Australian Journal of Agricultural Economics, Vol. 21, No. 1 (April 1977), pp. 13-25.

ESTIMATES OF COST FUNCTIONS FOR PRIMARY SCHOOLS IN RURAL AREAS*

IAN W. HIND

Riverina College of Advanced Education

Rural depopulation has put in question the economic viability of many rural institutions. This paper discusses a number of the important conceptual issues that need to be considered when examining the economics of one group of rural institutions, namely schools. Measurement of educational output in particular is fraught with many difficulties. The paper then presents some estimates of cost functions for primary schools in rural areas of New South Wales. The approach combines conventional statistical methods of deriving cost functions with synthetic procedures based on the technology of schooling. It is found that economies and diseconomies of size do exist with respect to certain costs, but it is not clear whether savings can be realized from consolidation of schools, due to the existence of extra pupil transport costs resulting from consolidation.

In recent years a number of research studies have been published relating to the economics of educational institutions, particularly schools. Some of these studies have been of a production-function orientation and amongst other considerations have examined the possibility of 'economies of scale' in schooling.[1] Other studies have involved estimates of cost functions in schools [see 18, 20, 21]. Agricultural economists have been involved in both types of studies [2, 19, 24]. A considerable amount of the research to date has been characterized by a lack of theoretical content with respect to the significance of the empirical estimates that have been produced. The main problem seems to be that there is no clear consensus as to the aims and objectives of schooling, thus rendering the measurement of educational output in any truly meaningful sense virtually impossible. Yet, without a clear understanding of what schools do (or what they produce), efficiency aspects of schools cannot be satisfactorily discussed.

A number of 'standard' models have been postulated. Thomas [22] has classified these as the administrator's model, the psychologist's model and the economist's model. In the administrator's model the unit of service is the index of output, e.g. a student year of primary schooling. Inputs are teachers, classrooms, instructional equipment, etc. In the psychologist's model the outputs are considered to be changes in the cognitive behaviour of students resulting from the provision of educational services. The output in the administrator's model is thus

* This paper presents the results of research undertaken by the author in the Department of Agricultural Economics at the University of Sydney. The work was supported by a grant from the Rural Credits Development Fund of the Reserve Bank of Australia. The author has benefited from the comments of an unknown referee and wishes to thank Professor Keith Campbell for his assistance at all stages of the project. The usual caveat remains.

[1] The terms 'scale economies' and 'size economies' have usually been treated as synonyms in the literature. For an indication of the type of studies that have been conducted see [1, 7, 14].

an input in the psychologist's model. Apart from problems of measurement, research based on this model suffers a number of basic difficulties. First, there is a significant deficiency in basic theory as to how learning occurs. Without a clear model of the learning process it is extremely difficult to sort out the individual effects of schooling. What is known is that non-school inputs are important in learning and the nature of interaction between the non-school inputs and school inputs is complex. A statistical model based on a series of simultaneous equations is likely to give more meaningful results than a simple single equation model. Second, even if a conceptually valid model of schooling could be established, Levin [16] has shown that it is highly unlikely that schools are operating technically efficiently, thus rendering it difficult to find economically efficient solutions to resource allocation problems. A more radical criticism of the psychologist's model is that it fails to accurately describe the schooling process and the role of schools in society. Dreeben [8] found that schools are more oriented towards the production of non-cognitive outputs. These characteristics of schools are supported by the findings of Gintis [9] that non-cognitive outputs are the major determinants of worker productivity. In a similar vein Griliches and Mason [11] have found that variations in cognitive output as measured by achievement tests account for only a small portion of differences in pecuniary success in the labour market. This leads to the so-called economist's model of schooling.

In the economist's model output is measured as the additional earnings accruing to the student as a result of his schooling. This model is thus based on the notion that expenditure on schooling is an investment in human capital. Calculations of rates of return on schooling have proliferated in the literature. However few explanations have been offered as to why extra schooling and supposedly higher quality schooling increases an individual's productivity and hence his earnings. As indicated earlier, there is growing evidence that the contribution of schooling to productivity operates primarily through a socializing influence and only partly via cognitive development.[2] Turning to more policy-oriented aspects, because of the long lead time between the schooling process and the additional earnings generated, the economist's model is of very limited use in planning resource management in schooling within a given level of the educational system.

In this paper a measure of output of schooling in the cognitive sense is not attempted. Rather, the study focuses on the costs of providing the schooling service itself, i.e. it is within the framework of the first model described. Specifically, the paper attempts to determine the extent to which the cost per pupil of primary schooling in New South Wales varies with school size. The extent to which the quality of educational programmes varies with school size is not subjected to empirical testing.[3] The information about the nature of the cost function

[2] The central theme of the recent study of Bowles and Gintis [6] is that the principle purpose of schools in capitalist societies is to reproduce the hierarchical social relations of production. This idea provides a serious challenge to neo-classical human capital theory.

[3] The literature relating to the effect of school size on educational outcomes is surveyed in Chapter IV of the author's Masters thesis. See Hind [12]. The evidence is indecisive because most studies have suffered from a number

for primary schools is very limited, most previous estimates of schooling cost functions being based on secondary schools. Also, previous studies have suffered from a number of methodological drawbacks. First, studies based on schools in the United States have used average district data rather than individual school data. Because the major proportion of education expenditures occur at the level of the individual school, the findings of district studies are of very limited application. Second, in few studies has there been a disaggregation of the various components of schooling costs. It is very unlikely that the relationship between size and cost is similar for each category of cost. Finally, the functional forms in previous studies do not appear to have been selected on the basis of any *a priori* economic reasoning. The research reported in this paper attempted to meet these limitations of previous studies.

There are two major categories of recurrent expenditure incurred in the provision of primary education at the school level. One of these is administrative and instructional expenditure, the majority of which is in the form of salaries and wages. The other category consists of maintenance expenditure on equipment, buildings and school grounds. In 1971 the former category accounted for about 93 per cent of recurrent expenditure on primary schooling in New South Wales and the latter about 7 per cent. It is not possible to satisfactorily disaggregate administrative and instructional costs.[4] In this study cost functions based on a cross-section of schools of different sizes were estimated via single equation least squares for each of the two main categories of primary schooling expenditure.

The Administrative and Instructional Expenditure Regressions

The structural model upon which the regression equations were based was as follows:

$$Y_1 = f(X_1, X_2, X_3, D_1, D_2, X_4, X_5)$$

The sign of the partial regression coefficient of X_1 was hypothesized to be negative and the sign of the coefficients of the other variables was hypothesized to be positive. The individual variables were defined as follows:

Y_1 = that component of expenditure per pupil paid out in salaries and wages to school principals, teachers, teachers' aides and auxiliary personnel, plus expenditure on instructional materials and equipment

X_1 = average pupil enrolment for the year

X_2 = ratio of teachers employed in promotion positions to the total number of full-time teachers on staff

X_3 = average annual salary of teachers who were not employed in promotion positions (referred to as 'teaching assistants')

---

of methodological shortcomings. However, Levin *et al.* [15] are of the opinion that, if anything, there is a negative relationship between educational output and school size.

[4] A fuller discussion of the method of calculating average unit cost of education for categories of expenditure in N.S.W. is given in [12], Chapter III.

D_1 = a dummy zero-one variable to test for significant differences in expenditure resulting from staffing procedures for school administration in large schools

$D_1 = 0$ if $X_1 < 600$

$D_1 = 1$ if $X_1 \geqslant 600$

D_2 = a dummy zero-one variable (shift factor)

$D_2 = 0$ if $X_1 < 35$

$D_2 = 1$ if $X_1 \geqslant 35$

$X_4 = D_2 \times X_1^{-1}$

$X_5 = D_2 \times X_1^{-2}$

It was decided to collect data relating to all of the schools in a group of inspectorates in the South Coast and Southern Highlands of N.S.W. because of the range of schools of different sizes in that area. However inadequacies in the data collected for some schools necessitated collecting data for a further group of schools in the Hunter Valley. A sample of 116 schools resulted, the smallest school having an enrolment of 9 pupils and the largest an enrolment of 928 pupils. The data were extracted from clerical records and computer files of the N.S.W. Departments of Education and Public Works and pertained to the year 1971.

The rationale for the selection of the independent variables and of the functional forms was as follows: The size of the school X_1 is measured by average pupil enrolment. The variable X_2 is an indicator of administrative structure. The number of staff in promotion positions is a function of school enrolment. Promotion positions carry higher salaries than assistant positions. As the number of years of experience of teaching assistants in a particular school increases, so will the total salary bill for that school. Average assistant's salary is measured by X_3.

Previous estimates of schooling cost functions have either specified a power function or a concave-from-above quadratic relationship between the dependent unit cost variable and the independent school size variable. In this study it was considered that the property of symmetry in the quadratic relationship was not appropriate, given the technology of schooling. Rather, it was considered that a hyperbolic relationship would be more appropriate because a large proportion of the costs of providing primary schooling are fixed costs, e.g. salaries paid to teaching staff. Thus, for fixed costs in the short run, the cost per pupil will vary in inverse proportion to the number of pupils. Cost per pupil will decrease as the number of pupils is increased until extra teachers (the fixed factor) are employed. For the individual school this would represent the transition from the short run to the long run. The transition from one short run position to another will result in a discontinuity in the average cost function. In the sample of schools selected, this first occurs when the enrolment reaches 35 pupils and at this enrolment level a school principal with teaching duties is employed. To take account of this discontinuity, the variables D_2, X_4 and X_5 were introduced into the equation to test for significant changes in the position and slope of the curve. For reasons of simplicity in interpretation and because the discontinuities rapidly decrease in size with an increase in enrolment, other dummy variables of this type were not included at higher enrolment levels.

The dummy variable D_1 was entered to test for diseconomies of size resulting from administrative structures in large schools. When a primary school reaches a certain size in New South Wales, the Department of Education decides that it is necessary to employ a number of staff in full-time administrative and auxiliary positions. When this study was conducted this practice occurred when the school enrolment reached 600 pupils.

Estimated Cost Functions for Administration and Instruction

Figure 1 is a scattergram and simple linear regression of administrative and instructional expenditure per pupil plotted against average pupil enrolment. The relationship is distinctly curvilinear and of the hyperbolic form. The simple linear regression equation indicates that approximately 80 per cent of variation in this category of expenditure is explained by the reciprocal of school size. Statistical results from this equation and from the multiple regressions are presented in Table 1. The equation which explains most variation in administrative and instructional expediture per pupil is equation 3. However, in this equation X_2 is not significant at the 5 per cent probability level. From the zero-order correlation matrix, $R_{D_2 X_2}$ is uncomfortably high, thus revealing a multicollinearity problem in the data (see Appendix 1). Two methods of remedying the problem were adopted. The first involved forming a cross-product hybrid variable from the two independent variables D_2 and X_2. Over the range of values of the variables in the sample, the hybrid variable thus formed was identical to the variable X_2. Thus this method, represented as equation 4, meant, in effect, deleting the dummy variable D_2 from the model. The second method (equation 5) involved treating the two intercorrelated variables as the one variable, thus deleting the variable with the lowest correlation with Y_1. This resulted in deleting X_2. The partial regression coefficients in equation 5 were all highly significant.

Equations for Maintenance Expenditure

The maintenance expenditure regressions were based on the following structural model:

$$Y_2 = f(X_6, X_7, D_3).$$

The signs of the partial regression coefficients of X_6 and X_7 were hypothesized to be negative and the sign of the coefficient of D_3 was hypothesized to be positive. The individual variables were defined as follows:

Y_2 = maintenance expenditure per pupil;
X_6 = average pupil enrolment for the period 1962-1971;
X_7 = average age of school buildings;
D_3 = a dummy zero-one variable which takes the value one only if there is a teacher's residence at the school.

Because maintenance expenditures are part of fixed costs accruing to overheads, it was considered that a continuous rectangular hyperbola would best approximate the relationship between school size and maintenance expenditure per pupil.

The average age of school buildings was included as a variable because it was hypothesized that maintenance costs increase due to wear and tear as buildings get older. The dummy variable, D_3, was

TABLE 1

Statistical Results for the Administrative and Instructional Cost Functions

Eqn. no.	Regressand	Regressor	Constant term	Partial reg. coeff.	Standard error	Sig. level	$\bar{R}^2$
1	Y_1	X_1^{-1}	172·85	3,451·55	157·69	0·001	0·81
2	Y_1	X_1^{-1}	−205·02	5,893·97	227·59	0·001	0·95
		D_1		24·14	7·35	0·01	
		D_2		168·22	16·18	0·001	
		X_4		610·35	593·63	n.s.	
		X_2		12·21	37·45	n.s.	
		X_3		0·04	0·01	0·001	
3	Y_1	X_1^{-1}	−197·82	5,758·44	214·64	0·001	0·95
		D_1		24·29	6·61	0·001	
		D_2		176·90	15·71	0·001	
		X_5		67,391·47	21,125·90	0·01	
		X_2		49·23	36·75	n.s.	
		X_3		0·04	0·01	0·001	
4	Y_1	X_1^{-1}	−45·02	3,910·99	202·62	0·001	0·90
		D_1		20·99	9·67	0·05	
		X_5		30,613·05	30,559·16	n.s.	
		X_2		186·25	44·25	0·001	
		X_3		0·03	0·01	0·001	
5	Y_1	X_1^{-1}	−199·72	5,752·89	215·39	0·001	0·95
		D_1		23·58	6·61	0·001	
		D_2		164·92	12·96	0·001	
		X_5		45,461·30	13,400·13	0·001	
		X_6		0·04	0·01	0·001	

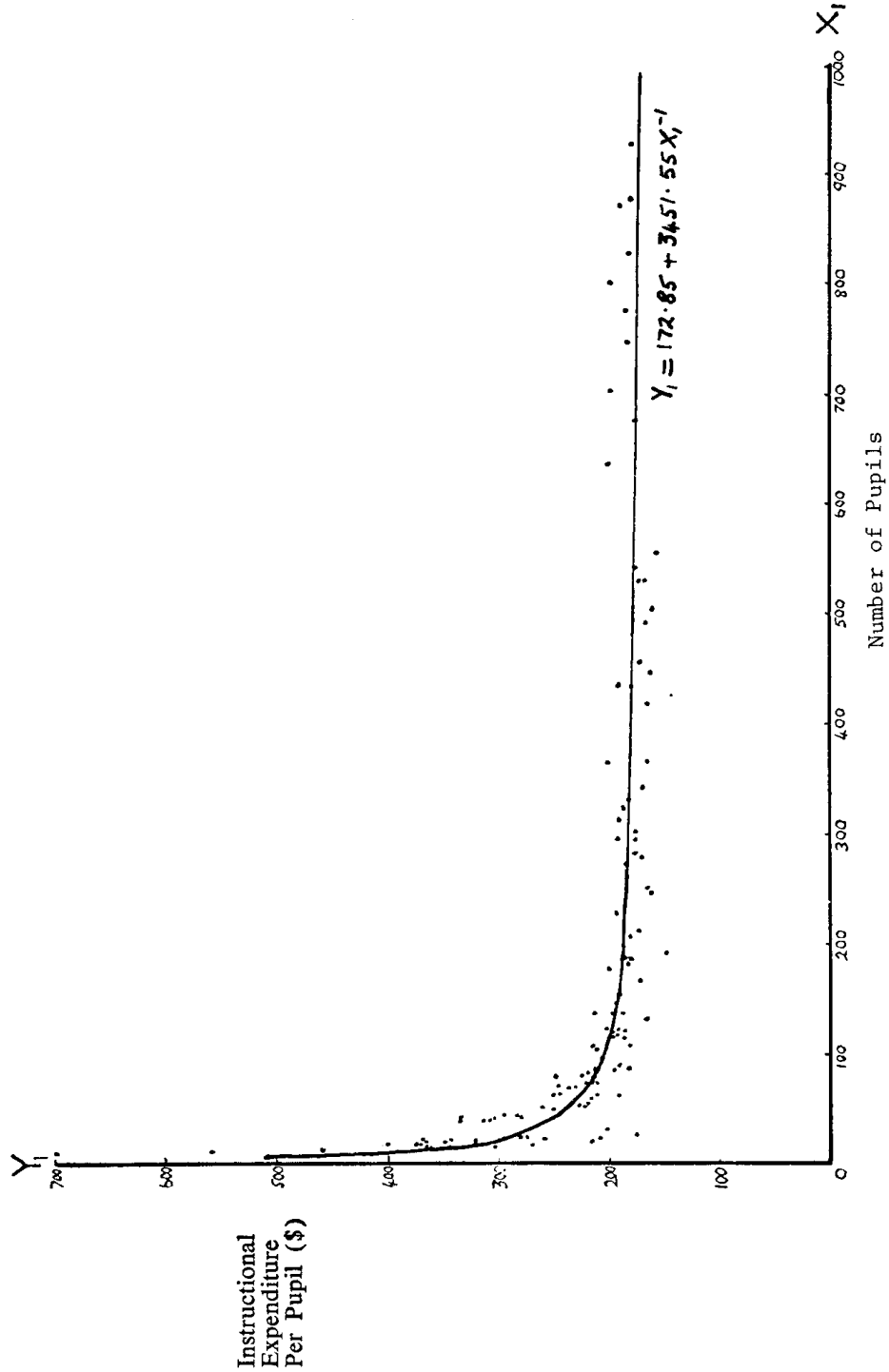

FIGURE 1—Relationship between instructional expenditure per pupil and school size.

included on account of the fact that the data available pertaining to maintenance expenditure were inclusive of all buildings on the school property. A number of school sites in the sample included a residence. Maintenance expenditure per pupil was calculated in a somewhat tedious fashion. Essentially there were two components. First, expenditure on recurring minor repairs, and second, expenditure on less frequent major renovations. Data pertaining to the former were extracted for the year 1971 and for the latter an average figure for a 10-year period was calculated, adjusted to 1971 prices.

Estimated Cost Function for School Maintenance

Again, a scattergram of maintenance expenditure per pupil plotted against school size revealed a curvilinear relationship (see Figure 2). School size alone explained about 60 per cent of variation in unit maintenance expenditure according to the simple transformed linear regression. Summary results of the statistical analysis are presented in Table 2. In equation 8, the coefficients of the variables describing school size and average age of buildings have the expected sign and are highly significant. The dummy variable D_3 has the expected positive sign but is not significant at the 5 per cent probability level. The variable is, however, significant at the 12 per cent level. It is worth noting that considerably less variation in unit maintenance expediture was explained by variations in the independent variables than was the case with the estimating equations relating to administrative and instructional expenditure. It is highly probable that measurement and stochastic error were far greater in the maintenance expediture equation. The measurement error in the calculation of unit maintenance expenditure itself was probably considerable due to the indexing procedures adopted, and with respect to the average age of school buildings it is possible that in the case of some of the older schools the data were not based on accurate information. Concerning stochastic error, there are several obvious factors which one would expect conceptually to account for some variation in unit maintenance expenditure. These factors would include physical phenomena such as weather and site factors and types of materials used in the construction of buildings.

The Economics of Rural School Consolidation

The statistical results confirm the hypothesis of the existence of economies of size in the provision of educational services at the primary school level. Unit maintenance costs were found to decrease continuously with an increase in school size. Most of the economies were exhausted by the 200 pupil enrolment level. With respect to administrative and instructional costs, both economies and diseconomies were found to exist. Most of the economies of size with respect to this category of costs were exhausted by the 100 pupil enrolment level. The diseconomies set in at the 600 pupil enrolment level due to an administrative structure with a higher labour component beyond that enrolment level. In each of the equations the dummy variable D_1 is positive and significant thus indicating diseconomies of size beyond the 600 pupil enrolment level.

TABLE 2

Statistical Results for Maintenance Cost Functions

Eqn. no.	Regressand	Regressor	Constant term	Partial Reg. coeff.	Standard error	't' Value	Sig. level	$\bar{R}^2$
6	Y_2	X_6-1	9·627	1,051·84	116·47	9·03	0·001	0·59
8	Y_2	X_6-1	−18·077	980·39	99·29	9·87	0·001	0·73
		X_7		0·44	0·11	4·11	0·001	
		D_3		10·76	6·68	1·61	n.s.	

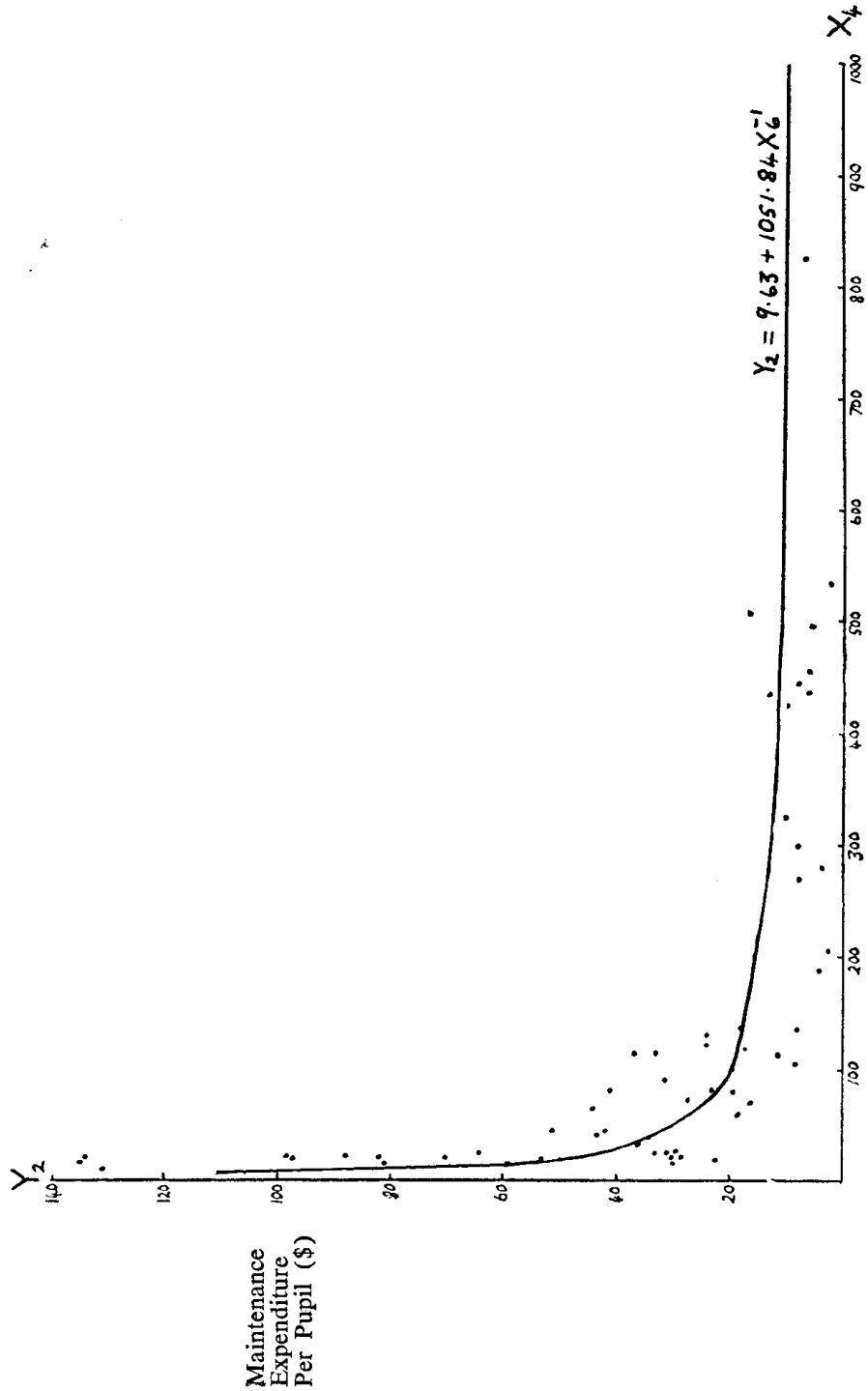

FIGURE 2—Relationship between maintenance expenditure per pupil and school size.

Do these economies of size provide the basis for an argument to consolidate schools in rural areas on efficiency grounds? There are really two aspects to this problem of the economics of rural school consolidation. First, the savings in unit costs resulting from the economies of size in the internal costs of schooling. Second, the increases in costs after consolidation associated with the necessity to transport students greater distances to and from schools. Earlier studies tended to ignore this second aspect. Indeed in studies relating to economies of size in tertiary institutions, transport costs have not been considered at all. Presumably this has been because such costs are usually borne privately and hence have no effect on the public purse.

This study raises a number of issues in relation to the former aspect. Because the major component of the cost of primary schooling is the salary paid to the classroom teacher, the regression estimates need to be interpreted with caution. Each time an additional teacher is employed with an increase in school size there will be a discontinuity in the unit cost function. This study explicitly took account of such a discontinuity at the 35 pupil enrolment level. Beyond that level, a smooth function was fitted. In reality, however, if the maximum pupil-teacher ratio was adhered to a discontinuity would occur in the unit cost function at each multiple of 35 pupils. Any conclusions about savings in cost from consolidation of schools based on a smooth function would therefore be to a certain extent misleading.

The existence of the discontinuities in the unit cost function indicates excess capacity in some size ranges. The implication of this is that for a given number of pupils total administrative and instructional costs will not necessarily decrease as the number of schools is increased. For any school working at capacity enrolment, irrespective of size, the marginal cost of a small increase in enrolment, necessitating the employment of an additional teacher, is high. On the other hand the marginal saving from a small reduction in enrolment is zero. This means that it is not really the size of the respective schools that is important when considering school consolidation. Rather, the savings in internal costs from consolidation depend on the number of schools that can be consolidated out of a given number of pupils. Any school can have excess teaching capacity ranging from 1/35 to 34/35 of a teacher. If it is assumed that within a range of schools the average excess capacity is half a teacher, then the greater the number of schools the greater the excess capacity. The elimination of this excess capacity is the saving resulting from school consolidation.

The calculation of the extra pupil transportation costs is fraught with many difficulties. It is known that there are economies in the transportation of pupils both with respect to the number of pupils conveyed and with respect to the number of miles travelled. However, transportation costs also vary considerably with population density, topographic conditons and road conditions. Using a separable programming technique a recent study in the United States found that 'while public schooling does seem to be characterized by size economies, consolidation cannot be counted on to provide large cost savings in sparsely populated rural areas'.[5]

[5]. See [13], p. 574.

In Australia most of the small schools that were established were in areas of relatively high population density such as the closely settled coastal dairying regions.[6] Many of these schools are now closed, with the small schools remaining being in sparsely populated areas and/or located considerable distances from other schools. The possibility of economies being realized would thus, in present circumstances, appear rather limited.

APPENDIX I

Zero-order Correlation Matrix for Administrative and Instructional Expenditure Regressions

	Y_1	X_1-1	D_1	D_2	X_4	X_5	X_2	X_3
Y_1	1·000							
X_1-1	0·899	1·000						
D_1	—0·171	—0·244	1·000					
D_2	—0·680	—0·887	0·157	1·000				
X_4	—0·001	—0·179	—0·239	0·487	1·000			
X_5	0·166	—0·007	—0·181	0·306	0·945	1·000		
X_2	—0·364	—0·573	0·028	0·808	0·819	0·722	1·000	
X_3	0·391	0·315	0·039	—0·492	—0·491	—0·412	—0·561	1·000

References

[1] Alkin, Malvin C., and Benson, Charles S., 'Economy of Scale in the Production of Selected Educational Outcomes', *Administrator's Notebook*, XVI, May 1968.

[2] Bieker, R. F., and Anschel, K. R., 'Estimating Educational Production Functions for Rural High Schools', *American Journal of Agricultural Economics*, 55, 1973, pp. 515-519.

[3] Bowles, S., 'Towards an Educational Production Function', in W. L. Hansen (ed.), *Education, Income and Human Capital*, New York: National Bureau of Economic Research, 1970.

[4] ———, 'The Determinants of Scholastic Achievement—An Appraisal of Some Recent Evidence', *Journal of Human Resources*, 3, 1968, pp. 3-25.

[5] ———, and Levin, H., 'More on Multicollinearity and the Effectiveness of Schools', *Journal of Human Resources*, 3, 1968, pp. 393-401.

[6] Bowles, S., and Gintis, H., *Schooling in Capitalist America: Educational Reform and the Contradictions of Economic Life*, New York: Basic Books, 1976.

[7] Cohn, E., 'Economies of Scale in Iowa High School Operations', *Journal of Human Resources*, 3, 1968, pp. 422-435.

[8] Dreeben, R., *On What is Learned in School*, Reading, Massachussetts: Addison-Wesley, 1968.

[9] Gintis, H., 'Education, Technology and the Characteristics of Worker Productivity', *American Economic Review*, 61, 1971, pp. 266-279.

[10] ———, 'Towards a Political Economy of Education', *Harvard Education Review*, 42, 1972, pp. 70-96.

[11] Griliches, Z., and Mason, W., 'Education, Income, and Ability', *Journal of Political Economy*, 80, 1972, pp. S74-S103.

[12] Hind, I. W., 'The Economics of Primary Schooling in Rural Areas of New South Wales', M.Sc.Agr. Thesis, University of Sydney, 1974.

[13] Holland, D. W., and Baritelle, J. L., 'School Consolidation in Sparsely Populated Rural Areas: A Separable Programming Approach', *American Journal of Agricultural Economics*, 57, 1975, pp. 567-575.

[14] Katzman, M., 'Distribution and Production in a Big City Elementary School System', *Yale Economic Essays*, 8, 1968, pp. 201-261.

[6] See Chapter II of the author's Masters thesis [12] for details regarding the history of the provision of primary schooling facilities in rural areas of New South Wales.

[15] Levin, H. M., *et al.*, 'Determinants of Post-School Success', *Review of Educational Research*, 41, 1971, pp. 1-18.
[16] ————, 'Measuring Efficiency in Educational Production', *Public Finance Quarterly*, 2, 1974, pp. 3-24.
[17] Michelson, S., 'Law and Economics in Hobson V. Hansen: For the Plaintiffs—Equal School Resource Allocation', *Journal of Human Resources*, 7, 1972, pp. 284-306.
[18] Osburn, D., 'Economies of Size Associated with Public High Schools', *Review of Economics and Statistics*, 52, 1970, pp. 113-115.
[19] Ricker, W. D., and Tyner, F. H., 'Efficiency in Educational Expenditures as a Criterion for School District Consolidation in North and West Florida', Department of Agricultural Economics, Report 37, University of Florida, 1972.
[20] Riew, J., 'Economies of Scale in High School Operation', *Review of Economics and Statistics*, 48, 1966, pp. 280-287.
[21] Shapiro, D., 'Economy of Scale as a Cost Factor in the Operation of School Districts in Alberta', *Canadian Journal of Economics*, VI, 1973, pp. 114-121.
[22] Thomas, J. Alan, *The Productive School*, New York: John Wiley & Sons, 1971.
[23] Wales, T. J., 'The Effect of School and District Costs on Education Costs in British Columbia', *International Economic Review*, 14, 1973, pp. 710-720.
[24] White, F. C., and Tweeten, L., 'Optimal School District Size Emphasizing Rural Areas', *American Journal of Agricultural Economics*, 55, 1973, pp. 45-53.