EVALUATING EXPENDITURE INEQUALITY USING
ALTERNATIVE SOCIAL WELFARE FUNCTIONS:
A CASE STUDY OF RURAL INDIA

Ranjan Ray

8503
EVALUATING EXPENDITURE INEQUALITY USING ALTERNATIVE SOCIAL WELFARE FUNCTIONS: A CASE STUDY OF RURAL INDIA

Ranjan Ray

8503
Abstract

This paper examines the sensitivity of the normative estimates of inequality to: (i) changes in the planner's 'inequality aversion', (ii) a-priori changes in specification of the individual welfare functions, (iii) incorporation of price variation, (iv) incorporation of relative deprivation, and (v) the type of inequality measure employed, viz 'income inequality' as proposed by Atkinson or 'utility inequality' as proposed by Muellbauer.

The empirical results, using time series of expenditure distribution for rural India [October, 1953 to July, 1974], show considerable sensitivity of the inequality magnitudes to the above factors. The paper presents theoretical evidence on the likely impact of price increases on inequality and confirms them with the empirical results. It, also, derives and applies formulae to identify items whose price increases have a relatively large impact on inequality.

The paper exploits the link between inequality measurement and welfare/consumer demand theory that is opened up by the normative approach to inequality. However, the extreme sensitivity of the inequality magnitudes and the wide disagreements between different evaluators, with different sets of prior beliefs and values, on the extent of inequality in rural India seems to limit the operational usefulness of such measures.
INTRODUCTION

There has, essentially, been two approaches to the measurement of economic inequality. The first employs statistical concepts e.g. range, relative mean deviation, Gini coefficient etc. to analyse income/expenditure distribution data, and present estimates of inequality which are purely "descriptive" in their meaning and content. These measures, referred to as "positive" in the literature, are not based on or derived explicitly from any social welfare function and are not sensitive to the "subjective" elements of the planner or policy maker in welfare evaluation, welfare prescription and decision making. The second approach, in contrast, is referred to as 'normative' and is based on an explicit formulation and use of social welfare. It views inequality as causing "welfare loss" to society and seeks to reduce and minimise such 'loss' via prescriptive statements based on inequality measures.

The normative approach to inequality measurement originated with Dalton's [1920] work, while the recent resurgence of interest in this area and approach owes largely to the contributions of Atkinson [1970] and Kolm [1969]. The normative approach has two principal advantages: first, it recognises a view that is now widely accepted, namely, that it is wrong to view inequality purely as a descriptive device and that the planner's perception of inequality should be somehow built into the measure itself; second, in linking the measurement of inequality with social and individual welfare, it opens up the full range of techniques, possibilities and scrutiny of modern welfare analysis and, as in the immediate context of the present paper, modern consumer demand theory as well. The positive/normative distinction between the alternative approaches to inequality measurement has, incidentally, two analogies in the literature on consumer demand - (a) the Laspeyre/Paasche price indices against the Konus-inspired 'true cost of living' index, and (b)
equivalence scale models of Prais-Houthakker against the Barten-inspired and
utility/demand theory based 'cost of children' models.

The normative approach is not without its drawbacks, however. The
Dalton-Atkinson-Kolm approach identifies 'optimality' with 'equity' although
in reality the two concepts are quite distinct. Moreover, as Sen [1978]
points out, under certain circumstances the Atkinson measure can violate the
minimal Pigou-Dalton criterion, namely, that an income transfer from poor to
rich should lead to an increase in inequality. Sen also gives an interesting
example of a contradiction that arises in the Atkinson framework in its
attempt to combine the descriptive and normative aspects of inequality
measurement.

As the title indicates, this paper takes the normative approach to
inequality measurement. Its principal motivation is to examine the
sensitivity of the Atkinson-type measures to (i) subjective values of the
planner or, more precisely, his inequality aversion, and (ii) alternative
specification of the individual's welfare function to allow for price
variation and relative deprivation. Prices become a particularly important
issue if one compares, as we do in this study, inequality across time
involving data with varying prices. There is now theoretical and empirical
evidence (see Iyengar and Bhattacharya [1965], Muellbauer [1974b], Roberts
[1980], Osmani [1982, pgs.50-57], Jorgenson and Slesnick [1983]) on the need
- to incorporate price movements in inequality measurement and welfare
prescription. This study presents additional evidence on the likely impact
of inflation on inequality. It, also, proposes and applies measures that
identify individual items whose prices are likely to exert significant impact
on inequality. The issue of robustness of inequality measures to changes in
functional specification, the planner's perception of individual welfare and
his attitude to inequality is an important policy issue for, as this study
reports later, the extreme sensitivity of the estimates in some cases casts
considerable doubt on the operational usefulness of the normative measures. The present study follows Muellbauer [1974a] in distinguishing between 'income' and 'utility inequality' and investigates the nature and extent of divergence between the two sets of estimates. While the former is defined on income space and is to do with allocating incomes across individuals so as to maximise social welfare subject to an aggregate income constraint, the latter is defined on utility space and involves allocation of utilities across individuals so as to maximise social welfare subject to an aggregate utility constraint. The Atkinson-Kolm idea of 'equally distributed equivalent income' can be used to define 'equally distributed equivalent utility' as that level of utility which, if enjoyed by all, yields the same level of social welfare as that actually obtained by the existing utility distribution. The parallel definitions of income and utility inequalities yield the respective concepts of income and utility inequality aversion. One does not necessarily imply the same amount as the other and only when the individual utility function is defined over money income only and the private marginal utility of income is constant, do the two measures coincide. Alternatively, if price variation is admitted and individual welfare is defined over money income and price, via the consumer's indirect utility function, the two measures coincide only if cost function is homothetic in utility, i.e. in the unlikely event of demand systems implying constant average and marginal budget shares. It is, incidentally, worth noting that the idea of 'welfarist inequality' on which the measure of utility inequality, just described, is based can be of types: one identifies 'equity' with equality of marginal utility for everyone or, what Sen [1977] calls, "Utilitarian Equality", while the other identifies equity with equality of total utility for everyone or, what Sen calls, "Total Utility Equality". Muellbauer's contribution, referred to earlier, and the present study adopt the former, while Tinbergen's [1978] study is an example of the latter.
There has, however, been relatively little interest in 'utility inequality' in spite of some of its obvious advantages over 'income inequality'. While the former is based on a social welfare function (SWF),

\[ W = G(U_1(y_1), U_2(y_2), ..., U_H(y_H)) \]

defined over individual utilities, the latter is based on SWF,

\[ W = F(y_1, ..., y_H) \]

defined over individual incomes. For either to generate inequality measures which are meaningful and scale independent (i.e. free of units of measurement), the corresponding SWF's \( G,F \) must be strictly quasi-concave, symmetric and homothetic in their respective arguments. As Muellbauer [1974a, p.494-495] points out, these assumptions are less restrictive when imposed on \( G(.) \) than when imposed on \( F(.) \). Moreover, utility inequality explicitly recognises that it is not money income, per se, but the 'utility' it generates that ultimately determines personal 'welfare'. In identifying 'equity' with equally distributed utility (marginal or total) rather than equally distributed income, it respects the 'weak equity axiom' of Sen [1973, p.18] and takes note of the fact that individuals differ in their ability to convert incomes into utilities or satisfaction. This is particularly true if households with differing demographic composition and family size, rather than individuals, are taken to be the unit of behaviour for welfare analysis.3

This paper also investigates the effect on inequality measurement of allowing the SWF to be non linear and non separable in individual incomes.4 One possible cause for such SWF is 'relative deprivation' (see Runciman [1966]) on the part of the individual which affects his welfare, which should be explicitly incorporated in his utility function and which causes the SWF to be non separable in individual incomes and possibly non linear as well. A further cause of non linearity could be non linear Engel curves which is usually present on budget data.5 The empirical implication on inequality measures of admitting such general SWF's is of considerable policy interest.

The empirical exercise is based on budget data from rural India. In this
study, "economic inequality" refers to inequalities in expenditure rather than income distribution. This is because there is no census data in India on household or individual income. As for sample surveys, the only nationwide surveys which have directly collected data on income are those of the National Council of Applied Economic Research (NCAER). However, the number of such surveys is too few to allow a meaningful time series study of inequality of the type attempted in this paper. Most of the time series studies on income inequality in India have used consumption expenditure data published periodically as National Sample Survey Reports (NSS). The NSS covers more households than NCAER. Moreover, as Srinivasan, Radhakrishnan and Vaidyanathan [1974] point out, "NSS is the only source which provides a more or less comparable time series information on the levels and pattern of consumption and distribution of population by per capita consumption levels. The information is gathered by a nationwide field organisation using a rather elaborate questionnaire". Moreover, as Mukherjee and Chatterjee [1972] observe, the NSS data serves as a satisfactory proxy for national income trends. The present study is based on NSS 7th-28th rounds (excluding the 26th and 27th rounds whose reports were not available) covering the period from 1953-54 (NSS 7th round) to 1973-74 (NSS 28th round). We ignore price variation within a survey and assume that households are identical with respect to their demographic characteristics.

The plan of this paper is as follows. The theoretical framework and the necessary concepts are introduced and discussed in Section 2. The empirical results are presented and discussed in Section 3, and we end with the concluding note of Section 4.

2. THEORETICAL FRAMEWORK AND ASSOCIATED CONCEPTS

2.1 Income Inequality and Utility Inequality

Let the SWF, \( W = G(U^S_1, U^S_2, \ldots, U^S_H) \) be defined to be increasing, quasi-
concave and symmetric in the 'social utilities' \( u^s_1 \) of the individual households \((i=1,\ldots,H)\). By 'social utility', we mean planner's evaluation of individual welfare, and is to be distinguished from 'private utility' \( u^p_1 \) which is the individual's welfare as perceived and evaluated by the individual himself. Let us, further, assume the individual's welfare to be an increasing function of his income. 

Following Atkinson, we assume the SWF to be linear and additive in 'social utilities' i.e.

\[ W = \sum_{i=1}^{H} u^s_i \quad (1) \]

and that 'social utility' is related to 'private utility' by

\[ u^s_i = A + \frac{1}{1-\varepsilon} u^p_i^{1-\varepsilon} \quad (2) \]

where \( A \) is an arbitrary constant. \( \varepsilon \) represents planner's inequality aversion, and must be non-negative for social utility to be concave in private utility, and greater than zero for strict concavity. We, further, assume the individual's utility function to be increasing in his income,

\[ u^p_1 = y^p_1 \quad (3), \quad \rho > 0 \]

\( \rho < 1 \) is needed for individual or private utility to be convex in income, although this is not an essential requirement. \( \rho - 1 \) measures the rate at which the private marginal utility of income changes with income. Though \( \rho \) is a behavioural parameter, it cannot be estimated from the budget data that is usually available, since it cannot be recovered from the estimable demand system implied by (3). To estimate or calculate \( \rho \), one needs special surveys which evaluate individual welfare by asking people directly - see Van Pragg's [1971] study on Belgian survey data. 

Equations (1)-(3) imply the following SWF in terms of individual income.
The 'equally distributed equivalent income' $y_e$ is, then, given by

$$y_e = \left[ \frac{1}{Y} \sum_{h=1}^{H} y_h \rho(1-\varepsilon) \right] \frac{1}{\rho(1-\varepsilon)}$$

(5)

which, in turn, yields the following measure of income inequality

$$I(\text{Inc}) = 1 - \left[ \frac{1}{\rho(1-\varepsilon)} \left( \frac{H}{\sum_{h=1}^{H} y_h \rho(1-\varepsilon)} \right) \right]$$

(6)

where $y = \frac{\sum_{h=1}^{H} y_h}{H}$. It is worth pointing out that if $\rho = 1$ i.e. if the marginal 'private' utility of income is constant, then (6) takes the Atkinson form. In general, however, (i.e $\rho \neq 1$), the planner's income inequality aversion is given by $\varepsilon^* = 1 - \rho(1-\varepsilon)$ which, of course, becomes $\varepsilon$ if $\rho = 1$. It, then, follows that even if $\varepsilon = 0$, the planner's income inequality aversion will be non-zero namely, $1 - \rho$ provided the marginal 'private' utility of income is non-constant.

The measure of 'utility inequality' is defined by

$$I(\text{Uti}) = 1 - \frac{U_e}{U}$$

(7)

where $U_e$ is 'equally distributed equivalent utility, and $\bar{U} = \frac{\sum_{i} U_i}{H}$ is 'mean utility'. Equations (1) and (2) imply the (Bergson) SWF defined over individual utilities to be

$$W = HA + \frac{1}{1 - \varepsilon} \sum_{i=1}^{H} U_i^{1-\varepsilon}$$

(8)
(8) implies the following expression for $U_e$

$$U_e = \left[ \frac{\sum_{i=1}^{H} 1^{1-\varepsilon}}{H} \right]^{\frac{1}{1 - \varepsilon}}$$

(9)

The measure of utility inequality is, then, given by

$$I(U_{\text{uti}}) = 1 - \left[ \frac{\sum_{i=1}^{H} \frac{1^{1-\varepsilon}}{y_i}}{H} \right]^{\frac{1}{1 - \varepsilon}}$$

(10)

Three points ought to be noted: (i) $\varepsilon$ is a measure of 'utility inequality' aversion and performs a role analogous to its counterpart in the Atkinson formulation; (ii) if $\rho = 1$, the two measures $I(\text{Inc.})$, $I(U_{\text{uti}})$ coincide, and (iii) if $\rho > 1$, then a given distribution in money income is likely to show a more unequal distribution in utility than in income, and vice versa in case of $\rho < 1$.

As already noted, the relation between income inequality aversion, $\varepsilon^*$, and utility inequality aversion, $\varepsilon$, is given by

$$\varepsilon^* = 1 + \rho(\varepsilon - 1)$$

(11)

It is, therefore, possible to find indifference contours between $\varepsilon$ and $\rho$ that keep income inequality aversion unchanged. (11) implies that, for a given $\varepsilon^*$,

$$\frac{\delta \varepsilon}{\delta \rho} = - \frac{\varepsilon^* - 1}{\rho^2} \begin{cases} 0 & \text{if } 0 < \varepsilon^* < 1 \\ < 0 & \text{if } 1 < \varepsilon^* < \infty \end{cases}$$

(12)
\[
\frac{\delta^2 \varepsilon}{\delta p^2} = \frac{2(\varepsilon - 1)}{\rho^3} \begin{cases}
< 0 & \text{if } 0 < \varepsilon^* < 1 \\
> 0 & \text{if } 1 < \varepsilon^* < \infty
\end{cases}
\tag{13}
\]

Fig. 1(a) - 1(b) show the indifference contours between \( \varepsilon, p \) for the two cases, \( \varepsilon > 1 \) and \( \varepsilon < 1 \).

2.2 Price Variation and Inequality

The framework of the previous section is inadequate if one considers inequality across time period involving price variation. In such a case, the individual utility function ought to be defined over money income and prices. This can be done using the indirect utility function, \( v(y_h, p) \), which measures the utility for an individual with money income \( y_h \) and facing a price vector \( p \). Here, we choose the following generalisation of the Gorman Polar Form introduced in Blundell and Ray [1982] and used in Ray [1983a]
\[ v_h = \frac{Y_h^\alpha - a(p,a)}{b(p,a)} \]  

(14)

where \( a(p,a) = \sum_i p_i^\alpha y_i \), \( b(p,a) = \prod_k p_k^{\alpha b_k} \), \( \sum_k b_k = 1 \), \( 0 < \alpha < 1 \).

It is worth noting that \((1 - \alpha)\) measures the non linearity or curvature of the Engel curves and that if \(\alpha = 1\), linear Engel curves are obtained. In the latter event, (14) yields the LES demand system with the parameters \([\gamma_i, \beta_i]\) taking on the familiar interpretation of subsistence quantity, and marginal budget share of the various items. Besides incorporating prices, (14) unlike (3) has the advantage that the functional parameters \(\beta_i, \gamma_i, \alpha\) can be estimated on available budget data as estimable parameters in the implied demand system and, hence, avoids the problem with knowing \(\rho\) noted earlier. We exploit this convenience in the empirical exercise reported later.

The SWF, now defined on individual incomes and prices is given by

\[
W = \frac{1}{1 - \varepsilon} \left[ \sum_h \left( \frac{Y_h^\alpha - a}{b} \right)^{1-\varepsilon} \right]
\]

(15)

which gives the following expressions for 'equally distributed equivalent income', \(y_e\), and 'equally distributed equivalent utility', \(U_e\).

\[
y_e = \left[ a + \left\{ \frac{1}{H} \sum_h \left( \frac{Y_h^\alpha - a}{b} \right)^{1-\varepsilon} \right\} \right]^{1\over\alpha}
\]

(16)

\[
U_e = \left[ \frac{1}{H} \sum_h \left( \frac{Y_h^\alpha - a}{b} \right)^{1-\varepsilon} \right]^{1\over(1-\varepsilon)}
\]

(17)

(16) and (17) give us the computing expressions for income inequality and utility inequality.\(^10\)
It is useful to note that if $e = 0$ i.e. if the planner is Benthamite on the utility spectrum with no aversion to utility inequality, then $I_{\text{Inc.}} = 0$, but

$$I_{\text{Inc.}} = 1 - \left[ a + \frac{1}{1 - (1 - e) \left( \sum_{h} y_{h} - a \right)^{1-e}} \right] \frac{1}{\alpha}$$

provided $\alpha \neq 1$. In such a case, the Atkinson measure is inversely related to $\alpha$ and becomes zero when $\alpha$ attains its maximum permitted value of unity i.e. when Engel curves are assumed linear. This suggests that enforcing the LES restriction of linearity in income, as done by Muellbauer [1974b] though widely rejected on budget data, introduces a downward bias into the income inequality measure and that this bias could be particularly serious if $\alpha$ is a long distance away from unity, as obtained in Blundell and Ray [1982], Ray [1983a] and in the present study.

From a policy point of view, it is important not only to take price movements into account in calculating inequality, but also to identify items whose prices are likely to have a relatively large impact on inequality. Indeed, if the government knew the items whose price increases are likely to affect inequality particularly adversely, a suitable egalitarian policy could be formulated in a period of accelerating inflation by subjecting them to price control. Let us consider, first, the affect of price rise on the
income-inequality measure, \( I^{(\text{Inc.})} = 1 - \frac{y_e}{y} \), where \( y_e \) is given by (16). Two points are worth noting: (i) \( y_e \) is homogeneous of degree one in prices and money incomes, and (ii) the inequality measure is homogeneous of degree zero in prices and money income.

Assuming that the prices considered here are 'consumer' prices, and that a price change leaves money incomes unchanged, we have

\[
\frac{\delta I^{(\text{Inc.})}}{\delta p_i} = - \frac{1}{y} \frac{\delta y_e}{\delta p_i}
\]

(21)

Hence, \( \frac{\delta I^{(\text{Inc.})}}{\delta p_i} \geq 0 \) according as \( \frac{\delta y_e}{\delta p_i} \leq 0 \)  \( \text{(21a)} \)

(16) yields, on differentiation and after some rearrangement,

\[
\frac{\delta y_e}{\delta p_i} = \frac{a}{\alpha y_e} \left[ 1 - \frac{1}{\alpha} H \sum_{h} \left\{ \frac{y_h - a}{y_e - a} \right\} \right]
\]

(22)

where \( a_i = \frac{\delta a(p,a)}{\delta p_i} \) and can be assumed to be non-negative \( \text{11} \) to ensure that the indirect utility function, given by (14), is non-decreasing in prices. (21) and (22) allow us to compute the effect of a price rise on inequality. It, then follows that

\[
\frac{\delta I^{(\text{Inc.})}}{\delta p_i} \leq 0 \quad \text{i.e.} \quad \frac{\delta y_e}{\delta p_i} \leq 0
\]

according as \( 1 \leq \frac{1}{\alpha} H \sum_{h} \left\{ \frac{y_h - a}{y_e - a} \right\} \)  \( \text{(23)} \)

and, moreover, the magnitude of the price effect on inequality depends crucially on the sensitivity of \( a(p,a) \) to the price of that item. In general, items of 'necessity' which are more prominent in 'subsistence' budgets are likely to be items whose prices have a relatively large impact on inequality. This is, indeed, borne out by the empirical evidence presented later. Using the expressions for indirect utility function assumed in
equation (14) and of $y_e$ derived and presented in equation (16), it can be shown that the right hand side of condition (23), i.e.

$$\sum_{h}^{1/H} \left( \frac{a}{y_h} - a \right) = \frac{1}{H} \sum_{h}^{1/H} \phi h - \varepsilon \tag{24}$$

where $\phi_h = \frac{U_h}{U_e}$, and $U_e$ is the 'equally distributed equivalent utility' given by (17). Defining $\theta_h = \phi_h^{-\varepsilon}$, one can interpret $\theta_h$ as the planner's welfare weight to individual $h$, where the weights are normalised at unity for the household whose utility level equals $U_e$.

For a particular distribution of money incomes, $\{y_h\}$, and a given vector of prices $p$, we have a distribution of utilities via the individual's expenditure patterns and the indirect utility function (14). Depending on the planner's aversion to utility inequality, $\varepsilon$, one can translate the distribution of utilities into a distribution of social welfare weights, $\theta_h$, where $\theta_j = 1$ for individual $j$ who happens to enjoy the 'equally distributed equivalent utility' level $U_e$. Condition (23) can, then, be re-written as:

$$\frac{\delta I(\text{Inc.})}{\delta p_1} \geq 0 \quad \text{according as} \quad \frac{1}{H} \sum_{h}^{1/H} \theta h \geq 1 \tag{25}$$

In other words, price rises will be inequality increasing if the arithmetic mean of the planner's welfare weights exceeds unity and the reverse, otherwise.

Several points are worth noting:

(i) Since the weights $\theta h$ depend, in part, on the planner's attitude
to inequality, it is possible though highly unlikely for two evaluators with sharply differing $\varepsilon$'s to disagree not only on the magnitude of $\frac{\delta I}{\delta p_1}$, but also on its sign as well!

(ii) $\varepsilon = 0 \Rightarrow \frac{\delta I}{\delta p_1} = 0$ - i.e. to a Benthamite, inflation has no effect on money inequality.

(iii) In general, for a skewed distribution of utilities with more individuals below, $j$, $(U_j = U_e)$ than those above him, namely, the sort of distribution that is usually likely to prevail, we can expect

$$\theta = \frac{1}{H} \sum_h \theta > 1.$$ Hence, inflation is likely to be inequality increasing.

(iv) The more unequal the utility distribution, the greater will be $\theta$ and, hence, the more inegalitarian will be the price increases.

Let us recall the measure of utility inequality given in equation (19).

$$I(U_{ti}) = 1 - \frac{U_e}{U} \quad (26)$$

where

$$U_e = \left[ \frac{1}{H} \sum_h \{y_h - a\} \right]^{1-\varepsilon} \cdot \frac{1}{(1-\varepsilon)} \quad (27)$$

$$U = \frac{1}{H} \sum_h (y_h - a)$$

Differentiating both sides with respect to price $p_1$ and after some rearrangement, we obtain

$$\frac{\delta I(U_{ti.})}{\delta p_1} = \frac{\alpha U_e}{(U)^2} \left[ \left\{ \frac{1}{H} \sum_h (y_h - a)^{-1} \right\} - 1 \right] - \frac{\delta I(U_{ti.})}{\delta p_1} = 0.$$ Also, an examination of the
terms on the RHS of (28) suggests that, generally, \( \frac{\delta I(\text{Uti.})}{\delta p_i} > 0 \) — in other words, price rise is likely to increase utility inequality as well.

2.3 Relative Deprivation and Inequality

The idea of relative deprivation largely originated with the classic study of W G Runciman [1966] who (p.10) defined its magnitude to be the extent of the difference between the desired situation and of the person desiring it. Sen [1973, p.41] suggested the possibility of linking this idea with social welfare evaluation and inequality measurement. Yitzhaki [1979] formulated the idea and re-interpreted the Gini coefficient in a manner consistent with the theory of relative deprivation.

Here, we identify the 'desired situation' as the geometric mean of the income distribution and interpret the meaning of 'relative deprivation' liberally, rather than literally, to encompass the feelings of 'envy' for an individual with income below that 'desired', and 'privileged' for one with income above that level. Returning to the framework of Section 2.1, the individual utility function is now defined to be an increasing function of 'augmented' income \( \bar{y} \), where \( \bar{y} \) augments money income, \( y_h \), to incorporate relative deprivation.

\[
U^I_i = \bar{y}_i \quad (29), \quad i=1,\ldots,H \quad \text{where}
\]

\[
\bar{y}_i = y_i + \bar{\theta} \log \left( \frac{y_i}{\bar{y}} \right), \quad \bar{y} = \left[ \frac{\sum_{i=1}^{H} y_i}{H} \right], \quad \bar{\theta} > 0, \quad \text{and} \quad \bar{\theta} \text{ is assumed unity to simplify calculations so that the measures of income and utility inequality coincide. Like} \bar{\theta} \text{ in Section 2.1, knowledge of} \theta \text{ requires special surveys of the sort carried out in Belgium and used extensively by Van Praag and his associates. In the absence of similar information for rural India, one needs to examine sensitivity of the inequality measure not only to the presence of relative deprivation (} \theta \neq 0\text{), but to the magnitude of} \theta \text{ itself.}
\]

The SWF, defined over money incomes, now becomes
\[ W = HA + \frac{1}{1 - \epsilon} \sum_{h=1}^{H} \left( y_h + \theta \log \left( \frac{y_h}{y} \right) \right)^{1-\epsilon} \]  

(30)

and the inequality measure, \( I \), is given by

\[ I = 1 - \frac{y_e}{y} \]  

(31)

where \( y = \frac{\sum y_h}{H} = \frac{\sum \bar{y}_h}{H} \), and

\[ y_e = \left[ \frac{1}{H} \sum_{h=1}^{H} \left( y_h + \theta \log \left( \frac{y_h}{y} \right) \right)^{1-\epsilon} \right] \frac{1}{1 - \epsilon} \]  

(32)

Since \( y \) does not depend on \( \theta \),

\[ \frac{\delta I}{\delta \theta} = - \frac{\delta y_e}{\delta \theta} \]  

(33)

Differentiating both sides of (32) with respect to prices and rearranging, we obtain

\[ \frac{\delta y_e}{\delta \theta} = \frac{1}{H} \sum_{h=1}^{H} \frac{y_h}{y_e} \left( \frac{y}{y} \right)^{-\epsilon} \log \left( \frac{y_h}{y} \right) \]  

(34)

(33)-(34) imply that the magnitude and direction of the impact of 'relative deprivation' on inequality depends on (i) the planner's inequality aversion, and (ii) the actual distribution of money incomes. It is useful to draw an analogy, here, with the discussion following equation (25) on the likely impact of price rise on inequality.

The conclusions are very similar to those noted in the previous section.
(a) To a Benthamite utilitarian \( (\varepsilon = 0) \), 'relative deprivation' does not affect the inequality measure - hardly surprising since, whatever the income distribution, the inequality to him is zero anyway.

(b) Denoting \( \lambda_h = \left( \frac{y_h}{y_e} \right) ^{-\varepsilon} \) to be the planner's welfare weight for individual \( h \), where \( \lambda_e \) is normalised at unity for the individual with income \( y_e \), it follows that

\[
\frac{\delta I}{\delta \varepsilon} \geq 0 \quad \text{according as } \quad \frac{1}{H} \sum_{h} \lambda_h \log \left[ \frac{y_h}{y} \right] \leq C \quad (35)
\]

In general, and as borne out by our results, the RHS of (35) is likely to be negative, while its absolute value rises with \( \varepsilon \) and with greater dispersion of incomes around the mean. The recognition of relative deprivation is likely to cause the planner to raise his inequality estimate upwards, and the required revision is larger, the more averse he is to inequality.

(c) An alternative way of presenting the above conclusion is to note that

\[
\frac{1}{H} \sum_{h} \lambda_h \log \left[ \frac{y_h}{y} \right] = \frac{1}{H} \sum_{h} \lambda_h ^* \log y_h \quad (36)
\]

where \( \lambda_h ^* = \lambda_h - \lambda \), \( \lambda = \frac{\sum \lambda_h}{H} \).

Condition (35) can, then, be rewritten as

\[
\frac{\delta I}{\delta \varepsilon} \geq 0 \quad \text{according as } \quad \frac{1}{H} \sum_{h} \lambda_h ^* \log y_h \leq 0 \quad (37)
\]

(37) implies that relative deprivation will not affect inequality calculations if (i) the planner does not care about inequality (i.e. \( \varepsilon = 0 \), \( \lambda_h ^* = 0 \)), or (ii) all money incomes are equal. Either of these cases is uninteresting, especially in the context of the present study.
3. RESULTS

The estimates of income and utility inequality, given by equations (6) and (10), are presented in Tables 1, 2 respectively. The calculations were performed for five NSS rounds spread widely over our sample period. For each round, the estimates were calculated at three levels of inequality aversion (\( \varepsilon \)) and using three a-priori values for \( \rho \). The calculations attempt to answer two basic questions: What is the overall trend in inequality in rural India over October 1953 to June 1974 covering a large part of India's post independence period? Are the inequality magnitudes, based on normative measures, unduly sensitive to the planner's inequality aversion, specification of the individual's welfare function, incorporation of prices and variation in relative prices, and the concept of inequality viz. income or utility that one chooses to employ?

The overall picture yielded by Tables 1, 2 is that of a general decline in money expenditure inequality over this period. This is consistent with NSS based evidence presented in Chatterjee and Bhattacharya [1974, Tables 2,3], Vaidyanathan [1974, Tables 1,3] and Bhatty [1974, Table 6]. Tables 1,2 generally agree on the overall decline in inequality over this period. Within the two approaches, however, and between the comparable magnitudes of the two measures, the estimates appear quite sensitive indeed. For low levels of \( \rho \) (0.5), income inequality exceeds utility inequality quite significantly, and the divergence is quite large for moderate and high levels of inequality aversion (\( \varepsilon = 1.0,3.0 \)). The reverse occurs at high levels of \( \rho \). In other words, for someone with a good deal of concern for inequality, the choice between \( I(\text{Inc.}) \) and \( I(\text{Uti.}) \) seems very important indeed. The measure, he ought to be using, depends crucially on the value of \( \rho \) he believes in. If, as is commonly supposed, individual utility is strictly concave in income (\( \rho < 1 \)), he ought to use 'income inequality' as defined by Atkinson, rather than 'utility inequality' as defined by Muellbauer: and the reverse if \( \rho > 1 \). However, in the absence of much scientific evidence on \( \rho \),
especially for rural India, the choice of the 'right' measure is an open issue. This, incidentally, points to the need for welfare surveys of the type mentioned earlier to give us some idea of \( \rho \). The importance of \( \rho \) is also evident from the large sensitivity of the utility inequality estimates presented in Table 2 - for example, an increase in the value of \( \rho \) from 0.5 to 1.5 leads to a more than five fold increase in the inequality estimate in many cases, and the pattern is maintained throughout the sample period.

Following our discussion earlier, we now admit variation in prices and relative prices. To implement formulae (18) and (19) of Section 2.2, we need to calculate, initially, \( a, b \) from the parameter estimates of the indirect utility function (14). These are obtained by estimating the implied demand system which, in budget share form, is

\[
\omega_{ih} = \sum_i \beta_i \left[ \frac{p_i}{y_i} \right]^a + \beta_i \left[ 1 - \sum_k \gamma_k \frac{p_k}{y_h} \right]^a \tag{38}
\]

where \( \sum \beta_i = 1 \), \( i \) denotes item, \( h \) denotes observation, and \( \{ a, \beta_i, \gamma_i \} \) are the estimable parameters. (38) is a one parameter generalisation of the widely used LES and admits both non linear Engel curves and non separable preferences via \( \alpha \neq 1 \). The following nine-item classification of aggregate commodity expenditure is adopted in this study: (i) Cereals (ii) Milk and Products (iii) Edible Oils (iv) Meat, Fish and Eggs (v) Sugar and Tea (vi) Other Foods (vii) Clothing (viii) Fuel and Light, (ix) Other Non-Food.

The parameter estimates from estimating (38) on the pooled NSS data are presented in Table 3. The \( \alpha \) estimate is significantly and considerably different from unity, and justifies using the present demand system as a significant improvement over the LES. The estimates also show that the restrictions of homothetic preferences and linear Engel curves

\( (\alpha = 1, \gamma_i = 0) \) needed for the income and utility measures to coincide are
decisively rejected by the data.

The extent of divergence between the income and utility measures of inequality can be seen from Table 4 which presents the two measures computed using (18) and (19) and the demand system parameter estimates reported in Table 3. The introduction of prices seems to open up considerable disagreements between the two measures not only on the magnitude of inequality but also on its trend over time. While the income measure shows an overall decline over the sample period, consistent with previous evidence, the utility measure suggests a stable estimate. Moreover, the utility measure not only shows considerably greater inequality than its income counterpart at each level of inequality aversion, its sensitivity to $\varepsilon$ is a good deal more as well. A planner with great concern for inequality should use the utility rather than income measure to register inequality. If one adopts the utility measures, the perceived inequality to a planner with only a 'moderate' aversion ($\varepsilon = 1.5$) is as large as that yielded by the income-measure only at very high levels of aversion ($\varepsilon > 7.0$). Also, the magnitudes of Table 4 are generally a good deal higher than those in Tables 1-2 which suggest, as we were led to expect earlier, that price rises have tended to be inequality increasing and that ignoring them is likely to bias the inequality estimates downwards quite severely, especially in a period of accelerating inflation.

Table 5 reports the estimates showing the percentage change in inequality due to a one percent rise in price of the individual item. The calculations, using formulae (21), (28), are for NSS 28th round. All the individual price rises are inequality increasing, but the magnitudes vary considerably between the various items and between the two measures. The ranking of the items, in terms of the magnitudes of their price effects on inequality, is, however, similar for the two measures. The price rise of Cereals is particularly inequality increasing. This suggests that, for an egalitarian policy to be effective, the price of Cereals must be controlled or, at least, that cereals
are made available to the rural poor at subsidised prices. Table 6 shows the impact on inequality measurement of allowing for relative deprivation. The estimates were obtained using formulae (31) and (32). The estimates confirm that incorporation of relative deprivation causes the planner to revise his inequality estimate upwards. The required revisions appear quite substantial in the earlier rounds. However, as the expenditure inequality began to fall in later years, the feeling of relative deprivation became less intense, so that by the end of our sample period, not only has inequality fallen but so also has the impact of relative deprivation on the inequality measure.
### TABLE 1

**Estimated Income Inequality**

<table>
<thead>
<tr>
<th>Round</th>
<th>Period</th>
<th>$\varepsilon = 0.5$</th>
<th>$\varepsilon = 1.5$</th>
<th>$\varepsilon = 3.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\rho = 0.5 \quad \rho = 1.0 \quad \rho = 1.5$</td>
<td>$\rho = 0.5 \quad \rho = 1.0 \quad \rho = 1.5$</td>
<td>$\rho = 0.5 \quad \rho = 1.0 \quad \rho = 1.5$</td>
</tr>
<tr>
<td>7</td>
<td>Oct.'53-Mar.'54</td>
<td>.10 .07 .04</td>
<td>.15 .22 .27</td>
<td>.25 .46 .45</td>
</tr>
<tr>
<td>11</td>
<td>Aug.'56-Feb.'57</td>
<td>.09 .06 .04</td>
<td>.13 .16 .25</td>
<td>.26 .41 .42</td>
</tr>
<tr>
<td>16</td>
<td>July'60-June'61</td>
<td>.08 .06 .04</td>
<td>.19 .17 .16</td>
<td>.26 .34 .43</td>
</tr>
<tr>
<td>21</td>
<td>July'66-June'67</td>
<td>.09 .05 .04</td>
<td>.14 .18 .19</td>
<td>.19 .32 .40</td>
</tr>
<tr>
<td>28</td>
<td>July'73-June'74</td>
<td>.08 .05 .03</td>
<td>.10 .23 .24</td>
<td>.18 .28 .35</td>
</tr>
</tbody>
</table>

### TABLE 2

**Estimated Utility Inequality**

<table>
<thead>
<tr>
<th>Round</th>
<th>Period</th>
<th>$\varepsilon = 0.5$</th>
<th>$\varepsilon = 1.5$</th>
<th>$\varepsilon = 3.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\rho = 0.5 \quad \rho = 1.0 \quad \rho = 1.5$</td>
<td>$\rho = 0.5 \quad \rho = 1.0 \quad \rho = 1.5$</td>
<td>$\rho = 0.5 \quad \rho = 1.0 \quad \rho = 1.5$</td>
</tr>
<tr>
<td>7</td>
<td>Oct.'53-Mar.'54</td>
<td>.02 .07 .14</td>
<td>.05 .22 .42</td>
<td>.13 .46 .62</td>
</tr>
<tr>
<td>11</td>
<td>Aug.'56-Feb.'57</td>
<td>.02 .06 .12</td>
<td>.02 .16 .40</td>
<td>.12 .41 .58</td>
</tr>
<tr>
<td>16</td>
<td>July'60-June'61</td>
<td>.01 .06 .12</td>
<td>.06 .17 .28</td>
<td>.15 .34 .59</td>
</tr>
<tr>
<td>21</td>
<td>July'66-June'67</td>
<td>.02 .05 .11</td>
<td>.04 .18 .33</td>
<td>.04 .32 .55</td>
</tr>
<tr>
<td>28</td>
<td>July'73-June'74</td>
<td>.00 .05 .10</td>
<td>.02 .23 .37</td>
<td>.09 .28 .48</td>
</tr>
</tbody>
</table>
TABLE 3

Parameter Estimates (with standard errors) of Demand System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>.020 (.045)</td>
<td>$\gamma_1$</td>
<td>1.500 (.150)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>.185 (.012)</td>
<td>$\gamma_2$</td>
<td>.120 (.025)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>.030 (.002)</td>
<td>$\gamma_3$</td>
<td>.072 (.005)</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>.035 (.002)</td>
<td>$\gamma_4$</td>
<td>.057 (.005)</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>.052 (.003)</td>
<td>$\gamma_5$</td>
<td>.061 (.007)</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>.110 (.010)</td>
<td>$\gamma_6$</td>
<td>.361 (.027)</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>.203 (.016)</td>
<td>$\gamma_7$</td>
<td>.119 (.028)</td>
</tr>
<tr>
<td>$\beta_8$</td>
<td>.011 (.006)</td>
<td>$\gamma_8$</td>
<td>.214 (.021)</td>
</tr>
<tr>
<td>$\beta_9$</td>
<td>.353 (.026)</td>
<td>$\gamma_9$</td>
<td>.252 (.049)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha$</td>
<td>.435 (.043)</td>
</tr>
</tbody>
</table>
### TABLE 4

**Estimated Inequality with Price Variation Incorporated**

<table>
<thead>
<tr>
<th>Round</th>
<th>Income Inequality, $i(\text{Inc.})$</th>
<th>Utility Inequality, $i(\text{UtI})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c=0.5$</td>
<td>$\epsilon=1.5$</td>
</tr>
<tr>
<td>7</td>
<td>0.13</td>
<td>0.38</td>
</tr>
<tr>
<td>16</td>
<td>0.15</td>
<td>0.42</td>
</tr>
<tr>
<td>28</td>
<td>0.09</td>
<td>0.31</td>
</tr>
</tbody>
</table>

### TABLE 5

**Effect of Individual Prices on Inequality**
*(Round 28, $c = 1.5$)*

<table>
<thead>
<tr>
<th>Item</th>
<th>$\frac{\delta I(\text{Inc.})}{\delta p_i}$</th>
<th>$\frac{\delta I(\text{UtI.})}{\delta p_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Cereals</td>
<td>0.17</td>
<td>2.15</td>
</tr>
<tr>
<td>2. Milk and Products</td>
<td>0.17</td>
<td>0.18</td>
</tr>
<tr>
<td>3. Edible Oils</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>4. Meat, Fish and Eggs</td>
<td>0.11</td>
<td>0.05</td>
</tr>
<tr>
<td>5. Sugar and Tea</td>
<td>0.16</td>
<td>0.09</td>
</tr>
<tr>
<td>6. Other Foods</td>
<td>0.15</td>
<td>0.48</td>
</tr>
<tr>
<td>7. Clothing</td>
<td>0.19</td>
<td>0.35</td>
</tr>
<tr>
<td>8. Fuel and Light</td>
<td>0.17</td>
<td>0.37</td>
</tr>
<tr>
<td>9. Other Non-Food</td>
<td>0.20</td>
<td>0.44</td>
</tr>
</tbody>
</table>
TABLE 6
Effect of Relative Deprivation on Estimated Inequality

<table>
<thead>
<tr>
<th>Round</th>
<th>(\varepsilon = 1.5)</th>
<th>(\varepsilon = 3.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\theta = 0.0)</td>
<td>(\theta = 1.5)</td>
</tr>
<tr>
<td>7</td>
<td>0.22</td>
<td>0.28</td>
</tr>
<tr>
<td>11</td>
<td>0.16</td>
<td>0.21</td>
</tr>
<tr>
<td>16</td>
<td>0.17</td>
<td>0.23</td>
</tr>
<tr>
<td>21</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>28</td>
<td>0.23</td>
<td>0.22</td>
</tr>
</tbody>
</table>

4. SUMMARY AND CONCLUSION

The 'normative' theory of inequality measurement has two inter-related advantages over its 'positive' counterpart: (i) it recognises that inequality ought not to be viewed as purely a descriptive device but should be flexible enough to reflect differences in the planners' attitudes to inequality and their evaluation/perception of individual welfare; and (ii) it allows the application of the techniques of modern welfare analysis and modern consumer demand theory. The normative approach combines the purely descriptive notion of 'how much inequality' with the ethical question of 'how bad is inequality' into a single measure. The idea is laudable but the resulting possibility of inequality magnitudes varying with a-priori beliefs raises serious policy questions about its operational usefulness. The ultimate test of an inequality measure must, surely, lie in its ability to generate widely agreed prescriptive statements that can guide in the formulation and implementation...
of egalitarian policies and in evaluating their success in achieving redistribution. The sensitivity of inequality estimates to a-priori beliefs is an important policy issue for, if evaluators disagree wildly in their assessment of economic inequality as the normative approach permits them to do, this can only be seen to limit its operational usefulness.

The possibility of such disagreements is still larger if one recognises that even 'behavioural' parameters like those that determine the individual welfare function are very rarely known with certainty. One requires specially designed surveys, of the type used by Van Praag and his associates, to estimate the individual welfare function. Since such surveys are very scarce and expensive, one has to rely on a-priori specification of the individual welfare function and this opens up the issue of sensitivity of the inequality estimate not only to the planner's ethics but also to what he believes the individual welfare function to be.

This paper examines the sensitivity of the inequality estimate to: (i) changes in the planner's 'aversion' to inequality, (ii) a-priori changes in the specification of individual welfare, (iii) incorporation of price movements and relative price variation, (i) incorporation of relative deprivation, and (v) the type of inequality concept employed, viz. 'income inequality' as proposed by Atkinson, or 'utility inequality' as proposed by Muellbauer. The empirical investigation is carried out on a time series of expenditure distribution data for rural India covering the period October, 1953 to July, 1974. We exploit the link between direct and indirect utility to introduce prices into individual welfare function and, thereby, incorporate price variation into inequality measurement. The indirect utility function, namely, a one-parameter generalisation of the LES allowing non linear Engel curves and non separable preferences, is initially estimated on the available disaggregated commodity expenditure data, and the parameter estimates then used to compute the inequality estimates. We also propose and use measures that identify items whose prices are likely to exert the greatest impact on
The principal results can be summarised as follows.

(i) If one ignores prices, then both the income and utility measures agree that there has been an overall decline in inequality in rural India over the period. This is seen to be consistent with previous studies.

(ii) The measures disagree, however, quite considerably on the actual magnitude of inequality. The disagreements increase sharply with departure from linearity in the assumed form for the individual welfare function. The question of which of the two measures one ought to be using depends crucially on what he believes the shape of the individual welfare function to be. A planner greatly concerned with inequality should use the income measure if he believes the individual utility to be strictly concave in income as is commonly supposed to be, and the utility measure otherwise. In the absence of much scientific and empirical evidence on individual welfare functions, the choice between the two measures remains an open issue.

(iii) With the introduction of prices, the differences widen still further and quite dramatically with the measures disagreeing not only on the magnitude but also on the trend in inequality. The choice between the two measures is now much less ambiguous, however, for the utility measure shows considerably greater inequality at all levels of inequality aversion. The empirical results confirm the theoretical arguments presented earlier and show price increases to be inequality increasing and, moreover, point to 'Cereals' prices as being the single most important factor. This suggests the need to control cereals prices or, at least, make cereals available to the rural poor at subsidised prices in a period of accelerating inflation.

(iv) The introduction of 'relative deprivation' causes the planner to revise his inequality estimated upwards, and the necessary revision is greater the more unequal the distribution of money incomes. In the context of rural
India, relative deprivation leads to a considerable upward revision in inequality in the earlier NSS rounds, but as the dispersion round the mean began to decline in the later rounds, so apparently did the feeling of 'relative deprivation' and its consequent impact on inequality measurement.

(v) The calculated inequality estimates are very sensitive to the planner's inequality aversion and, between the income and utility measures, the latter seems particularly so. Moreover, the sensitivity of inequality to its aversion increases with the introduction and incorporation of price variation.

The extreme sensitivity of normative conclusions to a-priori beliefs and values raises serious doubts about their operational usefulness. In the ultimate analysis, the very strength of the normative measures, namely, integrating inequality measurement with ethical beliefs and attitudes to inequality and social welfare seems to constitute the source of its inherent weakness as well. In India's context, in particular, the issues of inequality and poverty are inextricably linked. Unlike in more affluent societies, those at the bottom end of rural India's expenditure distribution are not simply the 'worse off'; they are the rural poor as well, with most of them living well below the 'poverty line'. The failure of the inequality evaluators to agree on their relative plight, because of sharp differences on issues and beliefs for which the rural poor couldn't care less, and the consequent lack of a coherent and consistent set of prescriptive responses to the government by its advisors must raise serious doubts about the relevance and usefulness of such measures in practical contexts.
FOOTNOTES


2. This is also the condition for welfare prescriptions to be price independent - see Roberts [1980].

3. See, for example, Ray [1983b], on the welfare implications of family size and composition, especially in relation to real expenditure comparisons, household utility and inequality.

4. Note, however, that we continue to assume that the SWF is linear and additively separable in individual utilities. Harsanyi [1955,1975] argues, for example, that a SWF must be a linear function of individual utilities, though not individual incomes (see, also, Broome [1983]). See, however, Diamond [1967] Rawls [1971] and Sen [1973] for dissenting views.

5. See Ray [1983a] for evidence on rural India's budget data.

6. See Bardhan [1974] for a survey of the literature on income distribution in India and, also, the volume edited by Srinivasan and Bardhan [1974].

7. See also Vaidyanathan [1974] for a vigorous defence of the use of NSS data to study inequalities in living standards in rural India.

8. Note that the terms 'income' and 'aggregate expenditure' are used synonymously in this paper.

9. In the absence of any similar information on \( p \) for rural India, the sensitivity of inequality measures to a-priori changes in \( p \) becomes an important policy issue.

10. Note the knowledge of \( \beta_i \) is not required for calculating either of the inequality measures.

11. Our parameter estimates show that \( a_i > 0 \) for all \( i \) and at all sample points.

12. Note that incorporating prices on the lines of Sec. 2.2 merely complicates calculations without any significant gain to justify it.

13. Bhaty also used NCAER based income data but the picture does not appear all that different, except over the period 1964/65 to 1967/68 when the income data shows a sharp rise in inequality.

14. In view of the wide regional variation in prices and expenditure patterns, however, the actual extent of government intervention would vary from State to State.
REFERENCES


J.C. Harsanyi (1975), 'Non Linear Social Welfare Functions: Do Welfare Economists have a Special Exemption from Bayesian Rationality?', Theory and Decision, 6, 311-332.


