Estimating Import Demand Functions in Major Beef Importing Countries by
Bayesian Hierarchical Linear Model

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Abstract

Trade elasticities are critical for price or structural change analysis. The Armington demand model is used to estimate trade elasticities, the country-of-origin bias, and the impact of the U.S. BSE outbreak on preferences. OLS estimation generates theoretically wrong signs, including a positive impact of U.S. BSE on other countries’ demand for imported beef. Results of a Bayesian hierarchical model show that import demand of countries that tend to import more beef from the U.S., such as Japan, Canada, Korea, and Russia, are affected more by the U.S. BSE outbreak than countries, such as the EU that tend not to import from the U.S.

Key words: Armington demand model, Bayesian hierarchical model, BSE, Country-of-origin bias.

JEL codes: F13, F14

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As the flow of trade increases, decision makers in more and more countries have witnessed how readily domestic producers and consumers to whom they are accountable are affected by the prices or structural changes in other countries. These effects are set to grow given the presence of not only the World Trade Agreement (WTO), but also new bilateral and multilateral trade agreements such as the Trans-Pacific Partnership (TPP), the Transatlantic Trade and Investment Partnership (T-TIP), and the Regional Comprehensive Economic Partnership (RCEP). To analyze trade impacts, various studies have estimated and developed trade models that we can obtain export and import elasticities. Trade elasticities are necessary for analyzing trade impacts of shocks in other markets, whether due to policy or market events or other causes. Therefore, estimating trade elasticity is important for analyzing trade policy.

There are many alternative methods to estimate trade elasticities, but there are drawbacks to each study. Reimer et al. (2012) estimate U.S. export demand elasticities for selected U.S. crops, showing short- and long-run elasticities and comparing these results with previous studies. The authors admit that there are some drawbacks, such as short data series and measurement errors. In addition, we are not aware of any cross-country meta-analysis of meat export demands or import supplies, although there has been some effort to compare domestic meat demand elasticities (e.g. Gallet 2010). Imbs (2010) argues that some estimated elasticities of imports from different countries with respect to the prices are not significantly different from zero probably because of econometric problems and lack of data information, and consequently judges that such estimates are often imprecise. Estimation methods can generate theoretically incorrect signs and unreasonable magnitudes of estimates. Using estimates that are imprecise and
potentially even invalid not only miscalculate values of imports and exports, but also
could mislead trade negotiators or other final users of trade economics. As an alternative
to direct estimation, analysts can calibrate parameters to values based on economic theory,
but imposing even an educated guess on parameters might be too subjective.

Bayesian methods can improve shortcomings from the uncertainty about whether
the magnitudes and the signs are met (Greene 2012). Hanrahan et al. (2001) estimate UK
beef import demand model using a Bayesian approach with theoretical restrictions
imposed on parameters. Kabe and Kanazawa (2014) estimate a Markov-Switching
Almost Ideal Demand System (MS-AIDS) over Japan beef market data using a Bayesian
approach. The authors conclude that imposing priors on the MS-AIDS model with a
Bayesian approach leads to better estimation than a maximum likelihood estimation
focusing on the mean square error (MSE). Montgomery (2002) uses economic theory for
priors in order to shrink estimates toward the theory when estimates are not met to the
theory. Griffiths (1998) shows the restriction of wrong signs and unexplainable
magnitudes by using the Bayesian methods. The author then illustrates a comparison of
the least square estimation resulting in unacceptable estimates to verify that Bayesian
estimates are better than OLS estimates. These articles conclude that Bayesian method
can improve estimation by addressing the uncertainty about the magnitudes and the signs.

The hierarchical model has been used to estimate parameters and to make
inference by using partially pooled estimations. With time series cross-country data,
Gelman (2006) shows that we can estimate over pooled and separated data. In each case,
he identifies disadvantages: if we have high price elasticity from estimation over pooled
data, then it is inappropriate to compare countries that have low price elasticities; and on
the other hand, if we estimate price elasticities over separated data, then there could be unreliable elasticities in some countries. The author, therefore, shows that a hierarchical model, in which parameters in each country are drawn from a population of parameters, can avoid problems associated with strictly time series approaches with pooled and separated data. The Bayesian hierarchical model has been introduced to estimate elasticities in marketing. Montgomery and Rossi (1999) estimate price elasticities for multiple brands and stores. The authors utilize the Bayesian hierarchical model imposing prior information based on the restrictions by additive utility models. The article demonstrates that the Bayesian hierarchical model based on economic theory provides more plausible and reliable estimates, compared to other econometric methods.

The purpose of this article is to employ the Bayesian hierarchical model to estimate Armington elasticities, country-of-origin bias, and the impact of the U.S. Bovine Spongiform Encephalopathy (BSE) outbreak for major beef importing countries, namely the Mexico, Japan, Canada, Korea, Russia, and EU. We build a hierarchical model and impose priors based on economic principles into the model. An Armington demand model is used. Relative prices are expected to play a critical role, reflecting the underlying arbitrage decisions of exporters and importers. By estimating import demand functions using the Bayesian hierarchical model, we can calculate changes of import quantities due to the effect of changes in relative prices in each country and assess the U.S. BSE outbreak in the U.S. Through the Armington demand model, we estimate the country-of-origin bias for beef. We also use a dummy variable indicating the BSE outbreak to assess structural changes. Our findings show the right sign on elasticities and permit us to generate more precise estimates of these key parameters, taking into account
both statistic and economic theory, and the magnitude of the U.S. BSE impacts, informing industry and policy makers about the benefits of efforts to prevent any future shock or this type.

The remainder of the article is set out as follows. The next section introduces the Armington demand model. The Bayesian inference is present in the third section. In the fourth section, the Bayesian hierarchical model is applied to estimate the Armington demand model in major beef importing countries. The last section concludes with reviewing the results of Bayesian estimation with a comparison to another estimation method.

Model Framework

The Armington demand model is applied to estimate Armington elasticities, meaning in general the degree of substitution between domestic and imported goods, and country-of-origin bias. Kawashima and Sari (2010) derive the Armington model under the assumption that budget is allocated in stages. In the first stage, total consumption is separated into two groups, namely domestic and imported good. These two groups are expected to be substitutes. In the second stage, the expenditures on imported good are divided by country-of-origin. Here, the import demand function is derived from the first stage in the Armington model. The import demand model is typically derived from the utility function:

\[ U = \left[ \delta_B \cdot D^{\frac{\sigma-1}{\sigma}} + (1 - \delta_B) \cdot M^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \text{ subject to } P_D \cdot D + P_M \cdot M = E, \]
where $U$ is the utility from the consumption of domestically produced good, $D$, and imported good, $M$. $E$ represents the budget that is allocated to purchasing domestic and imported good. $\sigma$ is the elasticity of substitution between domestically produced and imported goods. $\delta_B$ represents the preference for domestically produced good affected by the outbreak, as discussed below. To optimize the utility function subject to the budget constraint, the consumption ratio of domestically produced good to imported good is

$$
\frac{D}{M} = \left( \frac{\delta_B}{1-\delta_B} \right)^\sigma \left( \frac{p_M}{p_D} \right)^\sigma,
$$

where $p_M$ and $p_D$ are the prices of imported and domestically produced goods. The consumption ratio of the domestically produced good to imported good is determined by the ratio of the import price to domestic price. Through taking the natural logarithm on both sides, this equation becomes

$$
\log \left( \frac{D}{M} \right) = \sigma \cdot \log \left( \frac{\delta_B}{1-\delta_B} \right) + \sigma \cdot \log \left( \frac{p_M}{p_D} \right).
$$

The preference parameter $\delta_B$ is equal to $\delta_0 + B$, where $\delta_0$ is the preference for domestically produced good without the outbreak and $B$ is the impact of the outbreak on this preference. In the estimated equation, we cannot directly estimate the preference parameter or a change in the preference for domestically produced good. However, parameters of the estimated equation (3) can be linked to the fundamental parameters. The same is true of the shift in preferences associated with BSE, which is represented initially with a dummy variable that takes a value of zero before the outbreak and one during the outbreak. When the dummy variable is equal to zero, then the estimated functional form is
(4) \[ y_{t,i} = \beta_{0,i} + \beta_{1,i}p_{t,i} + \varepsilon_{t,i}, \]

where (4.1) \[ y_{t,i} = \log\left(\frac{D}{M}\right), \]
(4.2) \[ \beta_{0,i} = \sigma \cdot \log\left(\frac{\delta_o}{1-\delta_o}\right), \]
(4.3) \[ \beta_{1,i} = \sigma, \]
and (4.4) \[ p_{t,i} = \log\left(\frac{P_M}{P_D}\right). \]

In this notation, \( i \) is for major importing countries because import demands are estimated over cross-country trade data. \[ \frac{\beta_{0,i}}{\beta_{1,i}} = \log\left(\frac{\delta_o}{1-\delta_o}\right) \] from equation (4.2) and (4.3) shows the degree of the country-of-origin bias for domestically produced good over imported good in each country which the outbreak does not affect the preference. If there is no preference bias between domestic and imported good, \( \delta_o \) should be 0.5. Then, the ratio of \( \frac{\beta_{0,i}}{\beta_{1,i}} \) should be zero. If the ratio of \( \frac{\beta_{0,i}}{\beta_{1,i}} \) is greater than zero, then this result implies that there exists the country-of-origin bias. In other words, domestically produced good is preferred to imported good in case of \( \delta_o > 0.5 \) (Blonigen and Wilson 1999; Kawashima and Sari 2002).

To take account of the outbreak shock, a dummy variable is included that takes a value of one during the outbreak. The estimated functional form to derive the change in the preference is

(5) \[ y_{t,i} = \beta_{0,i} + \beta_{1,i}p_{t,i} + \beta_{2,i}D_t + \varepsilon_{t,i}, \]

where (5.1) \[ \beta_{0,i} + \beta_{2,i} = \sigma \cdot \log\left(\frac{\delta_B}{1-\delta_B}\right), \]
and \( D_t \) is the dummy variable that represents the occurrences of the outbreak. The degree of the country-of-origin bias for the domestically produced good over the imported good in each country is derived from \[ \frac{\beta_{0,i} + \beta_{2,i}}{\beta_{1,i}} = \]
log \left( \frac{\delta_B}{1 - \delta_B} \right). Estimated preference for domestically produced good in with presence of BSE as one result of equation (5) is \( \delta_B \).

The impact of the outbreak on the preference (B) can be derived when we subtract \( \delta_o \), which is the preference without the impact of the outbreak, from \( \delta_B \), which is the preference with the impact of the outbreak. The positive impact of the outbreak to the preference in each country represents that consumers might be more sensitive to the country where good is produced.

**Bayesian Inference**

Bayesian analysis has become a useful method to make precise inference if there are weaknesses from estimations of other methods (e.g. wrong sign or unreasonable magnitudes). While frequentists use data set to draw inferences about the population parameters given a random sample (data) distribution, Bayesian estimation utilizes data set with priors so that a posterior distribution for the inference can be generated. By Bayes’ theorem, the posterior distribution is proportional to conditional and marginal probability distributions (Gelman et al. 2014):

\[
(6) \quad p(\theta | y) = \frac{p(y|\theta) p(\theta)}{p(y)},
\]

where \( p(y) \) is the marginal density of the data over the sum of all possible values of \( \theta \) \( (p(y) = \sum_\theta p(\theta)p(y|\theta)) \). As we can consider that \( p(y) \) is constant, the unnormalized posterior density is

\[
(7) \quad p(\theta | y) \propto p(y|\theta) p(\theta).
\]
where \( p(y|\theta) \) is a likelihood function (or the sampling distribution) and \( p(\theta) \) is the prior distribution. So, the posterior distribution consists of those two distributions. The likelihood function deals with all information that is related with inferences. The prior distribution is assumed, based on beliefs and knowledge, including economic theories.

Greene (2012) mentions that Bayesian methodology is believed to address the uncertainty in terms of wrong signs and unreasonable magnitudes by imposing the prior. The author maintains that if the models and parameters do not represent truths to explain the unknown population, the subjectivity of Bayesian approach can be an alternative method to make more precise inferences.

**Empirical Application**

Mexico, Japan, Canada, Korea, Russia, EU are major beef importing countries. These six countries are used to estimate Armington elasticities, country-of-origin bias, and the impacts of the BSE outbreak on the preference.

**Data**

Table 1 describes major variables as to the consumption ratio of domestically produced beef to imported beef and the ratio of the import price to the domestic price. Consumption of domestically produced beef and imported beef are drawn from USDA-PSD. Domestic producer price is from OECD-MPS. Import price is calculated by multiplying the world beef price, exchange rate and 1 plus the ad valorem tariff rate. The
world beef price is the average of boxed beef cutout in U.S. and Australian and New Zealand 85% lean fores, CIF U.S. import price (IMF Primary Commodity Prices).

Each variable should be carefully interpreted. In case of the consumption ratio of domestically produced beef to imported beef, a low ratio demonstrates that the import share of total consumption is large. Mexico, Canada, Russia, and EU have high ratios, so that import share is relatively smaller. In contrast, Japan and Korea have low ratios among seven countries, so that Japan and Korea are more likely to depend on imported beef.

Turning to the ratio of the import price to the domestic price, a ratio close to one, as seen for Canada, suggests that these two pries are similar. The price ratios of Mexico, Russia, and EU are greater than one, which means that imported beef is more expensive than domestically produced beef. In contrast, the price ratios of Japan and Korea are well below one, showing that imported beef is relatively cheaper than domestically produced beef.

The BSE outbreak dummy variable is used in this Armington demand model. The BSE outbreak is based on an announcement of first BSE case in the U.S. and is equal to one after the announcement until the impact of U.S. BSE is sharply reduced. The BSE outbreak dummy variable is used to find differences of the preference for domestically produced beef in the period from 2003 to 2006. In particular, most importing countries banned U.S. beef import in 2003 and U.S. import beef recovered after 2006. Hence, the U.S. BSE outbreak dummy is defined the period from 2003 to 2006 (Taha and Hahn 2014). All data are annual from 1992 to 2014.
The Pooled and Separated Estimations

As an initial experiment, equation 5 is estimated over the pooled data representing the beef imports of the six countries. Estimates show the elasticity of substitution, country-of-origin bias, and the impact of the U.S. BSE outbreak on the preference assuming that this impact is the same in all importing countries. The elasticity of substitution is 0.49 when the BSE outbreak took place in 2003 and kept a fear of BSE to 2006. If the BSE dummy is excluded, then the country-of-origin bias is 0.46, so the results of the pooled data for all seven countries suggest that consumers prefer domestically produced beef to imported beef. If we impose the BSE outbreak dummy variable, then the country-of-origin bias is 0.79 which means that domestically produced beef is even more preferred to imported beef during the BSE outbreak.

These results are plausible because the sign of the substitution parameter and the impact of U.S. BSE outbreak appear correct in that the domestic bias increase in the presence of a BSE outbreak. However, the drawback of the pooled estimation is the underlying assumption that it is correct to apply these results to all of countries. For instance, each country has different preferences and the impact of the BSE outbreak can affect consumers in each country differently. Consumers in the six countries might have different responses to changes in the relative price, but the pooled estimates cannot differentiate impacts of each country. Therefore, using the results from the pooled estimation can be inappropriate to analyze trade impacts.

The Ordinary Least Square (OLS) estimation is used to estimate equation 5 over separated data (table 3). The results of the separated estimation are more complicated
than the results of pooled estimation. Because the elasticities of substitution and the impacts of the BSE outbreak for each country show different signs and magnitudes, some of which are inconsistent with theoretical expectations, interpretation of these results should be cautious. For example, Japan has negative sign on the elasticity of substitution, so that the consumption ratio of domestically produced beef to imported beef is decreased by either increasing import price or decreasing domestic price.

The OLS results of the separately estimated equations as regards the impact of the BSE outbreak are also contrary to expectations. In the absence of BSE, these estimates suggest that Mexico and Japan prefer imported beef to domestically produced beef, while the other countries prefer domestically produced beef. The U.S. BSE outbreak dummy leads Japan prefer to domestically produced beef, according to these estimates. In contrast, these OLS results suggest that South Korea and Russia prefer to more imported beef. From these estimation results, the country-of-origin bias is calculated (table 4).

The comparisons of the impacts of the BSE outbreak to the preference for domestically produced beef from the U.S. BSE outbreak are illustrated in table 5. While the preferences for domestically produced beef in Mexico, Japan, and Canada increase because of the fear about the BSE outbreak from 2003 to 2006, the preferences for domestically produced beef in the other countries decrease as if they do not care about the U.S. BSE outbreak. In fact, many countries banned imported beef from U.S. when the first case of BSE took place in 2003. As a consequence, although policy changes are not consumer preferences, the result of greater preference for imported beef seems suspicious.
The results of estimation of each country’s import-versus-domestic demand equation separately necessarily generate different results for each country. In principle, this method can find the trade characteristics of each country. However, based on economic perspectives, the signs and magnitudes of the elasticities of substitution and the impact of the BSE outbreak might be wrong if estimated using this econometric approach. For instance, the U.S. BSE outbreak dummy leads Japan prefer to domestically produced beef, according to these estimates. In contrast, these OLS results suggest that Korea and Russia prefer to more imported beef. Therefore, using the results presented thus far should be cautious or even discouraged if the goal is to analyze trade.

**Bayesian Estimation**

Bayesian approach is a method to address drawbacks of both pooled and separated estimation. Here, the Bayesian hierarchical model, which can be applied to partially pooled estimation among countries, is used (Gelman 2006; Montgomery and Rossi 1999). Each parameter, which in this case is the intercept, the elasticity of substitution, and the impact of the BSE outbreak, are in fact borrowed from parameters in the complete pooled estimation. For posterior distribution, priors and the likelihood function should be specified.
**Priors**

Figure 1 illustrates the directed acyclic graph structuring the priors. This structure expands the hierarchy so that each country’s parameters can be modeled as drawing from the distribution of the mean for six countries.

The following prior distributions are used for the parameters:

(8) \[ y_{t,i} \sim N(\beta_{0,i} + \beta_{1,i} p_{t,i} + \beta_{2,i} D_{BSE,t}, \sigma^2) \],

(9) \[ \beta_{0,i} \sim N(\beta_{0}, \tau_{0}^2), \beta_{1,i} \sim N(\beta_{1}, \tau_{1}^2), \text{and } \beta_{2,i} \sim Unif(0,1), \]

(10) \[ \beta_0 \sim N(\bar{\beta}_0, \bar{\sigma}_0^2), \beta_1 \sim N(\bar{\beta}_1, \bar{\sigma}_1^2), \]

(11) \[ \tau_0^2 \sim IG(a_0, b_0) \text{ and } \tau_1^2 \sim IG(a_1, b_1). \]

Parameters for each country, \( \beta_{0,i} \) and \( \beta_{1,i} \) where \( i \) represents the importing country, follow a normal distribution (9). \( \beta_{2,i} \) follows an uniform distribution between 0 and 1, partly in order to truncate from below zero, disallowing values below zero. The uniform distribution is assumed that the impact of U.S. BSE outbreak on the consumption ratio of domestically produced beef to imported beef cannot exceed 100% and be negative. We assume that the variance of \( \sigma^2 \) is unknown so that \( \sigma^2 \) follows a non-informative prior, namely the Jeffrey prior, \( p(\sigma^2) \propto 1/\sigma^2 \). Thus, in this experiment, \( \sigma^2 = 1000 \). We also assume that the hierarchical means, \( \beta_0 \) and \( \beta_1 \), follow the normal distribution (10) and the variances, \( \tau_0^2 \) and \( \tau_1^2 \), follow the inverse gamma distribution (11). \( \beta_0 \) and \( \beta_1 \) have zero for \( \bar{\beta}_0 \) and \( \bar{\beta}_1 \) as indicating the mean of hyper-prior. The precisions, \( \bar{\sigma}_0^2 \) and \( \bar{\sigma}_1^2 \), are equal to 0.001. Hence, \( \beta_0 \) and \( \beta_1 \) are a flat prior.
Corresponding to the super-population parameter distribution, deciding the priors for $\tau_k^2$ is not straightforward because the standard deviations are changed by different priors. The mean and variance of the Bayesian estimates could be sensitive to the mode of the standard deviations from the inverse gamma distributions, $E(\tau_k^2 | \cdot) = \frac{b_k}{a_k - 1}$ for $k = 0$ and 1. Increasing the variances of the prior distribution is a way to check if Bayesian estimates are correct. The Deviance Information Criterion (DIC) can be used to help determine which model is most appropriate (table 2). We rearrange $b_0$ and $b_1$ in distribution (11) in order to relax variances for $\beta_{0,i}$ and $\beta_{1,i}$ with fixed $a_k = 3$. Through these steps, we find appropriate priors, as $b_k = 1$ for $k = 0$ and 1, by selecting the model that has the smallest DIC.

_The Likelihood Function_

The other components of information needed to obtain the posterior distributions are the likelihood function. Information about the data set is summarized in the likelihood function. For classical approach, parameters can be estimated using the likelihood function. However, the Bayesian approach uses the likelihood function to incorporate other information, namely the priors. The model is linear, so that we can easily derive the likelihood function. Following the distribution given earlier (8), the likelihood function is

$$L(y_{t,i} | \beta_{k,i}, \beta_k, \sigma^2, \tau_k^2) = \prod_{t=1}^{T_J} f(y_{t,i} | \beta_{k,i}, \beta_k, \sigma^2, \tau_k^2)$$

$$= (2\pi\sigma^2)^{-\frac{T_J}{2}} \exp \left[ -\frac{1}{2\sigma^2} \sum_{t=1}^{T_J} \left( y_{t,i} - (\beta_{0,i} + \beta_{1,i}p_{i,t} + \beta_{2,i}D_{BSE,t}) \right)^2 \right].$$
Posterior Distributions and Gibbs Sampler

Based on the priors and the likelihood function introduced above, the joint posterior distributions are derived:

\[ p(\beta_{k,i}, \beta_k, \sigma^2, \tau_k^2 | y_{t,i}) \propto L(y_{t,i} | \beta_{k,i}, \beta_k, \sigma^2, \tau_k^2) p(\beta_{k,i} | \beta_k, \sigma^2, \tau_k^2) p(\beta_k, \sigma^2, \tau_k^2). \]

The mean of the marginal posterior distribution represents the Bayesian estimate. In order to derive the marginal posterior distribution, there are two possible options: direct sampling and Gibbs sampling (Gelman 2006). We use the Gibbs sampler to derive the mean and standard deviation of the marginal posterior distribution. To do so, the conditional posterior distributions of interest are used to implement the Gibbs sampler. The conditional posterior distributions of the interests are

\[ p(\beta_{k,i} | y_{t,i}, \beta_k, \sigma^2, \tau_k^2), \]
\[ p(\beta_k | y_{t,i}, \sigma^2, \tau_k^2), \]
\[ p(\sigma^2 | y_{t,i}, \beta_k, \beta_{k,i}, \tau_k^2), \]
\[ p(\tau_k^2 | y_{t,i}, \beta_k, \beta_{k,i}, \sigma^2), \]
\[ p(\beta_{k,i} | y_{t,i}, \beta_k, \sigma^2, \tau_k^2). \]

These conditional distributions are used to draw a sequence of values for each parameter by a Markov Chain. The Markov Chain creates a stationary distribution that can be made close to the marginal posterior distribution of the parameters. The following steps are necessary to apply the Markov Chain Monte Carlo simulation via Gibbs Sampler:

1) Give initial values for \( \beta_{k,i}^0, \beta_k^0, \sigma^2_0, \tau_k^0 \),

2) Draw \( \beta_{k,i}^{(t+1)} \), given \( \beta_k^t, \sigma^2_t, \tau_k^t \),

3) Draw \( \beta_k^{(t+1)} \), given \( \beta_{k,i}^{t+1}, \sigma^2_t, \tau_k^t \),

4) Draw \( \sigma^2_{(t+1)} \), given \( \beta_{k,i}^{t+1}, \beta_k^{t+1}, \tau_k^t \), and

5) Draw \( \tau_k^{2(t+1)} \), given \( \beta_{k,i}^{t+1}, \beta_k^{t+1}, \sigma^{2t+1} \).
After a large number of Gibbs sampling steps, the draws from the conditional distributions can be considered as draws from the corresponding marginal posterior distributions. The output should be checked to see if there is convergence to a stationary distribution. Figure 2 and 3 show trade plots to check convergence and consequently whether the MCMC algorithm is able to generate samples. Trace plots of each sample satisfy stationary and ergodic condition. For the Bayesian analysis, we generate 20,000 Gibbs sampler iterations and burn in 1000 draws to approximate the posterior distributions.

**Results of Bayesian Estimates**

The Bayesian estimates show correct signs and magnitudes, whereas the OLS estimates generally might not (table 3). Signs of the elasticity of substitution are consistent with OLS result, but the magnitude is difference except for Japan. The Bayesian hierarchical approach causes each parameter to tend toward the hyper-priors. The mean of hyper-prior for $\beta_1$ is 0.49. If estimates from OLS regressions on each country separately are generally smaller than 0.49, then Bayesian estimate is increased toward the hyper-prior.

Estimates of the impact of the U.S. BSE outbreak become positive in the Bayesian estimation, as compared to the impact of the BSE outbreak in the OLS results (table 3). The estimated impacts of the BSE outbreak on the preference for domestically produced beef over imported beef in Mexico, Canada, and EU are negative in the OLS results. However, Bayesian estimates show that the U.S. BSE outbreak has the positive
impact. The new results indicate that the consumers prefer domestically produced beef over imported beef more after the outbreak than before. We impose priors, namely a uniform distribution between 0 and 1, so the result is consistent with theory by construction.

The two estimation methods generate differences in the preferences for domestically produced beef (Table 4). If the value of the preference parameter is greater than 0.5, then consumers prefer domestically produced beef to imported beef. In the absence of BSE, for example, all six countries estimates suggest a preference for domestically produced beef over imported beef. The U.S. BSE outbreak causes the values to change. During the period assumed to correspond to the U.S. BSE outbreak, namely 2003 to 2006, domestic bias in all countries is increased.

Table 5 demonstrates how the preference for domestically produced of each country changes during the U.S. BSE outbreak from 2003 to 2006. The value \( B \) is calculated by subtracting the preference without the BSE dummy \( \delta_o \) from the preference with the BSE dummy \( \delta_B \). Hence, a positive value means that consumers increase their preference for domestically produced beef over imported beef. According to the results of the OLS, Mexico, Japan, and Canada tend to give greater preference to domestically produced beef over imported beef when the BSE outbreak took place from U.S. In principle, this seems like a valid result because consumers might prefer domestically produced beef instead of imported beef if there is a BSE crisis in a key exporting country. Thus, it is perhaps surprising that all other countries turn more to imported beef during the U.S. BSE outbreak, as indicated by the negative changes in this parameter at that time. Bayesian estimates impose priors that require the estimation
results will be consistent with theoretical expectations. The results of this method unsurprisingly show that countries prefer domestically produced beef to imported beef because of the U.S. BSE outbreak.

**Conclusion**

Trade elasticities are important to understand the impact of prices or structural changes. There are many alternative methods to estimate trade elasticities. However, many methods can give results that are flawed in terms of wrong signs or unreasonable magnitudes of estimated elasticities if economic theory is one source of criteria. Although methods such as OLS, Maximum likelihood estimation and even time series analysis are used to estimate trade elasticities, short data series or model specification problems could lead to unacceptable results.

The Armington model is used to estimate elasticities of substitution between domestically produced and imported beef, the country-of-origin bias, and the impact of the U.S. BSE outbreak on preferences. The OLS estimations over both pooled and separated data yield results that include parameters with incorrect signs and unreasonable magnitudes, based on an economic perspective. For example, even though the U.S. BSE outbreak started in 2003, Korea and Russia prefer imported beef to domestically produced beef in the period of this shock. Using estimates that are imprecise and potentially even invalid not only can cause miscalculated values of imports and exports, but also could mislead trade negotiators or other final users of trade economics.
A Bayesian hierarchical model is used to estimate the Armington model. This approach uses not only raw data, but also explicitly incorporates economic theory in the form of priors. Bayesian hierarchical models estimate parameters by using partially pooled estimation. A Bayesian hierarchical model can address drawbacks associated with least-square regressions, which generate wrong signs or unreasonable magnitudes over cross country trade data.

A main reason why the results are difference between OLS and Bayesian estimation is because this method combines available data with priors that take the form of fixed ranges or distributions from which parameter values must be drawn. While OLS is just estimated by data information, the Bayesian estimation uses not only data but also priors borrowed from suggested population priors. The Bayesian estimate could be too subjective, but the results can provide more reasonable trade analysis if assessed based at least in part on economic criteria.

There are some limitations to our model. First, the Armington model consists of two stages: (1) consumption is separated between domestic and imported goods and (2) imported good consumption is divided by country-of-origin. Here, the first stage is considered to focus on applying for Bayesian approach. Second, there have been other BSE, but this article only considers about the U.S. BSE outbreak. Third, the theoretical basis of consumer choice, as derived from the Armington model, is used to represent consumer choice between domestically produced beef and imported beef, but the U.S. BSE outbreak caused many or most of the countries in question to ban beef imports from the U.S. As such, what is being modeled as a consumer response to U.S. beef might be
motivated in part by a policy change, although the policy changes targeted a single supplier of beef rather than all suppliers of beef.

While the Bayesian approach is a controversial method when set against more common frequentist approaches (e.g. Allenby et al. 2005), our method has also been widely applied in similar settings and might be a better approach if economic criteria are paramount. For example, rather than the U.S. BSE causing more import demand among key beef importing countries, the Bayesian results necessarily lead to greater demand for domestically produced good, and the exact estimation results suggest larger effects for a countries that tend to import more U.S. beef, like Japan, Canada, Korea, and Russia, and less impact on others, like the EU. Our results relate to a major shock to the beef markets of the U.S. and U.S. trading partners in the recent past and can inform analysts about the impact of a rare shock whose effects have proven difficult to estimate.
Table 1. Descriptive Statistics of Explanatory Variables

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption Ratio of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic beef to</td>
<td>Mexico</td>
<td>7.97</td>
<td>6.23</td>
<td>3.41</td>
</tr>
<tr>
<td>Imported beef</td>
<td>Japan</td>
<td>0.68</td>
<td>0.13</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>2.52</td>
<td>1.05</td>
<td>1.71</td>
</tr>
<tr>
<td></td>
<td>Korea</td>
<td>0.94</td>
<td>0.43</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>Russia</td>
<td>2.37</td>
<td>1.44</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>EU</td>
<td>15.79</td>
<td>3.10</td>
<td>11.03</td>
</tr>
<tr>
<td>Ratio of Import price</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>to Domestic price</td>
<td>Mexico</td>
<td>1.83</td>
<td>0.19</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>Japan</td>
<td>0.30</td>
<td>0.07</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>1.00</td>
<td>0.09</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>Korea</td>
<td>0.31</td>
<td>0.07</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>Russia</td>
<td>1.39</td>
<td>0.40</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>EU</td>
<td>1.48</td>
<td>0.44</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Source: USDA-PSD and OECD-MPS
Table 2. DIC values for $b_0$ and $b_1$

<table>
<thead>
<tr>
<th></th>
<th>$b_k = 1$</th>
<th>$b_k = 2$</th>
<th>$b_k = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_D = 2\text{var}(\log p(y</td>
<td>\theta))$</td>
<td>67.8</td>
<td>68.6</td>
</tr>
<tr>
<td>$DIC = -2\log p(y</td>
<td>\theta) + 2P_D$</td>
<td>4.8</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Note: $a_0$ and $a_1$ are fixed as 3.
### Table 3. The Comparisons of OLS and Bayes Estimates, 2003 to 2006

<table>
<thead>
<tr>
<th></th>
<th>OLS estimate</th>
<th>Bayesian estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{0,MEX}$</td>
<td>0.95**</td>
<td>0.73 [0.16]</td>
</tr>
<tr>
<td>$\beta_{0,JPN}$</td>
<td>-0.08</td>
<td>0.02 [0.22]</td>
</tr>
<tr>
<td>$\beta_{0,CAN}$</td>
<td>0.34***</td>
<td>0.33 [0.06]</td>
</tr>
<tr>
<td>$\beta_{0,KOR}$</td>
<td>0.22</td>
<td>0.22 [0.21]</td>
</tr>
<tr>
<td>$\beta_{0,RUS}$</td>
<td>0.30***</td>
<td>0.27 [0.07]</td>
</tr>
<tr>
<td>$\beta_{0,EU}$</td>
<td>1.23***</td>
<td>1.17 [0.07]</td>
</tr>
<tr>
<td>$\beta_{1,MEX}$</td>
<td>0.41</td>
<td>0.32 [0.57]</td>
</tr>
<tr>
<td>$\beta_{1,JPN}$</td>
<td>-0.31</td>
<td>0.41 [0.42]</td>
</tr>
<tr>
<td>$\beta_{1,CAN}$</td>
<td>0.20</td>
<td>0.33 [0.64]</td>
</tr>
<tr>
<td>$\beta_{1,KOR}$</td>
<td>0.78</td>
<td>0.58 [0.40]</td>
</tr>
<tr>
<td>$\beta_{1,RUS}$</td>
<td>0.51</td>
<td>0.30 [0.38]</td>
</tr>
<tr>
<td>$\beta_{1,EU}$</td>
<td>0.49</td>
<td>0.01 [0.36]</td>
</tr>
<tr>
<td>$\beta_{2,MEX}$</td>
<td>-0.11</td>
<td>0.06 [0.05]</td>
</tr>
<tr>
<td>$\beta_{2,JPN}$</td>
<td>0.61</td>
<td>0.12 [0.08]</td>
</tr>
<tr>
<td>$\beta_{2,CAN}$</td>
<td>-0.19</td>
<td>0.20 [0.11]</td>
</tr>
<tr>
<td>$\beta_{2,KOR}$</td>
<td>0.04</td>
<td>0.07 [0.06]</td>
</tr>
<tr>
<td>$\beta_{2,RUS}$</td>
<td>0.15</td>
<td>0.06 [0.05]</td>
</tr>
<tr>
<td>$\beta_{2,EU}$</td>
<td>-0.11**</td>
<td>0.07 [0.06]</td>
</tr>
</tbody>
</table>

Note: *** 1%, ** 5%, and 10% level of significance.
Table 4. The Country-of-Origin Bias, 2003 to 2006

<table>
<thead>
<tr>
<th>Method</th>
<th>OLS result</th>
<th>Bayesian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta_o$</td>
<td>$\delta_B$</td>
</tr>
<tr>
<td>$\delta_{MEX}$</td>
<td>0.047</td>
<td>0.082</td>
</tr>
<tr>
<td>$\delta_{JPN}$</td>
<td>0.406</td>
<td>0.460</td>
</tr>
<tr>
<td>$\delta_{CAN}$</td>
<td>0.608</td>
<td>0.654</td>
</tr>
<tr>
<td>$\delta_{KOR}$</td>
<td>0.607</td>
<td>0.554</td>
</tr>
<tr>
<td>$\delta_{RUS}$</td>
<td>0.648</td>
<td>0.565</td>
</tr>
<tr>
<td>$\delta_{EU}$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table 5. The Comparison of the Impact of the U.S. BSE Outbreak on Preferences.

<table>
<thead>
<tr>
<th>Method</th>
<th>OLS</th>
<th>Bayesian</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEX</td>
<td>0.036</td>
<td>0.014</td>
</tr>
<tr>
<td>JPN</td>
<td>0.054</td>
<td>0.069</td>
</tr>
<tr>
<td>CAN</td>
<td>0.046</td>
<td>0.102</td>
</tr>
<tr>
<td>KOR</td>
<td>-0.053</td>
<td>0.028</td>
</tr>
<tr>
<td>RUS</td>
<td>-0.083</td>
<td>0.039</td>
</tr>
<tr>
<td>EU</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: Values represent $B$ calculated by $\delta_B - \delta_a$. 
Figure 1. Directed Acyclic Graph

Note: Dash circles and squares represent priors and hyper-priors, respectively.
Figure 2. Trace Plots in Case of No Impact of the U.S. BSE Outbreak
Figure 3. Trace Plots in Case of the Impact of the U.S. BSE Outbreak
References


IMF Primary Commodity Prices. Available at:


OECD Producer and Consumer Support Estimates database. Available at:


USDA Foreign Agricultural Service. Available at: http://apps.fas.usda.gov/psdonline/.

Accessed 10 May 2016.