Estimating Elasticities of Substitution Using Data Envelopment Analysis

Noah J Miller¹, Jason S. Bergtold², and Allen M Featherstone³

¹PhD Candidate and Graduate Research Assistant, Department of Agricultural Economics, Kansas State University, Manhattan, KS 66506, kpokharel@ksu.edu

²Associate Professor, Department of Agricultural Economics, Kansas State University, Manhattan, KS 66506, bergtold@ksu.edu

³Professor and Head, Department of Agricultural Economics, Kansas State University, Manhattan, KS 66506, afeather@ksu.edu


Copyright 2016 by Miller, Bergtold, and Featherstone. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided that this copyright notice appears on all such copies.
BACKGROUND

- Knowledge of the level of substitutability of inputs is crucial for farm management decision-making, but this knowledge is often limited and/or incomplete.
- The elasticity of substitution measures the substitutability of inputs - it shows the effect on outputs from a change in the marginal rate of technical substitution (production perspective) or a change in input prices (cost perspective), with output held constant.

TECHNICAL EFFICIENCY MODEL

- Variables (weights) correspond to the first and second constraints, with weight corresponding to \( w_i \) a free variable, the shadow price on the third constraint.
- The objective of the cost efficiency model is to choose \( a \) and \( b \) that minimizes the DMU's cost, relative to all other DMUs in the sample, where \( w_0 \) is the DMU's input cost and \( a \) is the cost-minimizing level of inputs.
- Cost efficient DMUs are those that are technically efficient, and also exhibit allocative efficiency. Cost efficiency is defined as the ratio of the cost-minimizing level of input used by the DMU to the observed input cost (\( a/w_0 \)).
- The constraints force composite inputs to be less than or equal to the technologically efficient output level, allow for variable returns to scale, and ensure non-negativity of weights.

OBJECTIVES

- Determine (Hicksian) elasticities of substitution for inefficient firms from the technical efficiency and cost efficiency DEA models assuming variable returns to scale.
- Illustrate the use of elasticities of substitution with an empirical example using enterprise data from the Kansas Farm Management Association (KFMA).

HICKSIAN TECH. EFFICIENCY ELASTICITIES

- The Hicksian elasticity of substitution for the production function shows the rate of change of the ratio of inputs divided by the rate of change of the marginal rate of technical substitution (\( e_H = \frac{\partial \ln \theta}{\partial \ln \psi} \)).
- Based on this measurement, the Hicksian elasticity for technical efficiency, for inefficient DMUs, can be derived directly from the Lagrangian of the BCC minimization problem:

\[
\frac{\partial \ln \theta}{\partial \ln \psi} = \left( \frac{\partial \ln \theta}{\partial \ln \psi} \right)_{\text{efficient}} - \left( \frac{\partial \ln \theta}{\partial \ln \psi} \right)_{\text{inefficient}}
\]

- This elasticity shows the degree of input substitutability that an inefficient DMU can make (at optimality) and remain on the technical efficient frontier.
- This measurement is only valid for inefficient DMUs. For efficient firms, that exist on the cost efficient frontier, continuous derivatives cannot be derived. Thus, different methods must be employed, but that was beyond the scope of the present research.

HICKSIAN COST EFFICIENCY ELASTICITIES

- The Hicksian elasticity of substitution for the cost minimization function is analogous to the technical efficiency function - it shows the rate of change of the ratio of input prices to the marginal rate of technical substitution.
- The elasticities derived from the production function can be used to calculate Hicksian production and cost elasticities as well as Morishima cost elasticities. The results of this estimation were used to compute Hicksian production and cost elasticities as well as Morishima cost elasticities in KFMA for each farm following the derivations shown earlier.

HICKSIAN TECH. EFFICIENCY ELASTICITIES

- The Hicksian elasticity of cost efficiency for inefficient DMUs, can be derived directly from the Lagrangian of the cost efficiency problem:

\[
\frac{\partial \ln \theta}{\partial \ln \psi} = \left( \frac{\partial \ln \theta}{\partial \ln \psi} \right)_{\text{efficient}} - \left( \frac{\partial \ln \theta}{\partial \ln \psi} \right)_{\text{inefficient}}
\]

- This elasticity shows the degree of input substitutability that an inefficient DMU can make (at optimality) and remain on the cost efficient frontier.
- This measurement is only valid for inefficient DMUs. For efficient firms, that exist on the cost efficient frontier, continuous derivatives cannot be derived. Thus, different methods must be employed, but that was beyond the scope of the present research.

HICKSIAN COST EFFICIENCY ELASTICITIES

- The Hicksian elasticity of cost efficiency for inefficient DMUs, can be derived directly from the Lagrangian of the cost efficiency problem:

\[
\frac{\partial \ln \theta}{\partial \ln \psi} = \left( \frac{\partial \ln \theta}{\partial \ln \psi} \right)_{\text{efficient}} - \left( \frac{\partial \ln \theta}{\partial \ln \psi} \right)_{\text{inefficient}}
\]

- This elasticity shows the degree of input substitutability that an inefficient DMU can make (at optimality) and remain on the cost efficient frontier.
- This measurement is only valid for inefficient DMUs. For efficient firms, that exist on the cost efficient frontier, continuous derivatives cannot be derived. Thus, different methods must be employed, but that was beyond the scope of the present research.

EMPIRICAL APPLICATION: DATA

- For an empirical illustration of the elasticities derived, the elasticity of dryland corn production under reduced tillage was examined for 119 corn-planting farms in Kansas in 2016.
- Enterprise-level input and output data was collected from the KFMA, an organization affiliated with Kansas State University that provides financial data and planning for farmers (KFMA 2015).
- Input data included total expenses for fuel, fertilizers, herbicides, seed, labor (both hired and unpaid labor, machinery (including machinery rental and repair) and land (in total costs). Output data was measured as total value of dryland crop production.
- Input variables were measured with a quantity index, with total input expenses divided by the lagrangian level input and output data was collected from the KFMA, an organization affiliated with Kansas State University that provides financial data and planning for farmers (KFMA 2015).
- The output variable was not transformed, since corn price was assumed to be the same for all farms in the study.

EMPIRICAL APPLICATION: RESULTS

- The technical and cost efficiency DEA models were estimated for each farm using the General Algebraic Modeling System (GAMS). The results of this estimation were used to compute Hicksian production and cost elasticities as well as Morishima cost elasticities in KFMA for each farm following the derivations shown earlier.
- The results of the estimation of Hicksian and Morishima elasticities show that, on average and for this set of farms, only a slight degree of substitutability (for complementarity) exists between inputs, with the result that changes in an input's relative marginal productivity or price does not substantially alter the proportion of inputs applied.
- Though the sample means of the elasticities indicate limited substitutability of inputs, individual farms exhibit a diversity of responses. Similarly, elasticity results differ across different estimation models.

CONCLUSION

- This study developed a procedure by which Hicksian production and cost elasticities and Morishima cost elasticities for inefficient DMUs can be derived using technical and cost efficiency DEA frameworks.
- An empirical example using corn enterprise data for Kansas farms under reduced tillage served as an illustration of these elasticities. The results of the empirical example indicate limited substitutability across inputs.
- Future research should focus on estimation of Hicksian and Morishima elasticities for efficient DMUs (i.e. DMUs that reside on the technical or cost frontier).

REFERENCES