US HFCS Price Forecasting Using Seasonal ARIMA Model

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Abstract

This paper focuses on forecasting US high fructose corn syrup (HFCS) prices using a seasonal autoregressive integrated moving average model. We use both monthly and quarterly data to forecast HFCS prices for the 1994–2015 period. We perform the Augmented Dickey–Fuller test for ensuring that the HFCS prices are stationary. We use mean absolute error, in–sample root mean square error, and out–of–sample root mean square error for evaluating the predictive accuracy of the models. Based on the out–of–sample performance, we found that the quarterly model performed well in predicting HFCS prices compared to monthly model. The results will help make better decision concerning the operation of corn-wet milling plant and HFCS production.
Introduction:

Sugar and HFCS have been major source of caloric sweetener use in the United States. However, since 2002, the total caloric sweetener consumption has consistently decreased, primarily due to a decrease in HFCS per-capita consumption. For example, the per-capita HFCS consumption declined from 37.7 pounds in 2002 to 26.8 pounds in 2014 (USDA, 2016a). On the other hand, HFCS prices have increased from 13.05 cents per pound in 2002 to 22.89 cents per pound in 2014 (USDA, 2016b). In 2015, the HFCS price was at 26.58 cents per pound (USDA, 2016b).

Due to a decrease in HFCS utilization, the corn-wet milling plants are at risk of closure. For instance, on September 16, 2014, Cargill announced its decision to shut its corn-wet milling facility in Memphis (US) due to its underutilization and increased operational expenses (Memphis Business Journal, 2014).

The motivation for forecasting HFCS prices comes from the current HFCS market situation with the HFCS utilization versus its price dynamics. The results of this study will help make better decision in the operation of corn-wet mill. For example, HFCS price forecast results in conjunction with HFCS export demand could be used to evaluate the amount of HFCS to be produced and the profitability of its production in a corn-wet mill facility.

Autoregressive integrated moving average (ARIMA) model is widely used in time series analysis for forecasting univariate time series. Box and Jenkins (1979) proposed ARIMA model and hence these models were popularly known as Box-Jenkins models.

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1 It is important to note that HFCS consumption (at consumer level) is different from the HFCS deliveries (at market level). USDA calculates the HFCS consumption at consumer level after accounting for losses at wholesale level, and retail level etc. The HFCS deliveries have decreased from 64.1 pounds per capita in 2002 to 45.6 pounds per capita in 2014.
The ARIMA model was later transformed to include seasonal autoregressive and moving average terms to become the seasonal ARIMA model.

This study uses seasonal autoregressive integrated moving average (SARIMA) method to forecast HFCS prices. Specifically, we use quarterly and monthly HFCS prices to design both quarterly and monthly models and evaluate the accuracy of the forecasts. Three forecast accuracy measures— in–sample mean absolute error (MAE), in–sample root mean squared error (IRMSE), and out–of–sample root mean squared error (ORMSE) —are used to compare the performance of various SARIMA models. Many argue that the reliability of any estimated model would be analyzed based on the out–of–sample prediction accuracy (Stock and Watson, 2003). Therefore, in our study, the SARIMA model that has the lowest ORMSE was preferred to forecast HFCS prices.

**Model:**

In this section, we represent the specifications of both non-seasonal and seasonal autoregressive integrated moving average models in addition to several test procedures before forecasting HFCS prices. The test procedures include Ljung-Box test for autocorrelation in the residuals and Jarque-Bera test for normality of the residuals of the fitted model.

We follow the backshift notation used in Box and Jenkins (1976) to describe the ARIMA model. Non-seasonal autoregressive integrated moving average (ARIMA) model: ARIMA(p,d,q) is represented as follows:

\[(1-\phi_1B- \cdots -\Phi_pB^p)(1-B)^d y_t = c + (1 + \theta_1B + \cdots + \Theta_qB^q) e_t\]
where $p$, $d$, $q$ are number of non-seasonal autoregressive lags, number of non-seasonal differences required to attain stationary, number of non-seasonal moving average lags, respectively.

For seasonal ARIMA model we have SARIMA($p,d,q$)($P,D,Q$)$_m$ as shown below (Athanasopoulos, and Hyndman, 2012):

\[
\begin{align*}
\text{ARIMA } & (p,d,q) \\
\left( \frac{\text{Non-seasonal part}}{\text{of the model}} \right) & \left( \frac{\text{Seasonal part}}{\text{of the model}} \right) \\
(P,D,Q) & _m
\end{align*}
\]

where $P$, $D$, $Q$ are number of seasonal autoregressive lags, number of seasonal differences required to attain stationary, number of seasonal moving average lags, respectively.

For example, a quarterly SARIMA (1,1,1)(1,1,1)$_4$ model is represented as follows (Athanasopoulos, and Hyndman, 2012):

\[
(1 - \phi_1 B) (1 - \Phi_1 B^4) (1 - B) (1 - B^4) y_t = (1 + \theta_1 B) (1 + \Theta_1 B^4) \epsilon_t.
\]

where $B$ is backshift operator representing the lag, and $\phi_1$, $\Phi_1$, $\theta_1$, $\Theta_1$ are the parameters of the model to be estimated.

Similarly, the monthly model is represented as follows:
Forecast Accuracy Measures:

We use three accuracy measures for selecting the best SARIMA model to forecast the HFCS prices. They are

Mean Absolute Error: \( MAE = \frac{1}{n} \sum_{i=1}^{n} |\hat{y}_i - y_i| = \frac{1}{n} \sum_{i=1}^{n} |e_i| \)

In–sample Root Mean Square Error: \( IRMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2} \)

Out–of–sample Root Mean Square Error: \( ORMSE = \sqrt{\frac{1}{k} \sum_{j=1}^{k} (\hat{y}_j - y_j)^2} \)

where \( n, k, \hat{y}_i, y_i, \hat{y}_j, y_j \) represent number of observation in the training set, number of observation in the test set, forecasts (predicted values) of HFCS prices in the training set used for estimation, actual HFCS prices of training set, forecasts of HFCS prices in test set, and actual HFCS prices in the test set, respectively.²

Diagnostic Tests:

1). Unit root test

² Training set is the set of observations used in the model estimation; while the test set (holdout set) is the set of out of sample observations used for testing the performance of the estimated model.
In time series analysis, a stationary (constant mean and constant auto-covariance) data series is a pre-condition in order maintain the stability of the model. There are many tests to perform unit root tests in the time series including Kwiatkowski–Phillips–Schmidt–Shin (KPSS) (1992), and Augmented Dickey-Fuller (ADF) test etc. We used ADF test (Dickey and Fuller, 1979) for testing unit root in the HFCS prices for both quarterly and monthly models. The ADF test for a differenced time series is shown below: 

\[ \Delta y_t = \alpha + \rho y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma_L \Delta y_{t-L} + e_t, \]

where \( \Delta y_t \) is the first non-seasonal difference, \( y_{t-1} \) is one period lag of prices, and \( \Delta y_{t-L} \) is the \( L \) period lag of differenced prices.

If the test statistic of the \( \hat{\rho} \) value is less than \(-2.9\), then we reject the null of non-stationarity. It is important to note that, when a trend variable (for a trend non-differenced data) is included in the ADF test, the critical value of the test statistic would be \(-3.5\).

2). Choice of Lag in Model Selection:

We used autocorrelation functions (ACF) and partial autocorrelation functions (PACF) of the stationary HFCS prices for a first guess model. However, various models are selected randomly and evaluated by comparing the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The model that gives the lowest AIC and/or BIC value is selected for the estimation.

3). Autocorrelation Test:

---

\(^3\) In quarterly model, HFCS prices are stationary only at the difference of differenced series: \( \Delta^2 y_t = \alpha + \rho y_{t-1} + \gamma_1 \Delta^2 y_{t-1} + \cdots + \gamma_L \Delta^2 y_{t-L} + e_t. \)
To detect any autocorrelations in the residuals of the fitted model, we carry out Ljung-Box test. The Ljung-Box test for autocorrelation in the residuals is defined as follows (Ljung and Box, 1978):

\[ X = n(n + 2) \sum_{l=1}^{l} \frac{\hat{\rho}_l^2}{n - l} \]

where \( n \) is the sample size, \( \hat{\rho}_l \) is the sample autocorrelation at lag \( l \), and \( h \) is the number of lags being tested. The null hypothesis is that the residuals are independently distributed or there is no autocorrelation in the residuals, while the alternate hypothesis is that there is autocorrelation in the residuals. The test statistic is a chi-square distribution with \( h \) degrees of freedom. The degrees of freedom (\( h \)) is computed based on autoregressive and moving average lags. For example, if we are fitting an SARIMA\((p,d,q)(P,D,Q)\) model, then the degrees of freedom equals \([h-(p+q+P+Q)]\).

4). Normality Test:

We perform the Jarque-Bera test for normality of the residuals as shown below (Jarque, and Bera, 1980):

\[ JB = \left( \frac{n}{6} S \right)^2 + \left( \frac{n}{24} (K - 3) \right)^2 \]

where \( n \) is the sample size, \( S \) is the sample skewness coefficient, \( K \) is the sample kurtosis coefficient, and \( JB \) is the Jarque-Bera statistic. The \( JB \) statistic has a chi-square (\( \chi^2 \)) distribution with two degrees of freedom. The null hypothesis of the normality of the residuals is rejected if the \( JB \) test statistic is larger than the critical value of the (\( \chi^2(2) \)) distribution. At 5% level of significance, the critical value of the (\( \chi^2 \)) distribution is 5.99 (Newbold, Carlson, and Thorne, 2013). Therefore, at \( p < 0.05 \), if the \( JB \) statistic is less than 5.99, we fail to reject the null hypothesis of normality and hence conclude that the
residuals of the fitted model exhibit normal distribution (Newbold, Carlson, and Thorne, 2013).

Data:
We collected the US HFCS–42 spot price data for the 1994–2015 period from the United States Department of Agriculture’s sugar and sweetener yearbook tables (USDA, 2016b). The HFCS–42 spot price data were measured in cents per pound. For both quarterly and monthly models, we used data from 1994–2013 period for the model estimation and the last two–year data were used as holdout sample for evaluating out–of–sample performance of the fitted model. Therefore, in quarterly model, we have 88 observations in total, out of which 80 observations were used in model estimation and remaining 8 observations were used as a holdout sample. Similarly, in monthly model, we have 264 observations in total, out of which 240 observations were used in model estimation, and the holdout set consists of remaining 24 observations.

Empirical Results and Discussion:
Quarterly Model:
In this section, we present and discuss the results comparing both quarterly and monthly models. We also discuss the results of different diagnostic tests before forecasting HFCS
prices. As mentioned earlier, the foremost step for forecasting HFCS prices is to test the data for stationarity using ADF test.

For quarterly model, we found that the non–seasonal first differenced HFCS prices are not stationary in addition to the raw data. Therefore, we transformed the data by taking the difference of differenced HFCS prices to ensure it is stationary. The double differenced HFCS prices are shown in Figure 1 (Panel, A). We performed ADF test to verify that the HFCS prices (after difference of the differenced series) are stationary. The Augmented Dickey–Fuller test statistic ($\hat{\rho}$) was found to be $-5.56$, which is less than the critical value ($-2.9$) at 5% level of significance.

Figure 1 (Panel A) shows the difference of differenced HFCS prices along with its ACF and PACF. Based on the ACF and PACF, our first guess model was SARIMA(3,2,1)(0,0,0)$_4$. We estimated the first guess model along with the other five models created randomly by changing both the seasonal and non–seasonal autoregressive and moving average lags. The AIC and BIC results of the models are shown in Table 1. Based on the results in Table 1, we chose the model SARIMA (0,2,1)(1,0,1)$_4$ as it has the lowest AIC and BIC values.

After estimation of the model SARIMA (0,2,1)(1,0,1)$_4$, we performed residual analysis as shown in Figure 2. Based on the residual analysis (figure 2), we conclude that there is no significant autocorrelation in the model residuals as we fail to reject the null hypothesis of the Ljung–Box test statistic ($p$–values are high).

In order to compare our chosen model with other competing models, we selected a total of 12 models (including the models shown in Table 1) to assess the performance of the out–of–sample prediction accuracy. Although our chosen model performed well
compared to most other models, we found SARIMA(0,2,0)(3,0,2)4 with the least out-of-sample RMSE as shown in Table 2.

We performed the diagnostic tests such as Jarque–Bera test for normality of the residuals for the SARIMA(0,2,0)(3,0,2)4. We found the Jarque–Bera test statistic to be 4.07, which is less than the critical value of chi–square distribution at 5% level of significance (5.99). Hence, we fail to reject the null hypothesis and conclude that the residuals of the SARIMA(0,2,0)(3,0,2)4 exhibit a normal distribution.

Finally, we performed Ljung–Box test for autocorrelation in the residuals of the SARIMA(0,2,0)(3,0,2)4. We found that the Ljung–Box test statistic is 33.06 and we conclude that there was no evidence of any autocorrelation in the residuals of the fitted model.

After performing residual analysis and the diagnostic tests, we believe that the SARIMA(0,2,0)(3,0,2)4 was correctly specified. Subsequently, it was used for forecasting HFCS prices. The forecasts generated from the SARIMA(0,2,0)(3,0,2)4 are shown in Figure 3 (Panel, A).

**Monthly Model:**

Alternatively, the monthly HFCS prices are found to be stationary after taking a non–seasonal first difference Figure 1 (Panel, B). The ADF test statistic ($\hat{\rho}$) was found to be –5.79 (less than the critical value, –2.9, at p < 0.05).

Figure 1 (Panel B) shows the differenced HFCS prices along with its ACF and PACF. Our first guess model was SARIMA(1,1,1)(1,0,1)12 based on ACF and PACF. Similar to the quarterly model, we estimated the first guess model along with the other
five models. Based on the lowest values of AIC and BIC, we chose the model SARIMA 
\((0,1,1)(2,0,1)_{12}\) for further estimation.

After estimation of the model SARIMA \((0,1,1)(2,0,1)_{12}\), we performed residual 
analysis as shown in Figure 2 (Panel B). The residual analysis do not show significant 
autocorrelation in the model residuals as we fail to reject the null hypothesis of the 
Ljung–Box test statistic (p–values are high).

Similar to the quarterly model, we selected a total of 12 models to evaluate the 
performance of the out–of–sample prediction accuracy. We found SARIMA 
\((3,2,1)(1,0,1)_{12}\) with the least out–of–sample RMSE as shown in Table 2.

We performed the diagnostic tests such as Jarque–Bera test for normality of the 
residuals for the SARIMA \((3,2,1)(1,0,1)_{12}\). We found that the Jarque–Bera test statistic to 
be 1211 (larger than the critical value of chi–square distribution at 5% level of 
significance [5.99]). Hence, we reject the null hypothesis and conclude that the residuals 
of the SARIMA \((3,2,1)(1,0,1)_{12}\) model do not exhibit a normal distribution. Therefore, a 
further investigation is required.

Finally, we also performed Ljung–Box test for autocorrelation in the residuals of 
the SARIMA \((3,2,1)(1,0,1)_{12}\). The Ljung–Box test statistic was 5.73 and hence we found 
no evidence of any autocorrelation in the residuals of the fitted model.

As a whole, the quarterly model performed well compared to monthly model. 
Additionally, the quarterly model also satisfied the normality of the residuals of the fitted 
model. The forecast results of both quarterly and monthly models show an increasing 
trend.
Historically, increase in HFCS prices during 2002–2014 may be due to an increase in export demand of HFCS especially from Mexico. For instance, US HFCS exports to Mexico have increased from 9,773 MT in 2002 to 1,147,289 MT in 2013 (USDA, 2016c). It will be interesting to see how the HFCS exports respond to a Mexican excise tax policy on beverages introduced in 2014. Recent studies have indicated that on an average the Mexican excise tax policy on beverages has decreased its purchases by 6% (Colchero, Popkin, Rivera & Ng, 2016). HFCS price would depend on the impact of the Mexican tax policy on its beverage sales in the near future and thereby affecting the US HFCS exports to Mexico.

Conclusions:

We used quarterly and monthly data to forecast HFCS prices for the 1994–2015 period. We used two–year (2014–2015) data as a holdout set for evaluating the performance of the models and the remaining data for estimation of the model. Based on the lowest out–of–sample root mean square error criterion, the quarterly model performed well compared to the monthly model. Quarterly model resulted in the SARIMA (0,2,0)(3,0,2)₄ model, while the monthly model resulted in the SARIMA (3,2,1)(1,0,1)₁₂. The forecasts of HFCS prices of both quarterly and monthly models show an increasing trend for two years ahead. Our study did not test for the presence of heteroscedasticity in the residuals of the fitted models.
References:


Colchero, MA., Popkin, BM., Rivera, JA., & Ng, SW. 2016. Beverage purchases from stores in Mexico under the excise tax on sugar sweetened beverages: observational study. bmj, 352, h6704 (last accessed on 22 January 2016) http://press.psprings.co.uk/bmj/january/sugartax.pdf


Figure 1. Panel (A): Double-Differenced HFCS prices of Quarterly Model. Panel (B): Differenced HFCS prices of Monthly Model.
Panel (A). Quarterly Model: ARIMA (0,2,1)(1,0,1)_{4}

Panel (B). Monthly Model: ARIMA (0,1,1)(2,0,1)_{12}

Figure 2. Residual Analysis of the respective models
Panel (A): Quarterly Forecasts of ARIMA(0,2,0)(3,0,2)_4

Panel (B): Monthly forecasts of ARIMA(3,2,1)(1,0,1)[12]

Figure 3. HFCS Price Forecasts of Two Models
Table 1. Identification of ARIMA Models

<table>
<thead>
<tr>
<th>S.No.</th>
<th>ARIMA order (p,d,q)(P,D,Q)ₘ</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quarterly Models</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>(3,2,1)(0,0,0)₄</td>
<td>–198.12</td>
<td>–186.33</td>
</tr>
<tr>
<td>2.</td>
<td>(3,2,1)(1,0,1)₄</td>
<td>–198.62</td>
<td>–182.12</td>
</tr>
<tr>
<td>3.</td>
<td>(2,2,2)(1,0,1)₄</td>
<td>–199.63</td>
<td>–183.14</td>
</tr>
<tr>
<td>4.</td>
<td>(1,2,0)(1,0,1)₄</td>
<td>–172.25</td>
<td>–162.82</td>
</tr>
<tr>
<td><strong>5.</strong></td>
<td><strong>(0,2,1)(1,0,1)₄</strong></td>
<td><strong>–202.50</strong></td>
<td><strong>–193.07</strong></td>
</tr>
<tr>
<td>6.</td>
<td>(3,0,0)(2,1,0)₄</td>
<td>–181.29</td>
<td>–167.31</td>
</tr>
<tr>
<td><strong>Monthly Models</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>(1,1,1)(1,0,1)₁₂</td>
<td>–986.68</td>
<td>–969.29</td>
</tr>
<tr>
<td>2.</td>
<td>(3,1,1)(1,0,1)₁₂</td>
<td>–983.36</td>
<td>–959.02</td>
</tr>
<tr>
<td>3.</td>
<td>(2,1,2)(2,0,1)₁₂</td>
<td>–984.98</td>
<td>–957.17</td>
</tr>
<tr>
<td>4.</td>
<td>(1,1,0)(2,0,1)₁₂</td>
<td>–986.72</td>
<td>–969.34</td>
</tr>
<tr>
<td><strong>5.</strong></td>
<td><strong>(0,1,1)(2,0,1)₁₂</strong></td>
<td><strong>–990.21</strong></td>
<td><strong>–972.83</strong></td>
</tr>
<tr>
<td>6.</td>
<td>(3,0,0)(2,1,0)₁₂</td>
<td>–891.32</td>
<td>–870.74</td>
</tr>
</tbody>
</table>

Notes: ARIMA models shown in *italics* are our first guess models based on ACFs and PACFs. ARIMA models shown in **bold** are the models chosen based on lower AIC and BIC values.
Table 2. Evaluation of Out-of-Sample Performance Across Different Models

<table>
<thead>
<tr>
<th>S.No.</th>
<th>ARIMA order (p,d,q)(P,D,Q)_m</th>
<th>Out–of–Sample RMSE</th>
</tr>
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<tr>
<td><strong>Quarterly Models</strong></td>
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</tr>
<tr>
<td>1.</td>
<td>(3,2,1)(1,0,1)_4</td>
<td>5.06</td>
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<td>2.</td>
<td>(2,2,2)(1,0,1)_4</td>
<td>3.93</td>
</tr>
<tr>
<td>3.</td>
<td>(0,2,0)(3,0,2)_4</td>
<td>1.29</td>
</tr>
<tr>
<td>4.</td>
<td>(0,2,1)(1,0,1)_4</td>
<td>4.03</td>
</tr>
<tr>
<td>5.</td>
<td>(3,0,0)(2,1,0)_4</td>
<td>5.81</td>
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<tr>
<td>6.</td>
<td>(3,0,0)(2,1,0)_4</td>
<td>5.81</td>
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<tr>
<td>7.</td>
<td>(3,0,1)(2,1,0)_4</td>
<td>5.69</td>
</tr>
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<td>8.</td>
<td>(3,0,2)(2,1,0)_4</td>
<td>5.69</td>
</tr>
<tr>
<td>9.</td>
<td>(3,0,1)(0,1,1)_4</td>
<td>5.84</td>
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<td>10.</td>
<td>(3,0,3)(0,1,1)_4</td>
<td>5.85</td>
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<tr>
<td>11.</td>
<td>(3,0,2)(0,1,1)_4</td>
<td>5.49</td>
</tr>
<tr>
<td>12.</td>
<td>(2,1,5)(0,1,1)_4</td>
<td>4.66</td>
</tr>
<tr>
<td><strong>Monthly Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>(3,2,1)(1,0,1)_{12}</td>
<td>3.98</td>
</tr>
<tr>
<td>2.</td>
<td>(2,1,2)(2,0,1)_{12}</td>
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<tr>
<td>4.</td>
<td>(0,1,1)(2,0,1)_{12}</td>
<td>4.81</td>
</tr>
<tr>
<td>5.</td>
<td>(3,0,0)(2,1,0)_{12}</td>
<td>6.04</td>
</tr>
<tr>
<td>6.</td>
<td>(3,0,1)(0,1,2)_{12}</td>
<td>5.95</td>
</tr>
<tr>
<td>7.</td>
<td>(3,0,1)(2,1,0)_{12}</td>
<td>6.02</td>
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<td>(3,0,2)(2,1,0)_{12}</td>
<td>6.11</td>
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<td>9.</td>
<td>(3,0,1)(1,1,0)_{12}</td>
<td>6.48</td>
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<td>10.</td>
<td>(4,0,3)(0,1,1)_{12}</td>
<td>7.55</td>
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<tr>
<td>11.</td>
<td>(3,0,2)(0,1,1)_{12}</td>
<td>6.02</td>
</tr>
<tr>
<td>12.</td>
<td>(2,1,4)(0,1,1)_{12}</td>
<td>5.78</td>
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</table>

Notes: Models are randomly chosen.
## Table 3. In sample and out–of–sample accuracy measures of top two models

<table>
<thead>
<tr>
<th>S.No.</th>
<th>ARIMA order (p,d,q)(P,D,Q)_m</th>
<th>MAE</th>
<th>IRMSE</th>
<th>ORMSE</th>
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<tr>
<td>Quarterly Models</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>(0,2,0)(3,0,2)_4</td>
<td>0.85</td>
<td>1.23</td>
<td>1.29</td>
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<td>2.</td>
<td>(0,2,1)(1,0,1)_4</td>
<td>0.63</td>
<td>1.07</td>
<td>4.03</td>
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<tr>
<td>Monthly Models</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>(3,2,1)(1,0,1)_{12}</td>
<td>0.256</td>
<td>0.539</td>
<td>3.980</td>
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<tr>
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<td>(0,1,1)(2,0,1)_{12}</td>
<td>0.238</td>
<td>0.522</td>
<td>4.810</td>
</tr>
</tbody>
</table>

Notes: MAE is Mean Absolute Error, IRMSE is in–sample Root Mean Square Error, and ORMSE is out–of–sample Root Mean Square Error.