analog to the minimization of average cost for long run equilibrium under risk aversion: the minimization of average cost plus an average risk premium. Minimization of this function leads to an output which is lower than that output which minimizes average cost. Further, relative factor demands are symmetric and downward sloping, regardless of risk preferences, if the distribution of price is additive such that entry and exit alters only expected price.

Whereas, under certainty, homotheticity of production implies that qualitative comparative static results are similar for the short and long run

factor demands, such a result is not implied under risk aversion. Finally, the multiplicative stochastic specification of price cannot guarantee that relative factor demands are symmetric and downward sloping. Unless one explicitly examines the stochastic environment of demand and supply and the effects of entry and exit on the probability distribution of price, no substantive conclusions can be deduced about the behavior of firm and industry relations.

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Footnotes

1/ It will be assumed throughout that the average cost curve is classically U-shaped (Hanoch).

2/ The short run implies that output price does not adjust but the firm is free to vary factors of production.

3/ Batra and Ullah have shown that cost minimization is consistent with the model employed here.

4/ The analyses could proceed in a similar manner if the firm maximizes $E[U(w + \pi)]$ where w is initial wealth.

5/ In the long run, profits are zero when output is zero. Since expected utility is defined only up to an increasing linear transformation, $U(0) = 0$ without loss of generality.

6/ Equation (4) implies $\bar{P}$ = SUAC. Yet equations (1) and (2) imply $\bar{P}$ = marginal cost plus the marginal risk premium. This can only occur at the minimum of a well behaved function as indicated in (5). Also, if $\bar{P}$ is to be an equilibrium price, then on average $\bar{P}$ must be generated by demand and supply conditions (Baron).

7/ Note that $AC'(q) + d[R(q)/q]/dq = 0$ at equilibrium.

8/ It is noted that the relevant comparison is not between the risk neutral and risk averse optimal outputs for a given distribution of price as in the short run analyses of Sandmo and others. Generally, the existence of the risk premium function generates a whole family of price distributions. Here, two members of this family are examined in terms of two different behavioral rules, $R > 0$ and $R = 0$. That is, the long run equilibrium expected price will be different for $R > 0$ than the case where $R = 0$. Hence,

equilibrium outputs will in general be different. This is in clear contrast to the short run analysis where optimal outputs are compared for $R > 0$ and $R = 0$ where the distribution of P (and hence, $\bar{P}$) is unique.

9/ See Pazner and Razin for a similar result. However, they do not develop the result using SUAC.

10/ In the risk neutral case equilibrium, (3) implies that expected profit, $E(\pi^*)$, equals zero. Differentiating $E(\pi^*)$ with respect to c_j gives (via Shepard's or McFadden's Lemmas)

$$\frac{\partial E(\pi^*)}{\partial c_j} = q^* \frac{\partial \bar{P}}{\partial c_j} - X_j^* = 0$$

or,

$$\frac{\partial \bar{P}}{\partial c_j} = \frac{X_j^*}{q} = \frac{\partial AC^*}{\partial c_j} .$$

11/ In order to simplify notation optimal quantities will not be starred.

12/ Equivalently using risky variants of the Shepard-McFadden Lemmas, Pope, Blair and Lusky have shown that

$$\frac{\partial E[U^*(\pi)]}{\partial c_j} = - X_j \; E[U'(\pi)]$$

$$\frac{\partial E[U^*(\pi)]}{\partial \bar{P}} = q \; E[U'(\pi)].$$

Since $E[U(\pi)]$ must remain at zero in long run equilibrium, it follows that

$$\frac{\partial E[U^*(\pi)]}{\partial c_j} = - X_j \; E[U'(\pi)] + q \; E[U'(\pi)] \frac{\partial \bar{P}}{\partial c_j} = 0;$$

or,

$$\frac{\partial \bar{P}}{\partial c_j} = \frac{X_j}{q} .$$

13/ It is assumed that L is strictly concave such that (13) holds with a strict equality.

14/ Note that $\partial(X_j/q)/\partial c_k = [q\ \partial X_j/\partial c_k - X_j\ \partial q/\partial c_k]\ q^{-2}$. If $\partial q/\partial c_k = 0$, then it follows from (13) that $\partial X_j/\partial c_j < 0$ and $\partial X_j/\partial c_k = \partial X_k/\partial c_j$ since $L_{jk} = L_{kj}$. Note that $\partial(X_j/q)/\partial c_j = \partial(X_j/q)/\partial c_j + \partial(X_j/q)/\partial \bar{P}\ \partial \bar{P}/\partial c_j$.

15/ Note that differentiation of (7) in this case yields $(\partial SUAC/\partial q)$. $(\partial q/\partial c_j) + \partial AC(q)/\partial c_j + \partial[R(q)/q]/\partial c_j = [C''(q) + R''(q)]\partial q/\partial c_j + \partial C'(q)/\partial c_j + \partial R'(q)/\partial c_j$. The first term vanishes from (7); $\partial AC(q)/\partial c_j = X_j/q$ and $\partial C'(q)/\partial c_j = \partial X_j/\partial q$ and $\partial R'(q)/\partial c_j = \partial[R(q)/q]/\partial c_j = 0$. Therefore (16) is obtained.