Two-sided Hedge Model: CI

- **Timeline:**
  - Buy $b_t$ input futures at $t$.
  - Buy a unit of input at $t$.
  - Sell $b_t$ input futures at $t$.
  - Sell one unit of output at $T$.

- **Cost:**
  - $\Pi = -\frac{1}{2} \sigma^2 \left( \frac{T}{t} \right) \int_0^T \left( \frac{S_t}{S_0} \right)^{\beta} \left( \frac{V_t}{V_0} \right)^{\delta} dt$.

- **Minimize:**
  - $Func = \sum \left[ \sigma_t \right]_t^T$.

- **Log spread:**
  - $\log \left( \frac{S_t}{S_0} \right) = \log \left( \frac{V_t}{V_0} \right)$.

Price Dynamics, $\beta_1$, and $\beta_2$

- Underlying price $S_t$ and convenience yield $\delta_t$ are assumed to follow a two-factor diffusion model (e.g., Schwartz, 1997):
  - $dS_t = \left[ \mu - \delta_t \right] S_t dt + \sigma_t \alpha_t \delta_t dz_t$.
  - $d\delta_t = \left[ \alpha_t - \delta_t \right] \delta_t dt + \sigma_t \beta_t \delta_t dz_t$.

- The log spread $\log \left( \frac{S_t}{S_0} \right)$ between the price of underlying ($S_t$) and price of related assets ($V_t$) satisfies (Brennan et al., 2009):
  - $dV_t = \left[ \alpha_t - \delta_t \right] dt + \sigma_t \beta_t \delta_t dz_t$.

- From (1) and (2), we have $\beta_1$ and $\beta_2$ as function of the deterministic parameters:
  - $\beta_1 \equiv \frac{\sigma_t \beta_t}{\sigma_t^2 - \sigma_t \alpha_t}$.
  - $\beta_2 \equiv \frac{\sigma_t \beta_t}{\sigma_t^2 - \sigma_t \alpha_t}$.

Application to a Jet Fuel Producer

- The firm uses light sweet crude oil to produce jet fuel and only has input futures contracts.
- Weekly data of futures prices for light sweet crude oil for delivery to Cushing, OK, and spot prices for New York Harbor jet fuel from Apr 4, 1990 to Aug 16, 2015 is used.
- Use Kalman Filtering to estimate parameters for (1) and (2):
  - $\theta_t = \chi_0 \left( \frac{S_t}{S_0} \right)^{\beta_1} \left( \frac{V_t}{V_0} \right)^{\beta_2}$.

Optimal Hedge: Other Cases

- The optimal hedge in other cases is derived via a similar procedure.
- For a CO firm, the optimal hedging policy is
  - $\beta_t' = \frac{\sigma_t \beta_t}{\sigma_t^2 - \sigma_t \alpha_t}$.

- For Demand-driven firms, PT suggests
  - $\beta_t' = \frac{\sigma_t \beta_t}{\sigma_t^2 - \sigma_t \alpha_t}$.

- For a Df firm, the optimal hedging policy is
  - $\beta_t' = \frac{\sigma_t \beta_t}{\sigma_t^2 - \sigma_t \alpha_t}$.

- For a DO firm, the optimal hedging policy is
  - $\beta_t' = \frac{\sigma_t \beta_t}{\sigma_t^2 - \sigma_t \alpha_t}$.

Comparisons of Hedging Models

- The jet fuel producer is a CI firm.
- Based on the estimated parameter values, we compute the direct and cross hedging ratio $\beta_1$ and $\beta_2$.
- The optimal two-sided hedge policy for the CI firm is $\delta_t = \left( \frac{S_t}{V_t} \right)$ and the one-sided hedge policy is $\delta_t = \delta_t$.

- Following Ederington (1979), we calculate the effectiveness of the two-sided and one-sided model as
  - $\varepsilon_{2,\delta} = \left( 1 - \frac{\text{var}(\Pi_{\text{CI}})}{\text{var}(\Pi)} \right)$.
  - $\varepsilon_{1,\delta} = \left( 1 - \frac{\text{var}(\Pi_{\text{CI}})}{\text{var}(\Pi)} \right)$.

- Where $\Pi_{\text{CI}}$ and $\Pi_{\text{CI}}$ stand for cash flow under one-sided model, two-sided model and unhedged positions, respectively.

Conclusions

- By embedding PT in the complete-market hedging model, we present a two-sided hedging strategy for firms in incomplete markets.
- Empirical analysis using a hypothetical jet fuel producer demonstrates the effectiveness of our two-sided hedge strategy.
- As PT is an important characteristic describing the overall operation of the market (Goodwin and Holt, 1999), the two-sided hedging strategy may be practical for many firms in multiple industries.

Reference

Corporate Hedging In Incomplete Markets: A Solution Under Price Transmission

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