Well Capacity and the Gains from Coordination in a Spatially Explicit Aquifer

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Introduction

Groundwater resources provide a valuable input to agricultural production, particularly in arid and semi-arid regions of the world. The value of groundwater resources used in agricultural production is directly determined by the profitability of irrigation. Groundwater pumping, however, imposes external costs on nearby groundwater uses, leading to inefficient use. The external costs arise due to the fact that groundwater use reduces the level of saturated thickness at nearby locations, which serves to both increase the cost of extraction and reduce the productivity of the water that is applied by other users. While the pumping cost externality has been well documented in the economics literature (e.g., Gisser and Sanchez 1980, Guilfoos et al., 2013), the productivity losses resulting from reduced saturated thickness have received less attention (Foster, Brozović, and Butler 2014 provide an exception). The nature and magnitude of these externalities have important implications for groundwater management, especially in situations where the local institutions governing groundwater use (e.g., groundwater management districts) do not coincide with the full spatial extent of an aquifer.

In this paper, we develop a spatially explicit hydro-economic model of groundwater use to address two research questions related to groundwater management. First, we highlight the theoretical and empirical relationship between changes in an aquifer’s saturated thickness at a given location and the net returns to irrigated agriculture. Second, we illustrate how this relationship influences the relative gains to dynamic behavior and the coordination of groundwater pumping. This latter question helps to provide feedback on the extent to which
groundwater use that accounts for dynamic linkages can support higher net returns, even when it does not involve the full coordination of all users.

We find that the gains from individual dynamic optimization and coordinated groundwater management critically depend on the sensitivity of crop yields to the level of saturated thickness. When lower levels of saturated thickness correspond to substantial yield reductions, even individual users stand to benefit from optimal management relative to behaving myopically. We also show that as the relationship between agricultural profits and saturated thickness becomes more inelastic, gains from management can only be achieved through coordination of multiple users. This result suggests that in some instances, local management efforts will only serve to enhance the value of scarce water resources if the spatial extent of the management efforts can be expanded to cover a sufficient number of resource users.

Background

Our study contributes to three strands of the economics literature related to groundwater resources. The first strand is the large set of research that investigates the finding by Gisser and Sanchez (1980) that groundwater management generates trivial social benefits. Researchers investigating the robustness of this result, known as the Gisser-Sanchez Paradox, include Allen and Gisser (1984), who show that it does not depend on the assumption of linear water demand. Similarly, Feinerman and Knapp (1983) evaluate groundwater management in Kern County, CA under a variety of assumptions related to demand parameters and aquifer characteristics and find that the returns to management are always less than ten percent. Brill and Burness (1994) investigate a wide range of assumptions and conclude that management can enhance value when the discount rate is low, when demand grows over time, and when well capacity diminishes with
lower aquifer levels. More recent attention in the literature has focused on evaluating spatially explicit models, where groundwater externalities are highest at the point of extraction and dissipate with distance from a well engaged in pumping. Under these more realistic conditions, Brozovic et al. (2008) show that when wells are spaced closely together, management gains can be relatively large. In this same vein, Guilfoos et al. (2013) implement a simulation model involving a spatially explicit aquifer in which external pumping effects concentrate in nearby farms and show that significant gains to management can exist.

A second strand of the literature that we contribute to explores the nature of groundwater pumping externalities. Most economic models of groundwater use represent pumping externalities as increasing pumping costs through lower aquifer levels (see Koundouri, 2004 for a review). Provencher and Burt (1993) identify two additional types of externalities that result from groundwater pumping; a stock externality as groundwater use in one period reduces the set of potential actions in future periods, and a risk externality that arises when groundwater use reduces the ability of risk-averse agents to respond to stochastic returns. Research by Negri (1989) also identified a strategic externality that exists as users compete to capture rents from limited water resources.

A more recent series of papers by Foster, Brozovic, and Butler (2014, 2015a, 2015b) points out another potential pumping externality related to the rate at which wells are physically able to pump water (referred to as well capacity). Groundwater use at one location reduces the saturated thickness at nearby locations, which leads directly to reduced well capacity at neighboring wells. Lower well capacity limits an irrigator’s ability to deliver water to crops when it creates the most value, thus reducing the productivity and profitability of water. In this study, we use a model of groundwater use to explicitly measure the relationship between
saturated thickness, well capacity, and productivity to provide an estimate of the magnitude of the externality resulting from diminished productivity.

The final strand of the economics literature that this study contributes to relates to the benefits of strategic behavior and coordination in common pool resource systems. This line of research has been motivated in large part by Ostrom’s case study and experimental research investigating the circumstances under which groups effectively come together to manage common pool resources (e.g., Gardner, Ostrom and Walker 1990; Ostrom et al. 1992; Ostrom et al. 1999). The research has been extended to cooperative behavior related to groundwater use (Walker, Gardner and Ostrom 1997). Researchers have also evaluated the theoretical implications of strategic and cooperative behavior with respect to groundwater resources (Rubio and Casino 2003). A study by Saak and Peterson (2007) captures the role that strategic behavior plays in groundwater use through implementation of a theoretical model of a two cell aquifer, where the groundwater dynamics are governed by Darcy’s Law. The model developed in this paper integrates a spatially explicit aquifer with both cost and production externalities to investigate the gains from local and coordinated groundwater management.

In the following section, we describe the hydro-economic model used to investigate the role of production externalities in determining the gain to coordinated groundwater management. The model is calibrated to wells in Eastern Colorado and solved numerically to demonstrate that returns to individual dynamic extraction behavior as well as coordination across users depend on the elasticity of production with respect to groundwater levels. To assess the importance of this production externality in practice, we use modeled production data on corn and wheat farms in the region to measure the relationship between farm revenue and saturated thickness. Finally, we
discuss the implication of the results for groundwater management and the need for coordination across space.

**Hydro-economic Model**

To investigate the gains from groundwater management, we model agricultural output at well $i$ in year $t$ as

$$y_{it} = \left( a_i w_{it} - \frac{1}{2} b_i w_{it}^2 \right) x_{it}^\eta, \quad i = 1, \ldots, N,$$

which can be sold for price, $p$. $w_t$ is the amount of water pumped, $a_i, b_i$ are positive parameters, $x_{it}$ is the saturated thickness of the aquifer at well $i$, and $\eta$ is the elasticity of production with respect to saturated thickness.

Production responses to saturated thickness are driven by the connection between saturated thickness and well capacity. $\eta \geq 0$ represents the decrease in production per unit of water as saturated thickness falls and well capacity drops. The marginal cost of water extraction is $c_0 - c_1 x_{it}$ where $c_0, c_1 > 0$. In this model, as saturated thickness falls, productivity decreases because of lower well capacity while marginal extraction costs increase as water must be lifted a greater vertical distance.

We model wells as cells situated along a linear array. The thickness of the aquifer at well $i$ adjusts according to Darcy’s Law and depends on pumping rates from wells in adjacent cells, $j_a$ as well as pumping at well $i$. Therefore,

$$\dot{x}_{it} = \frac{R_i + (\alpha - 1) w_{it}}{AS_i} - \sum_{j \in I_a} \frac{k_i A_{0i} (x_{it} - x_{jt})}{d_{ij} AS_i} \quad \forall i$$

(1)

where $R_i$ is the recharge into the cell of well $i$, $\alpha$ is the proportion of applied water that returns as recharge, $d_{ij}$ is the distance from well $i$ to well $j$, $A_{0i}$ is the cross-sectional area through which water flows, $AS_i$ is the surface area times aquifer storativity, and $k_i$ is hydraulic conductivity.
The second term on the right-hand side of equation 1 is summed across all adjacent cells, as indicated by the set, $j_a$.

We consider alternative coordination scenarios, including myopic pumping, individual dynamic optimization, and full optimization. As in Gisser and Sanchez (1980), we use the fully myopic pumping case as a benchmark for comparing gains to alternative management scenarios. When one well dynamically optimizes, this reflects a single (or group of) user’s decision to conserve water for future time periods. When making this decision, an individual accounts for the linkages across farms that are governed by Darcy’s Law but does not account for the cost that is imposed on the other users.

**Myopic Pumping Decisions**

To solve for the myopic pumping path, it is assumed that each individual well sets the marginal benefit of pumping to the marginal cost of pumping in year $t$. Specifically, for user $i$:

$$px_t^\eta (a_t - b_tw_{it}) = c_{0i} - c_{1i}x_{it}$$  \hspace{1cm} (2)

Solving for the quantity of groundwater used in a given period,

$$w_{it} = \frac{a_t}{b_t} - \frac{c_{0i} - c_{1i}x_{it}}{pb_t x_t^\eta}$$  \hspace{1cm} (3)

Finally, equation 3 can be plugged into equation 1 to determine how the aquifer saturated thickness at well $i$ changes over time. Solving the resulting system of ordinary differential equations (ODEs) for $N$ wells produces the myopic solution for saturated thickness, $x_{it}^m$, which is inserted into (3) to obtain $w_{it}^m$. Finally, $x_{it}^m$ and $w_{it}^m$ are plugged into the profit function to determine the net present value (NPV) of pumping at well $i$. Assuming discount rate, $r$, this becomes
The myopic value of pumping at well $i$ will be compared with the value generated from groundwater management under other scenarios. Because of the nonlinearity of the ODEs, the model does not have a closed-form solution and is solved numerically in the next section.

Socially Optimal Solution

Next, we characterize the socially optimal paths for pumping and saturated thickness. This represents the case in which wells coordinate and internalize all external pumping effects across space and time, including the increased costs and lower water productivity that result from decreasing aquifer levels. Solving this model numerically allows for an estimate of the gains to groundwater management as a function of $\eta$. In the socially optimal case, the planner’s objective is to

$$\max_{w_{it}} \int_0^T \sum_{i=1}^N p \left( a_i w_{it} - \frac{1}{2} b_i w_{it}^2 \right) x_{it}^{\eta} - (c_{0i} - c_{1i} x_{it}) w_{it} \right) e^{-rt} dt$$

(5)

s.t.

$$x_{it} = \frac{R_i + (\alpha - 1) w_{it}}{A S_i} - \sum_{j \in J_i a} k_i A_{0i}(x_{it} - x_{jt}) \frac{d_i j A S_i}{d_j AS_i} \quad \forall i$$

(6)

And $x_{i0}$ known. Defining $\lambda_{it}$ as the co-state variable associated with each of the $N$ state variables, the current value Hamilton for this problem becomes

$$H^{cv} = \sum_{i=1}^N p \left( a_i w_{it} - \frac{1}{2} b_i w_{it}^2 \right) x_{it}^{\eta} - (c_{0i} - c_{1i} x_{it}) w_{it}$$

$$+ \lambda_{it} \left( \frac{R_i + (\alpha - 1) w_{it}}{A S_i} - \sum_{j \in J_i a} k_i A_{0i}(x_{it} - x_{jt}) \frac{d_i j A S_i}{d_j AS_i} \right)$$

(7)
Assuming an interior solution, the Pontryagin conditions for this problem state that

\[ px_{it}^\eta (a_i - b_i w_{it}) - (c_{0i} - c_{1i} x_{it}) - \frac{\lambda_{it}(1 - \alpha)}{AS_i} = 0 \quad \forall \ i \]  

(8)

\[ \dot{\lambda}_{it} = r \lambda_{it} - \left[ \eta x_{it}^{\eta-1} p (a_i w_{it} - b_i w_{it}^2) + c_{1i} w_{it} \right. \\
\left. - \left( \sum_{j \in J_a} \lambda_{it} k_{ij} A_{0j} \right) \frac{d_{ij}}{d_{ij} AS_i} + \sum_{j \in J_a} \frac{\lambda_{jt} k_{ij} A_{0j}}{d_{ij} AS_j} \right] \quad \forall \ i \]  

(9)

And

\[ \dot{x}_{it} = \frac{R_i + (\alpha - 1) w_{it}}{AS_i} - \sum_{j \in J_a} \frac{k_i A_{0i} (x_{it} - x_{jt})}{d_{ij} AS_i} \quad \forall \ i \]  

(10)

Equation 8 can be solved for \( w_{it} \) so that:

\[ w_{it} = \frac{a_i}{b_i} - \frac{c_{0i} - c_{1i} x_{it}}{p b_i x_{it}^\eta} + \frac{\lambda_{it}(\alpha - 1)}{px_{it}^\eta b_i AS_i} \quad \forall \ i \]  

(11)

Plugging (11) into (9) and (10) produces a system of nonlinear ODEs, in \( \lambda_{it} \) and \( x_{it} \). Assuming \( T \) is finite, \( \lambda_{iT} = 0 \). Combining with \( x_{i0} \) known, the system of ODEs can be solved numerically to produce the optimal state and co-state paths, \( x_{it}^*, \lambda_{it}^* \). Using the solution in (11) produces \( w_{it}^* \).

Finally, the optimal paths can be plugged into (5) and separating the value from each well produces \( NPV_i^* \).

Single Dynamic Optimizer
In addition to the extreme cases of myopic producers and fully optimal coordination, we investigate single-user dynamic decision-making.\(^1\) If a given well optimizes value over time without coordination across wells, the objective function becomes:

\[
\max_{w_{it}} \int_0^T \left( p \left( a_i w_{it} - \frac{1}{2} b_i w_{it}^2 \right) x_{it}^\eta - (c_{0i} - c_{1i} x_{it}) w_{it} \right) e^{-\gamma t} dt
\]

s.t.

\[
x_{it} = \frac{R_i + (\alpha - 1) w_{it}}{AS_i} - \sum_{j \in I_a} \frac{k_i A_{0i} (x_{it} - x_{jt})}{d_{ij} AS_i}
\]

The main difference between the individual dynamic optimizer and the social planner in (5) is that the individual \(i\) considers profit only at well \(i\), and accounts only for how saturated thickness changes at well \(i\).

Following the same procedure as in the fully optimal scenario, the Pontryagin conditions for this dynamic optimization problem imply that

\[
w_{it} = \frac{a_i}{b_i} \left( \frac{c_{0i} - c_{1i} x_{it}}{pb_i x_{it}^\eta} + \frac{\lambda_{it} (\alpha - 1)}{px_{it}^\eta b_i AS_i} \right)
\]

And

\[
\dot{\lambda}_{it} = r \lambda_{it} - \left[ \eta x_{it}^{\eta - 1} p (a_i w_{it} - b_i w_{it}^2) + c_{1i} w_{it} - \left( \sum_{j \in I_a} \frac{\lambda_{jt} k_i A_{0i}}{d_{ij} AS_i} \right) \right]
\]

Note the apparent equivalence of (14) to (11). Individuals account for dynamic effects of pumping in both scenarios but the size of those effects differs. In the fully optimal case, the individual decision accounts for dynamic costs not only to an individual well, but to all wells in the model. In the individual case, only dynamic effects relevant at the same well are considered.

\(^1\) A range of cases is explored but here we focus on the single-optimizer decision. Theoretical results can also be made available in which a subset of wells coordinates while others continue to behave either myopically or rationally without coordination.
To solve the model with an individual optimizer, we assume that the individual correctly anticipates the behavior of others in the basin. As an illustration, if well $i$ behaves dynamically while all other wells behave myopically, equation (3) is used to determine pumping at other wells and $\lambda_{lt}$ with $l \neq i$ would equal zero. The resulting system of ODEs becomes

$$\dot{x}_{it} = \frac{R_i + (\alpha - 1)w_{it}^R}{AS_i} - \sum_{j \in J} \frac{k_i A_{0i}(x_{it} - x_{jt})}{d_{ij} AS_i}$$  \hspace{1cm} (16)$$

$$\dot{x}_{lt} = \frac{R_l + (\alpha - 1)w_{lt}^R}{AS_i} - \sum_{j \in J} \frac{k_i A_{0i}(x_{it} - x_{jt})}{d_{ij} AS_i} \quad l \neq i$$ \hspace{1cm} (17)$$

$$\dot{\lambda}_{it} = r\lambda_{it} - \left[ \eta x_{it}^{\eta - 1} p(a_i w_{it} - b_i w_{it}^2) + c_{1i} w_{it} - \left( \sum_{j \in J} \frac{\lambda_{it} k_i A_{0i}}{d_{ij} AS_i} \right) \right]$$ \hspace{1cm} (18)$$

Where $w_{it}^R$ comes from (14) while $w_{it}$ comes from (3). This system can be solved for $x_{it}^R$, $x_{lt}^R$, and $\lambda_{lt}^R$ to obtain the solution to the case where one individual dynamically optimizes while all others continue to behave myopically. Plugging this solution into $\int_0^T \left( p \left( a_i w_{it} - \frac{1}{2} b_i w_{it}^2 \right) x_{it}^{\eta} - (c_{0i} - c_{1i} x_{it}) w_{it} \right) e^{-rt} dt$ for all $i$ allows for a calculation of the NPV for each well in this scenario, $NPV_i^R$. Comparing $NPV_i^R$ for the individual who acts dynamically to $NPV_i^m$ describes the incentive that exists for individuals to individually conserve water given that others continue to behave myopically. By varying $\eta$ numerically, we investigate how the individual returns to dynamic optimization and coordination depend on the sensitivity of crop yields saturated thickness and well capacity. We then provide an empirical estimate for $\eta$ to explore the existence of conservation incentives in practice.

**Model Calibration**
The hydro-economic model is calibrated to three wells in the Republican River Basin in Eastern Colorado such that $i=1,2,3$. In this context, we compare the value generated from groundwater under various management scenarios. The portion of the Republican Basin that lies in the State of Colorado covers roughly 8,000 square miles and is administered by eight separate GWMDs. To provide appropriate context for our analysis related to the spatial proximity of individual wells, we focus on three GWMDs in the northern portion of the Basin. These three GWMDs, illustrated in Figure 1, contain a total of 1,442 wells that are actively engaged in pumping groundwater for irrigation.

![Image](image-url)  
Figure 1: Irrigation wells in three GWMDs in Republican River Basin

In Figure 2, we provide a graph of the cumulative distribution of the minimum distance between individual wells in the region. This distribution informs our parameterization of the distance between wells. Specifically, we utilize a distance between wells of 700 meters (2,297 feet), given that approximately 80 percent of wells have at least one additional well within that distance and
700 meters is approximately the modal distance to the nearest well. It is also interesting to note that the distribution of the minimum distance between wells is very similar in each of the three districts.

![Cumulative distribution of distance to nearest well for each GWMD](image)

**Figure 2**: Cumulative distribution of distance to nearest well for each GWMD

The parameters used to solve the model for three wells are provided in Table 1. The parameters were chosen so as to be reflective of aquifer and producer characteristics in the study area as well as to facilitate model convergence and coherence to previous literature. In particular, the marginal cost of pumping, $C_1 = 0.09$, return coefficient, $\alpha = 0.2$, and discount rate, $r = 5\%$, are taken from Guilfoos et al. (2013). Production parameters are derived from an assumption that the choke price for water is $500 and that the marginal product of water on the farms is zero beyond
525 acre-feet\textsuperscript{2}. $C_0$ was chosen to produce an initial demand for water in the myopic model that falls in the range of observed average use of approximately 180 acre-feet. Hydrologic parameters were obtained based on regional averages. Recharge was chosen such that recharge at each well corresponds to the regional average where annual groundwater pumping is roughly twice as high the annual quantity of groundwater used. Finally, $T$ is assumed to be 100 years.

### Table 1: Model Base Parameters

<table>
<thead>
<tr>
<th>Economics</th>
<th>Units</th>
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<tr>
<td>p</td>
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<tr>
<td>$a_1$</td>
<td>2.56</td>
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<tr>
<td>$a_2$</td>
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<tr>
<td>$a_3$</td>
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<tr>
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<tr>
<td>$c_{1i}$</td>
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<tr>
<td>discount rate</td>
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<tr>
<td>$\eta$</td>
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<tr>
<td>$x_{01}$</td>
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</tr>
<tr>
<td>$k_i$</td>
</tr>
<tr>
<td>$A_{0i}$</td>
</tr>
<tr>
<td>$A_i*S_i$</td>
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<tr>
<td>Distance</td>
</tr>
<tr>
<td>Recharge</td>
</tr>
</tbody>
</table>

We focus on three wells in the region, assumed to be located along a linear array as depicted in Figure 1. Note that while pumping by well 2 affects water levels in both cells 1 and 3, there is no direct connection between water levels in cells 1 and 3. An indirect linkage forms as pumping in

\textsuperscript{2} Note that the extensive margin is not necessarily held fixed. With more water, a given well could plant more acres as well as provide more water to existing acres planted.
cell 1 draws water from cell 2 in the following period, impacting the flow of water between cells 2 and 3 in subsequent periods. It should also be noted that the initial saturated thickness for each well, \( X_1 \), \( X_3 \), and \( X_3 \), are assumed to differ to allow for heterogeneity in initial water availability. The model results, however, are robust to variation in the assumed initial saturated thickness at each well.

The model is solved numerically under various management scenarios using the parameterization described here.

**Theoretical Model Results**
In this section, we describe the results of the parameterized theoretical model as presented in Table 2. To address our primary research questions, we generate results for three separate optimization conditions across a range of parameterizations for the elasticity between saturated thickness and crop yield between 0.08 and 1.5. By comparing myopic behavior to optimal behavior under each parameterization, we observe how changes in the production elasticity impact the returns to coordinating groundwater use across wells in the Basin. By comparing myopic behavior to outcomes when only one well is characterized by privately optimal dynamic behavior, we observe both the private returns to dynamic management as well as the external benefits generated at other wells from dynamic management.

The results in Table 2 clearly show that increases in the elasticity between saturated thickness and production can have a dramatic impact on the returns to groundwater management. For example, at an elasticity value of 1.5, all three producers achieve profits through optimal management that are more than 50 percent above myopic returns. By comparison, for relatively low elasticity values, the returns to coordination across wells are less than two percent. At an elasticity value of 0.1 the gains to optimal management shrink to less than three percent.

A similar relationship between the yield-saturated thickness elasticity and returns to privately optimal management by an individual well owner are also reflected in Table 2. Importantly, with a low yield-saturated thickness elasticity, the incentive to act dynamically is small, reaching only 1% with $\eta \leq 0.1$. This dynamic behavior by one individual does generate external benefits to neighboring wells that are more than 50 percent of the gain to the individual engaging in the privately optimal behavior. These benefits increase as $\eta$ increases but at low values, gains from groundwater management even of only two percent require coordination across multiple wells.
Table 2. Gains to Dynamic Decision-Making and Coordination

<table>
<thead>
<tr>
<th>Yield-Saturated Thickness Elasticity</th>
<th>PV Profit Myopic</th>
<th>Percent Change in NPV</th>
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<tr>
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<tr>
<td>0.08</td>
<td>$478,265</td>
<td>$476,716</td>
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<td>0.1</td>
<td>$472,833</td>
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<td>$424,969</td>
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<td>0.5</td>
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<tr>
<td>0.7</td>
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<tr>
<td>0.9</td>
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<tr>
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<tr>
<td>1.5</td>
<td>$298,995</td>
<td>$283,509</td>
</tr>
</tbody>
</table>

In practice, little data exist describing the empirical relationship between saturated thickness, well capacity, and yields. Therefore, we turn to a Basin-wide model of crop and irrigation decisions in Eastern Colorado.

**Empirical Estimates of Production Response to Saturated Thickness**

To understand the importance of the production elasticity, we use the results of a well-level model of planting, irrigation, and production in the Republican River Basin of Eastern Colorado. The model assumes that each of the 3,557 wells in the Basin is operated independently and that farmers choose the number of acres to plant in irrigated and dryland corn and wheat. Prior to planting, each farmer knows his well capacity and makes a planting decision to maximize expected profits. Based on a weather realization, irrigation decisions are made, producing irrigated yields, revenue and profit. We simulate the realization of an average weather year across the Basin to produce estimated revenue, $y_i$ and water use, $w_i$ at each well. To aggregate across multiple crops, we sum the dollar value. This allows the observed revenue produced from water to adjust to both intensive and extensive decisions that affect the use and value of water.
We also obtain data from the US Geological Survey (USGS) on the saturated thickness ($x_i$) and hydraulic conductivity ($k_i$) of the aquifer at each of the 3,557 wells. Finally, we obtain soil composition\(^3\) around each well from the USGS and use the majority soil type at the well. Let $s_i$ be a vector of dummy variables indicating the soil type at each well. Given this setup, we estimate equation 19 using OLS.\(^4\)

$$y_i = \alpha + \eta x_i + \beta_1 w_i + \beta_2 k_i + \beta_3 s_i + \epsilon_i$$  \hspace{1cm} (19)

Variables are as defined above but the revenue, water, and saturated thickness variables in equation 19 are expressed in logs. Therefore, the econometric estimate of $\eta$ represents the estimated yield-saturated thickness elasticity in the Basin. This estimate, combined with model results, provides an illustration of the potential gains to coordination and dynamic behavior for wells in the region.

**Econometric Results**

The coefficient estimates for equation 19 are presented in Table 3. Several variations of the model are presented to demonstrate the robustness of the estimate. Column 1 shows the coefficient estimates when soil quality is not controlled for. Because soil quality likely correlates with water used and its productivity, we include soil dummies in Columns 2-4. In the coefficients provided in columns 2-3, the estimated yield-saturated thickness elasticity is approximately 0.1. In column 4, we include an interaction term between water applied and saturated thickness because it is most likely that higher saturated thickness leads to higher well capacity, which enhances the ability of a farmer to time the delivery of water, raising its marginal

\(^3\) Categories include Clay Loam, Fine Sand, Find Sandy Loam, Gravelly, Loam, Gravelly Sandy Loam, Loam, Loamy Fine Sand, Loamy Sand, Sand, Silt Loam, Silty Clay, Silty Clay Loam, and Very Fine Sandy Loam.

\(^4\) Note that this econometric equation implies a production function that differs from that used in the theoretical section. Current modeling work is exploring sensitivity to production function specification for the dynamic results presented here.
productivity. When including the interaction term, the average marginal effect of the natural log of saturated thickness is 0.09 and does not differ significantly from 0.1. Therefore, it appears that the appropriate value for \( \eta \) in the theoretical model is approximately 0.1. In words, this implies that a ten percent decrease saturated thickness leads to a one percent decrease in annual revenue, due to lower well capacity.

### Table 3. Modeled Relationship between Saturated Thickness and Farm Revenue

<table>
<thead>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<td>LHS Variable is Farm Revenue</td>
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<td>OLS</td>
<td>OLS</td>
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</tr>
<tr>
<td>Natural Log of Saturated Thickness</td>
<td>0.0593*** (0.00703)</td>
<td>0.0971*** (0.00770)</td>
<td>0.0967*** (0.00768)</td>
<td>0.515*** (0.0633)</td>
</tr>
<tr>
<td>Natural Log of Water Applied</td>
<td>0.444*** (0.00790)</td>
<td>0.454*** (0.00795)</td>
<td>0.451*** (0.00797)</td>
<td>0.878*** (0.0646)</td>
</tr>
<tr>
<td>LN Saturated Thickness*LN Water</td>
<td>-0.0835*** (0.0125)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydraulic Conductivity</td>
<td>-0.00027*** (6.82e-05)</td>
<td>-0.00027*** (6.78e-05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>9.010*** (0.0471)</td>
<td>8.847*** (0.109)</td>
<td>8.935*** (0.111)</td>
<td>6.791*** (0.340)</td>
</tr>
<tr>
<td>FE</td>
<td>None</td>
<td>Soil Type</td>
<td>Soil Type</td>
<td>Soil Type</td>
</tr>
<tr>
<td>Observations</td>
<td>3,557</td>
<td>3,557</td>
<td>3,557</td>
<td>3,557</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.507</td>
<td>0.537</td>
<td>0.540</td>
<td>0.545</td>
</tr>
</tbody>
</table>

Note: Saturated thickness measured in feet. Water applied is in acre-feet. Higher saturated thickness increases well capacity and the ability to deliver water. Average marginal effect of saturated thickness in (4) is 0.087 (0.0078).

Model Results using Parameterized Elasticity

Given the empirical estimate of 0.10 for the saturated thickness-production elasticity, we now explore the difference in water use paths and value across management institutions in the theoretical model. Figure 4 shows the quantity of water used under the three management scenarios described in the theoretical model. At all three wells, coordination results in much less
water used across the planning horizon of 100 years. It is not until 80 years in the future that farms begin to use more water under the socially optimal trajectory than under myopic behavior.

Figure 4: Comparison of Water Use at three Wells under Alternative Groundwater Use Scenarios

Although less water is used in early periods under coordination, higher levels of saturated thickness mean that profits do not fall by as much over time. Figure 5 demonstrates that profits with management exceed myopic profits after less than 20 years. While Well 2 acting alone has very little effect on water use by Wells 1 and 3, Well 2 does generate a benefit that spills over into Wells 1 and 3. This benefit does not become large until several decades have passed. Nevertheless, this illustrates the spillover effect that was seen under higher elasticities in Table 2.
Discussion and Conclusion

The theoretical and empirical results presented here show a tight connection between the yield-saturated thickness elasticity and the gains to dynamic and coordinated groundwater use across time and space. Specifically, as this elasticity increases, the external and private future costs associated with pumping at a given well also grow. As a result, the gains to individual dynamic decision-making as well as the gains to coordination increase with $\eta$.

In practice, it appears that $\eta$ is quite inelastic, with a point estimate equal to approximately 0.1. Given this empirical estimate, there exists very little gain (approximately one percent) for an individual to unilaterally switch from myopic to dynamic decision-making. There does exist a gain from coordination, though these gains are just two percent at each well. Therefore, if there exists a policy goal to conserve water, this will likely require coordinated action across space. Stated another way, the long-run financial returns to groundwater
conservation policies are likely to be relatively small, especially if they are not widely undertaken.

The results presented here suggest that wells located at the border of management jurisdictions may lose from coordination efforts within a district. If the wells are close to non-regulated wells in a different district, the gains to management could be largely dissipated. Therefore, efforts to conserve groundwater by coordinating decisions across many farms should focus on clusters of wells that are hydrologically connected. Failure to do so will limit the gains for wells near management borders.

The model presented here abstracts away from some important features of groundwater use in practice. First, it focuses only on three wells and assumes that the connected portion of the aquifer covers just the three cells in which the wells are located. In practice, many wells could be connected hydrologically in complex ways that increase or decrease the gains from conservation. If the aquifer is large, the Gisser Sanchez (1980) effect is likely to hold as the marginal external cost of pumping at one well is spread more widely across space.

In our modelling efforts we have also held the distance between wells constant. As the distance between wells increases, however, the external effects of pumping decrease, creating greater incentives for an individual to dynamically optimize but reducing the gains from coordination across wells. We also have not accounted for the possibility that well owners typically operate few wells and occasionally operate many nearby wells. In these cases our model clearly shows the incentives that an individual well owner has to consider the dynamic linkages associated with groundwater use. We leave it to future work to evaluate specific areas
where well spacing is such that dynamic behavior may be particularly attractive for an individual well owner and how ownership of multiple wells varies across the study area.

Another promising area for future research concerns the separate identification of the pumping cost and production externalities described in the introduction. Our modelling efforts allow us to identify how profits change as a function of the elasticity between saturated thickness and production. This outcome however reflects the combined impact of productivity changes and changes in pumping costs. We also have not made an effort to quantify the marginal external cost as a function of aquifer conditions. Future work comparing the relative magnitudes of these separate external costs would represent a valuable contribution to the literature.

Despite its simplifications, the model presented here highlights the relationship between production externalities and the gains to management in groundwater systems. Our empirical results suggest that the response of revenue to saturated thickness is quite inelastic in practice, though future research should investigate if this result holds in other arid regions of the world. For example, in Eastern Colorado the ability to produce dryland crops mitigates some of the negative financial impact of low well capacity. In drier regions of the world, this may not be the case, leading to a more elastic relationship. Finally, the low elasticity estimate found here suggests minimal gains to efforts to unilaterally conserve groundwater, suggesting that meeting societal objectives to conserve water requires coordination across hydrologically connected groundwater users.

References


