A SIMULATION MODEL OF AN ARID ZONE SHEEP PROPERTY

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A simulation model of a pastoral property in the Western Division of New South Wales is described. Problems of applying the model to analysis of decision situations faced by graziers are reviewed, and the results of some early applications of the model are summarized.

Introduction

Two surveys of management practices and problems on pastoral properties in the West Darling region of New South Wales were conducted in 1968 and in early 1969.¹ The results of these surveys suggested that differences among management policies adopted by graziers were associated with a lack of consensus of opinion about the biological and financial consequences of alternative policies, as well as with interpersonal differences of the type discussed recently by Francisco and Anderson [8]. It was concluded that research into the biological and financial consequences of alternative management policies was desirable as a prerequisite for improvement in decision-making processes in the region.

Some management problems were initially examined with such techniques as partial budgets and parametric gross margin budgets.² However, the dominance of climatic effects, together with the complexity of the biological relationships involved, suggested the need for a methodology which allowed more flexible consideration of stochastic factors. In common with other researchers such as Anderson [1], Goodall [9], Moule [10] and Perry [12], the authors decided that the use of simulation models provided a promising approach to the analysis of pastoral management problems in an arid zone environment.

The authors chose to adopt a 'case study' approach, as the development of a simulation model for a particular property allowed testing of the model's performance under the decision policies and management goals specified by the operator, and the comparison of this simulated performance with the real performance of the property.³ The present

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¹ The West Darling region is defined as a socio-economic unit which covers about fifty thousand square miles in the Western Division of New South Wales, with the city of Broken Hill as the focus of economic activity. The detail of the surveys of pastoral properties is presented by Chudleigh [5], chapters 1 and 4.

² Problems examined included enterprise selection, choice of strain of sheep, and adoption of fleece weighing. The analyses are presented by Chudleigh [5], chapters 3 and 5.

³ The property chosen for the case study is of 75,000 acres and is located between Broken Hill and Wilcannia. It is intended that the basic model will later be modified for further case studies: most of the data required for modification for several other properties has already been collected.
paper reports on the development and use of this model up to the beginning of 1971.

Structure of the Model

Major components of the model were those factors which were thought to dominate the economic effects of alternative management policies. These major factors were considered to be:

- climatic conditions,
- feed production and disappearance,
- reproductive performance,
- wool production, and
- sheep and wool prices.

For most of these factors only limited documented data were available for the West Darling region. In construction of the model the authors drew heavily on historical records which were collected, concurrently with development of the model, from a variety of primary sources, principally local graziers and branches of woolbroking and stock and station agencies.

The system of representation of feed production and disappearance, which included a system for specification of the grazer's policy or set of decision rules on short-term adjustments to stocking rate, was based on a 'feed index' which has been described in detail by Chudleigh and Filan [6]. The index is measured in units corresponding to the feed requirements of a stated number of sheep for one month. Feed growth within a month is calculated as a function of current rainfall, temperature, and an index of vegetation condition, which is in turn a function of rainfall in preceding months. The parameters of the feed growth function were derived by regression analysis from a set of estimates which were secured in interviews with the co-operating grazer.

The general structure of relationships amongst the feed index, rainfall, and other elements of the simulation model is illustrated in Figure 1.

The basic stochastic element of the model is rainfall. A frequency distribution of monthly rainfall was available from records maintained for the case study property for a 78-year period. If months of zero rainfall are excluded, this frequency distribution may be closely approximated (for any calendar month) by an exponential function. Simulated rainfall data are therefore generated within the model in a two-stage procedure. First, a random number is drawn to decide whether rainfall for a particular month is zero or non-zero. As the second stage, for months of non-zero rainfall, a random number \( r \) is drawn from a uniform distribution in the interval 0 to 1, and rainfall for the month is estimated as

\[
x = -\frac{1}{u_m}(\log r)
\]

4 'Validation tests' on the model, reported later in this paper, indicated certain deficiencies in modelling of some financial aspects of the grazing system. Development of a revised model is now proceeding.

5 Note that Figure 1 does not represent a flow chart for the model. Of the factors represented, some are calculated monthly, some annually, and some only sporadically as influenced by abnormal seasonal conditions.

6 The computer model provides for reading of an historical sequence of rainfall observations as an alternative to the system described here for generation of a stochastic sample of rainfall observations.
where \( \mu_m \) is the average rainfall for month \( m \) for those years where rainfall in month \( m \) is non-zero. Chi-square tests comparing rainfall distributions generated in this way with the rainfall actually recorded over a 78-year period suggested that this procedure provided a realistic
distribution of rainfall levels. Successive values of the term \( r \) were generated independently, as it was assumed that autocorrelation of monthly rainfall observations was associated with a seasonal pattern, which was reflected in the term \( \mu_m \): this assumption may be compared with the rainfall analyses suggested by Phillips ([13] and [14]). A test for 'bunchiness', as outlined by Rutherford [15], indicated only a medium to low incidence of 'runs' of years of high or low rainfall on the case study property.

Other stochastic components of the model are lambing percentage, wool cut per head, wool quality (crimps per inch), clean yield of wool, and feed growth. Each of these components is calculated as a function of a number of other variables and an error term. The parameters of the relevant functions were estimated by regression techniques. The residuals from each of the regression functions appeared to be normally distributed with mean zero, to be independent of each other, and to be free from significant autocorrelation. Therefore the stochastic error terms in the simulation model are generated independently, using the Central Limit approach described by Naylor et al. ([11], pages 92-94).

A number of other components of the model were of a stochastic nature in that they were deterministic functions of the stochastic variables described above: wool price, for example, is calculated in the model as a function of wool quality and of the clean yield of wool.

The regression functions were estimated from various sets of historical data. In each case a priori concepts were used to define a number of possible functional forms; the parameters of each function were then estimated, and a particular function was selected for use in the simulation model after a review of \( R^2 \)-values, significance tests (\( t \)-tests) of individual parameters and an analysis of variance test for the significance of the function as a whole.\(^7\)

Lambing percentage records were collected for ten properties, and were analysed both in pooled form and for individual properties. The function selected was based only on data for the case study property and was estimated from records for a period of 28 years. Lambing percentage was estimated as a function of a trend variable and of rainfall over the sixth, seventh and eighth months after commencement of joining (in the computer simulation model the trend variable is treated as a constant corresponding to the year 1967). The stochastic lambing percentage figure predicted by this function was adjusted for the age composition of the breeding flock, the adjustment factors corresponding to results at Cunnamulla reported by Turner and Dolling [16].

Data on wool cut per head were analysed for nine properties, but, as in the case of reproductive performance, the function selected for the model was fitted to data from the case study property for the 28 years to 1967. This function predicted wool cut per head as a function of trend and of rainfall in each of the four preceding seasons (spring, summer, autumn, winter), with interaction effects between spring and summer rainfall, and a quadratic effect for autumn rainfall. The figure predicted by the function was adjusted for age composition of the flock, using relationships reported by Brown et al. [3] for Cunnamulla.

Functions for prediction of wool quality and of the clean yield of

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\(^7\) The relevant statistics are reported by Chudleigh [4], part II.
wool were estimated from pooled data from fifteen properties: number of observations per property varied slightly, so that a total of 136 observations were available. While a number of other variables, such as the incidence of wind and some qualitative attributes of the different stations, were considered, the prophetic functions used in the simulation model expressed both wool quality and yield as functions of seasonal rainfall and of dummy ('zero-one') variables for properties: the property effects, of course, were incorporated in the constant terms for the particular station being studied.

Wool prices were based on the average clean wool price at Adelaide for the period August, 1967 to January, 1969. Over this period the relationship between average clean wool price and wool quality (crimps per inch) was described by a linear function for the range of wool types produced in the West Darling region: this linear function was used to adjust the assumed clean wool price for the estimated fineness of the clip in each year simulated.

Prices for sheep of each age and sex class were treated as constants for off-shears sheep: adjustments were made for the amount of wool being carried at the time of purchase or sale. The assumed constant off-shears price for each class of sheep was based on the average price for each class at the Yelta saleyards over the years 1958 to 1968.8

**Operation of the Model**

The basic time-unit of the model is one month, corresponding to the time-unit of the 'feed index' system. This time unit serves as a suitable dating mechanism for the various management aspects of the model. Within each month, the first step is to check whether the time for normal selling of various classes of sheep has arrived. If so the number involved is recorded, inventories are adjusted, and income from sales is recorded. Next, if it is time for lambing, reproductive performance is estimated and sheep inventories are again adjusted. If it is time for mating, feed conditions are checked before a decision is made to proceed with mating. Feed conditions are also checked to provide a basis for the decision whether to increase or reduce the size of the flock. If it is the month for shearing, wool cuts, wool quality, clean yield of wool and clean wool price are estimated, and wool income and shearing costs are recorded. Similarly, the details of operations such as crutching, mulesing, jetting and dipping are generated and recorded, and sheep inventories are adjusted for culling, ageing of sheep, weaning and mortality.

Details of rainfall, the feed index, and a cumulative cash flow statement are printed for each month. Annual figures are reported for net income, taxable income, a number of sub-classes of cost and return items, and for a number of estimated variables such as lambing percentage and wool cut per head.

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8 Records of price, age and sex of individual mobs of sheep sold at Yelta (near Wentworth, N.S.W.) over the years 1958 to 1968 were obtained. A regression function using sex, mob size, age, greasy wool price, amount of wool carried and rainfall at Wentworth as independent variables explained forty-five per cent of the observed variance of sheep prices. However, considerations of programming time and computer memory requirements precluded the use of a sheep price generating function at this stage of the development of the model.
Validation of the Model

Assessment of the validity of the model was partly subjective. The manager of the case study property accepted as valid the implications of various aspects of the model (such as the form of the structural equations used) when these were discussed with him. For example, he agreed with the finding from regression analyses that rainfall at and immediately after the time of lambing has a critical effect on reproductive performance as indicated by the number of lambs marked.

An attempt was made to construct a more objective test of the validity of the model. A number of records from the case study property were available for the fifteen years from July, 1953 to June, 1968, so for validation purposes the model was run for a fifteen year period using actual rainfall data as input; other stochastic elements of the model were generated in the manner described above. Unfortunately, scarcity of data prevented testing of the model for a period other than that which provided the data for estimation of a number of the parameters of the model.

Since appropriate variance estimates were unavailable, it was not feasible to apply normal statistical tests of the hypothesis that the outcomes of the simulation could have come from a 'real' distribution of outcomes. An alternative test was adopted: thirty replications of a simulation of the fifteen-year period were used to provide estimates of the distribution of outcomes of the simulation model, so that we could test the hypothesis that the 'real' results recorded over 1953 to 1968 could have come from the population of results produced by the simulation for each financial year.

Let $X_i$ be an observation for a particular year from a population of possible observations for that year, with mean $(u)$ and variance $(\sigma^2)$. Then the estimates $(\bar{X}_t)$ of the mean $(u)$ will be normally distributed with mean $(u)$ and variance $(\sigma^2/30)$, as we have thirty observations to provide the estimates. The differences $(X_i - \bar{X}_t)$ will be normally distributed with mean zero and variance $(31\sigma^2/30)$, so that

$$\frac{[30^i(X_i - \bar{X}_t)]/[31^i(s)]}{[31^i(s)]}$$

will be distributed as Student's $t$ with twenty-nine degrees of freedom, where $(s)$ is the standard deviation of the thirty observations and is an estimate of $(\sigma)$. Consequently, $( (X_i - \bar{X}_t)/s )$, will be approximately distributed as Student's $t$-distribution for twenty-nine degrees of freedom.

If we enter as $(X_i)$ the 'real' result for year $(i)$, and as $(\bar{X}_t)$ and $(s)$ the mean and standard deviation of the thirty simulation results for year $(i)$, we have a test of the hypothesis that the real result could have come from the population of predicted or simulated results.

Results of application of this validation test for three variables are shown in Table 1. The simulation model appears to provide a valid representation of the physical operation of the property, but the combination of restrictive price assumptions with some simplified accounting procedures means that the model is unsuccessful in reproducing financial results actually recorded. The close agreement in regard to the basic physical measures of performance indicates that, even in its present form, the model may be useful in evaluation of the relative economic value of alternative management practices, although further development of the model is clearly desirable.
<table>
<thead>
<tr>
<th>Year to June 30</th>
<th>Number of Grown Sheep Shorn (No.)</th>
<th>Wool Cut per Head (Greasy) (lb.)</th>
<th>Sheep Trading Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual level</td>
<td>Mean predicted level</td>
<td>Standard deviation of predictions</td>
</tr>
<tr>
<td>1954</td>
<td>3719</td>
<td>3719</td>
<td>4.2</td>
</tr>
<tr>
<td>1955</td>
<td>2674</td>
<td>2898</td>
<td>401.0</td>
</tr>
<tr>
<td>1956</td>
<td>3616</td>
<td>2834</td>
<td>907.6</td>
</tr>
<tr>
<td>1957</td>
<td>3077</td>
<td>4517</td>
<td>1316.5</td>
</tr>
<tr>
<td>1958</td>
<td>4299</td>
<td>4255</td>
<td>1072.3</td>
</tr>
<tr>
<td>1959</td>
<td>4323</td>
<td>3938</td>
<td>745.7</td>
</tr>
<tr>
<td>1960</td>
<td>4952</td>
<td>4534</td>
<td>1108.1</td>
</tr>
<tr>
<td>1961</td>
<td>4349</td>
<td>4328</td>
<td>731.4</td>
</tr>
<tr>
<td>1962</td>
<td>4112</td>
<td>3967</td>
<td>457.6</td>
</tr>
<tr>
<td>1963</td>
<td>4847</td>
<td>5309</td>
<td>256.9</td>
</tr>
<tr>
<td>1964</td>
<td>4509</td>
<td>4943</td>
<td>411.2</td>
</tr>
<tr>
<td>1965</td>
<td>4153</td>
<td>4284</td>
<td>707.4</td>
</tr>
<tr>
<td>1966</td>
<td>3984</td>
<td>3755</td>
<td>370.2</td>
</tr>
<tr>
<td>1967</td>
<td>3411</td>
<td>3579</td>
<td>498.2</td>
</tr>
<tr>
<td>1968</td>
<td>3184</td>
<td>3146</td>
<td>647.4</td>
</tr>
</tbody>
</table>

Mean 3947 4000 13.2 13.1 7978 6973

\( t = 2.04 \) for 29 degrees of freedom and 95% level of significance.

* denotes significant difference between actual and mean predicted levels.
Some Applications of the Model

The structure of the model provides for a considerable amount of information to be read as input data; for example, factors for modification of long-run wool cut, mortality and reproductive performance levels, long term average sheep numbers, 'seeds' (starting values) for random number generators, specification of the grazier's policy on stocking rate adjustments to short-term changes in feed conditions, and the planned timing of the various flock management operations. A number of management alternatives (for example, the adoption of a fleece weighing programme) may be incorporated by relatively minor modifications to the computer programme.

In the design of experiments which use a model of this nature there is some problem in allocating the total number of 'years of farm operation' simulated for any one experimental treatment; the number of years simulated is the product of the number of replications and the number of years within a 'run'. As years within a run are not independent (performance in year (i) is affected by performance in year (i - 1)), the sampling efficiency of any given number of years of simulation is affected by the distribution between length of run and number of replications. In practice, the decision has been to treat a run as a sequence of thirty years of experience of rainfall and other stochastic factors, to provide one observation of the mean, and distribution through time, of 'dependent' variables such as net farm income. This decision has been based on the view that graziers recognize the variability of their environment and so base their decisions on the expected mean and variance of income, rather than on income for individual years. The period of thirty years was chosen, rather arbitrarily, as being sufficiently long to provide a reasonably consistent estimate of income variance, but short enough to have some relation to the 'planning horizon' of the grazier.

Within a replication of an experiment using the model there are several options available for treatment of stochastic elements. By appropriate control of the 'seeds' for the random number generator:

(a) Independent sets of stochastic elements may be used for each treatment;
(b) Identical sets of stochastic elements may be used for each treatment, so that all results or effects may be regarded as treatment effects, leaving no error term to be used in conventional analysis of variance (any 'error' will be a reflection of lack-of-fit of the statistical model, rather than variation in the normal sense of experimental error); or
(c) A 'partial blocking' policy may be adopted, in that uniform rainfall sequences are used for each treatment within the replication, but other stochastic elements are generated independently. In this case an analysis of variance may partition variance amongst treatment effects, 'block' or rainfall effects, and residual 'error' derived from the other stochastic elements.

Adoption of alternative (a) would result in very large error terms, so that detection of significant effects would require very large sample sizes—that is, a large number of replications. Alternative (b) eliminates
the scope for the conventional parametric statistical tests: however, since there is no 'unexplained' variance, one may simply scan the results to look for some magnitude of effect (selected a priori) that is regarded as being of practical interest (see, for example, Wright [17], chapter 14). Alternatively, non-parametric tests may be applied to the results. If alternative (c) is adopted, the form of the experiment will correspond closely to that of conventional field experiments, and both parametric and non-parametric statistical tests may be applied to the results.

Experimental uses of this type of model may be concerned with assessments of alternative decision policies for management; with an examination of certain aspects of business structure such as farm size; or with assessment of the sensitivity of performance indices like income to the level of certain structural parameters such as reproductive performance. The initial applications of the model described in this paper were in the third of these classes, and were intended to illustrate the potential contribution of the simulation model in assessment of research priorities for the arid zone.

In an early experiment the long-term average levels of wool cut per head, lamb marking percentage, and flock death rates were treated as variables. (In this experiment, identical stochastic element sequences were used for each treatment within a replication). The experiment indicated a linear relationship between average income and wool cut per head: average net income changed by $1700 for each one pound per head change in average wool cut, regardless of the series of seasonal conditions considered. The relationship between lambing percentage and average net income was non-linear and varied with seasonal conditions; however, in the region of primary interest (possible increases above the present 'real' average for the property of eighty per cent lambing), the relationship was approximately linear and interactions with seasonal effects were negligible, so that it was possible to conclude with some generality that an increase in reproductive performance from eighty to one hundred per cent lambing would be required to raise average net incomes by $1700. It was found that current death rates (about four per cent) were so low that only very small improvements in average net income could be achieved by reducing them.

From this assessment we have an indication of little potential gain from reduction in death rates, but significant potential gains from applied research which allows improvements in reproduction or in wool cut, an improvement of 20% in reproductive performance being equivalent to a gain of 7.7% in wool cut per head. If applied research workers have limited resources but are able to make some forecast of possible relative improvements in the two measures of flock performance achievable from research, the simulation study provides a basis for a method of selection of research programmes.

Other experiments using the simulation model were concerned with aspects of management policy. One experiment compared a range of 'normal' ages of culling of ewes.\textsuperscript{9} Results for nine replications of this experiment are summarized in Table 2. A non-parametric ranking test

\textsuperscript{9} The simulation model provides for variation from the "normal" culling age in situations where the grazier is forced to destock, or where he is trying to 'breed-up'. In the culling-age experiment the same sequence of stochastic elements was used for each treatment within a replication.
indicated significant differences between culling ages.\textsuperscript{10} If a minimum effect of $1000 on mean net farm income is adopted as a criterion, the results in Table 2 indicate that a culling age of 4½ years should be avoided, but that there is little to choose amongst the other culling ages considered. These results may be treated as results of an experiment based on a randomized block design, and subjected to analysis of variance, as shown in Table 3. However, since it is known \textit{a priori} from the structure of the simulation model and from the management of the experiment that all variation is associated with the experimental variable or with differences between blocks (replications), the 'residual' term in the analysis reflects the lack of fit of the linear model implied by the form of the analysis: it does not reflect 'experimental error'.

A similar experiment concerned with an aspect of management policy used the simulation model to compare the long-term average stocking rate sought by the grazier with stocking rates ten and twenty per cent,

\begin{table}
\centering
\caption{Experiment on Ewe Culling Age for Case Study Property}
\begin{tabular}{l|c|cccccc}
\hline
Replication & Average & \multicolumn{6}{c}{Average net income ($)} for culling at: \\
 & annual & 4½ years & 5½ years & 6½ years & 7½ years & 8½ years \\
 & rainfall & & & & & \\
(points) & & & & & & \\
\hline
1 & 749 & 8007 & 11567 & 12811 & 12676 & 12631 \\
2 & 651 & 2356 & 4778 & 5963 & 5934 & 5661 \\
3 & 699 & 8504 & 9314 & 9911 & 10007 & 10035 \\
4 & 783 & 12947 & 14075 & 14604 & 14720 & 14638 \\
5 & 673 & 7703 & 8424 & 8205 & 8398 & 8206 \\
6 & 715 & 6969 & 9100 & 10905 & 11417 & 10940 \\
7 & 689 & 6157 & 6779 & 6762 & 7117 & 7153 \\
8 & 700 & 7751 & 9733 & 10114 & 10051 & 9902 \\
9 & 629 & 1224 & 2988 & 1911 & 2225 & 2231 \\
\hline
Mean & 699 & 6846 & 8529 & 9021 & 9172 & 9044 \\
\hline
\end{tabular}
\end{table}

\begin{table}
\centering
\caption{Randomized Block Analysis of Experiment on Effects of Culling Age on Average Net Incomes}
\begin{tabular}{llll}
\hline
Source of variation & Sum of squares & Degrees of freedom & Mean square \\
\hline
Treatment (culling age) & 33,757,791 & 4 & 8,439,448 \\
Blocks (rainfall series) & 510,659,600 & 8 & 63,832,450 \\
Residual & 16,186,047 & 32 & 505,814 \\
Total & 560,603,438 & 44 & \\
\hline
\end{tabular}
\end{table}

\textsuperscript{10} The test used the Friedman test statistic as described by Bradley [2], pages 123-129.
respectively, below this level. Results of this experiment for five replications indicated that a long-term stocking rate reduction equivalent to ten per cent of the present level would reduce the standard deviation of net incomes by fifteen to twenty per cent, but would also reduce the mean level of net incomes by twenty to thirty per cent. Despite the limitation of the model imposed by lack of data on the effects of stocking rate on long-term vegetation composition, it is apparent that further work along these lines might be combined with analysis of graziers’ utility functions to provide an improved framework for decisions on management policies in the arid zone.\(^{11}\)

Conclusion

The applications of the simulation model which have been reported above provide illustrations of the potential use of this model in research into economic aspects of the structure and management of sheep properties in the West Darling region. However, as has been indicated in discussion of validation tests which were applied to the model, further development is required before confidence can be attached to the magnitude of financial results produced in operation of this simulation.

Despite this qualification, the tentative results of early experiments appear to be supported by, and further development of the model appears to be justified by, the results of the validation tests which indicate that the model which has been described provides a realistic simulation of the basic performance characteristics of the case study property.

References


\(^{11}\) The significance of the non-linear utility functions of graziers in the West Darling region has been indicated by Francisco and Anderson [8]. Possible approaches in the type of application indicated here are examined by Dillon [7].


