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# FACULTY OF APPLIED ECONOMICS 

# The golden rule in transfer pricing regulation 

## Wilfried Pauwels \& Marcel Weverbergh

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# The Golden Rule in Transfer Pricing Regulation 

Wilfried Pauwels and Marcel Weverbergh ${ }^{1}$<br>University of Antwerp, Faculty of Applied Economics<br>Antwerp


#### Abstract

In this paper we analyze the optimal regulation of an internationally integrated monopolist, producing in one country and selling in another country. The monopolist's pricing policy is constrained by transfer pricing regulations, and is subject to different tax rates on profits in the two countries. The governments of the two countries can use their tax rates as regulatory instruments, and they also determine an arm's length interval of acceptable transfer prices. The two governments can cooperate in order to maximize world welfare, or they can each try to maximize their own country welfare. It is shown that in several of the solutions governments apply a golden rule. This rule requires that the firm realizes all profits in the manufacturing country, while no profits are made in the retailing country. This can be obtained by choosing a sufficiently high (low) tax rate in the retailing (manufacturing) country, or by appropriately fixing the transfer price.


## JEL-Classification: F13, K2, C72

Key words: Transfer pricing, welfare analysis.
Address for correspondence: W.Pauwels or M.Weverbergh, University of Antwerp, Faculty of Applied Economics, Prinsstraat 13, B 2000 Antwerpen, Belgium
E-mail: Wilfried.Pauwels@ua.ac.be or Marcel.Weverbergh@ua.ac.be

[^0]
## Introduction

Since the seminal articles of Hirshleifer $(1956,1957)$ transfer pricing has been an active research area. See, e.g., the comprehensive monograph edited by Rugman (1985). The subject has attracted attention from accounting and management specialists, who were primarily interested in the internal efficiency and coordination aspects of transfer pricing. Important contributions were also made by public economists, who were more interested in regulation and welfare aspects.

In the literature focusing on economic regulation we can identify three key characteristics of the assumed regulatory setting. First, there is the question of which instruments are under the control of the regulators. More specifically, to what extent are transfer prices controlled by regulators? And what is the type of taxation used: profit taxes, import tariffs, or export tariffs? Secondly, there is the market structure in which the multinational firm is operating. Is this firm operating in a competitive market, or is it a monopolist? Finally, there is the objective function of the regulator. Is the regulator interested in social welfare, as traditionally defined by economists, or is he only interested in tax revenue? We will now elaborate briefly on each of these issues. This will allow us to make explicit the purpose and the contribution of the present paper.

With respect to the regulator's control of transfer prices, very different assumptions have been made in the literature. At one extreme, it can be assumed that the regulator does not control transfer prices at all. E.g., it may be that - on the basis of the arm's length principle - transfer prices must be equal to existing prices on the world market. Or it may be that there is asymmetric information which does not allow any direct control of transfer prices by the governments. See, e.g., Bond and Gresik (1996), and Elitzur and Mintz (1996). The other extreme assumption is that the regulators have direct control of transfer prices, in the sense that each government uses an 'accounting price', which determines the taxable profit in all countries involved. In such a setting double taxation can easily occur. See, e.g., Mansori and Weichenrieder (2001) and Raimondos-Möller and Scharf (2002).

In this paper we will assume that governments and multinational firms operate in an international context where transfer prices are regulated according to the standard guidelines. These guidelines aim to limit tax evasion, and to achieve bilateral (or multilateral) tax harmonization in a way that double taxation is avoided. Tax authorities require the integrated company to demonstrate that the transfer price applied is 'within the arm's length range', and therefore leads to an acceptable taxable profit base in the countries concerned.

If the traded commodity has an open market price, then this price must be the transfer price. If there exists an open market price for a very similar commodity, this price can be used as a 'comparable uncontrolled price'. If there is no comparable uncontrolled price, several methodologies have been developed to determine an acceptable transfer price. Examples are the 'resale price method' or the 'cost-plus method'. See, e.g., Halperin and Srinidhi $(1987,1991)$. These different methods, however, may lead to different results, so that quite often a whole range of transfer prices will be acceptable. The method that is actually chosen may be open for negotiation between the multinational and the tax authorities. In our model we will assume that each government has its own interval of acceptable transfer prices. The firm is then constrained to fix a transfer price in the intersection of these intervals. We define this intersection as the 'arm's length interval'. Within this arm's length interval the firm can freely choose its transfer price. This way of modelling the tax authorities' regulation is very realistic, and also very flexible. The tax authorities can enforce a smaller interval, implementing stricter rules, or they can take a less strict view with respect to the discretion allowed to the firm. In the extreme cases, either a specific transfer price is enforced, or no limits are imposed at all. This is also the approach taken by Kant (1990).

With respect to the instruments controlled by the regulator, there is also the question of which type of taxation the regulators apply. The impact of profit taxes is totally different from the impact of import or export tariffs. Import or export tariffs are based on quantities sold. They have a direct impact on the marginal cost of the firm, and hence also on the price-quantity decision of the monopolist. In Bond and Gresik (1996), and in Raimondus-Möller and Scharf (2002), taxes are calculated from administrative transfer prices. In effect, such taxes are similar to import or export tariffs, since they are based on quantities rather than on profits. In the present paper only profit taxes are assumed to apply.

Apart from the instruments controlled by the regulator, there is the market structure in which the firm is assumed to operate. In several contributions, e.g., in RaimondosMoller and Scharf (2002), the price in the final market is taken as given. This has important consequences. A major reason for vertical integration by the multinational firm is that the firm wants to take advantage of its price setting power, thereby avoiding the loss incurred by double marginalization. See, e.g., Tirole (1998, p.174). This loss occurs if the monopolistic firm operates in an independent (non-integrated) vertical channel framework, and is faced with a downstream firm that exercises monopoly power also. However, if the downstream firm is a price taker, the double marginalization issue does not arise, and integration would only lead to additional coordination costs, without tangible benefits. It is precisely the risk of double marginalization which gives rise to the coordination problem. In the management oriented part of the transfer pricing literature this risk of double marginalization is essential. In our model we assume that the multinational firm is a monopolist, who is a price maker in the final commodity market.

Finally, there is the specification of the regulator's objective function. In several contributions, social welfare is defined in the standard way as the sum of consumer surplus, producer surplus and tax revenue. In the literature on tax competition it is not uncommon to focus only on tax revenue. Examples in the context of transfer pricing are Elitzur and Mintz (1996), and Mansori and Weichenrieder (2001). It is obvious
that the implications of the two types of objective functions can be very different. Maximization of social welfare in the traditional sense may well lead to zero tax revenue. In our paper we use the traditional notion of social welfare. In the appendix we briefly indicate the implications of using tax revenue.

We can summarize the above discussion as follows. We consider a multinational monopolistic firm. This firm fixes the transfer price between the limits set by the regulators, and it also determines the final consumer price. The regulators control the boundaries of the arm's length interval, as well as the tax rates on profits. They use these instruments to promote social welfare. To simplify the analysis, we assume a linear demand function in the final market, and a constant marginal production cost. We further assume that the international division of the firm is given, so that problems of localization are ruled out. See, e.g., Levinsohn and Slemrod (1993).

The paper is organized as follows. In a first section we study the optimal pricing policy of the monopolist. This analysis allows us to predict how the monopolist will react to various possible actions of the regulators. In a second section we derive rules for optimal interventions by the regulators.

## 1. The firm's optimal behaviour ${ }^{2}$

We consider the following simple setting. A firm produces in one country, the manufacturer's country, M, and exports to another country, the retailer's country, R. The marginal cost in M is assumed to be constant, and is given by c. There is no marginal cost in R. The transfer price is denoted by $p_{M}$. We assume that the same transfer price $p_{M}$ is used is used for calculating profits in country R and in country M . The price charged to consumers in R is denoted by $p_{R}$. Consumer demand in R is given by $p_{R}=a-b q$, where $q$ is the quantity sold to consumers in R , and where a and b are positive constants. Tax rates may differ in the two countries, and are assumed between zero and one. A tax rate $t_{M}$ applies to the profits $\left(p_{M}-c\right) q$ realized in M , and a tax rate $t_{R}$ applies to the profits $\left(p_{R}-p_{M}\right) q$ realized in R . As long as the transfer price $p_{M}$ is accepted by both countries, there will be no double taxation. A change in $p_{M}$ then shifts profits from one country to the other.

The firm is an internationally integrated monopolist, taking all decisions centrally. These decisions concern the prices $p_{M}$ and $p_{R}$. The monopolist's after tax profits are given by
$\pi\left(p_{M}, p_{R}\right)=\left[\left(1-t_{R}\right)\left(p_{R}-p_{M}\right)+\left(1-t_{M}\right)\left(p_{M}-c\right)\right]\left(\frac{a-p_{R}}{b}\right)$

[^1]We simplify the notation by introducing a new variable $\alpha$, defined as $\alpha=\frac{1-t_{M}}{1-t_{R}}$. The profit function can then be written as

$$
\begin{equation*}
\pi\left(p_{M}, p_{R}\right)=\left(1-t_{R}\right)\left[\left(p_{R}-p_{M}\right)+\alpha\left(p_{M}-c\right)\right]\left(\frac{a-p_{R}}{b}\right) \tag{1}
\end{equation*}
$$

We now have to specify to which extent the monopolist is free to determine the prices $p_{R}$ and $p_{M}$. We consider two possibilities. A first possibility occurs when the only restriction on $p_{R}$ and $p_{M}$ is that profits on the two markets are nonnegative, i.e. that $p_{R} \geq p_{M} \geq c$. This means, in essence, that there is no transfer price regulation. A second possibility occurs when there is transfer price regulation, meaning that the transfer price should be within the "arm's length interval".

## a. No transfer price regulation.

In this section we consider the optimal pricing policy of the monopolist in case there is no transfer price regulation, or in case the regulation has no impact on the firm's pricing strategy. Even though the results for this case are intuitively obvious, we want to derive them formally here because they represent an important benchmark case. Consider then the following problem.

$$
\begin{equation*}
\operatorname{Max}_{p_{M}, p_{R}} \pi\left(p_{M}, p_{R}\right)=\left(1-t_{R}\right)\left[\left(p_{R}-p_{M}\right)+\alpha\left(p_{M}-c\right)\right]\left[\frac{a-p_{R}}{b}\right] \tag{2}
\end{equation*}
$$

s.t. $p_{R} \geq p_{M}$

$$
\begin{equation*}
p_{M} \geq c \tag{3}
\end{equation*}
$$

This problem allows the firm maximal freedom in determining its pricing policy, and in shifting profits from one country to the other. When solving this problem, we distinguish three cases.

Case 1: $\alpha=1$. As this is equivalent to $t_{M}=t_{R}$, the allocation of the firm's profits over R and M is without any consequences. In fact, $p_{M}$ disappears from the objective function, and the above problem reduces to a simple monopoly problem. The optimal price $p_{R}^{*}$ is given by the simple optimal monopoly price $p^{m o}=(a+c) / 2$, so that $p_{R}^{*}=p^{m o}$. Taking into account the constraints (3) and (4), $p_{M}$ can take any value between c and $(a+c) / 2$.

Case 2: $\alpha\left\langle 1\right.$. This is equivalent to $t_{R}\left\langle t_{M}\right.$. The solution of (2)-(4) is now given by
$p_{R}^{*}=p^{m o}$ and $p_{M}^{*}=c$. All profits are shifted to the low tax country R. Profits in M are zero.

Case 3: $\alpha>1$. This is equivalent to $\left.t_{R}\right\rangle t_{M}$. The solution of (2)-(4) is given by $p_{R}^{*}=p^{m o}$ and $p_{M}^{*}=p^{m o}$. All profits are shifted to the low tax country M. Profits in R are zero.

We can summarize these three cases as follows. Consumers are always charged the simple monopoly price $p_{R}^{*}=p^{m o}$. Maximal profits before taxes are always equal to the maximal monopoly profits $\left(p^{m o}-c\right)\left(\frac{a-p^{m o}}{b}\right)$. The firm manipulates $p_{M}$ in order to fully shift these monopoly profits to the low tax country. If $t_{R}\left\langle t_{M}\right.$, then $\left.p_{R}^{*}=p^{m o}\right\rangle p_{M}^{*}=c$. If $\left.t_{R}\right\rangle t_{M}$, then $\left.p_{R}^{*}=p^{m o}=p_{M}^{*}\right\rangle c$. The optimal transfer price $p_{M}^{*}$ is either c or $p^{m o}$. When $t_{R}=t_{M}$, all transfer prices between c and $p^{m o}$ are optimal.

## b. Transfer price regulation by the "arm's length range" principle.

In actual transfer price agreements the "arm's length principle" is used to restrict the transfer price $p_{M}$ to "reasonable" values. As argued in the introduction, we model the regulators' intervention by introducing an arm's length interval. All transfer prices in this interval are acceptable to both governments, and will not give rise to double taxation. Let us denote by $p_{L}$ and $p_{H}$ the lower and the upper bounds of this interval. Transfer price regulation will then require that $p_{L} \leq p_{M} \leq p_{H}$. Depending on the size of the difference $p_{H}-p_{L}$, the transfer price regulation can be said to be more or less strict. This regulation is extremely strict if $p_{L}=p_{H}$. It is least strict if $p_{L}=c$ and $p_{H}=p_{R}$. This is the case considered by Samuelson (1982). In fact, it coincides with the case considered in the previous section 2.a.

The monopolist, when choosing the prices $p_{M}$ and $p_{R}$, now faces the extra constraint $p_{L} \leq p_{M} \leq p_{H}$. The values of $p_{L}$ and $p_{H}$ are determined by the regulators, and are given for the monopolist. When only the restrictions (3)-(4) are imposed, we found that the optimal transfer price is $p_{M}=c$ when $\alpha\left\langle 1\right.$, and $p_{M}=p_{R}^{*}=p^{m o}$ when $\alpha>1$.Therefore, the restrictions $p_{L} \leq p_{M} \leq p_{H}$ can only be binding if $c \leq p_{L}$ for $\alpha<1$, and $p_{H} \leq p^{m o}$ for $\alpha>1$. In the following analysis we will, therefore, always assume that $c \leq p_{L} \leq p_{H} \leq p^{m o}$. As $p_{L} \geq c$, it follows that the inequality $p_{M} \geq c$ is implied by $p_{M} \geq p_{L}$.

This leads to the following problem of the monopolist.

$$
\begin{equation*}
\operatorname{Max}_{p_{M}, p_{R}} \pi\left(p_{M}, p_{R}\right)=\left(1-t_{R}\right)\left[\left(p_{R}-p_{M}\right)+\alpha\left(p_{M}-c\right)\right]\left[\frac{a-p_{R}}{b}\right] \tag{5}
\end{equation*}
$$

s.t. $p_{R} \geq p_{M}$

$$
\begin{equation*}
p_{M} \geq p_{L} \tag{6}
\end{equation*}
$$

$$
\begin{equation*}
p_{M} \leq p_{H} \tag{7}
\end{equation*}
$$

As with problem (2)-(4), we distinguish three cases.
Case 1: $\alpha=1$, or $t_{M}=t_{R}$. In this case, as in section 2.a, $p_{M}$ disappears from the objective function, and the remaining problem in $p_{R}$ is a simple monopoly problem. The solution is given by $p_{R}^{*}=p^{m o}$. The firm can then choose any value of $p_{M}$ such that $p_{L} \leq p_{M} \leq p_{H}$.

Case 2: $\alpha\left\langle 1\right.$, or $t_{R}\left\langle t_{M}\right.$. In this case the monopolist wants to shift profits from M to R. He will therefore choose $p_{M}$ equal to

$$
\begin{equation*}
p_{M}^{*}=p_{L} \tag{9}
\end{equation*}
$$

Given this value of $p_{M}$, the profit function can be written as

$$
\left(1-t_{R}\right)\left[p_{R}-\left[c+(1-\alpha)\left(p_{L}-c\right)\right]\right]\left[\frac{a-p_{R}}{b}\right]
$$

If we introduce the notation $M C\left(p_{L}, \alpha, c\right)=c+(1-\alpha)\left(p_{L}-c\right)$, this profit function can be written as

$$
\left(1-t_{R}\right)\left[p_{R}-M C\left(p_{L}, \alpha, c\right)\right]\left[\frac{a-p_{R}}{b}\right]
$$

$M C\left(p_{L}, a, c\right)$ can be interpreted as a constant "tax-adjusted" marginal cost: the marginal cost c is "adjusted" by $(1-\alpha)\left(p_{L}-c\right)$. With this notation the above problem is a standard problem of monopolistic profit maximization. Assuming for the moment that constraint (6) is not binding in the solution of this problem, the optimal value of $p_{R}$ is given by

$$
\begin{equation*}
p_{R}^{*}=\frac{a+M C\left(p_{L}, \alpha, c\right)}{2}=\frac{a+c}{2}+\frac{1-\alpha}{2}\left(p_{L}-c\right) \tag{10}
\end{equation*}
$$

The value of $p_{M}$ given by (9) satisfies the constraints (7) and (8). It is easy to see that the value of $p_{R}$ given by (10) also satisfies the constraint (6). Indeed,

$$
p_{R}^{*} \geq \frac{a+c}{2}=p^{m o} \geq p_{H} \geq p_{L}=p_{M}^{*}
$$

Case 3: $\alpha>1$, or $\left.t_{R}\right\rangle t_{M}$. The monopolist will then try to shift profits from R to M . He will therefore choose a value of $p_{M}$ equal to

$$
\begin{equation*}
p_{M}^{*}=p_{H} \tag{11}
\end{equation*}
$$

This value of $p_{M}$ satisfies constraints (7) and (8). For this value of $p_{M}$ the profit function can be written as

$$
\left(1-t_{R}\right)\left[p_{R}-\left[c+(1-\alpha)\left(p_{H}-c\right)\right]\right]\left[\frac{a-p_{R}}{b}\right]
$$

As in the previous case, we can introduce an "adjusted" marginal cost

$$
M C\left(p_{H}, \alpha, c\right)=c+(1-\alpha)\left(p_{H}-c\right)
$$

Note that, as $\alpha>1$, this marginal cost will be below c and can even be negative. If we neglect for the moment constraint (6), maximizing the profit function with respect to $p_{R}$ gives

$$
p_{R}^{*}=\frac{a+M C\left(p_{H}, \alpha, c\right)}{2}=\frac{a+c}{2}+\frac{1-\alpha}{2}\left(p_{H}-c\right)
$$

This value will only be a valid solution of (5)-(8) provided it is at least equal to $p_{M}^{*}=p_{H}$ (constraint (6)). If this is not the case, the optimal value of $p_{R}$ is equal to $p_{M}^{*}=p_{H}$. This result can be stated more formally as

$$
\begin{align*}
& \frac{a+c}{2}+\frac{(1-\alpha)}{2}\left(p_{H}-c\right) \geq p_{H} \Rightarrow p_{R}^{*}=\frac{a+c}{2}+\frac{(1-\alpha)}{2}\left(p_{H}-c\right)  \tag{12}\\
& \frac{a+c}{2}+\frac{(1-\alpha)}{2}\left(p_{H}-c\right)\left\langle p_{H} \Rightarrow p_{R}^{*}=p_{H}\right. \tag{13}
\end{align*}
$$

## c. Comparative statics

In section 2.b we derived optimal values of $p_{R}$ and $p_{M}$, taking as given the values of $\alpha$, $p_{L}$ and $p_{H}$. We now analyse how $p_{R}$ and $p_{M}$ react to changes in $\alpha$, for given
values of $p_{L}$ and $p_{H}$. We then study the impact of changes in $p_{L}$ and $p_{H}$, for a given value of $\alpha$.
i. The optimal values of $p_{R}$ and $p_{M}$ as functions of $\alpha$.

For values of $\alpha<1$, the optimal values of $p_{M}$ and $p_{R}$ are given by (9) and (10). For $\alpha=1$, the optimal value $p_{R}$ is given by $p_{R}^{*}=p^{m o}$, while the optimal value of and $p_{M}$ can be any value between c and $(a+c) / 2$. Finally, in case $\alpha>1$, the optimal value of $p_{M}$ is $p_{H}$, while the optimal value of $p_{R}$ is given by the optimality conditions (12) and (13). This solution depends on whether the no-loss constraint on $p_{R}\left(p_{R} \geq p_{M}\right)$ is binding or not. Conditions (12) and (13) can also be written as

$$
\begin{equation*}
\alpha \leq \frac{a-p_{H}}{p_{H}-c}=\alpha^{*} \Rightarrow p_{R}^{*}=\frac{a+c}{2}+\frac{1-\alpha}{2}\left(p_{H}-c\right) \tag{14}
\end{equation*}
$$

$\alpha\rangle \frac{a-p_{H}}{p_{H}-c}=\alpha^{*} \Rightarrow p_{R}^{*}=p_{H}$

The value $\alpha^{*}$ is defined by $\alpha^{*}=\frac{a-p_{H}}{p_{H}-c}$. We assumed before that $p_{H} \leq p^{m o}=(a+c) / 2$. It then follows that $\alpha^{*} \geq 1$.

The above results are illustrated on Figure 1. When $\alpha\left\langle 1\right.$, i.e., $t_{R}\left\langle t_{M}\right.$, the monopolist wants to shift his profits from M to R . However, as long as $\left.p_{L}\right\rangle c$, he cannot completely avoid the high tax rate $t_{M}$. This would require $p_{M}=c$, which is impossible if $\left.p_{L}\right\rangle c$. He will then adjust his pricing policy by charging a price $p_{R}$ which exceeds the simple monopoly price $p^{m o}$. As $\alpha$ increases (i.e. $t_{M}$ decreases and $/$ or $t_{R}$ increases) the difference between the optimal price $p_{R}^{*}$ and the simple monopoly price $p^{m o}$ decreases. When $\alpha>1$, i.e. $t_{R}>t_{M}$, the monopolist wants to shift profits from R to M . He could completely avoid the high tax rate $t_{R}$ by choosing $p_{R}=p_{M}=p_{H}$. However, if $\alpha$ is not too high, i.e., $\alpha\left\langle\alpha^{*}\right.$, this turns out not to be optimal. It is then better for the monopolist to realize a positive profit on the R-market by charging the price given by (14). For values of $\alpha$ exceeding $\alpha^{*}$, it is optimal to forego all profits on the R-market by putting $p_{R}^{*}=p_{M}^{*}=p_{H}$.

Figure 1. Optimal pricing as a function of relative tax rates

ii. The optimal values of $p_{R}$ and $p_{M}$ as functions of $p_{L}$ and $p_{H}$.

We now express the optimal values $p_{R}$ and $p_{M}$ as functions of $p_{L}$ and $p_{H}$. In case $\alpha<1$, the optimal value of $p_{M}$ is given by $p_{M}^{*}=p_{L}$. Equation (10) then allows us to draw $p_{R}^{*}$ as a function of $p_{L}$. This is shown on figure 2.a. For values of $\alpha>1, p_{M}^{*}$ is given by $p_{M}^{*}=p_{H}$. (12) and (13) can be rewritten as

$$
\begin{equation*}
p_{H} \leq \frac{a+\alpha c}{1+\alpha}=p_{H}^{*} \Rightarrow p_{R}^{*}=\frac{a+c}{2}+\frac{(1-\alpha)}{2}\left(p_{H}-c\right) \tag{16}
\end{equation*}
$$

$p_{H} \leq \frac{a+\alpha c}{1+\alpha}=p_{H}^{*} \Rightarrow p_{R}^{*}=p_{H}$

In these expressions $p_{H}^{*}$ is defined as $p_{H}^{*}=\frac{a+\alpha c}{1+\alpha}$. Relations (16) and (17) are illustrated on figure 2.b.

Figure 2: Optimal pricing as a function of $p_{M}$.

$$
\begin{aligned}
& \alpha<1 \\
& p_{M}^{*}=p_{L}
\end{aligned}
$$



Figure 2.a.
$\alpha>1$
$p_{M}^{*}=p_{H}$


Figure 2.b.

When $\alpha<1$, the optimal price $p_{R}^{*}$ exceeds the simple monopoly price $p^{m o}$, and it is increasing in $p_{L}$. When $\alpha>1$, then for sufficiently small values of $p_{H}$, i.e. $p_{H} \leq p_{H}^{*}$, the optimal price $p_{R}^{*}$ decreases as $p_{H}$ increases. The increase in $p_{H}$ allows the monopolist to realize more profits in the low tax country M . This induces the monopolist to decrease $p_{R}^{*}$. This will continue until $p_{R}^{*}=p_{M}^{*}=p_{H}$. Further decreases in $p_{R}^{*}$ would then violate the constraint $p_{R} \geq p_{M}$. Hence, for $p_{H} \geq p_{H}^{*}$, we have that $p_{R}^{*}=p_{M}^{*}=p_{H}$.

From the definition of the critical value $\alpha^{*}=\frac{a-p_{H}}{p_{H}-c}$ it is clear that an increase of $p_{H}$ decreases $\alpha^{*}$. Similarly, the critical value $p_{H}^{*}=\frac{a+\alpha c}{1+\alpha}$ depends on $\alpha$ : an increase of $\alpha$ decreases $p_{H}^{*}$. This illustrates the interdependency between the effects of a change in $\alpha$ and of a change in $p_{H}$. We return to this relationship between $\alpha$ and $p_{H}$ in the next section.

## 2. The governments' optimal policies

In the previous section we determined the monopolist's optimal reaction to changes in $\alpha, p_{L}$ and $p_{H}$. We can now use these results to determine how the governments of R and M should use these instruments in order to maximize welfare. Formally, we consider a two stage game. In the first stage the regulators fix the values of $\alpha, p_{L}$ and $p_{H}$. Taking these values as given, the firm then determines its optimal pricing policy in the second stage.

In the previous section we studied the second stage of the game. We now consider two variants for the first stage. It is possible that in this stage the two governments cooperate so as to maximize world welfare. Alternatively, the two countries may pursue their own interests, and play the game noncooperatively.

Both the cooperative and the non-cooperative games can be defined as tax games where governments set their tax rates, taking as given the arm's length interval of acceptable transfer prices. Alternatively, both types of games can also be defined as games in which the governments fix transfer prices, taking tax rates as given.

As stressed in the introduction, a country's welfare will be defined in the standard way of public economics, i.e., in terms of consumer surplus, producer surplus and tax revenue. Consumer surplus is only enjoyed in country R , and is given by

$$
\begin{equation*}
C S\left(p_{R}\right)=\frac{\left(a-p_{R}\right)^{2}}{2 b} . \tag{18}
\end{equation*}
$$

The tax revenues in countries R and M equal

$$
\begin{align*}
& T R_{R}\left(p_{R}, p_{M}, t_{R}, t_{M}\right)=t_{R}\left(p_{R}-p_{M}\right)\left(\frac{a-p_{R}}{b}\right)  \tag{19}\\
& T R_{M}\left(p_{R}, p_{M}, t_{R}, t_{M}\right)=t_{M}\left(p_{M}-c\right)\left(\frac{a-p_{R}}{b}\right) . \tag{20}
\end{align*}
$$

Total profits after taxes are

$$
\begin{equation*}
\pi\left(p_{M}, p_{R}, t_{R}, t_{M}\right)=\left[\left(1-t_{R}\right)\left(p_{R}-p_{M}\right)+\left(1-t_{M}\right)\left(p_{M}-c\right)\right]\left[\frac{a-p_{R}}{b}\right] \tag{21}
\end{equation*}
$$

We first consider the cooperative solution.

## a. The cooperative solution.

In this case the two governments cooperate with the objective of maximizing world welfare. World welfare is defined as the sum of (18), (19), (20) and (21). As the firms' tax payments cancel against the governments' tax receipts, this sum is equal to
$W_{W}\left(p_{R}\right)=\frac{\left(a-p_{R}\right)^{2}}{2 b}+\left(p_{R}-c\right) \frac{\left(a-p_{R}\right)}{b}$
Note that this expression does not depend directly on $p_{H}, p_{L}, t_{R}$ or $t_{M}$. These variables only affect world welfare to the extent that they affect $p_{R}$. If the two governments cooperate so as to maximize $W_{W}\left(p_{R}\right)$, it is clear that they should manipulate $t_{R}, t_{M}, p_{L}$ and $p_{H}$ such that $p_{R}$ is as close as possible to c. We now consider two cases, depending on the instruments the governments can control.

In the first case we assume that the two governments can only manipulate $\alpha$, taking $p_{H}$ and $p_{L}$ as given. These governments then realize the lowest possible price $p_{R}$ by choosing the tax rates $t_{R}$ and $t_{M}$ such that $\frac{1-t_{M}}{1-t_{R}}=\alpha \geq \alpha^{*}=\frac{a-p_{H}}{p_{H}-c}$. The value of $p_{R}$ is then $p_{R}=p_{H}$. This is clear from Figure 1. Note that choosing a value $\alpha \geq \alpha^{*}=\frac{a-p_{H}}{p_{H}-c}$ automatically implies that $p_{H}$ will satisfy $p_{H} \geq p_{H}^{*}=\frac{a+\alpha c}{1+\alpha}$.
As $\alpha^{*} \geq 1$, the inequality $\alpha \geq \alpha^{*}$ requires that $t_{R}$ sufficiently exceeds $t_{M}$. The inequality $t_{R}>t_{M}$ gives the firm an incentive to shift profits as much as possible to M , so that $p_{M}=p_{H}$. The profit margin in M is then $p_{M}-c=p_{H}-c \geq 0$. If, in addition, $\alpha \geq \alpha^{*}$, the profit margin in country R will disappear: $p_{R}-p_{M}=p_{R}-p_{H}=0$, and there is no tax revenue in R. Finally, note that $p_{R}=p_{H}\left\langle p^{m o}\right.$, so that part of the welfare loss of monopoly has disappeared, thanks to the good tax coordination of the two countries.

One may object to the above result that the value $\alpha$ * is company or industry specific, so that taxation differs from industry to industry. However, there need not be tax discrimination within the group of importing (or exporting) industries. Within a group of importing industries the regulator can choose a value of $\alpha$ exceeding the highest of all the industries' critical values. This value of $\alpha$ would then be welfare maximizing for all the industries in that group. Note that values of $\alpha$ exceeding $\alpha^{*}$ have no detrimental welfare effect. It is true, however, that for a given country the rule $\alpha \geq \alpha^{*}$ is discriminating between importing and exporting industries. Importing industries in a country should be taxed more than exporting industries in that country.

In the second case we assume that the two governments can only manipulate the prices $p_{L}, p_{H}$, and that they take $\alpha$ as given. We consider two possibilities.
If $\alpha\left\langle 1\right.$, the governments can put $p_{L}=c$, so that $p_{R}^{*}=\frac{a+c}{2}$. For any value of $\alpha$, with $\alpha<1$, this is the lowest value of $p_{R}$ that can be attained. See Figure 2.a. This same outcome will also obtain if the two governments decide not to intervene at all. Indeed, as seen in section 1.a, if there is no transfer price regulation, the monopolist will choose $p_{M}=c$ as part of his optimal pricing policy. His best value of $p_{R}$ is then $p^{m o}$.

If $\alpha>1$, the two governments can put $p_{H}=p_{H}^{*}=\frac{a+\alpha c}{1+\alpha}$. The resulting value of $p_{R}$ is then $p_{R}=p_{H}^{*}\left\langle p^{m o}\right.$. See Figure 2.b. Hence, part of the welfare loss of monopoly again disappears. Note that the optimal value $p_{H}=p_{H}^{*}$ exceeds the marginal cost c. Again, as in the previous case, it can be noted that by choosing $p_{H}=p_{H}^{*}$ the value of $\alpha$, considered fixed here, will be equal to its critical value, as $\alpha=\alpha^{*}=\frac{a-p_{H}^{*}}{p_{H}^{*}-c}$.

The results obtained in the foregoing two cases are summarized on Figure 3. The curve shown on this diagram is given by the equation
$\alpha=\frac{a-p_{H}}{p_{H}-c}$
or, equivalently, by the equation
$p_{H}=\frac{a+\alpha c}{1+\alpha}$

Assume the governments only manipulate $\alpha$. Then, for any given value of $p_{H}$ in the interval $\left(c, \frac{a+c}{2}\right), \alpha$ should be fixed at the value given by (23), or at any other larger value. Alternatively, assume the governments only manipulate the prices $p_{H}$ and $p_{L}$, and take tax rates as given. If then $\alpha<1$, the governments should not regulate $p_{M}$. If $\alpha>1, p_{M}\left(=p_{H}\right)$ should be fixed at the value given by (24). The bold part of the curve in Figure 3 then summarizes these optimal regulation rules. This relationship between $\alpha$ and $p_{H}$, as given by (23) or (24), can be called the golden rule of transfer price regulation.

The golden rule applies in case regulation operates through taxation, and in case it operates through transfer prices. It implies that world welfare will be maximal if the tax rate in the retailing country sufficiently exceeds that in the manufacturing country. This will drive the retail price down to the transfer price so that profits in the retailing country disappear. If the tax rate in the retailing country exceeds that in the manufacturing country, and if tax rates cannot be manipulated by the regulator, then the upper bound of the transfer price interval should be adjusted such that the equality between retail price and transfer price is obtained at the lowest possible transfer price. Profits in the retailing country will then again disappear.

Figure 3. The golden rule


Finally, we can also consider the case in which the governments can manipulate all the variables $\alpha, p_{L}$ and $p_{H}$ with the purpose of maximizing world welfare. From Figure 3 it is clear that, by fixing $p_{H}$ closer and closer to c , and by accordingly increasing $\alpha$ (by fixing $t_{R}$ closer and closer to 1 ), the consumer price $p_{R}$ will approach the value $p_{R}=c$. Monopoly profits disappear. This, of course, is the first best solution ${ }^{3}$.

## b. Noncooperative solutions.

We now consider the case where the governments of R and M no longer cooperate, but pursue their own interests. As a first step of this analysis we have to define the payoff functions of the two countries. In some contributions these payoffs are simply defined as the tax revenue of each country. See, e.g., Elitzur and Mintz (1996), and Mansori and Weichenrieder (2001). In our model these tax revenues are given by (19) and (20). As argued in the introduction, we will not follow this approach4. Instead, we will define the payoffs of the two countries in terms of social welfare as typically used in public policy analyses. In this view the welfare of country R must certainly include consumer surplus (18) and tax revenue in R (19). Similarly, the welfare of

[^2]country $M$ must include tax revenue in $M$ (20). There remains the problem of allocating the after tax profits of the monopolist (21) to the two countries. Several assumptions could be made here. A first possibility is to assume that after tax profits benefit a third country, different from the countries R and M . In this case after tax profits disappear from the analysis. This approach, however, is inconsistent with the cooperative solution of section 3.a. There we assumed that the monopolist's profits benefit countries R and M . A second approach is to assume that the part of these profits that originates in R , viz. $\left(1-t_{R}\right)\left(p_{R}-p_{M}\right) q$, benefits country R , while the remaining part, viz. $\left(1-t_{M}\right)\left(p_{M}-c\right) q$, benefits country $\mathrm{M}^{5}$. A third approach is to assume that after tax profits are allocated to the two countries $R$ and $M$ in the proportion determined by the distribution of the ownership over the two countries. This distribution can be assumed to be exogenously given. This is, e.g., the approach taken by Raimondos-Möller and Scharf (2002). We follow this last approach. In particular, we will assume that a fraction $\gamma$ of after-tax-profits contributes to the welfare of country R , while the remaining fraction $1-\gamma$ contributes to the welfare of country M.

The payoff $W_{R}$ of country R will then be equal to the sum of consumer surplus, tax revenue in R and the fraction $\gamma$ of after-tax-profits:

$$
\begin{equation*}
W_{R}=C S\left(p_{R}\right)+T R_{R}\left(p_{R}, p_{M}, t_{R}, t_{M}\right)+\gamma \pi\left(p_{R}, p_{M}, t_{R}, t_{M}\right) . \tag{25}
\end{equation*}
$$

The payoff $W_{M}$ of country M is equal to the tax revenue in M , plus the fraction $1-\gamma$ of after-tax-profits:

$$
\begin{equation*}
W_{M}=T R_{M}\left(p_{R}, p_{M}, t_{R}, t_{M}\right)+(1-\gamma) \pi\left(p_{R}, p_{M}, t_{R}, t_{M}\right) \tag{26}
\end{equation*}
$$

We now first analyze the case in which the two governments use the tax rates as instruments. We then consider the case in which they control the transfer prices.

## i. Tax rates as instruments.

In this section we take the values of $p_{L}$ and $p_{H}$ as given, and we study a tax competition game in which country R controls $t_{R}$, and country M controls $t_{M}$. The payoff functions of the two players are given by (25) and (26).

We first derive the reaction correspondence of country $R$. For each possible value of $t_{M}$ we determine the corresponding best possible value(s) of $t_{R}$. The exact dependence of payoff function (25) on $t_{R}$ depends on how $p_{R}$ is determined. As we have seen in section 1 , two cases are possible. If $\alpha \leq \alpha^{*}$, we have that

[^3]$p_{R}=p^{m o}+\frac{(1-\alpha)}{2}\left(p_{M}-c\right)$. The payoff (25) will then be a function only of $p_{M}, t_{R}$ and $t_{M}$. We denote this function as $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$. If $\alpha>\alpha^{*}$, then $p_{R}=p_{M}=p_{H}$, so that $W_{R}$ can be written as
\[

$$
\begin{equation*}
\bar{W}_{R}\left(t_{M}\right)=\frac{\left(a-p_{H}\right)^{2}}{2 b}+\gamma\left(1-t_{M}\right)\left(p_{H}-c\right)\left(\frac{a-p_{H}}{b}\right) . \tag{27}
\end{equation*}
$$

\]

Recall that in this case the tax revenue in R is zero.
We know from section 1 that, when $t_{R}<t_{M}$, the firm will choose $p_{M}=p_{L}$, so that the function $\tilde{W}_{R}\left(p_{L}, t_{R}, t_{M}\right)$ applies. If $t_{R}>t_{M}$, it will choose $p_{M}=p_{H}$, so that the function $\tilde{W}_{R}\left(p_{H}, t_{R}, t_{M}\right)$ applies. When $t_{R}=t_{M}$, the firm is indifferent between $p_{M}=p_{L}$ and $p_{M}=p_{H}$. It follows that the value of $W_{R}$ is undefined. We can then define country R's welfare as any value between $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right)$ and $\tilde{W}_{R}\left(p_{H}, t_{M}, t_{M}\right)$.

A detailed derivation of country R's reaction function is given in Appendix A. A value $t_{M}^{s w}$ of $t_{M}$ is defined by solving $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right)=\bar{W}_{R}\left(t_{M}\right)$ for $t_{M}$. If $t_{M}>t_{M}^{s w}$, we have that $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right)>\bar{W}_{R}\left(t_{M}\right)$. For these values of $t_{M}$ there is no optimal value of $t_{R}$ : country R can choose a value $t_{R}=t_{M}-\varepsilon$, where $\varepsilon$ is an arbitrarily small positive number. Country R is then "tax undercutting" country M , and realizes a payoff arbitrarily close to $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right)$. Country R's reaction correspondence is empty in this case. In the reverse case where $0 \leq t_{M} \leq t_{M}^{s w}$, the inequality $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right) \leq \bar{W}_{R}\left(t_{M}\right)$ holds, and country R will choose any value $t_{R}$ such that $\alpha \geq \alpha^{*}$.

More formally, the reaction correspondence of country R, denoted by $\varphi_{R}$, is given by

$$
\begin{gathered}
\varphi_{R}\left(t_{M}\right)=\left\{t_{R} \left\lvert\, \frac{1-t_{M}}{1-t_{R}} \geq \alpha^{*}\right.\right\} \text { for } 0 \leq t_{M} \leq t_{M}^{s w} \\
\varphi_{R}\left(t_{M}\right)=\varnothing \text { for } 1 \geq t_{M}>t_{M}^{s w} .
\end{gathered}
$$

This is illustrated on Figure 4. The unit square on this figure gives all the feasible combinations of $\left(t_{R}, t_{M}\right)$. For combinations above the $45^{\circ}$-line we have that $\alpha\langle 1$. For combinations below the $45^{\circ}$-line we have $\alpha>1$. The locus of combinations where $\alpha$ is constant is a straight line. Its equation is $t_{M}=1-\alpha+\alpha t_{R}$. Note, in particular, the line where $\alpha=\alpha^{*}$. The slope of this line is $\alpha^{*}=\frac{a-p_{H}}{p_{H}-c}$. This slope increases as $p_{H}$ approaches c. The reaction correspondence of country R is given by the shaded area on Figure 4. Note that in this area the tax revenue of $R$ is always zero. The dotted line segment on the $45^{\circ}$ line refers to the "tax undercutting" by country R.

Figure 4. Reaction correspondence R


We now turn to the reaction correspondence of country $M$. As was the case with (25), the exact dependence of the function (26) on $t_{M}$ depends on how $p_{R}$ is determined. If $\alpha \leq \alpha^{*}$, we have that $p_{R}=p^{m o}+\frac{(1-\alpha)}{2}\left(p_{M}-c\right)$. The payoff (26) will then be a function only of $p_{M}, t_{R}$ and $t_{M}$. We denote this function as $\tilde{W}_{M}\left(p_{M}, t_{R}, t_{M}\right)$. If $\alpha>\alpha^{*}$, then $p_{R}=p_{M}=p_{H}$, so that $W_{M}$ can be written as

$$
\begin{equation*}
\bar{W}_{M}\left(t_{M}\right)=\left(p_{H}-c\right)\left(\frac{a-p_{H}}{b}\right)\left[(1-\gamma)+\gamma t_{M}\right] . \tag{28}
\end{equation*}
$$

A detailed derivation of country M's reaction correspondence is given in Appendix B. The value of $t_{M}$ that maximizes the function $\tilde{W}_{M}\left(p_{M}, t_{R}, t_{M}\right)$ is given by

$$
\begin{equation*}
\frac{\left[a\left(1-t_{R}\right)+p_{M} t_{R}-c\right] \gamma}{\left(p_{M}-c\right)(1+\gamma)} . \tag{29}
\end{equation*}
$$

We denote this value by $\hat{t}_{M}\left(p_{M}, t_{R}\right)$. This function gives, for any value of $p_{M}$ and $t_{R}$, the corresponding value of $t_{M}$ that maximizes $\tilde{W}_{M}\left(p_{M}, t_{R}, t_{M}\right)$.

Figure 5 Reaction correspondence M


Now consider Figure 5. This figure gives the value of $\hat{t}_{M}\left(p_{M}, t_{R}\right)$ as a function of $t_{R}$, both in case $p_{M}=p_{H}$ and in case $p_{M}=p_{L}$. Define the values $t_{R}^{1}$ and $t_{R}^{2}$ as indicated on Figure 5. A value $t_{R}^{s w}$ of $t_{R}$ can be defined as the solution of the equation $\tilde{W}_{M}\left(p_{L}, t_{R}, \hat{t}_{M}\left(p_{L}, t_{R}\right)\right)=\tilde{W}_{M}\left(p_{H}, t_{R}, t_{R}\right) \quad$ for $t_{R}$. For values of $t_{R}$ in the interval $0 \leq t_{R} \leq t_{R}^{s w}$, country M follows the function $\hat{t}_{M}\left(p_{L}, t_{R}\right)$. For values of $t_{R}$ in the interval $t_{R}^{s w}<t_{R} \leq t_{R}^{1}$, country M will "tax undercut" country R by choosing a tax rate $t_{M}=t_{R}-\varepsilon$. This is indicated by the dotted line on Figure 5. Country M's reaction correspondence is empty in this case. For values of $t_{R}$ in the interval $t_{R}^{1}<t_{R} \leq t_{R}^{2}$, country M's reaction function is given by $\hat{t}_{M}\left(p_{H}, t_{R}\right)$. Finally, if $t_{R}^{2}<t_{R} \leq 1$, the best reply by country M is given by the tax rates $t_{M}$ for which $\alpha=\alpha^{*}$.
More formally, country M's reaction correspondence, denoted by $\varphi_{M}$, is given by

$$
\begin{gathered}
\varphi_{M}\left(t_{R}\right)=\left\{t_{M} \mid t_{M}=\hat{t}_{M}\left(p_{L}, t_{R}\right)\right\} \text { for } 0 \leq t_{R} \leq t_{R}^{s w} \\
\varphi_{M}\left(t_{R}\right)=\varnothing \text { for } t_{R}^{s w}<t_{R} \leq t_{R}^{1} \\
\varphi_{M}\left(t_{R}\right)=\left\{t_{M} \mid t_{M}=\hat{t}_{M}\left(p_{H}, t_{R}\right)\right\} \text { for } t_{R}^{1}<t_{R} \leq t_{R}^{2} \\
\varphi_{M}\left(t_{R}\right)=\left\{t_{M} \left\lvert\, \frac{1-t_{M}}{1-t_{R}}=\alpha^{*}\right.\right\} \text { for } t_{R}^{2}<t_{R} \leq 1
\end{gathered}
$$

We can now combine the reaction correspondences of the two countries to find the set of Nash equilibria. This set is given by the bold line segment on Figure 6. Note that this line segment is on the line defined by $\frac{1-t_{M}}{1-t_{R}}=\alpha^{*}$. Hence, all the

Figure 6. Noncooperative tax game:,Nash equilibira


Nash equilibria satisfy the golden rule. The retail price is equal to the transfer price, and there is no tax revenue in the retailing country. As we have seen in section 2.a, such tax combinations maximize world welfare, so that all our Nash equilibria are Pareto efficient. This can be confirmed by calculating the payoffs of the two countries in the Nash equilibria. These are given by

$$
\begin{gathered}
W_{R}=C S\left(p_{H}\right)+\gamma\left(1-t_{M}\right)\left(p_{H}-c\right)\left(\frac{a-p_{H}}{b}\right) \\
W_{M}=\left[1-\gamma\left(1-t_{M}\right)\right]\left(p_{H}-c\right)\left(\frac{a-p_{H}}{b}\right)
\end{gathered}
$$

The sum of these payoffs is equal to $C S\left(p_{H}\right)+\left(p_{H}-c\right)\left(\frac{a-p_{H}}{b}\right)$, which is exactly equal to maximal world welfare $W_{W}\left(p_{H}\right)$. See (22). From the above expressions it also follows that country R's payoff is decreasing in $t_{M}$ and increasing in $\gamma$, while country M's payoff is increasing in $t_{M}$ and decreasing in $\gamma$.

## ii. Transfer prices as instruments.

We now assume that each government regulates the transfer price to be applied in its own country, while it takes the tax rates $t_{R}$ and $t_{M}$ as given. There are then two intervals of acceptable transfer prices, one interval for each country. Both are determined independently, so that transfer prices applied in both countries can differ.

If each country then decides on its own interval of acceptable transfer prices, the monopolist will always choose the lowest acceptable price in country M, and the highest acceptable price in country R. Without loss of generality we can then assume that each government in fact decides on the exact transfer price to be applied to its own country. Let $p_{M}^{R}$ and $p_{M}^{M}$ be the transfer prices fixed by the governments in R and M , respectively. With two independent governments deciding on these transfer prices, it is quite possible that there is double taxation. This will be the case if $p_{M}^{R}\left\langle p_{M}^{M}\right.$. Conversely, if $\left.p_{M}^{R}\right\rangle p_{M}^{M}$, there is a price interval without taxation. The model we now analyze has some resemblance to the model developed by P.Raimondos-Moller and K.Scharf (2002). An important difference remains as they assume that the market in R is competitive, so that the price $p_{R}$ is given. In the model of Mansori and Weichenrieder (2001) the two governments use transfer prices to maximize their tax revenues.

Since we now have one transfer price for each country, we have to adjust slightly our model of section 1. The monopolist's profits before taxes are still given by $\left(p_{R}-c\right)\left(\frac{a-p_{R}}{b}\right)$. Tax revenues in countries R and M are now given by $T R_{R}=t_{R}\left(p_{R}-p_{M}^{R}\right)\left(\frac{a-p_{R}}{b}\right)$ and $T R_{M}=t_{M}\left(p_{M}^{M}-c\right)\left(\frac{a-p_{R}}{b}\right)$. Profits after taxes are then

$$
\begin{equation*}
\pi\left(p_{R}\right)=\left[\left(p_{R}-c\right)-t_{R}\left(p_{R}-p_{M}^{R}\right)-t_{M}\left(p_{M}^{M}-c\right)\right]\left(\frac{a-p_{R}}{b}\right) \tag{30}
\end{equation*}
$$

Given $p_{M}^{R}$ and $p_{M}^{M}$, the monopolist will maximize this profit function with respect to $p_{R}$. This optimal price is given by

$$
\begin{equation*}
p_{R}^{*}=\frac{1}{2\left(1-t_{R}\right)}\left(p_{M}^{M} t_{M}-p_{M}^{R} t_{R}+a\left(1-t_{R}\right)+c\left(1-t_{M}\right)\right) \tag{31}
\end{equation*}
$$

This price increases linearly in $p_{M}^{M}$, and decreases linearly in $p_{M}^{R}$.

We continue to assume that a fraction $\gamma$ of after tax profits benefits country R , while the remaining fraction $1-\gamma$ benefits country M . The payoff functions of R and M are then

$$
\begin{equation*}
W_{R}\left(p_{R}, p_{M}^{R}, p_{M}^{M}\right)=C S\left(p_{R}\right)+T R_{R}+\gamma \pi\left(p_{R}\right) \tag{32}
\end{equation*}
$$

$$
\begin{equation*}
W_{M}\left(p_{R}, p_{M}^{R}, p_{M}^{M}\right)=T R_{M}+(1-\gamma) \pi\left(p_{R}\right) \tag{33}
\end{equation*}
$$

Using (31) in (32) and (33), and differentiating these expressions with respect to $p_{M}^{R}$ and $p_{M}^{M}$, respectively, we obtain the following reaction functions

$$
\begin{align*}
& p_{M}^{R}=\frac{\left[c\left(1-t_{M}\right)+p_{M}^{M} t_{M}\right]\left[1-2 \gamma\left(1-t_{R}\right)\right]-a\left(1-t_{R}\right)\left[1-2 t_{R}-2 \gamma\left(1-t_{R}\right]\right.}{t_{R}\left[3-2\left[t_{R}-\gamma\left(1-t_{R}\right)\right]\right]}  \tag{34}\\
& p_{M}^{M}=\frac{c t_{M}+\gamma\left[a-c\left(1-t_{M}\right)-t_{R}\left(a-p_{M}^{R}\right)\right]}{t_{M}(1+\gamma)} \tag{35}
\end{align*}
$$

From (34) it follows that the sign of the dependence of $p_{M}^{R}$ on $p_{M}^{M}$ can be positive or negative. On the other hand, it follows from (35) that $p_{M}^{M}$ depends positively on $p_{M}^{R}$.

The Nash equilibrium is given by
$p_{M}^{R}=c+\frac{(a-c)\left(1-t_{R}\right)\left(1-2 \gamma-2 t_{R}\right)}{t_{R}\left(3-2 t_{R}\right)}$
$p_{M}^{M}=c+\frac{2 \gamma(a-c)\left(1-t_{R}\right)}{t_{M}\left(3-2 t_{R}\right)}$

We consider two interesting values of $\gamma$. First consider $\gamma=0$. All shareholders of the firm then live in country M . In the Nash equilibrium country M will then choose $p_{M}^{M}=c$, so that tax revenue in this country is zero. This may seem surprising. However, by doing so, it will induce country R to also choose a low transfer price (see (34)), which will increase profits of the shareholders.

A second interesting case occurs when $\gamma=1 / 2$. The retail price in the Nash equilibrium is now given by $p_{R}=\frac{2 a(1-t r)+c}{3-2 t_{R}}$. This price is equal to $p_{M}^{R}$, so that tax revenue in country R is zero. Moreover, country M's welfare in the Nash equilibrium is given by $W_{M}=\frac{3(a-c)^{2}\left(1-t_{R}\right)}{2 b\left(3-2 t_{R}\right)^{2}}$, which is independent of $t_{M}$. This value only depends on $t_{R}$. Hence, country R could drive country M's equilibrium welfare arbitrarily close to zero.

Returning now to the general case, we see that Nash equilibria are possible with $p_{M}^{R}\left\langle p_{M}^{M}\right.$, so that there is double taxation. The reverse, $\left.p_{M}^{R}\right\rangle p_{M}^{M}$, can also be true. In this case there is a price interval without taxation.

While the solution to this game is rather straightforward, an important reservation has to be made relating to possible institutional constraints which may apply. E.g., one may want to impose that $p_{R} \geq p_{M}^{R}$, so that the monopolist is not allowed to make losses in country R. Our above solution does not always satisfy this constraint. Indeed, the difference between the optimal consumer price and the transfer price in R is given by

$$
p_{R}-p_{M}^{R}=\frac{(a-c)(1-2 \gamma)\left(1-t_{R}\right)}{t_{R}\left(3-2 t_{R}\right)}
$$

This difference is positive if $\gamma\langle 1 / 2$ and negative for $\gamma\rangle 1 / 2$. Clearly for $\gamma\rangle 1 / 2$ the equilibrium involves losses in country R. This implies subsidies on the part of R, rather than taxes. This possibility was excluded previously on the basis of the no loss constraint in both countries All this leads to the conclusion that the game we are considering here is far removed from the idea of acceptable transfer prices which should belong to a reasonable arm's length interval. Instead, the solution is driven exclusively by the strategic considerations of the countries involved, and not by considerations of decent international trade.

## 3. Conclusion

In the literature on transfer pricing the implications for tax regulation have been a central issue. In the simple textbook case where a monopolist faces a single tax rate on profits, this tax rate will not affect the price-quantity decision of the monopolist. Profits after taxes are a given fraction of profits before taxes. In the case of a multinational firm, however, differences in tax rates between countries will affect the price and quantity decisions of the firm. The reason is that the firm can now use its price-quantity decision to shift profits from one country to the other. Profit maximization after taxes is then very different from profit maximization before taxes. This possibility of shifting profits from one country to the other is also larger in the case of profit taxes than in the case of import or export tariffs.

We have shown that the governments of the two countries can manipulate their tax rates such that the monopolist will decrease his retail price until it is equal to the upper bound of the transfer price interval. This realizes a clear welfare gain. A similar outcome can be obtained if the two countries appropriately manipulate the transfer price interval, for given tax rates. Both types of regulatory policies are derived from the same 'golden rule'. We also showed that the same golden rule applies equally in tax games, provided the arm's length principle remains applicable, where the two governments do not cooperate.

The implications of the golden rule for tax regulation can be criticized as they imply discriminatory profit tax rates. Indeed, profit taxes should be higher in importing industries than in exporting industries. This limits the effective use of tax rates, and leaves the manipulation of the arm's length interval as the only instrument to realize welfare improvements. It still remains true that, as long as the tax rate in the retailing country exceeds the one in the manufacturing country such welfare improvements can be realized.

Our results are highly dependent on the institutional context in the sense that we start from the assumption that there is 'tax coordination' between the two countries. By this we mean that both countries agree on an arm's length interval of transfer prices. Some contributions to the literature do not use this framework, and assume a 'noncoordinated regulation' situation, where countries set transfer prices independent of each other, and without consideration of double taxation implications. Whatever the
merits of this setup, we feel that our formulation better represents the actual practice. We also showed that the way welfare is defined in these models leads to fundamentally different outcomes.

Finally, our findings have clear implications for actual practice transfer price regulation. In practice, the arm's length interval is in principle derived from assessments based on 'comparable uncontrolled prices' or market prices. However, monopolistic situations by definition imply that a strict comparison is not possible. In practice, 'fair profit allocation' principles are then often applied. These imply that transfer prices should be such that both the retailing and the manufacturer's country should be entitled to their fair share of the profits generated. The tax (and arm's length price) regulation we derived is at odds with the 'fair profit principle': if appropriate taxes and transfer prices are implemented, the existence of profits in the retailing country would only imply that the regulation is ineffective, and zero profits would not point to unacceptable transfer prices.

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## Appendix A. Derivation of country R's reaction correspondence.

Recall that $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ applies when $\alpha \leq \alpha^{*}$, and that $\bar{W}_{R}\left(t_{M}\right)$ applies when $\alpha>\alpha^{*}$. We first derive two preliminary properties of the function $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$.

The first property concerns the sign of the derivative $\frac{\partial \tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)}{\partial t_{R}}$ at two particular values of $t_{R}$. We first evaluate this derivative at $t_{R}=0$. One finds that

$$
\begin{align*}
\frac{\partial \tilde{W}_{R}\left(p_{M}, 0, t_{M}\right)}{\partial t_{R}}=\frac{1}{4 b} & {\left[(a-c)-t_{M}\left(p_{M}-c\right)\right] }  \tag{A.1}\\
& {\left[\left(a-p_{M}\right)-\gamma\left[a+c-p_{M}\left(2-t_{M}\right)-c t_{M}\right]\right] }
\end{align*}
$$

The two terms in square brackets are positive ${ }^{6}$, so that the whole expression is also positive.

Let us denote by $t_{R}^{*}$ the value of $t_{R}$ such that, for a given value of $t_{M}, \alpha=\alpha^{*}$. This value is given by $t_{R}^{*}=1-\frac{\left(1-t_{M}\right)}{\alpha^{*}}$. One then finds that
$\frac{\partial \tilde{W}_{R}\left(p_{M}, t_{R}^{*}, t_{M}\right)}{\partial t_{R}}=\frac{\left(a-p_{M}\right)^{2}}{2 b}>0$.

The second property of $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ concerns the stationary points of this function with respect to $t_{R}$. These stationary points satisfy the equation $\frac{\partial \tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)}{\partial t_{R}}=0$. This equation can be written as
$t_{M}=1+\frac{\left.\left(a-p_{M}\right)\left(1-t_{R}\right)\left( \pm 1+\sqrt{1-4\left(1-t_{R}\right)(1-\gamma)\left(t_{R}-\left(1-t_{R}\right) \gamma\right.}\right)\right)}{2\left(p_{M}-c\right)\left[t_{R}-\left(1-t_{R}\right) \gamma\right]}$
If then
$1-4\left(1-t_{R}\right)(1-\gamma)\left(t_{R}-\left(1-t_{R}\right) \gamma\right) \geq 0$,
equation (A.3) describes two branches in $\left(t_{R}, t_{M}\right)$-space. These two branches meet at the point $\left(t_{R}, t_{M}\right)=(1,1)$, and each branch defines a real root of $\frac{\partial \tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)}{\partial t_{R}}=0$. One branch is increasing, and starts at some minimal value of $t_{R}$, and ends at the

[^4]point $\left(t_{R}, t_{M}\right)=(1,1)$. The other branch starts at the point $\left(t_{R}, t_{M}\right)=(1,1)$ and leads to values for $t_{M}$ outside the interval [0,1]. If (A.4) does not hold, then $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ has no stationary point in $t_{R}$.

We can now combine the two properties of $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ as follows. We consider two possibilities:
If (A.4) holds, then for each value of $t_{M}$ the exists exactly one real value of $t_{R} \in[0,1]$ where $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ has a stationary point. Because of properties (A.1) and (A.4) this stationary point cannot occur in the interval $\left[0, t_{R}^{*}\right]$. Hence, the function $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ must be increasing in this interval.
If (A.4) does not hold, then $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ has no stationary point in the interval $[0,1]$. As $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ is increasing at $t_{R}=0$ (see (A.1)), it must continue to increase for all $t_{R} \in[0,1]$.

It follows that the function $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ is always increasing in $t_{R}$ for all $t_{R} \in\left[0, t_{R}^{*}\right]$. For values $t_{R} \geq t_{R}^{*}$, the welfare of country R is given by $\bar{W}_{R}\left(t_{M}\right)$, as defined in (27). The typical shape of $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ as a function of $t_{R}$ is given in Figure A.1. On the horizontal axis we put $t_{R}$. On this axis we also indicate the value of $t_{M}$, which is an arbitrary fixed value. For any given value of $t_{M}$ we can determine $t_{R}^{*}$. In section 1 we have seen that the firm will choose $p_{M}=p_{L}$ if $t_{R}<t_{M}$, so that $\tilde{W}_{R}\left(p_{L}, t_{R}, t_{M}\right)$ applies. If the reverse inequality $t_{R}<t_{M}$ holds, the firm chooses $p_{M}=p_{H}$, and $\tilde{W}_{R}\left(p_{H}, t_{R}, t_{M}\right)$ applies.

Figure A 1 Welfare in R


The relevant segments of $\tilde{W}_{R}\left(p_{M}, t_{R}, t_{M}\right)$ are shown in bold on Figure A.1. At $t_{R}=t_{M}$ the difference $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right)-\tilde{W}_{R}\left(p_{H}, t_{M}, t_{M}\right)$ is equal to $\left(p_{H}-p_{L}\right) t_{M} \frac{\left(a-p^{m o}\right)}{b}$.The value of $W_{R}$ at $t_{R}=t_{M}$ can be defined as any value in the interval $\left(\tilde{W}_{R}\left(p_{H}, t_{M}, t_{M}\right), \tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right)\right)$.

Two cases can occur now. First, it may happen that $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right)>\bar{W}_{R}\left(t_{M}\right)$. From Figure A. 1 it is clear that in this case there is no optimal value of $t_{R}$. For any given value of $t_{M}$, country R can choose a value $t_{R}=t_{M}-\varepsilon$, where $\varepsilon$ is an arbitrarily small positive number. Country R is then "tax undercutting" country M , and realizes a payoff arbitrarily close to $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right)$. Country R's reaction correspondence is empty in this case. Second, the reverse inequality $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right) \leq \bar{W}_{R}\left(t_{M}\right)$ may hold. Country R will then choose any value $t_{R}$ satisfying $t_{R}^{*} \leq t_{R} \leq 1$.

We can now define a switching value $t_{M}^{s w}$ by solving $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right)=\bar{W}_{R}\left(t_{M}\right)$ for $t_{M}$. We then obtain $\tilde{W}_{R}\left(p_{L}, t_{M}, t_{M}\right) \leq \bar{W}_{R}\left(t_{M}\right) \Leftrightarrow t_{M} \leq t_{M}^{s w}$. For values $t_{M} \leq t_{M}^{s w}$, country R will set $t_{R} \geq t_{R}^{*}$. For values $t_{M}>t_{M}^{s w}$, no optimal value of $t_{R}$ exists.

The complete reaction correspondence $\varphi_{R}$, as given in the text and illustrated on Figure 4, then easily follows.

## Appendix B. Derivation of country M's reaction correspondence.

Consider the function $\tilde{W}_{M}\left(p_{M}, t_{R}, t_{M}\right)$ which applies if the tax rates satisfy $\alpha \leq \alpha^{*}$. As $\frac{\partial^{2} \tilde{W}_{M}\left(p_{M}, t_{R}, t_{M}\right)}{\partial t_{M}{ }^{2}}=-\frac{\left(p_{M}-c\right)^{2}(1+\gamma)}{2 b\left(1-t_{R}\right)}<0$, this function is strictly concave in $t_{M}$. The value of $t_{M}$ satisfying the first order condition $\frac{\partial \tilde{W}_{M}\left(p_{M}, t_{R}, t_{M}\right)}{\partial t_{M}}=0$ is given by

$$
\frac{\left[a\left(1-t_{R}\right)+p_{M} t_{R}-c\right] \gamma}{\left(p_{M}-c\right)(1+\gamma)}
$$

We denote this value by $\hat{t}_{M}\left(p_{M}, t_{R}\right)$. This function gives, for every combination of $p_{M}$ and $t_{R}$, the corresponding value of $t_{M}$ that maximizes $\tilde{W}_{M}\left(p_{M}, t_{R}, t_{M}\right)$. Figure 5 shows the graphs of $\hat{t}_{M}\left(p_{H}, t_{R}\right)$ and $\hat{t}_{M}\left(p_{L}, t_{R}\right)$.

We can now compare the maximal values of $\tilde{W}_{M}\left(p_{H}, t_{R}, \hat{t}_{M}\left(p_{H}, t_{R}\right)\right)$ and $\tilde{W}_{M}\left(p_{L}, t_{R}, \hat{t}_{M}\left(p_{L}, t_{R}\right)\right)$. It turns out that

$$
\begin{equation*}
\tilde{W}_{M}\left(p_{H}, t_{R}, \hat{t}_{M}\left(p_{H}, t_{R}\right)\right)-\tilde{W}_{M}\left(p_{L}, t_{R}, \hat{t}_{M}\left(p_{L}, t_{R}\right)\right)=\frac{t_{R}\left(p_{H}-p_{L}\right)\left[2(a-c)+t_{R}\left(p_{H}+p_{L}-2 a\right)\right]}{4 b\left(1-t_{R}\right)(1+\gamma)} \tag{B.1}
\end{equation*}
$$

which is positive, for all values of $t_{R}{ }^{7}$. It follows that country M prefers the tax rate $\hat{t}_{M}\left(p_{H}, t_{R}\right)$ to the tax rate $\hat{t}_{M}\left(p_{L}, t_{R}\right)$, provided $t_{R}>\hat{t}_{M}\left(p_{H}, t_{R}\right)$. This inequality must be satisfied to make sure that the firm chooses $p_{M}=p_{H}$. On Figure 5 it is seen that this is the case for $t_{R}^{1}<t_{R} \leq t_{R}^{2}$. Hence, for values of $t_{R}$ in this interval, country M's best reply is the tax rate $\hat{t}_{M}\left(p_{H}, t_{R}\right)$.

When $t_{R} \leq t_{R}^{1}$ country M will no longer choose $\hat{t}_{M}\left(p_{H}, t_{R}\right)$, as then $t_{R}<\hat{t}_{M}\left(p_{H}, t_{R}\right)$ giving a payoff $\tilde{W}_{M}\left(p_{L}, t_{R}, \hat{t}_{M}\left(p_{H}, t_{R}\right)\right)$ which is smaller than $\tilde{W}_{M}\left(p_{L}, t_{R}, \hat{t}_{M}\left(p_{L}, t_{R}\right)\right)$. Country M then has the choice between the tax rate $\hat{t}_{M}\left(p_{L}, t_{R}\right)$ and the tax rate $t_{M}=t_{R}-\varepsilon$ where $\varepsilon$ is a arbitrarily small positive real number. This tax rate is as close as possible to $\hat{t}_{M}\left(p_{H}, t_{R}\right)$, while it guarantees that $p_{M}=p_{H}$ applies. Country M is "tax undercutting" country R . It realizes a payoff which is arbitrarily close to $\tilde{W}_{M}\left(p_{H}, t_{R}, t_{R}\right)$. We now define
$\Delta\left(t_{R}\right)=\tilde{W}_{M}\left(p_{L}, t_{R}, \hat{t}_{M}\left(p_{L}, t_{R}\right)\right)-\tilde{W}_{M}\left(p_{H}, t_{R}, t_{R}\right)$.
This function is quadratic in $t_{R}$. It is easy to see that
$\Delta(0)=\tilde{W}_{M}\left(p_{L}, 0, \hat{t}_{M}\left(p_{L}, 0\right)\right)-\tilde{W}_{M}\left(p_{H}, 0,0\right)=\frac{(a-c)^{2} \gamma^{2}}{4 b\left(p_{L}-c\right)(1+\gamma)}>0$.
It follows that for $t_{R}=0$ country M will choose $\hat{t}_{M}\left(p_{L}, t_{R}\right)$.

Since at $t_{R}=t_{R}^{1}$ the undercutting strategy coincides with $\hat{M}_{M}\left(p_{H}, t_{R}\right), \Delta\left(t_{R}^{1}\right)$ is equal to the negative of expression (B.1), evaluated at $t_{R}=t_{R}^{1}$. It follows that $\Delta\left(t_{R}^{1}\right)<0$. As then $\Delta(0)>0$ and $\Delta\left(t_{R}^{1}\right)<0$, there must be exactly one root $t_{R}^{s w}$ where $\Delta\left(t_{R}^{s w}\right)=0$, with $0<t_{R}^{s w}<t_{R}^{1}$. It follows that $\Delta\left(t_{R}\right)>0$ for $0 \leq t_{R}<t_{R}^{s w}$, while $\Delta\left(t_{R}\right)<0$ for $t_{R}^{s w}<t_{R} \leq t_{R}^{1}$. If then $0 \leq t_{R}<t_{R}^{s w}$, the optimal reaction by country M is $\hat{t}_{M}\left(p_{L}, t_{R}\right)$. For $t_{R}^{s w}<t_{R} \leq t_{R}^{1}$, there is no optimal value of $t_{M}$ so that country M's reaction correspondence is empty.

We still have to consider country M's optimal reaction to values of $t_{R}$ in the interval $t_{R}^{2} \leq t_{R} \leq 1$. For values of $t_{M}$ such that $\left(t_{R}, t_{M}\right)$ is below the line $\alpha=\alpha^{*}$, $\bar{W}_{M}\left(t_{M}\right)$ applies which is increasing in $t_{M}$. See (28). For values of $t_{M}$ such that $\left(t_{R}, t_{M}\right)$ is above the line $\alpha=\alpha^{*}$, depending on whether $t_{R}<t_{M}$ or $t_{R}>t_{M}$ holds, either

[^5]$\tilde{W}_{M}\left(p_{L}, t_{R}, t_{M}\right)$ or $\tilde{W}_{M}\left(p_{H}, t_{R}, t_{M}\right)$ applies,. Welfare of country M when $t_{R}=t_{M}$ can be defined as any value between $\tilde{W}_{M}\left(p_{L}, t_{R}, t_{R}\right)$ and $\tilde{W}_{M}\left(p_{H}, t_{R}, t_{R}\right)$. Consider now an arbitrary value of $t_{R}$ in the interval $t_{R}^{2}<t_{R} \leq 1$. For values of $t_{M}$ satisfying $t_{M}>t_{R}$, country M's welfare increases as $t_{M}$ decreases because these points are above $\hat{t}_{M}\left(p_{L}, t_{R}\right)$. At $t_{R}=t_{M}$ country M's welfare jumps from $\tilde{W}_{R}\left(p_{L}, t_{R}, t_{R}\right)$ to $\tilde{W}_{R}\left(p_{H}, t_{R}, t_{R}\right)$. This represents an increase by $\frac{t_{R}(a-c)\left(p_{H}-p_{L}\right)}{2 b}$. If $t_{M}$ decreases further, $\tilde{W}_{M}\left(p_{H}, t_{R}, t_{M}\right)$ increases until $t_{M}$ hits the line $\alpha=\alpha^{*}$. (We move closer to the $\operatorname{line} \hat{t}_{M}\left(p_{H}, t_{R}\right)$.). We conclude that, for values of $t_{R}$ in the interval $t_{R}^{2} \leq t_{R} \leq 1$, the reaction function of country M coincides with the line $\alpha=\alpha^{*}$.

## Appendix C Tax competition when countries only care about tax revenue.

In this Appendix we briefly look at the Nash equilibria of a tax competition game in which the two countries are only interested in their tax revenues as given by (19) for country R, and by (20) for country M. Assuming that the tax rates satisfy $\alpha \leq \alpha^{*}$, and substituting $p_{R}=p_{R}^{m o}+\frac{(1-\alpha)}{2}\left(p_{M}-c\right)$ in (19) and (20), we obtain the tax revenues $T R_{R}=\frac{t_{R}\left[\left(1-t_{R}\right)^{2}\left(a-p_{M}\right)^{2}-\left(1-t_{M}\right)^{2}\left(p_{M}-c\right)^{2}\right]}{4 b\left(1-t_{R}\right)^{2}}$
$T R_{M}=\frac{\left(1-t_{M}\right)\left(p_{M}-c\right)\left(\left(1-t_{R}\right)\left(a-p_{M}\right)+\left(1-t_{M}\right)\left(p_{M}-c\right)\right)}{2 b\left(1-t_{R}\right)}$
Maximizing these expressions with respect to $t_{R}$ and $t_{M}$, respectively, we obtain the following first order conditions:
$t_{M}=1-\frac{\left(1-t_{R}\right)^{3 / 2}\left(a-p_{M}\right)}{\sqrt{1+t_{R}}\left(p_{M}-c\right)}$
$t_{M}=\frac{a-c-t_{R}\left(a-p_{M}\right)}{2\left(p_{M}-c\right)}$

The reaction function of country R is obtained by inverting the first equation, writing $t_{R}$ as a function $t_{R}\left(p_{M}, t_{M}\right)$ of $t_{M}$. The second equation gives the reaction function $t_{M}\left(p_{M}, t_{R}\right)$ of country M.

Figures C. 1 and C. 2 give typical shapes of these reaction functions. The reaction function of country R typically starts at some minimal value of $t_{R}$, and rises to 1 as $t_{M}$ increases. This minimal value of $t_{R}$ is 0 for $p_{M}=\frac{a+c}{2}$, and it increases to 1 as $p_{M}$ approaches c. For any given $p_{M}$ the reaction function typically follows a trajectory starting with $t_{R}>t_{M}$, at some point crossing the $45^{\circ}$ line into to the region where $t_{R}<t_{M}$. It can be shown that the reaction function of R is entirely to the left of
the line $\alpha=\alpha^{*}$. This is not surprising: this line implies zero profits in R , and therefore no tax revenue. Consequently, no Nash equilibria are possible in the region where $\alpha \geq \alpha^{*}$.

The reaction function of M is linear and decreasing in $t_{R}$. The optimal tax rate of country M is $t_{M}=1$ for sufficiently small values of $t_{R}$. If $p_{M}$ increases, country M's optimal tax rate decreases for each value of $t_{R}$.

Figure C 1. An equilibrium in the $p_{L}$ region


The possible Nash equilibria have the following characteristics ${ }^{8}$. For a sufficiently high transfer price $p_{L}$, an equilibrium in the $p_{M}=p_{L}$ region occurs where $t_{R}<t_{M}$, with a tax rate for country M in the interval $0.5<t_{M} \leq 1$. On the other hand, if the transfer price $p_{H}$ is sufficiently low, country R will always opt for a tax rate in the $p_{M}=p_{H}$ region (excluding the region where $\alpha \geq \alpha^{*}$ ). These possibilities are illustrated on figures C1 and C2

[^6]Figure C 2. An equilibrium in the $p_{H}$ region

$t_{R}$

In the tax game of section 2.b all Nash equilibria were located on the line $\alpha=\alpha^{*}$. See Figure 6. This property was the essence of the Golden Rule. If countries only care about tax revenue, this type of equilibrium does not occur. In this case the arms length interval will determine which country applies the highest tax rate, and therefore, whether $p_{M}=p_{L}$ or $p_{M}=p_{H}$ applies. It can be noted that, contrary to some of the tax competition games found in the literature (Raimondos-Möller, Scharf, 2002), the system analysed here does not lead to a 'race to the bottom' or the a 'race to the top'.


[^0]:    ${ }^{1}$ The authors want to thank M.Cools and B.De Borger for their useful comments on an earlier version of the paper. The authors remain responsible for all remaining errors.

[^1]:    ${ }^{2}$ Historically important contributions to the literature on the microeconomics of the multinational firm are Copithorne (1971) and Horst (1971).

[^2]:    ${ }^{3}$ If the governments fix $p_{H}=c$ and $t_{R}=1$, the value of the monopolist's profit function (1) is trivially equal to zero. The optimal price $p_{R}$ to be charged by the monopolist is then indeterminate. We can assume that the monopolist then puts $p_{R}=c$.
    ${ }^{4}$ In Appendix C of this paper we briefly examine the case where the payoffs of the two countries are given by their tax revenues. It is shown there that the resulting Nash equilibria differ significantly from the Nash equilibria obtained in our paper. In particular, if countries only care about tax revenue, the golden rule no longer applies.

[^3]:    ${ }^{5}$ The analysis of this case is very similar to the analysis given here, and leads to highly similar conclusions.

[^4]:    ${ }^{6}$ The first term in square brackets is clearly positive as $a-c>p_{M}-c$. The second term in square brackets is equal to $a-p_{M}>0$ if $\gamma=0$. For $\gamma=1$ it is equal to $\left(1-t_{M}\right)\left(p_{M}-c\right)>0$. As this second term is linear in $\gamma$, it must be positive for all values of $\gamma$ between 0 and 1 .

[^5]:    ${ }^{7}$ For $t_{R}=0$ the expression between square brackets is equal to $2(a-c)>0$. For $t_{R}=1$ this expression is equal to $\left(p_{H}-c\right)+\left(p_{L}-c\right)>0$. As the expression is linear in $t_{R}$, the conclusion follows.

[^6]:    ${ }^{8}$ Switching behavior from the reaction function applicable in the $p_{M}=p_{L}$ regime to the $p_{M}=p_{H}$ regime (on, or just below the diagonal) by country R is possible, close to the intersection with the diagonal. This is not discussed here.

