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Working paper
~~Reprint Series~~ No. 8204

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ABSTRACT

This paper presents the Mean-Gini (MG) approach to analyze risky prospects and construct optimum portfolios. The method possesses the simplicity of the mean-variance model with the efficiency of stochastic dominance. Hence, Gini's mean difference is superior to the variance for evaluating the variability of a prospect. The analysis is further extended with the concentration ratio that permits to classify different securities with respect to their relative riskiness. The MG approach is then applied to capital markets and the security valuation theorem is derived as a general relationship between average return and risk. This is further extended to include a degree of risk aversion that can be estimated from capital market data.

(*) This research was supported by a grant from the Government of Israel, the National Committee for Research and Development, Grant No. 5107.

We are grateful to Haim Levy for useful comments. All the remaining errors are ours.

MEAN-GINI, PORTFOLIO ANALYSIS, AND THE PRICING OF RISKY ASSETS

The formal similarity between modelling decision-making under uncertainty and evaluating and interpreting income inequality has been noted and used by several economists. An example is Samuelson who used the same method to prove that it pays to diversify risky investment (1967) and that equal distribution of income among identical Benthamites maximizes the sum of social utility (1966). Atkinson (1970) showed that the rules for stochastic dominance, which were developed as criteria for evaluating risky investments, can be translated into Lorenz-curve terms in evaluating income inequality.

The purpose of our paper is to interpret some of the recent results on income inequality and apply them to portfolio analysis. In particular, we establish that Gini's mean difference, the Lorenz curve, and the concentration ratio, which are extensively used in the field of income inequality, can also be used to characterize risky prospects and construct optimum portfolios.

The superiority of Gini's mean difference over the variance as an index of variability has been demonstrated by Yitzhaki (1982). In particular, it was shown that using the mean and Gini's mean difference as summary statistics of the distribution of a risky investment permits the user to derive necessary conditions for stochastic dominance, enabling him to discard from the efficient set prospects that are stochastically dominated

by others. This property means that Gini's mean difference is a better candidate for evaluating the variability of a distribution. In the present paper, we argue that Gini's mean difference can replace the variance and that the concentration ratio based on it can replace the covariance needed in portfolio theory.

The mean-Gini portfolio selection rule is superior to the mean-variance selection rule in the sense that the mean-Gini efficient set consists of portfolios that, according to stochastic dominance criteria, cannot be dominated by any other portfolio. Furthermore, as with the mean-variance model, the mean-Gini ranking can be used to derive the pricing of risky assets in an equilibrium framework. But it is easier to compute efficient portfolios by mean-Gini rules than by mean-variance rules and much simpler than using stochastic dominance rules. Moreover, Gini's mean difference can be extended into a family of coefficients of variability, differing from each other by a parameter, ν, that represents the investor's risk aversion. It turns out that each member of this family can be used in portfolio theory for constructing capital-asset pricing models.

In the first section, we define Gini's mean difference and justify its use. In the second, the properties of a portfolio which is specified by its mean and its Gini coefficient are developed, while the third section presents some extensions of the Gini coefficient to portfolio theory, such

as the concentration ratio (Kakwani, 1977, Pyatt, Chen, and Fei, 1980). The fourth section is devoted to the capital-asset pricing model and contains a discussion of the properties of the approach, whereas the last section presents the extended Gini's mean difference (Yitzhaki, 1980) and applies it to portfolio analysis.

Much of the discussion in this paper is based on a reinterpretation of results on income inequality in the portfolio context. Thus, whenever possible we do not prove the propositions but refer the reader to the original papers.

I. GINI'S MEAN DIFFERENCE

Gini's mean difference is an index of the variability of a random variable. It is based on the expected value of the absolute difference between every pair of realizations of the random variable. That is, let $F(y)$ and $f(y)$ respectively represent the cumulative distribution and the density function of prospect y and assume that there exist $a > -\infty$ and $b < \infty$ such that $F(a) = 0$, $F(b) = 1$, then Gini's mean difference is defined as follows:[1]

$$(1) \qquad \Gamma = \frac{1}{2} \int_a^b \int_a^b |y - x| f(x) f(y) dx dy \ .$$

This definition is not easy to handle and one finds at least eight different formulations of Gini's mean difference in the literature. For our purposes it will be useful to deal with two of them. The first is

$$(2) \qquad \begin{aligned} \Gamma &= \int_a^b [1 - F(y)] dy - \int_a^b [1 - F(y)]^2 dy \\ &= \mu - a - \int_a^b [1 - F(y)]^2 dy \ . \end{aligned}$$

Equation (2) presents Gini's mean difference in terms of the expected value of the distribution, μ, and its cumulative distribution function, $F(y)$.[2] This equation is important when dealing with stochastic dominance criteria.[3]

The other Gini formula which is useful for analysing portfolios is[4]

$$(3) \qquad \Gamma = 2 \int_a^b y[F(y) - \tfrac{1}{2}] f(y) dy$$

that is, Gini's mean difference is twice the covariance of variable y and its cumulative distribution. For completeness, we state the following properties of Gini's mean difference:

1. The Gini coefficient is non-negative and bounded from above by $\mu - a$. If y is a given constant, $\Gamma = 0$. Furthermore, as is shown in Yitzhaki (1980), its maximum value is reached for the distribution

$$f(y) = \begin{cases} \frac{1}{2} & y = a \\ \frac{1}{2} & y = b \\ 0 & \text{otherwise.} \end{cases}$$

2. The Gini coefficient is sensitive to mean-preserving spreads (Atkinson, 1970).

3. Let $y_2 = \alpha y_1$, where α is a constant; then $\Gamma_2 = |\alpha|\Gamma_1$.

4. Let $y_2 = y_1 + c$ where c is a constant; then $\Gamma_2 = \Gamma_1$.

5. Let $y_3 = y_1 + y_2$, where y_1 and y_2 are two independently distributed variables; then $\Gamma_3 \leq \Gamma_1 + \Gamma_2$

6. Let y be a normally distributed variable with μ and σ^2; then $\Gamma = \sigma/\sqrt{\pi}$ (Nair, 1936).

Properties 2 to 5 are similar to those attributed to the standard deviation. Hence it is not surprising that the Gini coefficient can be used to derive the efficient set of uncertain prospects in the same way as is done using the mean-variance criterion. This feature is implied by the behavior of investors who rank uncertain prospects by their mean and the dispersion of their returns. Efficient sets of uncertain prospects are

constructed such that no other feasible prospect will be included in the set unless it has a lower dispersion for a given mean or a higher mean for a given dispersion. Usually the standard deviation is used as the measure of dispersion; we propose to use instead Gini's mean difference. Hence, the efficient set that answers the mean-Gini (MG) criterion is obtained by finding, for each given mean μ_o, that prospects which are not in the efficient set have at least larger or equal Gini's mean difference. If combinations of uncertain prospects are allowed to be held, the efficient set is obtained by constructing, for each given mean, a mix of prospects that minimize the Gini coefficient of that portfolio. We advocate the use of the mean-Gini (MG) method first because, if prospects are normally distributed, the efficient set of the mean-Gini is identical to the efficient set of the mean-variance (MV) method (see property 6 above). Secondly, it is justified by the superiority of the mean-Gini over the mean-variance approach in ranking uncertain prospects according to stochastic dominance (SD) rules as we will now bring forward.

Proposition 1: Let y_1 and y_2 be two uncertain prospects. The condition $\mu_1 - \Gamma_1 \geq \mu_2 - \Gamma_2$ is a necessary condition for y_1 to dominate y_2 according to first and second stochastic dominance rules (Yitzhaki, 1982). (See Appendix for a proof.)

Define S_{MG} the efficient set obtained by the MG method, S_{SD} the efficient set obeying the stochastic dominance first and second rules and S_1 the set obeying proposition 1, we assert the following: S_1 is a subset of S_{SD} and S_1 is also a subset of S_{MG}.

Hence, applying proposition 1 to the efficient set constructed by the MG method enables us to obtain an efficient set which is a subset of the efficient set according to first and second SD rules. This subset is not liable to the

criticism usually advanced against the efficient set constructed by the MV rule (See Rothschild and Stiglitz, 1970; Hanoch and Levy, 1969). The use of proposition 1 can be demonstrated by an example: assume that y_1 is uniformly distributed between 0 and 1 while y_2 is uniformly distributed between 2 and 4. Both y_1 and y_2 are in the efficient set according to mean-variance and mean-Gini rules, with $\mu_1 = \frac{1}{2}$, $\Gamma_1 = \frac{1}{6}$, $\sigma_1 = \frac{1}{\sqrt{12}}$ and $\mu_2 = 3$, $\Gamma_2 = \frac{1}{3}$, $\sigma_2 = \frac{1}{\sqrt{3}}$. But clearly all investors prefer y_2 over y_1. Applying proposition 1 to the set obtained by the mean-Gini criterion enables us to discard y_1 from the efficient set.

The superiority of the mean-Gini approach over the stochastic dominance (SD) criteria results from its similarity to the MV method. As far as we know, there is no method for constructing optimum portfolios by SD rules. Users of the mean-Gini method may minimize the Gini coefficient of a linear combination of prospects subject to a given required mean return. Changing the mean permits the user to construct the efficient set corresponding to the mean-Gini criterion. This set can be used for portfolio analysis and capital asset pricing equilibrium according to MG rules. Furthermore, by applying proposition 1 to that efficient set, a subset is obtained which <u>is contained</u> in the efficient set of portfolios according to SD rules.

II. PORTFOLIO ANALYSIS

The properties of a portfolio whose performance is summarized by the mean and the Gini coefficient are similar to those of the regular mean-standard deviation model. These properties can be illustrated by the familiar textbook diagram in which Γ, the Gini coefficient of the return on the portfolio, is depicted on the horizontal axis, while, μ, the mean of the one-period return, is on the vertical axis, as in Figure 1.

The performance of two prospects, A and B, is denoted by A and B in the mean-Gini space. The return on portfolio y_p is given by the convex combination of the return on A and B. Hence $y_p = \alpha x_A + (1 - \alpha)x_B$ where α is the share of wealth invested in A and x_A and x_B are the one-period returns.

As in the ordinary mean-standard deviation model the performance of the portfolio y_p depends also on the correlation between A and B and on α. To show this, consider the three special cases as drawn in Figure 1. First, if prospects A and B are linearly dependent, the coefficient of correlation (ρ_{AB}) is equal to unity and, following from the properties of Gini's mean difference, the line ACB represents all the possibilities of a portfolio mix composed of A and B. Second, if A and B are independent ($\rho_{AB} = 0$), the curve ADB expresses the performance of the portfolio, y_p, showing intuitively that diversification improves that performance. Furthermore, the portfolio returns would be much improved if A and B were negatively correlated as shown by the broken line AFB for the extreme case of $\rho_{AB} = -1$.

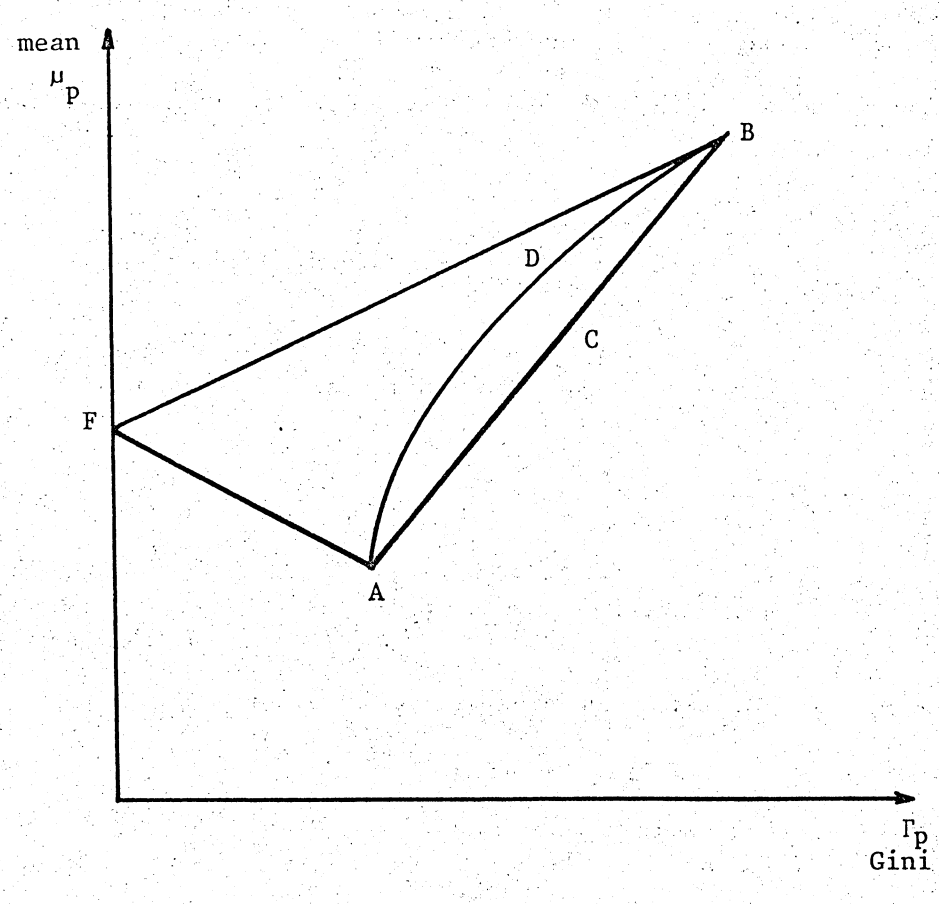

Figure 1

The effect of the variability of a prospect on the variability of the portfolio can be presented much as it is in the mean-standard deviation model. By equation (3), the Gini coefficient of a portfolio is

$$(4) \quad \Gamma_p = 2 \int_a^b y[F_p(y) - \tfrac{1}{2}]dF_p(y) = 2\mathrm{cov}[y_p, F_p(y_p)],$$

where F_p is the cumulative distribution of the portfolio. Since

$$(5) \quad y_p = \sum_{i=1}^{N} \alpha_i x_i , \qquad \text{for } \alpha_i \geq 0 , \quad \sum_{i=1}^{N} \alpha_i = 1$$

where x_i is the return on prospect i, we obtain

$$(6) \quad \Gamma_p = 2 \sum_{i=1}^{N} \alpha_i \mathrm{cov}[x_i, F_p(y_p)] ;$$

that is, the risk of the portfolio can be decomposed into a weighted sum of the covariance between the variables x_i and the cumulative distribution of the portfolio, p.

It is worth mentioning that the variance of the portfolio can be written as

$$(7) \quad \mathrm{var}(y_p) = \sum_{i=1}^{N} \alpha_i \mathrm{cov}(x_i, y_p) ,$$

and the difference in the decomposition of the nondiversifiable risk by the two methods is that in (6) the portfolio is represented by the cumulative distribution of its returns, F_p, while in (7) it is represented by its returns, y_p .

By multiplying and dividing each component of (6) by $\Gamma_i = 2\mathrm{cov}[x_i, F_i(x_i)]$ where $F_i(x_i)$ is the cumulative distribution of prospect i, we obtain

$$(8) \qquad \Gamma_p = \sum_{i=1}^{N} \alpha_i R_i \Gamma_i ,$$

where $R_i = \mathrm{cov}[x_i, F_p(y_p)]/\mathrm{cov}[x_i, F_i(x_i)]$ which represents the ratio of non-diversifiable risk tothe variability of prospect i .

III. THE CLASSIFICATION OF PROSPECTS BY RELATIVE RISK

One can classify a security by its risk by using $\mathrm{cov}[x_i, F_p(y_p)]$ as an index of the undiversified risk carried by a security i whereas the diversified risk is the share of total risk that can be reduced by including security i in the portfolio y_p. However, this classification is silent about the possibility that security i can be riskier than security j in one situation and less risky in another.

One way to improve the classification is to use concentration curves that enable us to determine the security's risk according to the return on the portfolio.[5] This classification enables us to determine, for any two prospects A and B within portfolio p, whether all investors agree that prospect A is riskier than B with regard to portfolio p or that they may disagree among them about the relative riskiness of A. Assume two securities with identical positive expected rates of return; i. e., $E_i(x_i) = E_j(x_j) = \mu_j$. Now define the function $g_j(y_p)$ as the conditional expectation of rate of return x_j, given the portfolio y_p; that is,

$$(9) \quad g_j(y_p) = E_j(x_j/y_p) .$$

We assume that $y_p \geq 0$, $g_j(y_p) \geq 0$ and that the first derivative of g( ) exists.[6] If $E[g_j(y_p)] = \mu_j$, one can define the concentration curve of security j as the relationship between

$$\phi_j[g_j(y_p)] = \frac{1}{\mu_j} \int_0^{y_p} g_j(y) f_p(y) dy$$

(10)

$$F_p(y_p) = \int_0^{y_p} f_p(y) dy .$$

Concentration curves are plotted in Figure 2 below. It can easily be seen that if $x_j = y_p$, equation (10) represents the Lorenz curve of portfolio y_p (curve B in the figure). The relative riskiness of the securities in a portfolio can be compared according to the following proposition:

<u>Proposition 2</u>:[7] The concentration curve for the function $g_j(y_p)$ will be above (below) the concentration curve for the function $g_i(y_p)$ if $\eta_i(y_p)$ is less (greater) than $\eta_j(y_p)$, for all y_p, where η is the elasticity of g with respect to y.

Two corollaries follow from proposition 2 :

<u>Corollary 1</u>: The concentration curve for the function $g_j(y)$ will be above (below) the egalitarian line (45^0 line) if $\eta_j(y)$ is less (greater) than zero, for all y.

That is, in Figure 2, the 45^0 degree line represents the risk free asset. Stocks which are always negatively correlated with the portfolio have a concentration curve which is above the 45^0 degree line.

The second corollary permits comparison between the securities and the portfolio they make up.

Corollary 2: The concentration curve for the function $g_j(y)$ lies above (below) the Lorenz curve for the distribution $F_p(y_p)$ if $\eta_j(y_p)$ is less (greater) than unity for all $y_p \geq 0$.

Corollary 2 permits us to distinguish between two kinds of securities in a portfolio. First, we observe aggressive securities, with concentration curves below the Lorenz curve of the portfolio, whose high degree of responsiveness to the portfolio leads to considerable instability. Second, we have defensive securities, with concentration curves above the Lorenz curve of the portfolio which reduce instability because they are less responsive. These results are summarized in Figure 2.

Let OAB be the Lorenz curve of the portfolio. The aggressive stock will be represented by OCA while the defensive stock is represented by the concentration curve ODA. The 45^0 line portrays the risk-free asset (if it exists) while OFA represents a stock which is negatively correlated with the portfolio.

By construction and definition, the relative Gini coefficient is 1 - 2 (area under the Lorenz curve) and the relative concentration ratio for security j is 1 - 2 (area under the concentration curve for security j). Hence,

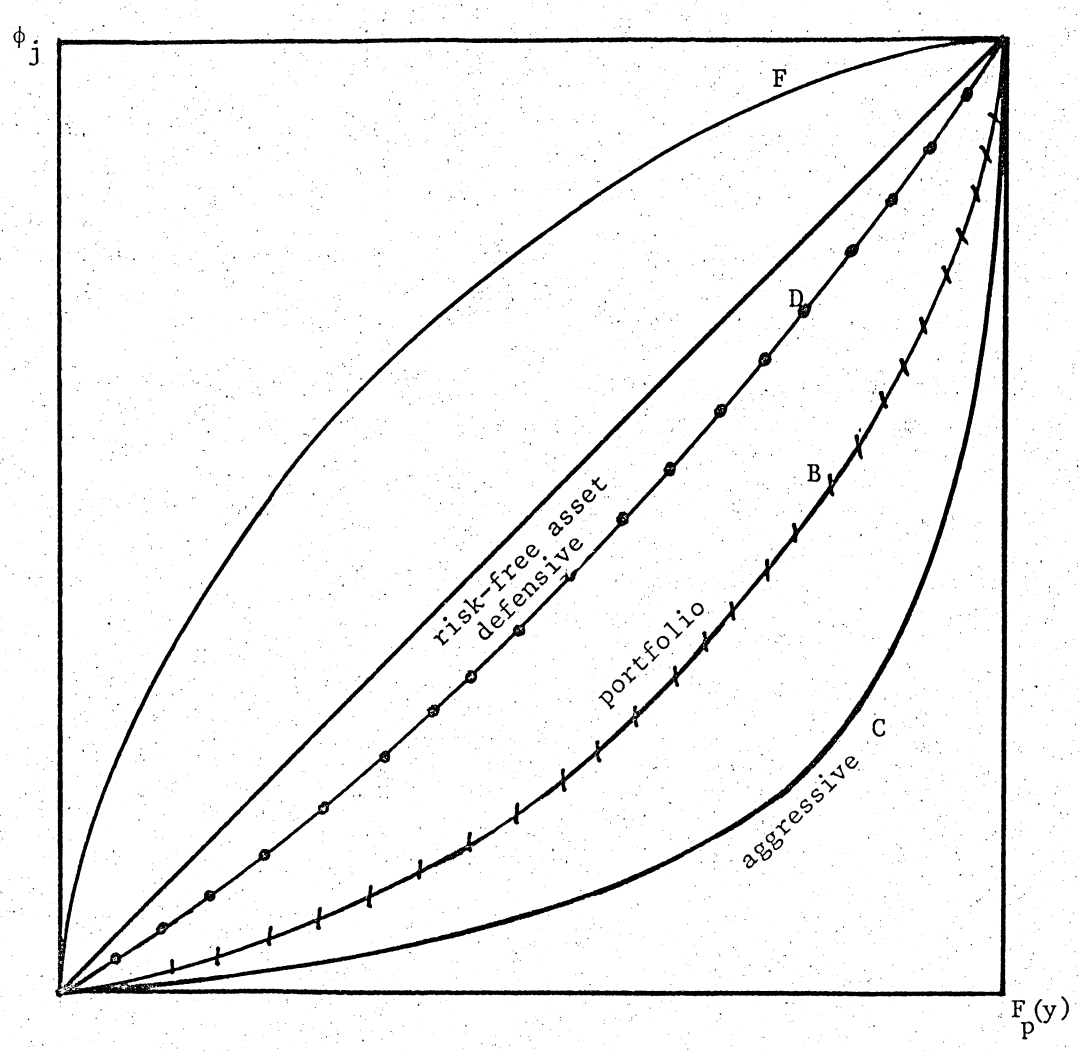
ϕ_j
F
D
B
C
risk-free asset
defensive
portfolio
aggressive
$F_p(y)$

Figure 2

whenever concentration curves do not intersect, the classification of securities by the Gini coefficient and the concentration ratio truly represents their relative riskiness. That is, if $\mathrm{cov}[x_i, F_p(y_p)] > \mathrm{cov}[y_p, F_p(y_p)]$, security i is said to be <u>aggressive</u>, and it is said to be <u>defensive</u> if the inequality is reversed. However, when concentration curves intersect, the relative riskiness of security i depends on the investor aversion towards risk. Thus, different investors may disagree about the relative riskiness of securities.

IV. THE PRICING OF RISKY ASSETS

In this section we develop the security valuation theorem for investors holding mean-Gini efficient portfolios. The CAPM market-equilibrium relationship has been formulated for MV efficient portfolios by Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966). The theorem states that for any security, the higher its nondiversifiable risk, the higher will be its expected return. Nondiversifiable (or systematic) risk is that part of the security's total risk that cannot be reduced by diversifying a portfolio without reducing its expected rate of return. The theorem is stated in the context of competitive financial markets without taxes and without restrictions on short selling and borrowing. In these markets investors trade risky assets whose quantities are known and fixed, to build efficient portfolios that answer their preferences. By doing so, they act in the securities market, building forces that influence and determine the value of these securities. Assuming that investors build their portfolios according to a MV utility, the familiar CAPM relationship between expected return and risk is expressed as

$$(11) \qquad \mu_i = r_f + [\mu_m - r_f]\,[\mathrm{cov}(x_i, y_m)]/\sigma_m^2 \quad ,$$

where $\mu_i = E(x_i)$ is the expected rate of return on security i

r_f' = is the rate of return on a risk-free security

$\mu_m = E(y_m)$ is the expected rate of return on the market portfolio

σ_m^2 = variance of the rate of return on the market portfolio.

Equation (11) was derived under several assumptions of which the most important are (a) single-period analysis, (b) the existence of a risk-free asset, and (c) perfect competition in the securities market.

Since then, most efforts in modern financial theory have been directed to adapting the model to different contexts and testing it empirically.

Much less attention has been devoted to whether its theoretical foundations were sound, with the notable exception of stochastic dominance theory (SD). Unfortunately, the SD approach was found to be computationally cumbersome and cannot provide answers to investors' questions on security valuation in terms of portfolio diversification.

The valuation theorem proposed here retains the main assumptions of the classical CAPM. However, instead of holding MV efficient portfolios, investors construct market portfolios which are SD efficient. This is done by the mean-Gini approach.

Each investor determines his optimum portfolio by choosing a securities mix that minimizes the Gini's mean difference of the portfolio given its expected rate of return. Investors are permitted to borrow and lend at the riskless rate, r_f. The rate of return of investor j's portfolio is given by

$$(12) \quad y_j = \sum_{i=1}^{N} \alpha_i^j x_i + (1 - \sum_{i=1}^{N} \alpha_i^j) r_f, \qquad \Sigma\alpha_i^j \leq 1 ,$$

where x_i is the rate of return on security i (i = 1, ..., N) and α_i^j is the share of investor j's wealth invested in security i. Hence, the investors' problem is to minimize Γ_j subject to

$$(13) \quad E(y_j) = \sum_{i=1}^{N} \alpha_i^j \mu_i + (1 - \sum_{i=1}^{N} \alpha_i^j) r_f .$$

Recall that the Gini coefficient of the portfolio can be written as

$$(14) \quad \Gamma_j = 2\mathrm{cov}[y_j, F_j(y_j)] = 2 \sum_{i=1}^{N} \alpha_i^j \mathrm{cov}[x_i, F_j(y_j)] .$$

Thus the necessary conditions for a minimum are simply:

$$(15) \quad 2\ \mathrm{cov}[x_i, F_j(y_j)] + 2 \sum_{k=1}^{N} \alpha_k^j \frac{\partial\ \mathrm{cov}[x_k, F_j(y_j)]}{\partial \alpha_i} = \lambda_j(\mu_i - r_f) \quad \text{for all } i = 1, \ldots, N$$

and equation (13), where λ_j is the Lagrange multiplier associated with investor j .

As we saw earlier, $\mathrm{cov}[x_i, F_j(y_j)]$ is the concentration ratio[8] between security i and the portfolio of individual j. It represents the degree by which the risk of security i cannot be diversified by including it in j's portfolio.

By property 3 of the Gini coefficient, we know that Γ_j is homogeneous of degree one in α_i^j. Therefore

$$(16) \quad \Gamma_j = \sum_{i=1}^{N} \alpha_i^j \frac{\partial \Gamma_j}{\partial \alpha_i}$$

or

$$(16a) \quad \Gamma_j = 2 \sum_{i=1}^{N} \alpha_i^j \ \mathrm{cov}[x_i, F_j(y_j)] + 2 \sum_{i=1}^{N} \sum_{k=1}^{N} \alpha_i^j \alpha_k^j \frac{\partial \, \mathrm{cov}[x_k, F_j(y_j)]}{\partial \alpha_i} ,$$

implying by equation (14) that the double sum on the right-hand-side vanishes.

By multiplying each of the conditions in (15) by its share α_i^j and summing over all the securities N, we obtain for every investor

$$(17) \quad \Gamma_j = \lambda_j \sum_{i=1}^{N} \alpha_i^j (\mu_i - r_f)$$

or

$$(18) \quad \Gamma_j = \lambda_j [\sum_{i=1}^{N} \alpha_i^j \mu_i - r_f + (1 - \sum_{i=1}^{N} \alpha_i^j) r_f]$$

and we have obtained the relation between risk (expressed by the Gini) of the portfolio and its expected return as

$$(19) \quad \Gamma_j = \lambda_j [E(y_j) - r_f] .$$

Equation (19) represents the highest feasible straight line in the (μ, Γ) space given an efficiency frontier constructed by portfolio combinations of risky assets. Since the efficiency frontier is concave within that space due to properties 3 and 5, the second-order conditions for a minimum are satisfied. In that context $1/\lambda_j$ is the investor subjective price of risk since it relates the expected rate of return of the chosen portfolio to its risk. The investor will chose a portfolio mix along that line that maximizes his utility.

Now assume a market of similar investors who are risk averse, have identical investment opportunities, and minimize the Gini coefficients of their portfolios subject to their expected rates of return. In that case, equation (19) will be identical for all the investors in that market.

For a given risk-free rate of return, the unit price of risk will be equal and determined by the slope of the market line in the (μ, Γ)space (see Figure 3).

For an investor which does not borrow nor lend, all his wealth will be invested in risky securities whose portfolio is the market portfolio, pictured by m in Figure 3. Thus for all investors, the price of risk will be

$$(20) \quad 1/\lambda = (\mu_m - r_f) / \Gamma_m$$

where μ_m is the expected return on the market portfolio and Γ_m is its Gini coefficient. In that case, the Gini coefficient of the portfolio held by investor j is equal to Γ_m since the optimum ranking, $F_j(y_j)$ of investor j, remains unchanged whether or not a risk-free security is added and thus is equal to $F_m(y_m)$ for all investors. Thus, from (15)

$$(21) \quad 2 \sum_{k=1}^{N} \alpha_k^j \frac{\partial \operatorname{cov}[x_k, F_m(y_m)]}{\partial \alpha_i} = 0$$

and the equilibrium condition for every security i and investor j becomes

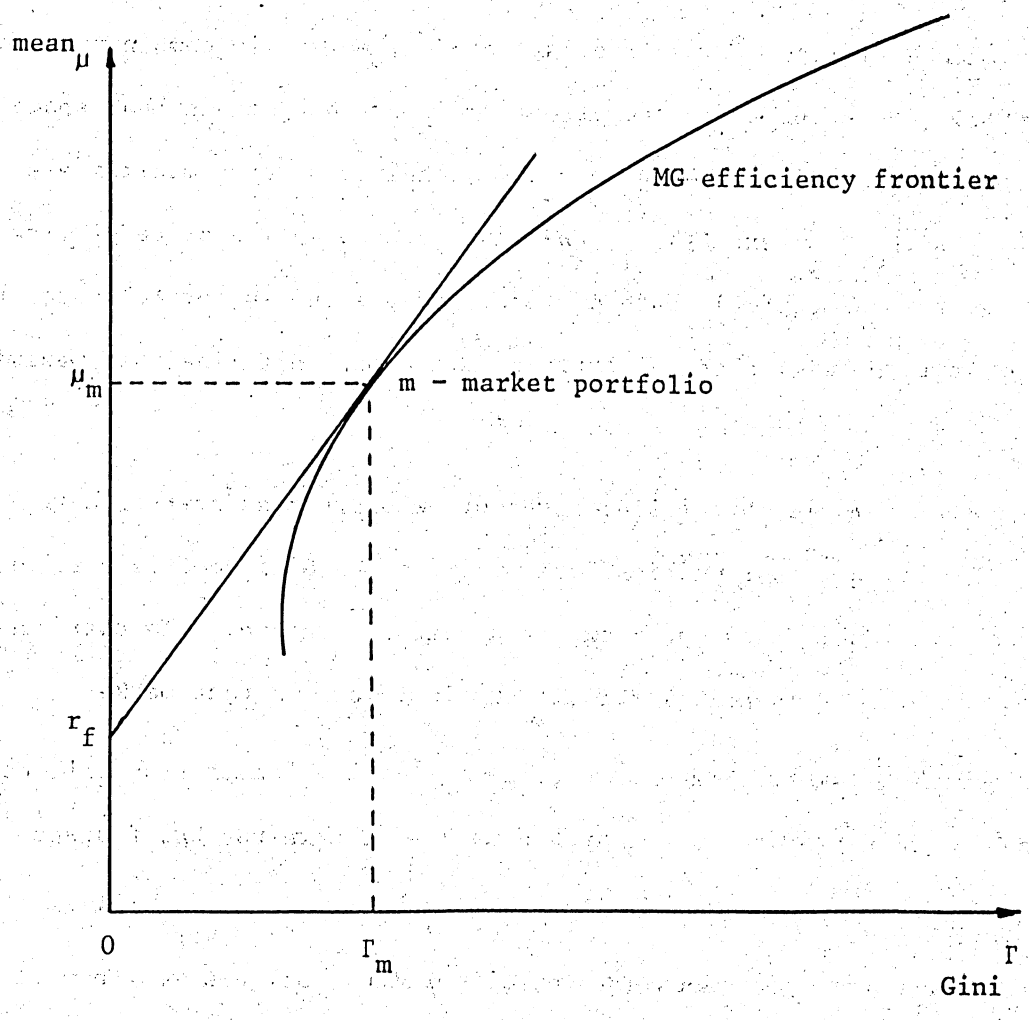
mean μ
MG efficiency frontier
μ_m
m - market portfolio
r_f
0
Γ_m
Γ
Gini

Figure 3

(22) $$\mu_i = r_f + (\mu_m - r_f) \cdot 2\mathrm{cov}[x_i, F_m(y_m)] / \Gamma_m .$$

This is essentially the CAPM valuation relationship for a market of investors using the mean-Gini approach. To understand the dependence between systematic risk and expected return, let us rewrite $2\mathrm{cov}[x_i, F_m(y_m)]$ as $R_{im}\Gamma_i$ where

$$R_{im} = \frac{\mathrm{cov}[x_i, F_m(y_m)]}{\mathrm{cov}[x_i, F_i(x_i)]}$$

is a sort of rank correlation coefficient and $\Gamma_i = 2\ \mathrm{cov}[x_i, F_i(x_i)]$.

Thus

(23) $$\mu_i = r_f + (\mu_m - r_f)R_{im}(\Gamma_i/\Gamma_m) .$$

Therefore, security i's beta is simply

(24) $$\beta_i = R_{im}(\Gamma_i/\Gamma_m) .$$

As is well known, β_i represents the degree of responsiveness of the rate of return of security i to changes in the market, and R_{im} is the proportion of total risk (expressed by the Gini coefficients of security i, Γ_i) that cannot be eliminated by the market without reducing the expected rate of return.

At this point, it is important to draw the analogy between the β_i in (24) and the β_i derived from the MV-CAPM. It can be shown that for normally distributed securities, $\Gamma_i = \sigma_i/\sqrt{\Pi}$ and $\Gamma_m = \sigma_m/\sqrt{\Pi}$, where σ_i and σ_m are the standard deviations of i and m .

Therefore, R_{im} does converge to the Pearson correlation coefficient between security i and the market m . This assertion is intuitively deduced since in the case of normally distributed prospects, the MV and MG 'betas' coincide for the same set of observations. However this is not always true for prospects that are <u>not</u> normally distributed. In general, MV and MG betas will be different; with the MG betas corresponding to SD efficient securities markets. Furthermore when investors use the mean-extended-Gini (MEG) approach to evaluate risk and construct efficient portfolios, the computation and estimation of betas will depend on their attitude towards risk, as will now be shown, when we extend the analysis with the MEG.

V. THE EXTENDED GINI COEFFICIENT

In this section we develop the extended Gini coefficient and apply it to portfolio analysis and the CAPM. Gini's mean difference may be extended into a family of coefficients of variability differing from each other in the decision-maker's degree of risk aversion, which is reflected by the parameter ν. Rewrite equation (2) as

$$\Gamma(\nu) = \int_a^b [1 - F(y)]dy - \int_a^b [1 - F(y)]^\nu dy$$

$$(25) \qquad = \mu - a - \int_a^b [1 - F(y)]^\nu dy \ ,$$

where $1 < \nu < \infty$ is a parameter chosen by the user of the method. We define (25) as the extended Gini coefficient whose properties are (Yitzhaki, 1980):

1. The extended Gini coefficient is non-negative and bounded from above for all ν. That is, $0 \leq \Gamma(\nu) \leq \mu - a$. It is a non-decreasing function of ν. The proof of this property is immediate if we remember that $1 \geq F(x) \geq 0$ for $a \leq x \leq b$.

2. For $\nu = 1, 2, 3, 4 \ldots$ integers, the extended Gini coefficient is simply $\Gamma(\nu) = \mu - E[\min(x_1, x_2, \ldots, x_\nu)]$ since

$$F_{\min(x_i)}(y) = \text{Prob}[\min_{i=1,2\ldots\nu}(x_i) \leq y] = 1 - \text{Prob}[\min(x_i) > y] = 1 - [1-F(y)]^\nu .$$

Thus, $E[\min_{i=1,2,\ldots\nu.}(x_i)] = a + \int_o^b [1-F(y)]^\nu dy$ for ν integer

3. The extended Gini coefficient is sensitive to mean-preserving spreads.

4. If $y_2 = \alpha y_1$, then $\Gamma_2(\nu) = |\alpha|\Gamma_1(\nu)$.

<u>Proof</u>: Since $F_2(y_2) = F_1(\alpha y_1)$ for $\alpha > 0$ we get

$$\int_{\alpha a}^{\alpha b} \{[1 - F_2(y_2)] - [1 - F_2(y_2)]^{\nu}\}\, dy_2$$

$$= \alpha \int_a^b \{[1 - F_1(y_1)] - [1 - F_1(y_1)]^{\nu}\} dy_1 \ .$$

5. Let $y_2 = y_1 + c$; then $\Gamma_2(\nu) = \Gamma_1(\nu)$.

6. Let y_1, y_2 be two prospects; then $[\mu_1 - \Gamma_1(\nu)] - [\mu_2 - \Gamma_2(\nu)] \geq 0$

for $\nu = 1, 2, 3, \ldots$ is the necessary condition for first and second degree stochastic dominance (Yitzhaki 1982).

7. Let y_1, y_2 be two prospects with cumulative distributions that intersect at most once. Then $[\mu_1 - \Gamma_1(\nu)] - [\mu_2 - \Gamma_2(\nu)] \geq 0$ for all $\nu = 1, 2, 3$ is a sufficient condition for y_1 to stochastically dominate y_2 (Yitzhaki, 1982).

The interpretation of ν can best be seen by looking at $\mu - \Gamma(\nu)$. Its value for different values of ν are

$$\mu - \Gamma(\nu) = \begin{cases} \mu & \nu = 1 \\ \mu - \Gamma & \nu = 2 \\ a & \nu \to \infty \end{cases}$$

and it is a non-increasing function of ν . Thus one can view $\Gamma(\nu)$ as the risk premium that should be substracted from the expected value of the distribution. The case $\nu = 1$ represents the risk-neutral investor while $\nu \to \infty$ represents the investor who is interested in the minimum value of the distribution, and maximizes this minimum.[9]

The extended Gini coefficient of a portfolio can be decomposed into an equation similar to (6). That is,

$$(26) \quad \Gamma_p(\nu) = -\nu \sum_{i=1}^{N} \alpha_i \mathrm{cov}\{x_i, [1 - F_p(y_p)]^{\nu-1}\} .$$

<u>Proof</u>: Let $y_p = \sum_{i=1}^{N} \alpha_i x_i$; then

$$\Gamma_p(\nu) = \mu_p - a - \int_a^b [1 - F_p(y)]^{\nu} dy .$$

Defining v and u as

$$v = [1 - F(y)]^{\nu} \quad ; \quad \frac{\partial v}{\partial y} = -\nu[1 - F(y)]^{\nu-1} f(y)$$

$$u = y \quad ; \quad \frac{\partial u}{\partial y} = 1$$

and applying integration by parts, we get

$$\Gamma_p(\nu) = \mu_p - a - [1 - F(y)]^{\nu} y \Big|_a^b - \nu \int_a^b [1 - F(y)]^{\nu-1} y f(y) dy$$

$$= \mu_p - \nu \int_a^b [1 - F_p(y)]^{\nu-1} y f_p(y) dy ;$$

but

$$\int_a^b [1 - F_p(y)]^{\nu-1} f_p(y)dy = -[1 - F_p(y)]^{\nu}/\nu \Big|_a^b = \frac{1}{\nu} ,$$

thus

$$\Gamma_p(\nu) = \mu_p - \nu \int_a^b \{[1 - F_p(y)]^{\nu-1} - \frac{1}{\nu}\} yf_p(y)dy - \int_a^b yf_p(y)dy$$

$$= -\nu \int_a^b \{[1 - F_p(y)]^{\nu-1} - \frac{1}{\nu}\} yf_p(y)dy$$

$$= -\nu \mathrm{cov}\{y_p, [1 - F_p(y)]^{\nu-1}\} .$$

Therefore

$$(27) \quad \Gamma_p(\nu) = -\nu \sum_{i=1}^{N} \alpha_i \mathrm{cov}\{x_i, [1 - F_p(y)^{\nu-1}\} ,$$

since $y_p = \sum_i \alpha_i x_i$.

Q.E.D.

That is, the nondiversified risk of a prospect is its covariance with the portfolio rank to the power ν . The higher ν, the greater the weight given to the performance of the prospect when the yield of the portfolio is low. Note that if $\nu = 2$, then we have the regular Gini coefficient.

We derive the CAPM, using the extended Gini of degree ν. For investor j , the extended-Gini of his portfolio is given by

$$\Gamma_j(\nu) = -\nu \sum_{i=1}^{N} \alpha_i^j \mathrm{cov}\{[1 - F_j(y_j)]^{\nu-1}, x_i\} .$$

Investor j chooses the α_i^j that minimize $\Gamma_j(\nu)$ subject to the expected rate of return on the portfolio given by (12). As shown, ν reflects the degree of risk aversion. Hence, it is possible to model a securities market that will exhibit different market portfolios because of different degrees of risk aversion. For the present, we require that investors are similar and that they display identical ν. Therefore, for all j, the necessary conditions for a minimum are

$$(28)\quad \begin{aligned} -\nu\mathrm{cov}\{[1 - F_j(y_j)]^{\nu-1}, x_i\} &= \lambda_j(\mu_i - r_f) \\ 1/\lambda_j &= (\mu_m - r_f)/\Gamma_m(\nu) , \end{aligned}$$

where $\Gamma_m(\nu)$ is the market portfolio's extended Gini coefficient of degree ν, and the market-equilibrium relation becomes

$$(29)\quad \mu_i = r_f - [(\mu_m - r_f)/\Gamma_m(\nu)]\ \nu\mathrm{cov}\{[1 - F_m(y_m)]^{\nu-1}, x_i\}$$

If the 'rank' correlation ratio of degree ν between security i and the market is

$$R_{im}(\nu) = \frac{\mathrm{cov}\{[1 - F_m(y_m)]^{\nu-1}, x_i\}}{\mathrm{cov}\{[1 - F_i(x_i)]^{\nu-1}, x_i\}}$$

the market-equilibrium valuation for every i and any ν is given by

$$(30) \quad \mu_i = r_f + (\mu_m - r_f) R_{im}(\nu) \frac{\Gamma_i(\nu)}{\Gamma_m(\nu)}$$

Thus,

$$(31) \quad \beta_i(\nu) = R_{im}(\nu) \frac{\Gamma_i(\nu)}{\Gamma_m(\nu)}$$

Thus, even if investors have the same attitude towards risk as expressed by ν, different efficient market portfolios and different systematic risks for each security will be obtained. This feature must be borne in mind when estimating betas. It must be added that if securities are normally distributed, the MEG β's will be identical to the MV betas, independently of ν. But our concern for the existence for different efficient market portfolios is principally directed towards distributions other than normal such as the log normal and the uniform distribution. For these distributions, the MV approach is not consistent with expected utility maximization and stochastic dominance, whereas the mean-Gini is.[10]

VI. CONCLUSION

We have presented a new approach to analyze risky prospects and construct optimal portfolios. This method owns the simplicity of a two-parameter model with the efficiency of stochastic dominance. To that extent, we claim that Gini's mean difference is superior to the variance for evaluating the variability of a security. Furthermore, the concentration ratio based on the Gini coefficient permits us to classify different securities with respect to their relative riskiness. Finally we have applied the MG approach to capital markets and the security valuation theorem was derived on a general relationship between average return and risk. By extending the analysis with the mean-extended-Gini method, we have explicitly introduced the degree of risk aversion as a parameter that can determine the specific composition of market portfolios.

The main implications of the model reside as whether investors, in general, behave more in a MG or MEG framework rather than follow the MV approach. These implications can be empirically tested by estimating the performance of CAPM for different degrees of ν , and comparing these with the results obtained from MV-CAPM. Two-parameter portfolio models were tested using regression techniques. This was notably exemplified by Fama and MacBeth (1973) who supported the hypothesis that risk-averse investors hold efficient portfolio in terms of mean and standard-deviation of the returns. Recently, certains doubts were

raised as to whether the different regression procedures were to test the CAPM (see Ross, 1980). By proposing the mean-Gini and mean-extended-Gini models as means of evaluating capital market data, we add a new dimension to modern finance theory, suggesting that we should return to the drawing board.

Appendix

Proposition 1: (Yitzhaki, 1982). The condition $\mu_1 - \Gamma_1(\nu) \geqq \mu_2 - \Gamma_2(\nu)$ is a necessary condition for y_1 to dominate y_2 according to first and second stochastic rules for $\nu \geqq 0$.

Proof: For y_1 to dominate y_2 according to FSD and SSD rules it is necessary that

(A.1) $$F_1(y) \leqq F_2(y) \quad \text{for all } y$$

and

(A.2) $$\int_a^y F_1(t)dt \leqq \int_a^y F_2(t)dt \quad \text{for all } y.$$

The condition $\mu_1 - \Gamma_1(\nu) \geqq \mu_2 - \Gamma_2(\nu)$ implies

(A.3) $$\int_a^b [1-F_1(y)]^\nu dy \geqq \int_a^b [1-F_2(y)]^\nu dy \quad \text{for } \nu > 1.$$

If $\nu = 1$, the proposition is proved directly since (A.3) holds whenever (A.1) and (A.2) exist. For $\nu > 1$ we know that since the function z^ν is strictly convex for positive z, $z^\nu > z_o^\nu + \nu z_o^{\nu-1}(z-z_o)$. Thus

(A.4) $$[1-F_1(y)]^\nu > [1-F_2(y)]^\nu + \nu[1-F_2(y)]^{\nu-1}[F_2(y) - F_1(y)]$$

and

(A.5) $$\int_a^b \{[1-F_1(y)]^\nu - [1-F_2(y)]^\nu\}dy > \nu\int_a^b [1-F_2(y)]^{\nu-1}[F_2(y)-F_1(y)]dy .$$

since $$\int_a^y [F_2(t) - F_1(t)]dt \geqq 0 \quad \text{for all } y \text{ by (A.2)}$$

and

$[1-F_2(y)]^{\nu-1}$ is non negative and non increasing in y,

(A.6) $$\int_a^b [1-F_2(y)]^{\nu-1}\cdot[F_2(y) - F_1(y)]dy \geqq 0 \quad \text{for } \nu > 1.$$

by the following lemma.

Thus from (A.5) we have

(A.7) $$\int_a^b [1-F_1(y)]^{\nu}\, dy \geqq \int_a^b [1-F_2(y)]^{\nu}\, dy$$

whenever y_1 stochastically dominates y_2 .

<u>Lemma</u>: Let $h(z)$ be a non-negative and non-increasing function of z, and let $g(z)$ be a function with the property that $\int_a^x g(z)dz \geqq 0$ for all x. Then $\int_a^x h(z)\cdot g(z)dz \geqq 0$ for all x.

<u>Proof</u>: Assume $g(x)$ changes signs n times between a and b at $x_1, x_2, \ldots, x_n$. Then $g(x) > 0$ for $a < x < x_1$ and $g(x) < 0$ for $x_1 < x < x_2$ and so on. Thus $\int_a^{x_2} g(z)\cdot h(z)dz \geqq h(x_1)\int_a^{x_2} g(z)dz \geqq 0$.

Since this argument can be repeated for x_i when $i = 2,\ldots n$,

$$\int_a^{x_i} g(z)\cdot h(z)dz \geqq h(x_{i-1})\int_a^{x_i} g(z)dz \geqq 0 ,$$

we have

$$\int_a^x g(z)\cdot h(z)dz \geqq 0 \quad \text{for all } x.$$

FOOTNOTES

[1]For simplicity of presentation, continuous and bounded random variables will be used throughout the paper, keeping in mind that most of the results can be applied to discontinuous and unbounded distributions. The index i will be omitted whenever it is not necessary to distinguish between two variables. Note that we use half of Gini's original mean difference. Note too that we here use the absolute forms of Gini's mean difference and the concentration ratio; that is, we do not follow the more usual practice of dividing by the mean (see Kendall and Stuart, 1977).

[2]For derivation of equation (2) for continuous, discrete and unbounded distribution see Dorfman (1979).

[3]For the definitions of the stochastic dominance rules see Hanoch and Levy (1969) and Rothschild and Stiglitz (1970).

[4]For derivation of formula (3) see Kendall and Stuart (1977) for the continuous case and Pyatt, Chen, and Fei (1980), in the case of discrete distributions.

[5]For a discussion of the properties of the concentration curve see Kakwani (1977) and Pyatt et al. (1980).

[6]The restrictions $g_j(\ ) \geq 0$ and $y_p \geq 0$ should be interpreted as a shift of the origin.

[7]This proposition and the two corollaries were proved by Kakwani (1977).

[8]Note that to obtain the concentration ratio as defined by Pyatt et al. (1980), $\mathrm{cov}[x_j, F(y_j)]$ should be divided by $E_j(x_j)$.

[9]Values of $0 \leq \nu < 1$ represent the case of the risk lover, the extreme case $\nu = 0$ representing the investor interested in the maximum value of the distribution, b (the max-max investor). In this paper, we restrict ourselves to risk-averse investors.

[10]See Yitzhaki (1982) for a consistency analysis of the mean-Gini approach for different distributions.

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