RELATIVE COST-EFFECTIVENESS OF INPUT AND OUTPUT SUBSIDIES*

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A subsidy on a single input is compared with an output subsidy as a means of stimulating output, and the conditions under which the single input subsidy is (a) more treasury cost-effective and (b) overall the more socially efficient measure, are explored. Rationalisations for input subsidies, particularly fertiliser subsidies, are examined in the light of the results.

Introduction

According to conventional wisdom, an output subsidy is more efficient than an input subsidy as a means of encouraging output of a good. The argument runs that, irrespective of whether the effect on output of either subsidy is corrective or distorting, the input subsidy does, and the output subsidy does not, distort the choice of inputs away from the least-cost combination.  

Apart from such obvious exceptions as the case of fixed input coefficients, or an input-market distortion corrected by the input subsidy, the argument is valid, provided that either:

(a) the treasury cost of stimulating a given increase in output is no greater for the output than for the input subsidy; or
(b) this cost is met by non-distorting taxes.

But if, as is the realistic case, the raising of funds to pay subsidies itself gives rise to dead-weight losses elsewhere in the economy, and if the input subsidy is more cost-effective than the output subsidy in stimulating output, the possibility exists that the input subsidy will be, overall, the more efficient measure. The smaller losses associated with its smaller claim on public funds may more than offset the additional losses caused by the input distortions engendered in the subsidised industry.

* We are indebted to Roger Mauldon and an anonymous referee whose resistance to our arguments as initially presented induced us to remove some ambiguities and, we hope, generally clarify our exposition. The basic argument in the paper was developed during Parish’s involvement in the 1975-76 IAC inquiry into assistance for the consumption of phosphatic fertilisers, and it is stated in the final report (Industries Assistance Commission, 1976, p. 38). Ted Sieper, in a submission to the inquiry (Sieper 1976) and in subsequent discussion, contributed substantially to the development of these ideas.

† Barker and Hayami’s (1976) preference for a fertiliser subsidy over an output subsidy as a means of encouraging food self-sufficiency in the Philippines does not depart from the conventional wisdom. As they say, ‘It has to be remembered, . . . that we are dealing with an economy characterized by suboptimality in farmers’ application of inputs and that our benchmark is not free market equilibrium.’ (p. 625).
In what circumstances will an input subsidy cost the treasury less than an output subsidy to achieve a given expansion of output? This is the principal question we address here. So far as we are aware, it is not a question that has received much attention from economists. However, we are aware of one piece of evidence which suggests that differences in the treasury cost-effectiveness of different types of subsidy can be quite marked. Using empirically-derived production functions for two types of farms, and assuming profit-maximising behaviour by farmers, Mauldon (1967) estimated that the production response per dollar of subsidy disbursed was about 2.25 times as great for input subsidies (on fertiliser or labour) as for an output subsidy.

In the ensuing discussion, three types of subsidy will be considered:
(a) a subsidy on output;
(b) an equi-proportional (uniform) subsidy on all inputs used in the production of the output; and
(c) a subsidy on one (or a subset of the inputs) used in the production of the output.

Our main interest is in (a) and (c). The uniform input subsidy is introduced into the discussion mainly for analytical convenience, since it provides a useful intermediate case with which (a) and (c) can be compared in turn.

A brief heuristic argument and illustrations of why subsidies can differ in their cost-effectiveness are provided in the first section of the paper. Geometric proofs of some of our propositions are provided in the second section, with a more formal mathematical presentation being provided in the Appendix. A discussion of implications and applications of the results completes the paper.

Reasons for Differences in Cost-Effectiveness

Any subsidy does its work at the margin by opening a gap between marginal cost and marginal revenue. Subsidy payments to intramarginal units of input or output are wasted from the point of view of encouraging expanded production, but cannot, ordinarily, be avoided. Subsidies having the same effect and involving the same cost at the margin may, nevertheless, differ in the amounts absorbed by intramarginal units of the item subsidised, and hence may differ in their cost-effectiveness. These differences arise in the presence of increasing or decreasing returns to scale, and because of changes in input intensities as production expands.

The effect of returns to scale can be illustrated by comparing an output subsidy with a uniform input subsidy paid to a competitive industry subject to decreasing returns. To be equally effective in encouraging an expansion of output, each subsidy would have to open the same gap between the price and the marginal cost of a unit of output; that is, the subsidy paid per unit of output would have to be identical with the subsidy paid on the bundle of inputs comprising its marginal cost. However, the total cost of the uniform input subsidy would be less than that of the output subsidy.
put subsidy. With decreasing returns, inputs are more productive on the average than at the margin. Hence, the total payments made under the input subsidy, if spread over the total output, would represent a lower rate of subsidy per unit of output (and a lower total payment) than under the output subsidy. Another way of making the point is as follows: with decreasing returns, payments to inputs do not exhaust the total product; the output subsidy is paid on the 'surplus' output, as well as that part of output the value of which is equal to the total cost of inputs, whereas a uniform input subsidy is paid on the latter only.

Yet another way of explaining the superior cost-effectiveness of a uniform input subsidy is in terms of the fact that, under decreasing returns, production is more input-intensive at the margin than on average. This leads naturally to the conjecture that the subsidisation of a single input that enters more intensively into the marginal than into the average input mix (and hence represents a higher proportion of marginal than of average costs) might be more treasury cost-effective than the subsidisation of all inputs uniformly. An obvious—if extreme—example is provided by a fixed factor that does not enter into marginal cost at all: subsidising the fixed factor is pure waste, so that a subsidy restricted to the variable inputs is more cost-effective than either a uniform input subsidy or (if increasing returns are ruled out) an output subsidy.4

In the general case of non-radial expansion paths (where the changing relative input intensities result either from technical conditions of production, or from differences in the supply elasticities of the inputs), subsidising the input that is used more intensively at the margin will tend to be more treasury cost-effective than a uniform input subsidy, since savings will be effected on subsidy payments with respect to intramarginal production. However, producers will also be induced to substitute the subsidised for the unsubsidised inputs. The new input mix will thus attract more subsidy than if the substitution had not occurred, and this will partly, wholly, or more-than-wholly offset the savings in the cost of subsidy with respect to intramarginal production. Hence, it is uncertain whether a single-input subsidy will cost the treasury less than a uniform subsidy on all inputs. Even if it does cost less, the outcome may be socially more costly, since the input mix will be distorted in favour of the subsidised input, and the resulting increase in real costs of production may exceed the reduction in dead-weight losses elsewhere in the economy associated with the decrease in the amount of subsidy required.

**Geometric Analysis**

In this section we show, geometrically, how, in the case of a two-input production function characterised by decreasing returns and non-radial expansion paths, a subsidy on one input may be more cost-effective than a uniform subsidy on both inputs. Since, with decreasing returns, the latter is more cost-effective than an output subsidy, it follows *a fortiori* that the one-input subsidy may be more cost-effective than an output subsidy. The simpler case of zero input substitutability is considered first, followed by the case of input substitution.

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4 With constant returns, an output subsidy is exactly equivalent to a uniform subsidy on fixed and variable inputs: part of the output subsidy is captured by the fixed factor as higher rents—just as, in the case of decreasing returns with all inputs variable, part of the output subsidy goes to enhance entrepreneurial surplus.
In Figure 1 is shown part of the expansion path (the arrowed line) for a representative firm using two inputs, \( l \) and \( k \), to produce an output, \( q \). Zero substitutability between the inputs is assumed—hence the expansion path is unique, and not dependent upon the relative prices of \( l \) and \( k \). Two isoquants, \( q \) and \( q + dq \), are shown as dotted lines.

The lines whose intercepts on the \( l \) axis are labelled \( T_1 \) and \( T_2 \) are the isocost lines, at prevailing market prices, which are tangent to the isoquants \( q \) and \( q + dq \), respectively. The increment of total cost (i.e. the marginal cost) of moving from output \( q \) to \( q + dq \) is \( T_1T_2 \) (=\( MN \)), using input \( l \) as numeraire.

It is desired to lower this marginal cost, as perceived by producers, from \( MN \) to \( RN \). (Suppose this cost reduction is needed in order to induce producers to expand output by \( dq \).) This can be done by means of a subsidy on input \( k \) alone: such a subsidy will lessen the slope of the
isocost lines. The requisite subsidy is that which will generate the isocost lines shown on the diagram with \( l \)-axis intercepts \( S_1 \) and \( S_2 \); for the vertical distance between these lines, \( S_1S_2 \), represents the costs to producers of expanding output by \( dq \), and this cost equals \( RN \), the desired level. The total cost at unsubsidised prices of producing \( q + dq \) is \( OT_2 \), while its cost at subsidised prices is \( OS_2 \). This cost difference is made up by government subsidy. The total cost to the government of the \( k \)-subsidy is, therefore, \( T_2S_2 \).

Now suppose that the marginal cost, as perceived by producers, was reduced from \( MN \) to \( RN \) by means of an equiproportional subsidy on both inputs. Average and total cost would be reduced by the same proportion as marginal cost, that is, by \( MR/MN \). The cost to the government of the subsidies paid would be \( OT_2(MR/MN) \).

This cost can be depicted on the diagram as follows. Draw the ray \( ON \) and project it to meet the \( T_2 \) isocost line at \( W \). Join \( W \) and \( R \) and project the line to meet the \( l \) axis at \( X \). \( T_2X \) now represents the cost of the equiproportional subsidy.\(^5\)

It is evident that, in the situation depicted in Figure 1, the cost to the government of bringing about a given reduction in the marginal cost of the product is less if \( k \) alone is subsidised, than if both \( k \) and \( l \) are subsidised in an equiproportional manner. It is also obvious that this result follows from the fact that production of \( q \) is more \( k \)-intensive at the margin than on average. This is shown by the fact that the slope of an expansion path (which shows the ratio in which \( l \) and \( k \) are combined at the margin) is flatter than the ray \( ON \) (which shows the average factor ratio at \( N \)). Subsidising \( k \) thus lowers marginal cost more than it lowers average cost.

It can also be seen from the diagram that, if marginal and average factor intensities were the same, the cost of the two types of subsidy would be identical. For in this case the expansion path would be radial, points \( V \) and \( W \) would collapse to a single point, and the lines \( VS_2 \) and \( WX \) would collapse to a single line.

The results apply to the case of zero elasticity of factor substitution. Do they hold under conditions of factor substitutability?

Figure 2 is analogous to Figure 1 and is labelled analogously, but it incorporates isoquants that allow for factor substitution. The expansion path at unsubsidised, or at equiproportionately subsidised, market prices is labelled \( E \). However, if \( k \) alone is subsidised, the firm will shift to a new expansion path. For the rate of subsidy implicit in the isocost lines \( S_1 \) and \( S_2 \), \( E \), is the appropriate expansion path. Our problem now is to compare the costs to the government of lowering the cost faced by producers of moving from output \( q \) to output \( q + dq \), either along \( E \) or along \( E \).

As perceived by producers, the cost at \( k \)-subsidised prices of moving along \( E \), from \( N' \) to \( V' \) is \( N'R' \), while the cost at unsubsidised prices of moving along \( E \) from \( N \) to \( V \) is \( MN \). The latter cost could be reduced to \( NR (=N'R') \) by the payment of a uniform percentage subsidy on \( l \) and \( k \) at the rate \( MR/MN \).

The costs to the government of the two schemes can be compared in the same manner as for Figure 1. The cost of the \( k \) subsidy is \( T_2S_2 \), while the cost of the uniform subsidy is \( T_2X \). The cost of the \( k \) subsidy is

\[^5\] It can be shown, by similar triangles, that \( T_2X/OT_2 = MR/MN \).
greater than it would have been had the isoquants been right-angled: in
that case \( V \) and \( V' \) would coincide, as would \( T_2 \) and \( T_3 \), thus reducing the
subsidy cost by \( T_2T_3 \); also, since the isocost line defining \( S_3 \) would be
tangent to \( V \), not \( T' \), \( S_2 \) would lie above its present position, and this
would further reduce the subsidy cost. However, the cost of a uniform
subsidy is the same as it would have been with right-angled isoquants.

The real costs of the resources used by producers in expanding output
are \( T_2T_1 \) in the case of the single-input subsidy, and \( T_2T_1 \) for the
uniform subsidy.

The four propositions listed below are illustrated in Figures 1 and 2.
(a) With zero factor substitutability, subsidising the factor that is used
more intensively at the margin requires a smaller subsidy to achieve a
given output expansion than does subsidising both factors uniformly.
Since the input mix cannot be distorted, the single-input subsidy is
socially more efficient.
(b) When factor substitution is allowed for, it is no longer assured that the single-input subsidy is more cost-effective from the point of view of the treasury; $T_2S_2$ could be larger than $T_2X$.

(c) Even if it is more treasury cost-effective, the single-input subsidy may not be the socially more efficient measure, since production costs are higher than with a uniform subsidy. Suppose that the dead-weight losses and administrative costs associated with the raising of government funds amount, at the margin, to some fraction, $\gamma$, of each dollar raised. Then the total real costs of achieving the given increment of output amount to $\gamma(T_2S_2) + T_2T_1$ for the single-input subsidy, and to $\gamma(T_2X) + T_2T_1$ for the uniform subsidy. The difference in total cost is thus $\gamma(T_2S_2 - T_2X) + T_2T_1$. For the single-input subsidy to be socially more efficient, the first term has to be negative and larger in absolute value than the (positive) second term.

(d) Since, with decreasing returns, a uniform-input subsidy is more treasury cost-effective than an output subsidy, the conditions required for a single-input subsidy to be more cost-effective than an output subsidy (both with respect to the treasury and society as a whole) are less stringent than those stated in (b) and (c) above.

Implications and Applications

Despite their potential to engender inefficient production, input subsidies, direct and indirect, are widely employed, particularly in agriculture. Examples include favourable tax treatment of investments; provision of cheap credit; provision, free of charge, or below cost, of various services, including research, extension, and product grading and certification; transport subsidies; and, probably most ubiquitous of all, fertiliser subsidies.

Various rationalisations of these interventions are offered. Public-good aspects of the subsidised activity may be appealed to, as in the case of research and extension. Macroeconomic demand management may provide the motive for investment incentives. Fertiliser subsidies are frequently said to be corrective of a tendency of farmers to use fertiliser in sub-optimal quantities. However, both farmers and taxpayers tend to perceive these subsidies as a means of income transfer to farmers. Even if the belief in farmer ignorance or irrationality with respect to fertiliser use were true, it is doubtful whether a fertiliser subsidy is the appropriate corrective device. For example, if sub-optimal inputs of fertiliser result from ignorance, the subsidisation of information, soil-testing services etc., is likely to be a more effective form of intervention. For some calculations on this point, with respect to maize production in South Africa, see Nieuwoudt (1979).

Our argument suggests an alternative explanation and/or normative justification for input subsidies: from the government's point of view, they may be a more cost-effective way of encouraging output; and if they are, they may also (but need not) be more cost-effective from the point of view of society as a whole. To provide a base for a cost-effective subsidy it would be desirable for an input:

(a) to have a high elasticity of supply;

(b) to be substitutable for factors that are fixed or in relatively inelastic supply; and
(c) to have low substitutability for other inputs the supply of which is elastic.

Points (a) and (b) would ensure that it were used more intensively on the margin than on average, while (c) would limit the input-mix distortion engendered by the subsidy.

In our view, fertiliser would seem to meet these desiderata rather well, being an elastically-supplied land substitute and a complement of most other inputs. However, we are aware that this view of fertiliser's role in production is controversial, it having been challenged by Sieper (1976) and by Mauldon (private communication). Mauldon argues that, in Australian conditions, some 60 per cent of the phosphatic fertiliser and 80 to 90 per cent of the nitrogenous fertiliser used serves as a land complement and, together with land, as a substitute for other inputs. While admitting that fertiliser substitutes for land during a land development phase (when the nutrient status of soils is being built up), Mauldon argues that in the soil maintenance phase yields per hectare are a high proportion of maximum attainable yields, and fertiliser application rates are not sensitive to the price of fertiliser. In these circumstances, any expansion of output would come from product substitution and from the substitution of non-land inputs (including management) for fertilisers.

In characterising fertiliser as a land substitute and a complement of most other inputs we had in mind the historical development of agriculture, in Australia and elsewhere, in which large increases in output have been obtained from a largely fixed land base, with the aid of increased inputs of fertiliser and other inputs. Further, rather dramatic, evidence of the substitutability of fertiliser for land is provided by the fact that for two crops subject to stringent area controls in Australia, sugar and rice, our yields are the highest in the world.

On Mauldon's view, virtually all of the increase in production of a particular product on developed land takes place on the extensive margin (i.e. by displacing other products), and very little if any on the intensive margin. If this is the case, then land and fertiliser would behave as complements in the production of, say, wheat, or of any other particular crop or livestock product. However, this would not characterise their relationship with respect to agricultural output as a whole from developed land. If, for example, wheat's fertiliser 'requirements' exceeded the average, an increase in the proportion of wheat in the product mix would result in an increase in fertiliser input, land input remaining constant. A very substantial shift from livestock to crop production in Australia would involve abandonment of the clover-ley farming system currently in use and a big increase in the use of nitrogenous fertiliser. For these reasons we maintain our view that, in the context of agricultural output in the aggregate, fertiliser is a land substitute, but concede that, for particular products, land and fertiliser may well be economic complements.

Input subsidies may be used to ensure that some factor suppliers benefit selectively (or are denied benefits) from the subsidisation program in a way that is not possible with output subsidies. In particular, fertiliser subsidies are likely to be less beneficial, and may be detrimental, to landowners. On the other hand, a major limitation of their usefulness is the fact that many inputs are not narrowly output-specific. Thus input subsidies can more feasibly be used to encourage the expansion of particular
activities (e.g. investment in general or particular types of investment) or whole sectors using some specific input (e.g. agriculture via use of fertiliser) than to stimulate output of particular products.

The aim of an efficient tax system is to raise revenue at least cost in terms of distortions to the pattern of production and consumption. Factors in fixed or relatively inelastic supply are thus good objects for taxation. This is the essence of Henry George's argument for taxing the unimproved value of land. By contrast, subsidies are normally intended to change production or consumption patterns and are wasted if spent on inelastically-supplied or inelastically-demanded factors. The argument of this paper is thus the obverse of the Henry George argument for land taxation.

In our analysis we have adopted the traditional standpoint of welfare economists in assuming the only real costs of transfers to be the dead-weight losses and administrative costs associated with the raising and disbursement of the transferred funds, the transfer itself netting out to zero. However, contributors to the modern rent-seeking literature, such as Tullock (1967) and Krueger (1974), assume that transfers (and surpluses generally) are obtained and retained as a result of political activities in which real resources are expended. Both the recipients of transfers, and those from whom they are made, have an incentive to engage in lobbying activities, and the larger the transfer, the greater the incentive. From this viewpoint, the 'little triangles' of dead-weight loss are only a part—and possibly only a small part—of the real cost of transfers. The transferred revenue or subsidy 'rectangle' is also dissipated, at least in part (and possibly more than wholly) by political activities of interested parties. Thus, in comparing the real costs of different policies, the size of the respective transfers, and hence their treasury cost-effectiveness, assumes an added significance.

APPENDIX

We now turn to a more formal analysis of relative costs of input and output subsidies. Let $q$ denote the level of output, with price $p$, and assume two inputs, $k$ and $l$ with prices $c$ and $w$, respectively. Assume perfect competition in the product and factor markets, so that the producer chooses $q$, $k$, and $l$ to maximise the surplus:

\[ \pi = pq - wl - ck, \]

subject to a production function:

\[ q = F(k, l). \]

The only conditions imposed on the production function are decreasing returns to scale and convex isoquants. The following are the first-order conditions for profit maximisation:

\begin{enumerate}
  \item $q = F(k, l);$
  \item $pF_s(k, l) = c;$ and
  \item $pF_t(k, l) = w.$
\end{enumerate}

The unrestricted profit-maximising point is a solution to (A3). In the
following, the symbols $q$, $k$, $l$ will indicate the solution of these equations, and $F$, $F_k$ etc. will be assumed to be evaluated at these points, where $F_k$ etc. represent first partial derivatives, and $F_{kl}$ etc. represent second partial derivatives.

To secure an increase in output from $q$ to $q+dq$, where $dq$ represents an infinitesimal change in output, three possible types of subsidy will be considered:

(a) a subsidy on output of $dp$;
(b) an equi-proportional (uniform) subsidy on both inputs, that is, $dc$ and $dw$ with $wdc = cdw$; and
(c) a subsidy on one input only, say a subsidy on $k$ of $dc$.

Our main interest is in (a) and (c). The uniform input subsidy is introduced into the discussion for analytical convenience, since it provides a useful intermediate case with which (a) and (c) can be compared.

To find the required subsidies, differentiate equations (A3) totally to give the displacement equations:

\[(A4) \quad \begin{align*}
(a) & \quad dq = F_k dk + F_l dl; \\
(b) & \quad F_k dp + pF_{kl} dk + pF_{kl} dl = dc; \text{ and} \\
(c) & \quad F_l dp + pF_{kl} dk + pF_{kl} dl = dw.
\end{align*}\]

These can be written compactly as:

\[(A5) \quad \begin{align*}
(a) & \quad dq = f'dM; \text{ and} \\
(b) & \quad dp \cdot f + pGdM = dm,
\end{align*}\]

where $dM = [dk \quad dl]'$; $f = [F_k \quad F_l]' = (1/p)[c' \quad w]$;

$dm = [dc \quad dw]'$; and $G = \begin{bmatrix} F_{kk} & F_{kl} \\ F_{lk} & F_{ll} \end{bmatrix}$

Then:

\[(A6) \quad dq = (1/p)f'H dm - (1/p)f'Hf dp,\]

where $H = G^{-1}$, provides the profit maximising relationship between the output response, $dq$, and changes in output price or input costs, $dp$, $dc$ and $dw$.

The relative costs can now be considered.

**Output subsidy**

Set $dc = dw = 0$. Then by (A6) $dp = -pdq/f'Hf$ is the required output subsidy, with a total cost to the treasury, $C_o$, given by:

\[(A7) \quad C_o = qdp + dqdp = -[pqdq + p(dq)^2]/f'Hf.\]

**Uniform input subsidy**

Set $dp = 0$ and $dw = wdc/c$. Then, by (A6), and using (A3) (b) and (c), $dc = c dq/f'Hf$, with total cost to the treasury, $C_o$, given by:

\[(A8) \quad C_o = -[ck + ldw + dkdc + dldw],
\]

\[-[(ck + wh)dc + pdqdc]/c,
\]

\[-[(ck + wh)dq + p(dq)^2]/f'Hf,\]
since \[ dk \frac{dl}{1 w/c} = f' dM p/c = dq \frac{p/c}{c} \] by (A5) (a).

**Single input subsidy**

Set \( dp = dw = 0 \). Then, by (A6), \( dc = pdq / f' He \), where \( e = [1 \ 0]' \) is the required subsidy, with total cost to the treasury, \( C_s \), given by

\[
(C_s = -[kdc + dkdc].
\]

Now the profit maximising response of inputs to the subsidy \( dc \) is, by (A5) (b), \( dM = Hedc / p \) and so \( dk = e' Hedc / p = H_{1c} dc / p \). Thus the total cost of the subsidy is:

\[
A10 \quad C_s = -[p dq + p H_{11}(dq)^2 / f' He]/f' He.
\]

Introducing the notation \( A = c^2 H_{11} + 2cwH_{12} + w^2 H_{22} \), \( B = c^2 H_{11} + cwH_{12} \) and \( E = A - B \), and using the profit maximising conditions (A3) (b) and (c), the three costs can be written:

\[
A11 \quad C_o = -p^2 [(pq + p(dq)^2) / A];
\]

\[
A12 \quad C_u = -p^2 [(ck + w) dq + p(dq)^2] / A; \quad \text{and}
\]

\[
A13 \quad C_s = -p^2 [(ckdq + p c^2 H_{11}(dq)^2 / B) / B].
\]

Let \( D \) denote the determinant of \( G \). By the assumption of decreasing returns to scale, \( D > 0 \). Now we note that along a convex isocount:

\[
(dL/dK)_q = -F_k / F_t, \quad \text{and}
\]

\[
(d^2L/dk^2)_q = -F_{kk} / (F_{tt} - 2F_tF_{kt} + F_{tt} F_{kt} + F_{tt}^2) / F_{kk}^2 > 0,
\]

by the assumption of convex isocounts. Thus, \( A = -F_{tt} p^2 / D \) \( (d^2L/dk^2) \) \( q \) and hence \( A < 0 \). Similarly, \( B \) and \( E \) can be shown to be negative provided \( F_{kk} \), \( F_{tt} < 0 \) and \( F_{kt} > 0 \).

Equations (A11) through (A13) represent compact expressions for the treasury cost of each of the three policies under consideration. Each of these costs is made up of two parts, a coefficient of \( dq \) corresponding to the subsidy paid on intramarginal units, and a coefficient of \( (dq)^2 \) corresponding to subsidy payments to marginal units.

Consider first the comparison of an output subsidy with an equi-proportional input subsidy. Comparing (A11) and (A12), the \( (dq)^2 \) terms are equal and the excess cost of an output subsidy over a uniform input subsidy (i.e. \( C_o - C_u \)) is given by:

\[
A14 \quad C_o - C_u = -[p^2 / A] (pq - (ck + w) dq).
\]

Thus, an output subsidy is more costly than a uniform input subsidy provided the value of output exceeds the cost of inputs at unsubsidised prices—a condition that is fulfilled under decreasing returns.

To compare (A12) and (A13), let \( e_1 \) denote the excess of (A12) over (A13) considering only the coefficients of \( dq \) and let \( e_2 \) denote the excess considering only the coefficients of \( (dq)^2 \), with the net excess, \( C_o - C_s \), being the sum of \( e_1 \) and \( e_2 \). Then:

\[
e_1 = -p^2 [(ck + w) / A] - (ck / B)
\]

\[
= -p^2 [kw / A] [(l / k) - (cE / wB)].
\]
Now the relationship between \( dk \) and \( dl \) along an expansion path can be derived from (A5) (b) by setting \( dc = dw = 0 \). Then \( dM = -Hf \, dp/p \) and, hence, the slope of the expansion path is:

\[
\frac{dl}{dk} \bigg|_{x} = \frac{(F_{i}H_{12} + F_{i}H_{11})/(F_{i}H_{11} + F_{i}H_{12})}{cE/wB},
\]

where we have used (A3) (b) and (c). Hence, the above difference is positive or negative as \( \frac{dl}{dk} \bigg|_{x} \geq l/k \) and, for this component of cost, subsidising \( k \) only is less expensive if the production process is more \( k \)-intensive at the margin than on average.

Considering \( e_2 \) now, this is given by:

\[
e_2 = -p^2[1/A - c^2H_{11}/B^2],
\]

\[
= -p^2[B^2 - c^2H_{11}/A]/AB^2,
\]

\[
= -p^2c^2w^2[H_{12}^2 - H_{22}H_{11}]/AB^2,
\]

\[
= p^2c^2w^2/DA^2B^2,
\]

\[
= -p^2cckwl/(ck + w)^2 B^2,
\]

where \( \sigma \), the elasticity of substitution, is defined as (Henderson and Quandt 1971, p. 62):

\[
\sigma = -F_i F_j (F_i k + F_j l)/kl(F_{jk}F_{ij}^2 - 2F_i F_{jk}F_{il} + F_{ij}F_{il}^2).
\]

The cost difference is negative for this component, that is, the uniform subsidy is less costly than the \( k \)-only subsidy. The difference is greater, the greater is the elasticity of substitution, and is clearly zero when the elasticity of substitution is zero.

To sum up, \( e_1 \) can be positive or negative, while \( e_2 \) is strictly negative. If the process is more \( k \)-intensive at the margin than on average, there is a potential cost saving, and a potential excess cost, of subsidising \( k \) alone as compared with subsidising \( k \) and \( l \) proportionately. The net effect (i.e. the sign of \( C_\cdot - C_\cdot \)) will depend on the relative contributions of the two components.

The analysis can be extended to include the case of imperfectly elastic supply of inputs. Let \( w = w(l) + w_0 \) and \( c = c(k) + c_0 \). The terms \( w_0 \) and \( c_0 \) can be thought of as subsidies. The corresponding first-order conditions are:

(a) \( \dot{q} = F(k, l) \);

(b) \( \dot{p}F_i = c'(k).k + c(k) + c_0 \); and

(c) \( \dot{p}F_l = w'(l).l + w(l) + w_0 \).

Total differentiation and rearrangement as before leads to:

\[
dq = (f' H [dc_0 \, dw_0] - f' H dp)/p,
\]

except that:

\[
H = \begin{bmatrix}
F_{kk} - 2c'(k) & F_{ki} \\
F_{ik} & F_{ii} - 2w'(l)
\end{bmatrix}^{-1}
\]

and terms involving \( c''(k) \), \( w''(l) \) have been deleted. (Thus, supply curves have been represented by linear approximations about the initial point
$k,l).$ With this new interpretation of the elements of $H$, expressions (A11) through (A13) again represent the three costs.

Concentrating on costs $C_u$ and $C_v$, we compare the coefficients of $dq$. $A$ is still negative, so again the important result is the sign of $z$ defined as:

$$z = (l/k) - (cE/wB),$$

$$= (l/k) - (c/w)(w^2H_{22} + cwH_{12})/(c^2H_{11} + cwH_{12}),$$

$$= l/k - (wF_{xx} - cF_{xx} - 2wc'(k))/(cF_{xx} - wF_{xx} - 2cw'(l)).$$

The second term involves the slope of the expansion path and the elasticity of supply of factors. This expression is positive if:

$$l(cF_{xx} - wF_{xx}) - k(wF_{xx} - cF_{xx}) < 2clw'(l) - 2wkc'(k).$$

(Nota that $cF_{xx} - wF_{xx} - 2cw'(l) < 0$.) To simplify the interpretation, let us assume a homothetic production function; then by (A13) $(dl/dk)|_x = l/k$, and the difference in costs is positive if $w'(l)/w > c'(k)k/c$, that is, if $\epsilon_1 < \epsilon_4$, where $\epsilon_1$ is the elasticity of supply of $l$; and $\epsilon_4$ is the elasticity of supply of $k$.

Thus for this component of cost, subsidizing $k$ only is less expensive if the elasticity of supply of $k$ exceeds the elasticity of supply of $l$.

Finally, turning to the coefficients of $(dq)^2$, the signs of $A$, $B$ and $D$ are unaffected, so for this component of cost, subsidising $k$ only is at least as costly as uniform subsidies. Thus, as in the perfectly competitive case, the two cost components may be of opposite signs.

References

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