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Sustainable Water Policies and the Optimal Development of Desalination Technologies

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Yacov Tsur[a] and Amos Zemel[b]

Abstract: In many arid and semi-arid regions whether or not to use desalinated water has long been a non-issue and policy debates are focused on the timing and extent of the desalination activities. We offer a model to analyze how water scarcity and demand structure, on the one hand, and cost reduction via R&D activities, on the other hand, affect the desirable development of desalination technologies and the optimal time profiles of fresh and desalinated water supplies. The optimal R&D policy is found to follow a Non-Standard Most Rapid Approach Path (NSMRAP): The state of desalination technology—the accumulated knowledge from R&D activities—should approach a prespecified target process as rapidly as possible and proceed along it forever. The NSMRAP property enables a complete characterization of a comprehensive water policy in terms of a simple and tractable set of rules.

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1. Introduction

Population and economic growth worldwide have lead to exploitation of natural resources beyond nature's reproduction capacity. While obvious for nonrenewable resources, such as mineral deposits, this phenomenon has been noticeable also for renewable resources, as forest areas, fishing grounds, wildlife populations and fresh water stocks (aquifers, lakes, rivers) have begun to diminish. This raises the *sustainability* question of whether growth can be supported in the long run by the natural stock on which it is based. If not, sustainability requires finding substitutes for those resources that are bound to be depleted. Two prominent examples are fossil energy and fresh water. The first is a nonrenewable resource for which solar energy has been proposed as a backstop substitute; the second is a renewable resource with desalinated seawater as a backstop resource.

The development of backstop technologies requires R&D investments, which itself consumes resources and takes time to bear fruits. The sustainability issue therefore involves a delicate balance between (a) the temporal exploitation of the primary resources on the one hand, and (b) the proper scheduling of R&D investment to have a backstop resource ready in time to substitute the primary resource. In this paper we formulate the optimal policy rules for the exploitation of fresh water (the primary resource) and the development of desalinated technologies (the backstop resource).

In many arid and semi-arid regions whether or not to use desalinated water has long been a non-issue and policy debates are focused on the timing and extent of the desalination activities. At stake here is not subsistence water (this relatively small quantity can often be supplied from local fresh sources), but rather water used as input of agricultural, industrial and environmental production, for which the usual economic considerations apply. Currently desalinated water is expensive (ranging between \$0.8/m^3 to \$1.0/m^3, 1992 prices, Lithwick et al., 1998), hence attracts only small demand. However the various technologies considered, such as distillation, Reverse Osmosis and Electrodialysis (Spiegler and Laird, 1980), or the recently proposed Water Towers (Zaslavsky, 1997), leave a large room for cost reducion, pending appropriate investment in R&D.

As noted above, R&D activities take time before they accumulate via learning processes to advance the state of technology and reduce desalination cost. Hence, policies that consider transition to desalinated water must include the time profile of R&D in desalination technologies as a principal decision variable. This work offers such a framework, drawing heavily on Tsur and Zemel (1998b), who analyzed the development of solar technologies as a substitute for fossil energy. The main difference stems from the fact that fresh water stocks are typically renewable, whereas fossil fuel deposits are not. The presence of recharge processes changes the specification of the optimal policy in a number of ways, particularly in that it allows the stock to be depleted during the process and refill at a later date when technological progress reduces the cost of desalination. With a nonrenewable resource, this option is, of course, not available. Nevertheless, the underlying structure of the solutions for both cases is otherwise similar.

We show that the optimal desalination R&D process admits a *nonstandard* Most Rapid Approach Path (MRAP) (Spence and Starrett, 1975), in that the state of desalination technology (the net accumulation of learning from R&D) approaches as rapidly as possible a pre-specified *root process* (rather than a *stationary state*) and proceeds along it thereafter. The parameters of the optimal policy are tuned so as to ensure that the transition from fresh to desalinated water supply takes place in a continuous manner, avoiding sudden cuts in fresh water supply rates that must follow premature depletion of the fresh water stock. These observations justify substantial early engagement in desalination R&D programs that should precede future shortage of fresh water supply.

The next section sets up the problem and defines the set of feasible water policies. Section 3 characterizes the optimal policy, focusing attention on the R&D aspects and avoiding formal proofs. Section 4 considers non-stationary water demand (due, say, to population growth). Section 5 concludes, and the appendix presents the technical derivations.

2. Water policy formulation

The economy under consideration can derive water from two sources: a renewable fresh water stock of finite size, and desalinated seawater. The use of the latter source is practically limited only by its cost. The cost of fresh water supply does not change over time. The cost of desalination can be reduced as a result of technological progress due to R&D.

Demand

The derived demand for water at each point of time, $D(p)$, is a decreasing function of the price of water p. The inverse demand, $D^{-1}(q)$, represents the price along the demand curve corresponding to any rate of water supply q (Figure 1). The case of a non-stationary demand $D(p,t)$ is discussed in Section 4.

Conventional supply of fresh water

Let $C(q^c)$ represent the instantaneous cost of conventional fresh water supply (pumping, conveyance) at the rate q^c (for simplicity, a single source is considered). We assume that $C(q^c)$ is increasing and strictly convex, hence the marginal cost $M_c(q^c) \equiv dC(q^c)/dq^c$ increases with the supply rate.

The fresh water stock at time t, X_t, evolves over time according to

$$\dot{X}_t \equiv dX_t / dt = R(X_t) - q_t^c \tag{1}$$

where $R(X) \equiv \xi(\overline{X} - X)$ is the fresh water rate of replenishment, which vanishes when the stock is at full capacity, i.e. when $X = \overline{X}$. Integrating (1) gives

$$X_t = \bar{X} + (X_0 - \bar{X})e^{-\xi t} - \int_0^t q_t^c e^{-\xi(t-s)} ds \tag{2}$$

Supply of desalinated water

The unit cost of desalination is assumed independent of the supply rate of desalinated water q^s, but depends on the state of desalination technology, which we call *knowledge* and denote by K_t. Given K_t, the desalination technology at time t admits constant returns to scale and can be characterized by the unit (or marginal) cost function $M_s(K_t)$ which decreases with knowledge. The latter, in turn, accumulates due to the learning associated with the R&D investments I_τ, $\tau \leq t$, that had taken place up to time t.

The balance between the rate of R&D investment, I_t, and the rate at which existing knowledge is lost or becomes obsolete due to aging or new discoveries determines the rate of knowledge accumulation

$$\dot{K} \equiv dK/dt = I_t - \delta K \tag{3}$$

where the knowledge level K is measured in monetary units and the constant δ is a knowledge depreciation parameter. Integrating (3), we obtain

$$K_t = \int_0^t I_\tau e^{\delta(\tau - t)} d\tau + K_0 e^{-\delta t}. \tag{4}$$

Social benefit

The surplus to water users (excluding the water bill, which is just a transfer from consumers to suppliers) generated by $q = q^c + q^s$ is given by the area below the (inverse) demand curve to the left of q: $G(q) = \int_0^q D^{-1}(z)dz$. The cost of supplying $q^c + q^s$ is $C(q^c) + M_s(K_t)q^s$. The net consumer and supplier surplus generated by $q^c + q^s$ is therefore $G(q^c + q^s) - [C(q^c) + M_s(K_t)q^s]$. Subtracting the R&D cost, the instantaneous net social benefit at time t is given by

$$G(q_t^c+q_t^s) - C(q_t^c) - M_s(K_t)q_t^s - I_t\,. \tag{5}$$

Water policy

A water policy consists of three control (flow) and two state processes. The flow processes are q_t^c (supply rate of fresh water), q_t^s (supply rate of desalinated water) and I_t (R&D investment rate). The state variables are X_t (stock of fresh water) and K_t (desalination knowledge). A policy $\Gamma = \{q_t^c, q_t^s, I_t | t \geq 0\}$ determines the evolution of the state variables via (1–4) and gives rise to the instantaneous net benefit (5). The optimal policy is the solution to

$$V(X_0, K_0) = Max_\Gamma \int_0^\infty [G(q_t^c + q_t^s) - C(q_t^c) - M_s(K_t)q_t^s - I_t]e^{-rt}dt$$
$$\text{subject to (1), (3), } q_t^c, q_t^s \geq 0,\ 0 \leq I_t \leq \bar{I},\ X_t \geq 0 \text{ and } X_0, K_0 \text{ given.} \tag{6}$$

In (6), r is the time rate of discount and $\bar{I}$ is an exogenous bound on the affordable R&D effort which implies, in view of (3), the upper bound $\bar{K} = \bar{I} / \delta$ on desalination knowledge.

3. Optimal policy characterization

The optimal policy $\Gamma^* = \{q_t^{c*}, q_t^{s*}, I_t^* | t \geq 0\}$ and the associated state processes X_t^* and K_t^* are characterized in two steps. First, the supply rates q_t^{c*} and q_t^{s*} are specified conditional on the state of K_t and on the scarcity rent of the fresh water stock. In the second step, the optimal R&D policy (investment and knowledge processes) and the fresh water scarcity rent process are determined. The supply rates are determined in much the same way as they would in a static situation, by equating supply and demand, where the dynamics enter through the effects of fresh water scarcity on the marginal costs of fresh water supply. The optimal R&D policy, it turns out, admits a non-standard Most Rapid Approach Path (MRAP) (Spence and Starrett, (1975)). This property enables a complete characterization of the optimal water policy, as presented below. The technical derivation is relegated to the appendix.

Step 1: Fresh and desalinated water supplies

The optimal supply rule states that, at each point of time, an additional unit of water is to be supplied from the cheapest available source. The marginal cost of fresh water supply consists of the direct supply cost $M_c(q^c)$ ($\equiv dC(q^c)/dq^c$) plus the *scarcity rent* λ_t (also known as user cost or shadow price) associated with the remaining stock of fresh water. The optimal supply rule requires that fresh water is supplied up to the rate a satisfying

$$M_c(a) + \lambda = M_s(K) \tag{7}$$

and desalination plants supply the residual demand (see Figure 1). Since a negative supply is not permitted, a is defined as $a(K,\lambda) = Max\{M_c^{-1}(M_s(K) - \lambda)\, , 0\}$.

Given K and λ, the marginal cost of water supply is specified as

$$M(q \mid K, \lambda) = \begin{cases} M_c(q) + \lambda & \text{if } q \leq a(K,\lambda) \\ M_s(K) & \text{otherwise} \end{cases} \tag{8}$$

and total supply is given by the rate $q(K,\lambda)$ at which supply and demand intersect, i.e.,

$$q(K,\lambda) = \{q \mid M(q|K,\lambda) = D^{-1}(q)\}, \tag{9}$$

(see Figure 1). If $M_s(K) > D^{-1}(q(K,\lambda))$, desalination is too expensive, $q(K,\lambda) < a(K,\lambda)$ and the entire demand is provided by fresh water. While our general formulation permits the analysis of this case, (which is relevant for locations where fresh water abundance leaves no room for desalination), we consider situations in which some degree of desalination is worthwhile. Therefore, we assume that $M_s(K) = D^{-1}(q(K,0))$ for all relevant knowledge levels. Thus, given K_t and λ_t, $q^c = a(K,\lambda)$ and desalination plants generate the residual demand:

$$\textit{(fresh water)} \qquad q^c(K_t,\lambda_t) = a(K_t,\lambda_t) \tag{10a}$$

$$\textit{(desalination)} \qquad q^s(K_t,\lambda_t) = D(M_s(K_t)) - q^c(K_t,\lambda_t)\,. \tag{10b}$$

Figure 1

A difficulty with implementing (10) may arise if the fresh water stock is empty and

$q^c(K_t,\lambda_t)$ exceeds $R(0)$. Fortunately, this situation cannot occur under the optimal policy. This is so because the optimal processes K_t^* and λ_t^* are so chosen that at the time of depletion, and while the fresh water stock is empty thereafter, it is not desirable to supply fresh water beyond the recharge rate $R(0)$. This property is derived in the appendix as

Claim 1 (continuity of fresh water supply at depletion): If it is optimal to deplete the fresh water stock, the following conditions hold at the optimal depletion time T^*:

$$M_s(K_{T^*}^*) = M_c(R(0)) + \lambda_{T^*}^* \tag{11}$$

and

$$\int_0^{T^*} q^{c*}(K_t^*,\lambda_t^*)e^{-\xi(T^*-t)}dt = \overline{X} + (X_0 - \overline{X})e^{-\xi T^*}. \tag{12}$$

Equation (11) implies that at the depletion time the optimal fresh water supply rate equals the (empty stock) replenishment rate $R(0)$ and will not undergo a discontinuous change. Equation (12) is just a restatement of the depletion event at time T^*, i.e. that $X_{T^*} = 0$ (cf. 2).

Step 2: The optimal R&D policy (I_t^* and K_t^*) and fresh water scarcity rent (λ_t^*)

We show now that the optimal R&D policy follows a *nonstandard* Most Rapid Approach Path (NSMRAP). A *standard* MRAP is defined by the K-process that approaches as rapidly as possible some prespecified steady state level and remains there forever. To formulate such a policy, let K_t^m denote the K-process initiated at K_0 and driven by the R&D policy that invests at the maximal rate $I_t = \bar{I}$. Recalling (4),

$$K_t^m = (1-e^{-\delta t})\overline{K} + K_0 e^{-\delta t}. \tag{13}$$

The standard MRAP initiated below the steady state level $\hat{K}$ is given by $K_t = Min\{K_t^m, \hat{K}\}$.

A *nonstandard* MRAP (NSMRAP) involves, rather than a fix steady state, a prespecified target K-process. Initiated below the target process, the NSMRAP begins as K_t^m, but if K_t^m crosses the target process before the latter arrives at its own steady state $\hat{K}$, then the NSMRAP switches to the target process and cruises along it to $\hat{K}$. A NSMRAP, therefore, is specified in terms of K_t^m and some target K-process such that the most rapid approach is to the target process rather than to a target steady state. Of course, if the target process settles at its steady state $\hat{K}$ before being reached by K_t^m, the NSMRAP reduces to the standard MRAP. We now introduce the target process corresponding to the optimal R&D policy. We refer to it as the *root process* for a reason soon to become obvious.

Define

$$L(K,\lambda) \equiv -M_s'(K)q^s(K,\lambda) - (r+\delta). \tag{14}$$

This function (which is a generalization of the *evolution function* used to determine steady states of infinite-horizon dynamic problems by Tsur and Zemel (1996, 1998a)) can be viewed as the derivative (with respect to K) of the utility to be maximized by the optimal R&D process (see the appendix). Thus, we seek the root $K(\lambda)$ of $L(K,\lambda)$, i.e., the solution of $L(K(\lambda),\lambda) = 0$, in the region $\Re(\lambda) = \{K \mid \partial L(K,\lambda)/\partial K<0\}$ in which $L(K,\lambda)$ decreases in K. Using (10), we see that q^s does non-decreasing in λ. Recalling that $M_s'(K) < 0$, we find that $K(\lambda)$ is non-decreasing and bounded above by the root of the limit function $-M_s'(K)D(M_s(K)) - (r+\delta)$, corresponding to a vanishing fresh water supply rate (e.g., when λ is very large).

$K(\lambda_t)$ is the *root process* corresponding to the scarcity rent process λ_t and bears a simple economic interpretation. Increasing knowledge by the infinitesimal amount dK reduces the cost of desalination by $-M_s'(K)q^s(K,\lambda)dK$ but incurs a cost of $(r+\delta)dK$ due to interest payment on the investment and the increased depreciation. The root $K(\lambda)$ represents the balance between these conflicting effects. To rule out corner solutions, we assume that the evolution function possesses a unique root in $\Re(\lambda)$, and that $K_0 < K(\lambda) < \bar{K}$ for all λ.

Initiated at $K_0 < K(0)$, the NSMRAP with respect to $K(\lambda_t)$ is $K_t = Min\{K_t^m, K(\lambda_t)\}$ associated with the following R&D investment process:

$$I_t = \begin{cases} \bar{I} & \text{if} \quad K_t < K(\lambda_t) \\ K'(\lambda_t)\dot{\lambda}_t + \delta K(\lambda_t) & \text{if} \quad K_t = K(\lambda_t) \end{cases} \tag{15}$$

Implicit in (15) is the assumption that $\bar{I}$ is large enough to allow K_t to cruise along $K(\lambda_t)$, i.e., the NSMRAP is feasible, which we assume to hold for the optimal λ_t^*. We can now formulate the MRAP property:

Claim 2: Given the optimal scarcity rent process λ_t^*, the optimal R&D policy is a NSMRAP with respect to the root process $K(\lambda_t^*)$.

We turn now to determine the optimal fresh-water scarcity process λ_t^*. Prior to depletion, the scarcity process is of the form $\lambda_t^* = \lambda_0^* e^{(r+\delta)t}$ (see appendix), thus we need only determine the nonnegative parameter λ_0^*. To this end, the following notational convention is introduced: a ^ symbol above a state K indicates a root of the evolution function with some specification of the supply rates; the superscript 0 signifies evaluation at $\lambda_0 = 0$; the superscript R signifies evaluation with $q^c = R(0)$.

Let $\hat{K}^0 = K(0)$ be the root of $L(K,0)$, i.e., $-M_s'(\hat{K}^0)q^s(\hat{K}^0,0) = (r+\delta)$. This level is the lower bound of the non-decreasing function $K(\lambda)$. Let $K^{cr} = M_s^{-1}(M_c(R(0)))$ be the critical knowledge level above which the fresh water stock becomes inessential: $K \geq K^{cr}$ implies $M_c(q^c) \leq M_s(K) \leq M_s(K^{cr}) = M_c(R(0))$, hence $q^c \leq R(0)$ for all nonnegative λ. Using (13), we find that the MRAP K_t^m passes through the states $\hat{K}^0$ and K^{cr} at the dates

$T^0 = \dfrac{1}{\delta}\log[(\overline{K} - K_0)/(\overline{K} - \hat{K}^0)]$ and $T^{cr} = \dfrac{1}{\delta}\log[(\overline{K} - K_0)/(\overline{K} - K^{cr})]$ respectively.

For any nonnegative date T define

$$Q(T) = \int_0^T q^c(K_t^m, 0)e^{\xi t}dt - \overline{X}(e^{\xi T} - 1) \qquad (16)$$

where $q^c(K_t^m,0)$ is the (not necessarily feasible) fresh water supply rate defined in (10a). It follows from (2) that if $Q(T)$ exceeds the initial stock X_0 at some date T, then $X_T < 0$ and $q^c(K_t^m,0)$ is not feasible up to T. We can now state (see proof in the appendix):

Claim 3: $\lambda_0^* = 0$ *if and only if* $\{\hat{K}^0 \geq K^{cr}$ and $X_0 \geq Q(T^{cr})\}$, in which case $\hat{K}^0$ is the optimal knowledge steady state level, and the optimal R&D policy is the standard MRAP $K_t^* = Min\{K_t^m, \hat{K}^0\}$.

Claim 3 appeals to economic intuition. A steady state above the critical level K^{cr} implies a fresh water supply rate below $R(0)$ and corresponds to a non depleted stock and a vanishing scarcity rent. The latter implies that the root process is fixed at its lowest level $\hat{K}^0$, which must be approached as rapidly as possible. It only remains to ensure that the initial fresh water stock is large enough to support this policy all the way to $\hat{K}^0$. When $X_0 \geq Q(T^{cr})$, this is indeed the case, since for $t > T^{cr}$, while $K_t^m > K^{cr}$, the stock will surely not be depleted.

The situation is somewhat more involved when $\hat{K}^0 \geq K^{cr}$ but $X_0 < Q(T^{cr})$. The former condition favors, as explained above, vanishing scarcity rent and a nonempty stock. Yet, the latter condition implies that the initial fresh water stock cannot support $q^c(K_t^m,0)$: somewhere along the way, before K^{cr} is reached, the stock must be depleted. If the scarcity rent were zero, this would imply that q^c undergoes a discontinuous drop to $R(0)$ at the depletion date, violating Claim 1; hence, initially λ_t^* must be positive.

The reasoning underlying the characterization of this situation is as follows (a formal proof is presented in the appendix). Since depletion must take place, the initial scarcity rent must be positive and it should be chosen, together with the depletion date T^*, so that depletion and the rate equality $q^c = R(0)$ occur simultaneously, in accordance with Claim 1. On the depletion date, $K_{T^*} < K^{cr}$, for otherwise λ_{T^*} in (11) cannot be positive. The post depletion period, therefore, divides into two sub-periods: $(T^*,T^{cr}]$ and $(T^{cr},\infty]$. In the first sub period, the optimal process K_t^* follows K_t^m from K_{T^*} to K^{cr}, while the fresh water stock is empty and q^c is restricted to $R(0)$. The scarcity rent $\lambda_t^* = M_s(K_t^m) - M_c(R(0))$ (c.f. 7), *decreases* in time during this period, reflecting the decreasing value of fresh water as desalination knowledge accumulates. On the closing date T^{cr} of this sub period, $K_t^m = K^{cr}$ and $M_s(K^{cr}) = M_c(R(0))$, the fresh water stock becomes inessential and λ_t^* vanishes from that time on.

If $\hat{K}^0 > K^{cr}$, then during $T^{cr} < t \le T^0$, the optimal process continues to follow K_t^m towards $\hat{K}^0$, further reducing the price of desalinated water. Consequently, the fresh water supply rate q_t^c falls short of $R(0)$ and the fresh water stock gradually fills up again. If $\hat{K}^0 = K^{cr}$, the system enters the steady state K^{cr} at T^{cr} with an empty fresh water stock. (This is a singular case in which both the stock and the scarcity rent vanish.) We summarize the above in

Claim 4: When $\hat{K}^0 \ge K^{cr}$ and $X_0 < Q(T^{cr})$, then $K_t^* = Min\{K_t^m, \hat{K}^0\}$ and the time evolution of the optimal scarcity rent process λ_t^* is divided into three stages: (*i*) an initial exponential increase $\lambda_t^* = \lambda_0^* e^{(r+\xi)t}$ until depletion with λ_0^* (> 0) and T^* (< T^{cr}) defined implicitly by (11)-(12); (*ii*) a decreasing scarcity rent period during $T^* \le t \le T^{cr}$, with an empty fresh water stock and $\lambda_t^* = M_s(K_t^m) - M_c(R(0))$; (*iii*) a vanishing scarcity rent period with $\lambda_t^* = 0$ for $t \ge T^{cr}$, during which the fresh water stock refills if $\hat{K}^0 > K^{cr}$ or remains empty if $\hat{K}^0 = K^{cr}$.

The unique feature characterizing this case, namely the non-monotonic behavior of the fresh water stock and the scarcity rent processes, is traced to the renewable nature of the fresh water resource. Indeed, a related study of a nonrenewable resource (Tsur and Zemel, 1998b) yields only non-decreasing scarcity rents. Also note that although the root process is non monotonic, the knowledge process does not follow it but reduces to the standard MRAP.

We turn now to the case $\hat{K}^0 < K^{cr}$. Claim 3 requires a positive scarcity rent, suggesting that the fresh stock must be depleted. The continuity condition on the depletion date (Claim 1) requires the fresh water supply rate to equal the replenishment rate $R(0)$ on that date. It turns out (see the appendix) that during the post depletion period the fresh water supply remains at the rate $R(0)$, so that the fresh water stock remains empty and the desalinated water supply rate equals $D(M_s(K))-R(0)$. Accordingly, define

$$L^R(K) = -M_s'(K)[D(M_s(K)) - R(0)] - (r+\delta) \tag{17}$$

and let $\hat{K}^R$ be the root of L^R. It is verified in the appendix that $\hat{K}^R \in (\hat{K}^0, K^{cr})$ (see also Figure 2) and that this root is the steady state of the optimal K-process.

Figure 2

Let $\lambda^R = M_s(\hat{K}^R) - M_c(R(0))$ be the scarcity rent consistent with fresh water supply rate of $R(0)$ and $K = \hat{K}^R$. Clearly, $\hat{K}^R < K^{cr}$ implies $\lambda^R > 0$ (since $M_c(R(0)) = M_s(K^{cr})$ and M_s is decreasing). Let $T^R = \frac{1}{\delta}\log[(\bar{K} - K_0)/(\bar{K} - \hat{K}^R)]$ denote the date K_t^m passes through $\hat{K}^R$. Further, let $\lambda_0^m = \lambda^R e^{-(r+\xi)T^R}$ and $\lambda_t^m = \lambda_0^m e^{(r+\xi)t}$. Thus, $K(\lambda_{T^R}^m) = K(\lambda^R) = \hat{K}^R = K_{T^R}^m$ and the processes $K(\lambda_t^m)$ and K_t^m intersect at the date T^R. Since the root process is assumed to be slower than K_t^m, the two processes can cross only once. It follows that $\lambda_0 \geq \lambda_0^m$ implies that $K(\lambda_0 e^{(r+\xi)t}) \geq K_t^m$ for all $t \leq T^R$, while if $\lambda_0 < \lambda_0^m$ the two processes cross prior to T^R.

The optimal K-process, by virtue of Claim 2, is of the form $K_t^* = Min\{K_t^m, K(\lambda_t^*)\}$. One possibility is that K_t^m lags behind $K(\lambda_t^*)$ prior to the date T^R and the optimal process is a standard MRAP to $\hat{K}^R$. The alternative is that K_t^m overtakes the root process at an earlier date, and the optimal K-process is a NSMRAP, evolving along with the root process during its final stage. Whether the optimal process is a standard or nonstandard MRAP (i.e., whether

$\lambda_0^* \geq \lambda_0^m$ or $\lambda_0^* < \lambda_0^m$) depends on the initial fresh water stock in the following way:

Let $q^c(K_t^m, \lambda_t^m)$ be the (not necessarily feasible) fresh water supply rate corresponding to the knowledge and scarcity rent processes K_t^m and λ_t^m (as specified in 10a) and set $Q^m = \int_0^{T^*} q^c(K_t^m, \lambda_t^m)e^{\xi t}dt - \bar{X}(e^{\xi T^*} - 1)$. If $Q^m > X_0$, then $q^c(K_t^m, \lambda_t^m)$ is not feasible since according to (2), it yields a negative fresh water stock prior to T^R. The optimal rate q^{c*} would therefore have to be smaller and this can be achieved by setting $\lambda_0^* > \lambda_0^m$, implying that $K_t^m \leq K(\lambda_t^m) < K(\lambda_t^*)$ for $t \leq T^R$ and $K_t^* = Min\{K_t^m, \hat{K}^R\}$. A similar argument reveals that $Q^m < X_0$ entails $\lambda_0^* < \lambda_0^m$, so that K_t^m crosses $K(\lambda_t^*)$ at some date prior to T^R, giving rise to a NSMRAP policy. We summarize the above in

Claim 5: (*i*) When $\hat{K}^0 < K^{cr}$ and $Q^m \geq X_0$, then $K_t^* = Min\{K_t^m, \hat{K}^R\}$ is a simple MRAP to $\hat{K}^R$ and λ_0^* ($\geq \lambda_0^m$) is determined together with the depletion date T^* ($\leq T^R$) by (11)-(12).
(*ii*) When $\hat{K}^0 < K^{cr}$ and $Q^m < X_0$, then K_t^* follows a NSMRAP to $\hat{K}^R$. The parameters λ_0^*, T^* and $\tau < T^R$ (where τ is the time K_t^m overtakes the root process) are obtained by solving (11)-(12) and $\tau - \log[(\bar{K} - K_0)/(\bar{K} - K(\lambda_\tau^*))]/\delta = 0$.

A detailed proof of Claim 5 is given in the appendix. We only note here that in case (*i*) depletion occurs at or before the entry to steady state, whereas in the latter scenario involving the NSMRAP, the fresh water stock is depleted exactly at the steady state entrance date, so that $K_{T^*}^* = \hat{K}^R$. In this case depletion occurs after T^R, because the root process, which is slower than K_t^m, arrives at the steady state $\hat{K}^R$ at a later date. This delay is designed to take advantage of the large initial fresh water stock. So long as this relatively cheap resource can be exploited above the recharge rate, it does not pay to arrive too early at the knowledge steady state, which is optimal only when fresh water supply is restricted to the recharge rate.

The optimal water policy

We close with a summary of the optimal policy:

(i) If $\hat{K}^0 \geq K^{cr}$, then the optimal R&D policy is a standard MRAP to $\hat{K}^0$, i.e., $K_t^* = Min\{K_t^m, \hat{K}^0\}$. If, in addition, $X_0 \geq Q(T^{cr})$, then the fresh water stock is *not scarce*, i.e., $\lambda_t^* = 0$ and the stock will never be depleted, whereas $X_0 < Q(T^{cr})$ gives rise to the following scarcity rent process: (*a*) $\lambda_t^* = \lambda_0^* e^{(r+\xi)t}$ for $0 \leq t \leq T^*$, where λ_0^* (>0) and T^* are determined by (11)-(12); (*b*) $\lambda_t^* = M_s(K_t^m) - M_c(R(0))$ for $T^* \leq t \leq T^{cr}$; and (*c*) $\lambda_t^* = 0$ for $t > T^{cr}$. The steady state fresh water stock is positive.

(ii) If $\hat{K}^0 < K^{cr}$ and $Q^m \geq X_0$, then the optimal R&D policy is a standard MRAP to $\hat{K}^R$, i.e., $K_t^* = Min\{K_t^m, \hat{K}^R\}$ and $\lambda_t^* = \lambda_0^* e^{(r+\xi)t}$ for $t \leq T^*$ and $\lambda_t^* = M_s(K_t^*) - M_c(R(0))$ for $t > T^*$, where λ_0^* is determined together with the depletion date T^* by (11)-(12). Depletion will occur no later than the entrance date to the steady state $\hat{K}^R$.

(iii) If $\hat{K}^0 < K^{cr}$ and $Q^m < X_0$, then the optimal R&D policy is a NSMRAP to $\hat{K}^R$ and the optimal scarcity process is $\lambda_t^* = \lambda_0^* e^{(r+\xi)t}$ for $t \leq T^*$ and remains at the level $\lambda_0^* e^{(r+\xi)T^*}$ thereafter. The parameters λ_0^*, T^* and the transition date $\tau < T^R$ are obtained by solving (11)-(12) and $\tau - \log[(\bar{K} - K_0)/(\bar{K} - K(\lambda_\tau^*))]/\delta = 0$. Depletion of the fresh water stock and entrance to the steady state occur simultaneously.

(iv) Given K_t^* and λ_t^*, the optimal time profiles of fresh and desalinated water supply rates are specified in (10).

Tsur and Zemel (1998b) derived similar properties for nonrenewable resources. The knowledge steady states corresponding to the renewable and nonrenewable models differ, however, because the steady state supply of the nonrenewable case is derived exclusively from the backstop resource, whereas in the renewable case it can be derived from both sources. The nonrenewable case, therefore, admits a unique steady state corresponding to the upper bound of $K(\lambda)$. In contrast, the present renewable situation yields lower knowledge steady states because the lower equilibrium supply rates of desalinated water cannot justify such high investment rates on R&D. Indeed, when $\hat{K}^0 \geq K^{cr}$, the optimal steady state corresponds to the lower bound of $K(\lambda)$. This result is in accord with the economic intuition that the irreversible nature of the depletion of a nonrenewable resource calls for higher investments in the development of an alternative technology.

The time profiles of the primary resource stocks in both cases are also contrasted: While the stock of a nonrenewable resource decreases monotonically, that of a renewable resource may initially decrease to depletion and later on increase back, as technological progress reduces exploitation of the primary resource below its maximal recharge rate.

4. Time-dependent demand

Water demand increases with time due to the rising standard of living and population growth. This feature is described by allowing the demand $D(p,t)$ to depend explicitly on time. In this section we examine the effects on the optimal water policy caused by the explicit time-dependence of water demand. We find that the NSMRAP nature of the optimal R&D policy is preserved, although the associated root process is quite different. We present below the main results; the derivations follow closely the analysis in the appendix and are omitted.

An increasing demand induces a corresponding time dependence on the optimal supply rates, but the nature of the relation between these rates and the intersection points of the supply and demand curves, as presented by equations (7)–(10) and Figure 1, remains unaltered. When some desalinated water is desirable at the outset (i.e., when $M_s(K_0)$ lies below the intersection of $M_c(q)$ and $D^{-1}(q,0)$), the fresh water supply rate $q^c(K_t,\lambda_t)$ remains as specified in (10a), but the desalination rate $q^s(K,\lambda,t) = D(M_s(K_t),t) - q^c(K_t,\lambda_t)$ attains an explicit time argument.

The continuity property of the fresh water supply rate at the depletion time, as expressed by condition (11), remains valid. Thus, given the optimal knowledge and scarcity rent processes, the determination of the optimal supply rates is essentially the same as in the reference model for which demand is stable in time.

The modifications needed for the optimal knowledge process are subtler. We find that the optimal knowledge process is again a NSMRAP but the associated root processes obtain explicit time dependence. The root process $K(\lambda,t)$ is defined by the solution of

$$-M_s'(K)q^s(K,\lambda,t) - (r+\delta) = 0$$

and inherits the explicit time dependence from q^s. The steady states $\hat{K}^0$ and $\hat{K}^R$, which were *constant* in the reference model, also depend on time because $D(M_s(K),t)$ introduces a time dependence to the evolution functions that define them. In contrast, the critical level $K^{cr} = M_s^{-1}(M_c(R(0)))$ depends on the cost structure alone and is therefore unaffected by incorporating time dependence to water demand.

With $q^s(K,\lambda,t)$, $K(\lambda,t)$, $\hat{K}^0(t)$ and $\hat{K}^R(t)$ substituting their corresponding counterparts of the reference case, the characterization of the optimal water policy follows that of the reference model. The above substitutions, however, have some important consequences.

First, the relative location of the parameters $\hat{K}^0(t)$ and K^{cr}, which plays a major role in the classification of the reference model, now changes with time. Therefore, only if $\hat{K}^0(t)$ exceeds K^{cr} early enough, and if the initial fresh stock is large enough, can we expect a non-depletion (and zero scarcity) policy. Under such a policy, demand increases will be exclusively met by cheap desalinated water.

Second, the sharp distinction made in the reference model between the *standard* MRAP (where R&D is carried out at the maximum feasible rate until the steady state is reached) and the NSMRAP (where the approach to the steady state includes a ride along the root process) no longer holds. This is so because the steady state turns now into a process on its own, to be followed by the optimal process at a reduced investment rate during the singular stage.

Nonetheless, the characteristic property of the optimal knowledge process in the reference model—of a NSMRAP to the appropriate root process—follows directly from Spence and Starrett's (1975) analysis and remains intact. The optimal knowledge policies under stationary and non-stationary demands differ only inasmuch as the respective root processes are different. We note, in this context, that although the non-stationary demand does not permit the supply policy to settle at a steady state since the rate of desalinated water supply has to meet the increasing demand, the bounded knowledge process must eventually settle at some stationary level. Knowledge will stabilize either at the level which cannot contribute to further decreases in the cost of desalination (which must have some lower bound that cannot be improved upon), or at the exogenous upper bound $\overline{K}$, whichever comes first.

5. Closing Comments

Water scarcity can induce responses of various kinds. First, it might lead to conflicts and competition among nations, regions or sectors (see, e.g., the collection of works edited by Biswas, 1994, by Dinar and Loehman, 1995, and by Just and Netanyahu, 1998). Alternatively, it can encourage steps towards more efficient use of water via improved irrigation and distribution systems, quality-differentiated supplies and efficient pricing (see Tsur and Dinar, 1997, and works in Parker and Tsur, 1997). Finally, when the futility of the first approach is recognized and the potential of the second approach is realized, one may turn to the development of alternative sources, namely desalination technologies. This work is concerned with the third approach, focusing attention on its intertemporal aspects, particularly on the optimal scheduling of the R&D activities.

In an earlier paper (Tsur and Zemel, 1998b), we have derived optimal rules for the development of solar technologies in light of the environmental costs of fossil energy and its finite reserves. Both works consider the optimal development of backstop substitutes for a limiting primary resource--fossil energy then, fresh water now. The difference between the two stems from the fact that fresh water is typically renewable whereas fossil deposits are not. The presence of recharge processes renders the scarcity of the primary resource less crucial, but at the same time it changes the optimal policy quite substantially. For example, it is possible in the present case that the fresh water stock will be first depleted and eventually refill (fully or partly) as technological progress reduces the cost of desalination to the extent that fresh water exploitation decreases below the recharge rate. Such behavior is, of course, impossible for nonrenewable deposits. The time profile of the primary resource stock has far reaching implications for the optimal R&D policy, since the latter depends crucially on the scarcity (shadow) price of the former.

Sure enough, many regions around the Globe have all the water they need from local, fresh sources. But the number of water-scarce regions is growing by the year and in many desalinated seawater is (or will be) cheaper than fresh water conveyed from remote sources. When desalination is (eventually) desirable, our analysis lends support to the view that its development should be made well in advance times of water shortage.

Long-term development programs are fraught with uncertainty. Examples include uncertainty concerning the availability of the primary resource (due, e.g., to uncertain initial stock or quality degrading processes) and future trends and fluctuations in demand. Tsur and Zemel (1995) studied the effects of the first type of uncertainty on fresh water exploitation policies. Extending this model to the present context (of exploitation and R&D policies), as well as the consideration of other uncertain sources, remains a challenge for future research.

APPENDIX: Derivation of the optimal policy

We present below the formal proofs of Claims 1-5 of Section 3, characterizing the optimal supply rule and R&D policy.

Preliminaries: Let T denote the time at which the stock of fresh water is first depleted. The optimization problem (6) is recast as

$$V(X_0, K_0) = Max_{\Gamma,T} \int_0^T [G(q_t^c + q_t^s) - C(q_t^c) - M_s(K_t)q_t^s - I_t]e^{-rt}dt + e^{-rT}V(0, K_T) \qquad \text{(A1)}$$

subject to the same constraints. The current-value Hamiltonian for (A1) is of the form

$$H_t = G(q_t^c + q_t^s) - C(q_t^c) - M_s(K_t)q_t^s - I_t + \lambda_t[R(X_t) - q_t^c] + \gamma_t(I_t - \delta K_t),$$

where λ_t and γ_t are the current-value costate variables corresponding to X_t and K_t, respectively. Incorporating the Lagrange multipliers associated with the constraints on q^c, q^s and I_t, the Lagrangian $\Im_t = H_t + \alpha_t^c q_t^c + \alpha_t^s q_t^s + \alpha^0 I_t + \alpha^I(\bar{I} - I_t)$ is obtained.

Necessary conditions include (see Leonard and Long (1992); all variables are evaluated at their optimal values):

(a) $Max_{q^c,q^s}\{\Im_t\} \Rightarrow D^{-1}(q_t^c + q_t^s) - M_c(q_t^c) - \lambda_t + \alpha_t^c = 0$ and $D^{-1}(q_t^c + q_t^s) - M_s(K_t) + \alpha_t^s = 0$, hence

$$M_s(K_t) = M_c(q_t^c) + \lambda_t \qquad \text{(A2)}$$

$$q_t^c + q_t^s = D(M_s(K_t)) \qquad \text{(A3)}$$

hold along the optimal plan whenever q^c and q^s are both positive and the corresponding Lagrange multipliers vanish. This establishes the optimal supply rule (10).

(b) Maximizing the Lagrangian with respect to I_t reveals that I_t equals 0 or $\bar{I}$ whenever $\gamma_t \neq 1$. Thus, I_t can undergo a discontinuity only at the singular value $\gamma_t = 1$.

(c) $\dot{\lambda} - r\lambda = -\partial H / \partial X = \lambda\xi$ whenever X does not vanish, yielding $\lambda_t = \lambda_0 e^{(r+\xi)t}$.

(d) $\lambda_0 X_T = 0$ is the transversality condition associated with $X_T \geq 0$, implying that $\lambda_0 = 0$ if the fresh water stock is never empty.

(e) $\gamma_T = \partial V(0,K_T)/\partial K$ is the transversality condition associated with the free value of K_T.

(f) $H_T = rV(0,K_T)$ is the transversality condition associated with the free choice of T.

Proof of Claim 1: Let $q_-^c = lim_{t\uparrow T}\, q_t^c$; $q_+^c = lim_{t\downarrow T}\, q_t^c$ be the limiting pre- and post depletion supply rates of fresh water (the subscripts + and – denote the corresponding pre- and post depletion limits of other quantities as well.) We need to show that

$$q_-^c = q_+^c = R(0). \tag{A4}$$

This means that the fresh water stock will *not* be depleted before the marginal cost of fresh water is high enough to exclude its supply above the natural recharge rate.

Since γ_+ is the initial knowledge shadow price for the post depletion problem, it follows that $\partial V(0,K_T)/\partial K = \gamma_+$. Moreover, for the pre depletion problem, condition (e) above reads $\gamma_- \equiv \gamma_T = \partial V(0,K_T)/\partial K$. Thus, the costate variable γ_t evolves smoothly as the pre depletion problem turns into the post depletion problem at the depletion time T. In view of condition (b), the quantity $I_t(\gamma_t - 1)$ is also continuous on that date.

The Bellman equation for the post depletion value reads

$$\begin{aligned} &rV(0,K_T) = G(D(M_s(K_T))) \\ &-M_s(K_T)[D(M_s(K_T)) - q_+^c] - C(q_+^c) - I_+ + \gamma_+(I_+ - \delta K_T) \end{aligned} \tag{A5}$$

where we have used again the fact that $\partial V(0,K_T)/\partial K = \gamma_+$.

The transversality condition (f), $H_- \equiv H_T = rV(0,K_T)$, where

$$\begin{aligned} &H_- = G(D(M_s(K_T))) - M_s(K_T)[D(M_s(K_T)) - q_-^c] \\ &-C(q_-^c) - I_- + \gamma_-(I_- - \delta K_T) + \lambda_-[R(0) - q_-^c] \end{aligned} \tag{A6}$$

is compared with (A5), using the continuity of γ_t and $I_t(\gamma_t - 1)$ at $t = T$. We find that

$C(q_-^c) - C(q_+^c) - M_s(K_T)(q_-^c - q_+^c) + \lambda_-(q_-^c - R(0)) = 0$, or

$C(q_-^c) - C(q_+^c) - M_s(K_T)(q_-^c - q_+^c) + \lambda_-(q_-^c - q_+^c) + \lambda_-(q_+^c - R(0)) = 0$, which reduces, using

(A2), to $C(q_-^c) - C(q_+^c) - M_c(q_-^c)(q_-^c - q_+^c) + \lambda_-(q_+^c - R(0)) = 0$.

Now, to deplete the stock we require $q_-^c \geq R(0)$ while following depletion $q_+^c \leq R(0)$. Thus, we write $C(q_-^c) - C(q_+^c) = M_c(\tilde{q}^c)(q_-^c - q_+^c)$, where $q_+^c \leq \tilde{q}^c \leq q_-^c$, hence $[M_c(\tilde{q}^c) - M_c(q_-^c)](q_-^c - q_+^c) = \lambda_-(R(0) - q_+^c) \geq 0$. However, since $M_c(q^c)$ increases with q^c, the left hand side cannot be positive, implying that these conditions must hold as equalities, as (A4) states. Condition (11) follows from (A2) whereas (12) restates that $X_{T^*} = 0$. □

Proof of Claim 2: We first show that given the optimal scarcity rent λ_t^*, the optimal R&D policy (I_t^*, K_t^*) can be obtained as the solution of the one-dimensional problem

$$V(K_0) = K_0 + Max_{\{I_t\}} \int_0^\infty \vartheta(K_t, t) e^{-rt} dt \tag{A7}$$

subject to $\dot{K} = I_t - \delta K$, $0 \leq I_t \leq \bar{I}$ and K_0 given, where

$$\vartheta(K,t) = G(q^c + q^s) - C(q^c) - \lambda_t^* q^c - M_s(K)q^s - (r+\delta)K$$

and $q^c = q^c(K, \lambda_t^*)$, $q^s = q^s(K, \lambda_t^*)$ are given by the optimal supply rule. The integrand $\vartheta(K,t)$, denoted the *equivalent utility*, is independent of the control I and its explicit time dependence enters through the scarcity rent $\lambda_t^* = \lambda_0^* e^{(r+\xi)t}$. Consider first

$$V(K_0) = Max_{\{I_t\}} \int_0^\infty \tilde{\vartheta}(K_t, I_t, t) e^{-rt} dt \tag{A8}$$

subject to the same constraints and supply rule as in (A7), where

$$\tilde{\vartheta}(K,I,t) = G(q^c + q^s) - C(q^c) - \lambda_t^* q^c - M_s(K)q^s - I.$$

It is verified that the necessary conditions corresponding to (A8) coincide with the necessary conditions associated with I_t, K_t and the costate variable γ_t of the original problem (6). Following Spence and Starrett (1975), we use (3) to remove I from $\tilde{\vartheta}$. Integrating the resulting $\dot{K}$ term by parts, we obtain (A7).

Differentiating $\vartheta(K,t)$ with respect to K, noting that (A2)-(A3) imply that the terms involving $\partial q^c/\partial K$ and $\partial q^s/\partial K$ vanish, gives $\partial\vartheta/\partial K = L(K,\lambda)$ (c.f. (14)), and the unique root $K(\lambda_t)$ maximizes $\vartheta(K,t)$ at any time t. Now, the analysis of Spence and Starrett (1975) shows that the MRAP to the maximum of the equivalent utility is the optimal process for this type of problems, characterized by utilities which do not depend explicitly on the controls. Indeed, the problem at hand is not autonomous due to the time dependence introduced by the scarcity rent λ_t^*. However, these authors have established (see their footnote, p. 394), that the same result applies when the MRAP process follows the root process rather than a stationary maximum. Once the root process has been reached, $\vartheta(K,t)$ must be maintained at its maximum by tuning I_t so as to ensure that $K_t = K(\lambda_t)$, as specified in (15). □

Two immediate corollaries follow:

Corollary 1: The optimal process K_t^* cannot decrease.

Proof: Initiated below the root process, K_t^* can only increase towards the root process but not exceed it. Once on the root process, K_t^* can decrease only if the latter decreases. This cannot happen before the fresh water stock is depleted, since before depletion the scarcity rent is either zero or increases exponentially, and $K(\lambda)$ increases with λ. For a period of vanishing stock, with fresh water extraction at the recharge rate $R(0)$, λ may decrease (see below). However, K_t^* must differ from the root process during that period through which, according to (A2), $M_s(K_t^*) = M_c(R(0)) + \lambda_t$, and a decrease in λ_t implies that K_t^* must increase. □

Corollary 2: The optimal process K_t^* must converge to a steady state.

The corollary follows from Corollary 1 and the fact that K_t^* is bounded. □

Proof of Claim 3: For convenience we repeat the notation introduced in the text:

$\hat{K}^0 = K(0)$ is the root of $L(K,0)$ satisfying $-M_s'(\hat{K}^0)q^s(\hat{K}^0,0) = r+\delta$.

$K^{cr} = M_s^{-1}(M_c(R(0)))$; $K > K^{cr}$ implies $q^c(K,\lambda) < R(0)$ for any λ.

$K_t^m = (1-e^{-\delta t})\overline{K} + K_0 e^{-\delta t}$ is the standard MRAP initiated at K_0.

$T^{cr} = \frac{1}{\delta}\log[(\overline{K}-K_0)/(\overline{K}-K^{cr})]$ is the time when the process K_t^m passes through K^{cr}.

$Q(T) = \int_0^T q^c(K_t^m,0)e^{\xi t}dt - \overline{X}(e^{\xi T}-1)$ (if $Q(T)>X_0$ then $X_T<0$ and $q^c(K_t^m,0)$ is not feasible).

Suppose that $\lambda_0 = 0$. Then, the root process reduces to the stationary point $\hat{K}^0$ and, according to Claim 2, $K_t^* = Min\{K_t^m, \hat{K}^0\}$ and $q^c(K_t^*,0)$ is the optimal fresh water supply. If $\hat{K}^0 < K^{cr}$ then $K_t^* < K^{cr}$ and $q^c(K_t^*,0) > R(0)$ at all times t. The stock will therefore be depleted on a finite date, at which time q^c must undergo a discontinuous drop to $R(0)$, violating Claim 1. Thus, $\hat{K}^0 \geq K^{cr}$ and the optimal process must pass through K^{cr}. However, if $X_0 < Q(T^{cr})$, then $q^c(K_t^*,0)$ is not feasible and the fresh water stock will be depleted prior to T^{cr}, implying again a discontinuity in q^c. Indeed, the second condition of Claim 3 is required to ensure that the initial stock suffices to support $q^c(K_t^*,0)$. Otherwise, a positive scarcity rent is called for.

To see that the conditions of Claim 3 suffice, suppose that both $\hat{K}^0 \geq K^{cr}$ and $X_0 \geq Q(T^{cr})$ hold. Then, $X_0 \geq Q(T)$ for all T. (If $X_0 < Q(T)$ for $T < T^{cr}$, then $X_0 < Q(T^{cr})$ since $q^c(K_t^m,0) > R(0)$ for $K_t^m < K^{cr}$, which holds for $T < t < T^{cr}$, violating the assumed condition. Similarly, if $X_0 \geq Q(T)$ holds at T^{cr} it must hold for any $T > T^{cr}$ since $q^c(K,0) < R(0)$ for $K > K^{cr}$). Thus, the fresh water stock is never depleted using $q^c(K_t^m,0)$. Now, assume that $\lambda_0^* > 0$. Then $K_t^* \geq Min\{K_t^m, \hat{K}^0\}$ since the root process always lies above $\hat{K}^0$. But $q^c(K,\lambda)$ decreases in both arguments, hence $q^c(K_t^*,\lambda_t^*) < q^c(K_t^m,0)$ for all $t < T^{cr}$ and the fresh water stock is never depleted under the optimal policy $q^c(K_t^*,\lambda_t^*)$, violating the transversality condition (d) $X_T\lambda_0 = 0$. □

Proof of Claim 4: To verify that $K_t^* = Min\{K_t^m, \hat{K}^0\}$, note that $K(\lambda) \geq \hat{K}^0$ for any nonnegative λ. Claim 2, then, requires that K_t^* must follow the K_t^m at least up to $\hat{K}^0$. Since $\hat{K}^0 \geq K^{cr}$, the fresh water stock cannot vanish upon arrival at $\hat{K}^0$ or thereafter (with a positive λ). Hence, the shadow price must vanish upon arrival at $\hat{K}^0$, implying that the root process reduces to $\hat{K}^0$ from that time on, hence $K_t^* = Min\{K_t^m, \hat{K}^0\}$.

(*i*) From Claim 3 we know that $\lambda_0^* > 0$, hence the fresh water stock must be depleted at or before T^{cr} (after T^{cr}, $K_t^* > K^{cr}$ and depletion cannot occur). The values of λ_0^* and T^* must conform to Claim 1.

(*ii*) Following depletion, the fresh water supply rate is restricted to $R(0)$. Equation (A2), then, gives $\lambda_t^* = M_s(K_t^*) - M_c(R(0)) = M_s(K_t^*) - M_s(K^{cr})$ as long as this quantity is not negative, i.e., during the period $T^* \leq t \leq T^{cr}$. The supply mix is $R(0)$ and $D(M_s(K_t^*)) - R(0)$ for fresh and desalinated water, respectively.

(*iii*) At T^{cr}, $K_t^m = K^{cr}$, the shadow price vanishes and the third phase begins. As knowledge accumulates, $K_t^* \geq K^{cr}$, $q^c(K_t^*,0)$ decreases below $R(0)$ and desalination makes up the remaining demand. The fresh water stock fills up, eventually to enter a steady state at the stock level $\hat{X} = \bar{X} - q^c(\hat{K}^0,0)/\xi \geq 0$, equality holding only if $\hat{K}^0 = K^{cr}$. □

We turn to the case $\hat{K}^0 < K^{cr}$. This case involves a different steady state, namely the root $\hat{K}^R$ of $L^R(K) = -M_s'(K)[D(M_s(K)) - R(0)] - (r+\delta)$, (cf. (17)). In terms of the root process, this steady state can be written as $\hat{K}^R = K(\lambda^R)$, where $\lambda^R = M_s(\hat{K}^R) - M_c(R(0))$ is shown in Lemma 1 below to be positive. It is useful to distinguish between the date $T^R = \frac{1}{\delta}\log[(\bar{K} - K_0)/(\bar{K} - \hat{K}^R)]$ when the MRAP K_t^m passes through $\hat{K}^R$ and the time T^{*R} when the optimal process K_t^* enters $\hat{K}^R$: $K^*_{T^{*R}} = \hat{K}^R$. Since no process can proceed faster than K_t^m, it must be that $T^{*R} \geq T^R$.

We recall the benchmark scarcity rent $\lambda_0^m = \lambda^R e^{-(r+\xi)T^*}$, the corresponding process $\lambda_t^m = \lambda_0^m e^{(r+\xi)t}$ and the benchmark quantity $Q^m = \int_0^{T^R} q^c(K_t^m, \lambda_t^m) e^{\xi t} dt - \bar{X}(e^{\xi T^R} - 1)$. The proof of claim 5 is presented via a series of Lemmas:

Lemma 1: When $\hat{K}^0 < K^{cr}$, then $\hat{K}^R \in (\hat{K}^0, K^{cr})$.

Proof: Suppose that $\hat{K}^R \geq K^{cr}$, so that $M_s(\hat{K}^R) \leq M_s(K^{cr}) = M_c(R(0))$ and $q^c(\hat{K}^R, 0) \leq R(0)$. Hence, $q^s(\hat{K}^R, 0) = D(M_s(\hat{K}^R)) - q^c(\hat{K}^R, 0) \geq D(M_s(\hat{K}^R)) - R(0)$ and, since $M_s(K)$ is decreasing, $L(\hat{K}^R, 0) \geq L^R(\hat{K}^R) = 0 = L(\hat{K}^0, 0)$, hence $\hat{K}^R \leq \hat{K}^0 < K^{cr}$, violating the assumption that $\hat{K}^R \geq K^{cr}$. Indeed, with $\hat{K}^R < K^{cr}$, we verify that $\lambda^R \equiv M_s(\hat{K}^R) - M_c(R(0)) = M_s(\hat{K}^R) - M_s(K^{cr}) > 0$. Moreover, the definition of λ^R implies that $q^s(\hat{K}^R, \lambda^R) = D(M_s(\hat{K}^R)) - R(0)$, hence both $L^R(K)$ and $L(K, \lambda^R)$ vanish at $\hat{K}^R$ and $\hat{K}^R = K(\lambda^R) > K(0) = \hat{K}^0$. The situation is depicted in Figure 2. □

Lemma 2: When $\hat{K}^0 < K^{cr}$, the optimal steady state is at $X = 0$, $K = \hat{K}^R$ and $\lambda = \lambda^R$.

Proof: Assume that the fresh water steady state stock is not empty. The corresponding scarcity rent must vanish, implying that $\hat{K}^0$ is the steady state knowledge level. But when $\hat{K}^0 < K^{cr}$ the fresh water supply rate $q^c(\hat{K}^0, 0)$ exceeds $R(0)$ and the finite stock must be depleted. Thus, when $\hat{K}^0 < K^{cr}$ the steady state occurs with an empty stock and $\lambda_t^* > 0$. The fresh water supply rate at the steady state must therefore equal $R(0)$, implying, in view of Claim 2, that $\hat{K}^R$ is the knowledge steady state. Since $\hat{K}^R = K(\lambda^R)$, it follows from Claim 2 again that λ^R is the scarcity rent at the steady state. □

Lemma 3: When $\hat{K}^0 < K^{cr}$, the optimal processes K_t^* and λ_t^* enter their respective steady states $\hat{K}^R$ and λ^R at or after the fresh water stock depletion date, i.e., $T^{*R} \geq T^*$. During $0 \leq t \leq T^*$, λ_t^* increases exponentially. If $T^{*R} > T^*$, then during $T^* \leq t \leq T^{*R}$ the scarcity rent decreases back to its steady state level λ^R and the process $K(\lambda_t^*)$ is non-monotonic.

Proof: Suppose $K_t^* = \hat{K}^R$ at some $t < T^*$ and recall that $q_t^c > R(0)$ prior to depletion. Then, using (A2), $M_c(q_t^c) + \lambda_t = M_s(\hat{K}^R) = M_c(R(0)) + \lambda^R$, implying that $\lambda_t < \lambda^R$. It follows that $K(\lambda_t) < K(\lambda^R) = \hat{K}^R = K_t^*$. But the optimal knowledge cannot exceed the root process and we conclude that $\hat{K}^R$ cannot be entered prior to depletion, so that $T^{*R} \geq T^*$ and $K_{T^*}^* \leq \hat{K}^R$. Using (11) we find $M_c(R(0)) + \lambda^R = M_s(\hat{K}^R) \leq M_s(K_{T^*}^*) = M_c(R(0)) + \lambda_{T^*}^*$, hence $\lambda_{T^*}^* \geq \lambda^R$. If the strong inequality holds and $T^{*R} > T^*$, the shadow price (and the corresponding root process) must decrease after T^* until they reach λ^R and $\hat{K}^R$, respectively, at T^{*R}. □

By Claim 2, $K_t^* = Min\{K_t^m, K(\lambda_t^*)\}$. One possibility is that K_t^m lags behind $K(\lambda_t^*)$ before $\hat{K}^R$ is reached and the optimal process is a standard MRAP to $\hat{K}^R$. The alternative is that K_t^m overtakes the root process at an earlier date, and the optimal K-process is a NSMRAP, following the root process at its final stage. To identify the conditions under which either of these cases hold, we need

Lemma 4: Suppose $\hat{K}^0 < K^{cr}$. Then, (*i*) if $T^* < T^R$ then $T^{*R} = T^R$ and K_t^* follows the standard MRAP to $\hat{K}^R$; (*ii*) if $T^* > T^R$ then $T^{*R} = T^*$ and K_t^* follows the NSMRAP before arriving at $\hat{K}^R$; (*iii*) if $T^* = T^R$ then $T^{*R} = T^*$ and K_t^* follows the standard MRAP as in (*i*).

Proof: (*i*) Suppose $T^* < T^R \leq T^{*R}$. According to Lemma 3, the root process is non-monotonic, exceeding $\hat{K}^R$ at the depletion date and returning to it at T^{*R}. If the optimal process were to follow the root process before T^{*R}, it must also be non-monotonic, contradicting Corollary 1. Thus, $K_t^* = K_t^m$ all the way to $\hat{K}^R$.

(*ii*) Suppose $T^* > T^R$. Then $T^{*R} \geq T^* > T^R$ and K_t^* departs from K_t^m to follow $K(\lambda_t^*)$ before arriving at $\hat{K}^R$. But the optimal process is monotonic, hence the root process must also be monotonic, which according to Lemma 3 can occur only if $T^{*R} = T^*$.

(*iii*) Suppose $T^* = T^R$ but $T^{*R} > T^*$. According to Lemma 3, the root process is non-monotonic and cannot be followed by K_t^*, which must therefore proceed with the standard MRAP all the way to $\hat{K}^R$. It follows that K_t^m and K_t^* reach $\hat{K}^R$ on the same date, contradicting our assumption that $T^{*R} > T^R$. Thus, $T^{*R} = T^* = T^R$ and $K_t^* = Min\{\hat{K}^R, K_t^m\}$. □

Whether or not K_t^m overtakes $K(\lambda_t^*)$ depends on the initial scarcity rent λ_0, as $K(\lambda_t)$ increases with $\lambda_t = \lambda_0 e^{(r+\xi)t}$. With the benchmark process λ_t^m as defined above, $K(\lambda_{T^R}^m) = K(\lambda^R) = \hat{K}^R = K_{T^R}^m$ and $K(\lambda_t^m)$ meets K_t^m at $t = T^R$. The root process is assumed to be slower than K_t^m and the two processes cannot cross twice. It follows that for any $\lambda_0 \geq \lambda_0^m$, $K(\lambda_0 e^{(r+\xi)t}) \geq K_t^m$ for all $t \leq T^R$, whereas if $\lambda_0 < \lambda_0^m$ the two processes must cross prior to T^R. In view of Claim 2, this observation implies

Lemma 5: Suppose $\hat{K}^0 < K^{cr}$. (*i*) If $\lambda_0^* \geq \lambda_0^m$ then $K_t^* = K_t^m$ until T^R; (*ii*) if $\lambda_0^* < \lambda_0^m$ then $K_t^* = K_t^m$ for $t \leq \tau$ and $K_t^* = K(\lambda_t^*)$ for $t > \tau$, where $0 < \tau < T^R$ is the date K_t^m crosses $K(\lambda_t^*)$.

To establish which of the two cases in Lemma 5 applies, we need

Lemma 6: Suppose $\hat{K}^0 < K^{cr}$. Then $\lambda_0^* \geq \lambda_0^m$ if and only if $Q^m \geq X_0$.

Proof: Assume first that $Q^m \geq X_0$. This implies that under the (K_t^m, λ_t^m) policy the stock is depleted before or at time T^R. Suppose that $\lambda_0^* < \lambda_0^m$. The process K_t^m is not slower than any feasible policy hence $K_t^* \leq K_t^m$ for all $t \leq T^R$. Moreover, since $q^c(K,\lambda)$ decreases in both arguments, $q^c(K_t^m, \lambda_t^m) < q^c(K_t^*, \lambda_t^*)$ and the optimal policy yields $T^* < T^R$. However, according to Lemma 5, $\lambda_0^* < \lambda_0^m$ implies that the root process is adopted before T^R, which entails, according to Lemma 4, $T^* > T^R$, contradicting our previous assumtion. Thus, $Q^m \geq X_0$ implies $\lambda_0{}^* \geq \lambda_0{}^m$.

Suppose now that $\lambda_0{}^* \geq \lambda_0{}^m$, hence the optimal policy is the standard MRAP $K_t^* = K_t^m$ until T^R. Thus, $q^c(K_t^m, \lambda_t^m) \geq q^c(K_t^*, \lambda_t^*)$ and depletion under the optimal policy cannot precede depletion under the (K_t^m, λ_t^m) policy. From Lemma 4 we know that $T^* \leq T^R$, hence depletion under the (K_t^m, λ_t^m) policy cannot occur later then T^R, so that $Q^m \geq X_0$. □

Proof of Claim 5: (*i*) When $\hat{K}^0 < K^{cr}$ and $Q^m \geq X_0$, then according to Lemma 6, $\lambda_0{}^* \geq \lambda_0{}^m$. Lemma 5, in turn, implies that $K_t^* = Min\{K_t^m, \hat{K}^R\}$ is a simple MRAP to $\hat{K}^R$. (*ii*) When $\hat{K}^0 < K^{cr}$ and $Q^m < X_0$, then according to Lemma 6, $\lambda_0{}^* < \lambda_0{}^m$. Lemma 5, in turn, implies that K_t^* follows a NSMRAP to $\hat{K}^R$. □

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Figure 1: *Right panel*: Water supply and demand at time t, given K_t and λ_t. The area ABCD represents the sum of consumer and producer surpluses. *Left panel*: Marginal cost of desalination as a function of knowledge.

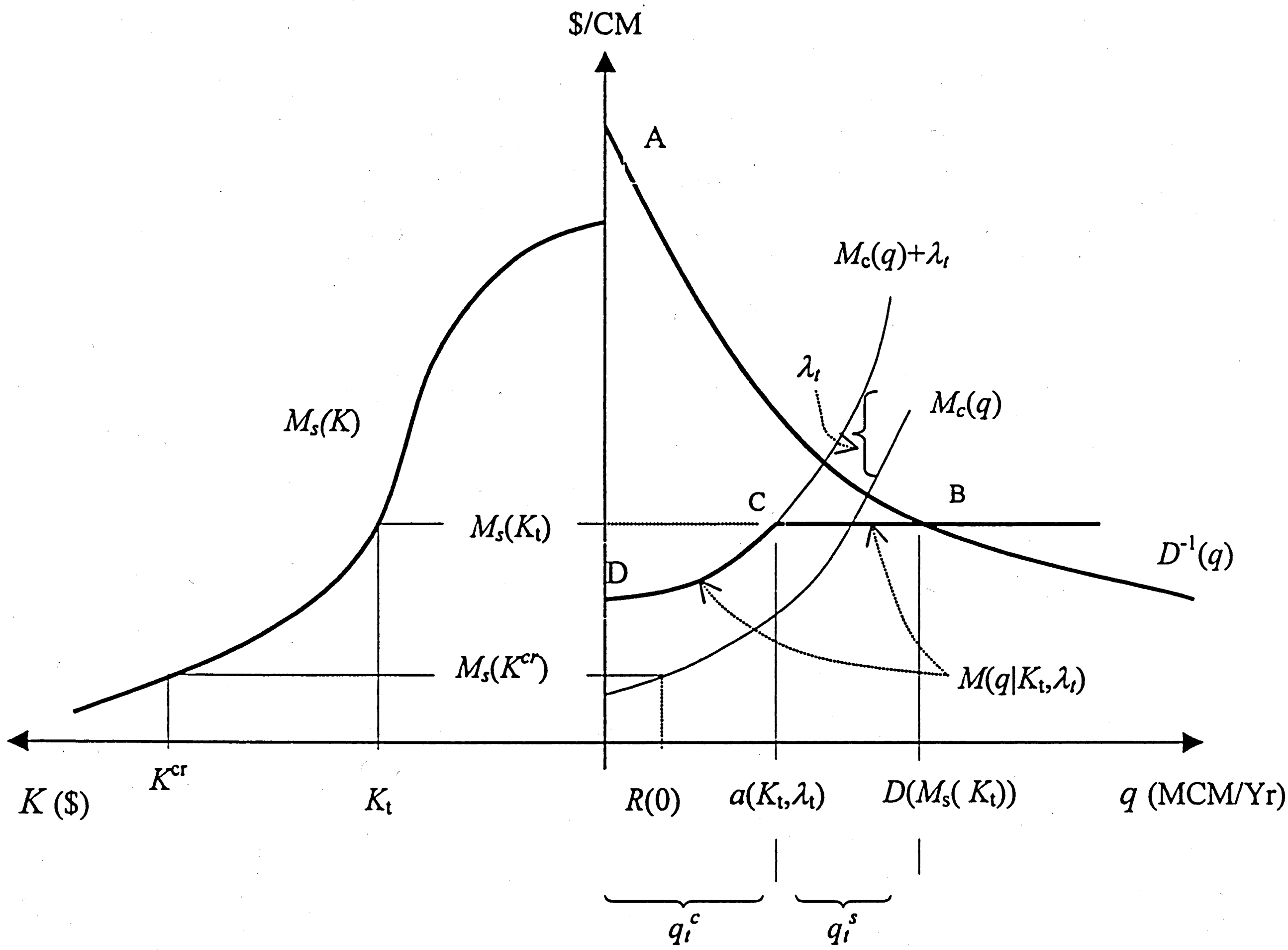
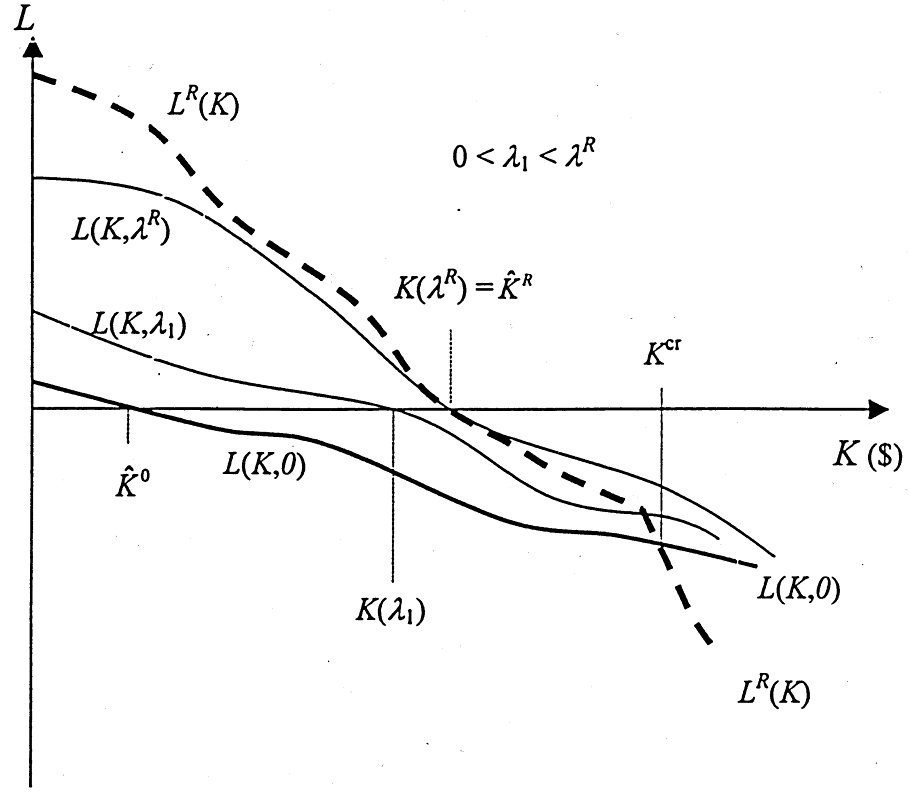


Figure 2: The evolution functions $L(K,\lambda)$ (Equation 14) and $L^R(K)$ (Equation 17) vs. the knowledge level K when $K^{cr} > \hat{K}^0$. $\hat{K}^0$ is the root of $L(K,0)$, K^{cr} is the critical knowledge level in which $M_s(K^{cr}) = M_c(R(0))$ and is also the intersection of $L^R(K)$ and $L(K,0)$. Both $L^R(K)$ and $L(K,\lambda^R)$ vanish at $\hat{K}^R$.



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