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ON THE EQUALIZATION OF PROFIT RATES IN MARXIAN GENERAL EQUILIBRIUM*

by

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1. Introduction

It is usually taken as a postulate in Marxian discussions that the rate of profit is equal, at equilibrium, for all capitalists. Such a phenomenon should not, however, be a postulate, but rather a theorem, for what capitalists try to do is maximize profits, and any macroeconomic phenomenon (such as an economy-wide unique rate) should be derived as a consequence of individual capitalist accumulation behavior.

In another paper (Roemer (1978)) the author has formulated a Marxian general equilibrium model. The question whether profit rates are equalized was discussed in only a special case. In general, profit rates among capitalists are not equalized in that model, as shall be pointed out below. This is due to the non-existence of a market for finance capital in that model: that is, capitalists were not able to borrow and lend. In this paper, a finance capital market is appended to the model of Roemer (1978), and it is shown that Marxian equilibria exist for the more general model: and furthermore, profit rates are always equalized at equilibrium.

This sounds like a familiar neoclassical story--the existence of a capital market will allow investment funds to be efficiently allocated, so that the rate of return on the marginal dollar is everywhere the same. There is, however, another type of profit-rate equalization which is not driven by the existence of a capital market, but rather by the requirement that the system reproduce itself. Furthermore, in the general case, the argument here shows that it is not "competition" in some vague sense which equalizes profit rates,

but precisely the existence of a capital market. This is a point which has, perhaps, not been made sharply enough in Marxian discussions.

There is another reason to prove theorems which show when a Marxian equilibrium will enjoy profit-rate equalization. Suppose that at given prices p, the rates of profit in different sectors differ. A process of capital movement may then begin: we think of capital leaving low-profit rate sectors and entering high-profit rate sectors. The presumption has been that such a dynamic process leads to equalization of profit rates, as a consequence of changing prices. Nikaido (1978) has shown that such dynamic processes do not necessarily converge. Thus, the dynamic foundations of equal-profit-rate equilibrium seem very shakey. It is, therefore, important to understand precisely which postulates of the Marxian model give rise to a static equilibrium with equal profit rates.

The paper builds upon the model of Roemer (1978); hence, the discussion of that paper will not be repeated here, nor will proofs of theorems here be written in detail when they follow closely proofs provided in that paper.

2. The Non-Equalization of Profit Rates in a Model Without Capital Market

In the general model of Roemer (1978), at equilibrium, profit rates may differ for different capitalists. This can happen in two ways: (1) if the production sets P^ν are cones, and differ among capitalists, then profit rates can differ; (2) if the production set $P^\nu \equiv P^1$ is the same for all capitalists and is not a cone (but is convex), then profit rates can differ. The situation in (2) arises because of diminishing returns: capitalists with more capital will operate "farther out" in P^1, thus generating greater total profits but at

a lower profit rate. (If, however, all capitalists face the same conical production set, then profit rates will be equalized at equilibrium.)

With the introduction of a capital market, it will be shown below that profit rates among capitalists are necessarily equal at equilibrium. The inefficiencies which can arise due to (1) imperfect information or imperfect entry (which is what occurs when capitalists face different production sets, as in (1) of the above paragraph), and (2) rents (which can be thought of as case (2) of the above paragraph) are overcome by a finance capital market.

It is worthwhile to review the one important case where profit rates are equalized for all production activities at Marxian reproducible solutions even without a capital market. Suppose all capitalists face the same conical production set, and it is generated by an indecomposable Leontief technology. Then the only price vector capable of reproducing the system is one which equalizes the profit rates for all production processes, even without a capital market. (See Roemer (1978), Section 1.) Briefly the argument is this: capitalists will only invest in maximal profit rate processes. If all processes do not generate the maximal profit rate, then some processes will not operate. But by indecomposability, the economy cannot reproduce itself unless all processes operate. In this case, then, the requirement of reproducibility drives profit-rate equalization across production activities independent of the existence of a capital market. This point will be returned to at the end of the paper.

3. Marxian Equilibrium with Finance Capital Market

We use the notation of Roemer (1978). There are N capitalists indexed by ν. Capitalist ν processes an endowment $\omega^\nu \geq 0$ of goods. He faces a production set P^ν whose points are $(-\alpha_0^\nu, -\underline{\alpha}^\nu, \overline{\alpha}^\nu)$ where $\overline{\alpha}^\nu, \underline{\alpha}^\nu \in R_+^n$, $\alpha_0 \geq 0$; α_0 is the direct labor input, $\underline{\alpha}^\nu$ is the input vector of commodities; $\overline{\alpha}^\nu$ is the output vector of commodities. Assumptions A1-A4 hold for P^ν. (See Roemer (1978).)

3A. Capitalist Behavior

A capitalist might borrow funds in amount D^ν. His feasible set, with such borrowing, at prices p, is:

$$B^\nu(p, D^\nu) = \{\alpha^\nu \in P^\nu | p\underline{\alpha}^\nu + \alpha_0^\nu \leq p\omega^\nu + D^\nu\}$$

(Negative borrowing, of course, is lending.)

Capitalists, facing prices (p,r), where p is the commodity price vector and r is the interest rate, maximize profits. Profits are the value of what the capitalist possesses at the beginning of the next period minus the value of current endowments. Thus, at borrowings D^ν, profits will be:

$$\Pi^\nu(p,r; D^\nu) = \max_{\alpha^\nu \in B^\nu(p,D^\nu)} \{[p\overline{\alpha}^\nu] + [D^\nu + p\omega^\nu - (p\underline{\alpha} + \alpha_0^\nu)] - [(1+r)D^\nu] - p\omega^\nu\}$$

where the terms are, respectively, income from production, the value of assets not used in production but held over until next period, the value of lendings repaid, and the value of today's endowments.

We may simplify:

$$\Pi^\nu(p,r; D^\nu) = \max_{\alpha^\nu \in B^\nu(p, D^\nu)} \{[p\overline{\alpha}^\nu - (p\underline{\alpha}^\nu + \alpha_0^\nu)] - rD^\nu\}$$

Let:

$A^\nu(p,r;\ D^\nu) = \{\alpha^\nu \in B^\nu(p,\ D^\nu) | \Pi^\nu(p,r;\ D^\nu)$ is achieved$\}$

Let:

$\mathcal{D}^\nu(p,r) = \{D^\nu | \Pi^\nu(p,r;\ D^\nu)$ is maximized$\}$. ($\mathcal{D}^\nu(p,r)$ may be empty for some values of (p,r).)

Let

$$\mathcal{A}^\nu(p,r) = \bigcup_{D^\nu \in \mathcal{D}^\nu(p,r)} A^\nu(p,r;\ D^\nu).$$

Capitalist behavior, formally, is: given prices (p,r), to choose any action in $\mathcal{A}^\nu(p,r)$. That is, capitalist ν chooses an amount to borrow (lend) at which his profits are maximized at those prices; he then chooses any production action which realizes those maximal profits.

3B. Equilibria

Definition 1: (p,r) is a reproducible solution with finance capital market if:

(1) $(\forall\nu)(\exists D^\nu \in \mathcal{D}^\nu(p,r)(\Sigma D^\nu = 0)$ (feasibility of optimal borrowing)

(2) $(\exists \alpha^\nu \in A^\nu(p,r;\ D^\nu))(\Sigma\underline{\alpha}^\nu + \alpha_0^\nu b \leqq \omega)$ (feasibility)

(3) $\Sigma\overline{\alpha}^\nu - \Sigma\underline{\alpha}^\nu \geqq \Sigma\alpha_0^\nu b$ (reproducibility)

(4) $pb = 1$ (subsistence wage)

Conditions (2), (3), (4) are familiar from before. Condition (1) states that there is a set of individually optimal borrowings for capitalists which is socially feasible: since capitalists can borrow only from each other, net borrowings must sum to zero.

As was shown in Roemer (1978), condition (3) should be thought of as a supplementary condition. If (p,r) exists for which (1), (2) and (4) are satisfied, (p,r) shall be called a competitive equilibrium with finance capital

market. It shall require additional investigation to determine whether reproducible solutions exist.

We next show that a competitive equilibrium exists, and that all capitalists' "profit rates" are equalized at such an equilibrium.

3C. Existence of Competitive Equilibrium with a Finance Capital Market, and Profit-Rate Equalization

We first introduce a simpler equilibrium concept: joint profit maximization. What is the distribution of capital values which would produce maximal joint profits at prices p--in the absence of a capital market?

Definition 2. Let total endowments ω be given. Let p be given. Let a distribution of numbers $(C^1, \ldots, C^N)$ be given, $C^\nu \geq 0$. Define

$$\overline{B}^\nu(p,C^\nu) = \{\alpha^\nu \in P^\nu | p\underline{\alpha}^\nu + \alpha_0^\nu \leq C^\nu\};$$

Let

$$\overline{\Pi}^\nu(p,C^\nu) = \max_{\alpha^\nu \in B^\nu(p,C^\nu)} \{[p\overline{\alpha}^\nu] + [C^\nu - (p\underline{\alpha}^\nu + \alpha_0^\nu)] - C^\nu\};$$

or

$$\overline{\Pi}(p,C^\nu) = \max_{\alpha^\nu \in B^\nu(p,C^\nu)} [p\overline{\alpha}^\nu - (p\underline{\alpha}^\nu + \alpha_0^\nu)]$$

$\overline{\Pi}^\nu(p,C^\nu)$ is the function which assigns to (p,C^ν) the maximum achievable profits with prices p and wealth endowment C^ν.

Let:

$$\overline{A}^\nu(p,C^\nu) = \{\alpha^\nu \in \overline{B}^\nu(p,C^\nu) | \overline{\Pi}^\nu(p,C^\nu) \text{ is achieved}\}$$

Define $\overline{\Pi}(p; C^1, \ldots, C^N) = \sum_\nu \overline{\Pi}^\nu(p,C^\nu)$.

Definition. p is a joint profit maximizing (JPM) competitive equilibrium if:

(1) $(\exists C^\nu)(\Sigma C^\nu \leq p\omega)$

(2) $\overline{\Pi}(p; C^1, \ldots, C^N)$ is maximized (for p fixed) over all distributions satisfying condition (1);

(3) $(\forall \nu)(\exists\, \alpha^{\nu} \in \overline{A}^{\nu}\,(p, C^{\nu}))(\Sigma \underline{\alpha}^{\nu} + \Sigma \alpha_0^{\nu} b \leqq \omega)$.

(4) $pb = 1$.

This says that if capital values are distributed in the manner $(C^1, \ldots, C^N)$, then an individually optimal, socially feasible solution exists, and that joint profits can never be greater at prices p. (Note: Recall there is no borrowing in this model.)

By virtue of the next Lemma, in our study of the economy with the finance capital market, we shall be able to limit our investigation to JPM equilibria.

Lemma 1. Let $\{(p,r);\ D^1, \ldots, D^N\}$ be a competitive equilibrium with finance capital market. Then $\{p;\ C^1, \ldots, C^N\}$ is a JPM equilibrium, where $C^{\nu} \equiv p\omega^{\nu} + D^{\nu}$.

This Lemma says that the equilibria with finance capital market are all JPM equilibria--re-interpreted without the capital market. Hence, to find all of the former, we may limit our search to the latter. Conversely, it will be shown below that all JPM equilibria can be re-interpreted as equilibria with a finance capital market.

Proof:

Note that joint profits in the two economies $\{(p,r);\ D^1, \ldots, D^N\}$ and $\{p;\ C^1, \ldots, C^N\}$ are identical. They differ only by the interest and loan charges $\sum_{\nu} (1+r)D^{\nu}$, which sum to zero, since by hypothesis $\Sigma D^{\nu} = 0$.

Suppose, then, that $\{p;\ C^1, \ldots, C^N\}$, as defined, were not a JPM equilibrium. Then it is possible to redistribute capital values so that $\{p;\ C'^1, \ldots, C'^N\}$ yield greater joint profits. But by the above paragraph, the borrowing induced by:

$$D'^{\nu} \equiv C'^{\nu} - p\omega^{\nu}$$

would yield greater joint profits in the economy with capital market. Hence, for at least one capitalist, borrowing D'^{ν} must be superior to borrowing D^{ν}, at prices (p,r). Hence $\{(p,r); D^1, \ldots, D^N\}$ was not an equilibrium with finance capital market. q.e.d.

Theorem 1. A JPM competitive equilibrium p exists.

Proof of Theorem 1: (sketch)

1. We assume, for simplicity, $b > 0$. Hence p may range over the simplex $S = \{p | pb = 1\}$.

2. For $p \in S$, there exists a feasible distribution of capital values $(C^1, \ldots, C^N)$ which maximizes joint profits. This is true because individual maximal profit functions $\Pi^{\nu}(p,C^{\nu})$ can be shown to be continuous in C^{ν}, by the assumptions on P^{ν}. Hence joint profits are a continuous function defined on the compact domain of feasible capital value distributions.

3. Define the correspondence:

$$Z(p) = \{(\Sigma\underline{\alpha}^{\nu} + \Sigma\alpha_0^{\nu}b) - \omega | \alpha^{\nu} \in \overline{A}^{\nu}(p,C^{\nu}), \text{ where } (C^1, \ldots, C^N) \text{ maximizes joint profits at } p\}$$

It follows from the assumptions on P^{ν} that $Z(p)$ is upper-hemi-continuous and convex-valued. Furthermore, $pZ(p) \leqq 0$ by the definition of $\overline{A}^{\nu}(p,C^{\nu})$.

4. It follows from the Gale-Nikaido lemma that p exists for which $Z(p) \leqq 0$, which provides the required JPM equilibrium q.e.d.

Theorem 2. (A) An equilibrium with finance capital market exists, (p,r).

(B) If (p,r) is any such equilibrium, then r is equal to the marginal rate of profit in production for each capitalist ν for whom the marginal rate of profit is well-defined.

We require a well-known result:

Lemma 2. A continuous, concave function f of a real variable possesses right and left derivatives at all points, and $\frac{df^+}{dx} \leq \frac{df^-}{dx}$.

Proof of Theorem 2:

Let $\xi = (p;\ C^1, \ldots, C^N)$ be a JPM equilibrium which exists by Theorem 1. Recall $\overline{\Pi}^\nu(p;\ C^\nu)$ is the function which assigns to the value C^ν the maximal profits capitalist ν makes when restricted to his feasible set $\overline{B}^\nu(p;\ C^\nu)$. $\overline{\Pi}^\nu(p;C^\nu)$ can be shown to be continuous (as has been remarked), and concave, by the convexity of P^ν. We fix p, and from now on, speak simply of $\overline{\Pi}^\nu(C^\nu)$.

Since ξ is JPM, it follows that for all sufficiently small positive numbers δ:

$$\forall \mu,\nu \quad \overline{\Pi}^\mu(C^\mu + \delta)) - \overline{\Pi}^\mu(C^\mu) \leq \overline{\Pi}^\nu(C^\nu) - \overline{\Pi}^\nu(C^\nu - \delta) \qquad (1);$$

for if (1) failed for some δ, then funds in amount δ could be transferred from capitalist ν to capitalist μ, and joint profits would increase, an impossibility.

Dividing inequality (1) by δ and passing to the limit as $\delta \to 0$ gives:

$$\frac{d^+\ \overline{\Pi}^\mu}{dC^\mu} \leq \frac{d^-\ \overline{\Pi}^\nu}{dC^\nu} \qquad \forall \mu \neq \nu \qquad (2)$$

where the derivatives are evaluated at C^μ and C^ν (a little notational abuse). (This limit operation is legitimate since, by Lemma 2, the right and left derivatives exist.)

However, by Lemma 2 also it follows that:

$$\frac{d^+\ \overline{\Pi}^\nu}{dC^\nu} \leq \frac{d^-\ \overline{\Pi}^\nu}{dC^\nu} \qquad \forall \nu = 1, N \qquad (3).$$

From (2) and (3):

$$M \equiv \max_\nu \frac{d^+\ \overline{\Pi}^\nu}{dC^\nu} \leq \min_\nu \frac{d^-\ \overline{\Pi}^\nu}{dC^\nu} \equiv m \qquad (4).$$

Choose r, therefore, so that $r \in [M\ ,\ m\ ]$.

It is now shown r is the appropriate interest rate which allows one to re-interpret the existing JPM equilibrium as an equilibrium with finance capital market. Define

$$D^{\nu} = C^{\nu} - p\omega^{\nu}$$

which shall be the borrowings of capitalist ν. Since $r \leq \frac{d^{-}\,\overline{\Pi}^{\nu}}{dC^{\nu}}$ for all ν, it follows that within a small neighborhood of D^{ν}, no capitalist wishes to borrow less than he does at D^{ν}, given prices (p,r). (That is, profits on his "last" dollar of capital are at least as great as the interest rate.) But since $r \geq \frac{d^{+}\,\overline{\Pi}^{\nu}}{dC}$ for all ν, within a small neighborhood of D^{ν}, no capitalist wishes to borrow more than he does at D^{ν}. Since the profit functions are concave, this local argument shows the borrowings D^{ν} are in fact a global optimum for the capitalists.

The argument in the last paragraph is slightly intuitive and not formally precise, since it employs the (fictitious) profit functions $\overline{\Pi}^{\nu}(p,C^{\nu})$--the actual profit functions for capitalists are $\Pi^{\nu}(p,r;\ D^{\nu})$ in the economy with finance capital market. To be formally precise, we observe, from the simplified definitional expressions for $\Pi^{\nu}(p,r;\ D^{\nu})$ and $\overline{\Pi}^{\nu}(p,C^{\nu})$ that:

$$\frac{d^{\pm}\,\Pi^{\nu}(p,r;\ D^{\nu})}{dD^{\nu}}\,(C^{\nu}) \equiv \frac{d^{\pm}\,\Pi^{\nu}(p,r;\ D^{\nu})}{dD^{\nu}}\,(p\omega^{\nu} + D^{\nu}) = \frac{d^{\pm}\,\overline{\Pi}^{\nu}(p,C^{\nu})}{dC^{\nu}}\,(C^{\nu}) - r$$

Hence, from choice of r, it follows that:

$$\frac{d^{-}\Pi^{\nu}(p,r;\ D^{\nu})}{dD^{\nu}}\,(p\omega^{\nu} + D^{\nu}) \geq 0 \qquad \forall\nu$$

and

$$\frac{d^{+}\,\Pi^{\nu}(p,r;\ D^{\nu})}{dD^{\nu}}\,(p\omega^{\nu} + D^{\nu}) \leq 0 \qquad \forall\nu$$

from which it follows rigorously that the point $(p,r;\ D^1, \ldots, D^N)$ constitutes an equilibrium.

It has been shown that the JPM equilibrium is an equilibrium with finance capital market. If, in addition, the profit functions are in fact <u>differentiable</u> then:

$$\forall \nu \qquad 0 = \frac{d\Pi^\nu}{dD^\nu}.$$

It follows from examination of the definition of $\Pi^\nu(p,r;\ D^\nu)$ that the <u>rate of profit from productive activity</u> is equalized to the interest rate for all capitalists at such a point:

$$\frac{d}{dD^\nu} \max_{\alpha^\nu \in B^\nu(p,D^\nu)} [\, p\bar{\alpha}^\nu - (p\underline{\alpha}^\nu + \alpha_0^\nu)] = r$$

It has, finally, to be shown that <u>any</u> equilibrium (p,r) with finance capital market has the stated property (B) of the <u>Theorem</u>. If $\{(p,r);\ D^1, \ldots, D^N\}$ is such an equilibrium then by <u>Lemma 1</u> it is a JPM equilibrium also. Hence, by the argument here, inequalities (2) and (3) hold, and hence the interval $[\,M\ ,\ m\,]$ of (4) can be defined. If $r \notin [\,M\ ,\ m\,]$, then the argument given shows that the equilibrium in question is not a JPM one: because some capitalist could profitably borrow (lend) more. Hence $r \in [\,M\ ,\ m\,]$, and the conclusion follows. q.e.d.

3D. <u>Existence of Reproducible Solutions with Finance Capital Market</u>

It is now necessary to demonstrate that <u>reproducible solutions</u> with finance capital market exist. As in the model without finance capital market, we can show such solutions exist for any wealth distribution--but

not for any initial distribution of endowment vectors. We can prove:

Theorem 3. Let $(W^1, \ldots, W^N)$ be any vector of wealths. Then there exists a set of initial endowment vectors

$$\bar{\omega} = \{\omega^1, \omega^2, \ldots, \omega^N\}$$

with the property that there exists a reproducible solution with finance capital market $\{(p,r); D^1, \ldots, D^N\}$ with the property that $p\omega^\nu = W^\nu$ for all ν.

Since a reproducible solution is a special kind of competitive equilibrium, it follows from Theorem 2 that the rates of profit are equalized in all production sets P^ν at the reproducible solution.

A sketch of the method of proving Theorem 3 follows.

Definition 3. Let W be a positive real number. We define $(p; C^1, \ldots, C^N)$ to be a quasi-reproducible joint-profit maximizing solution (QRJPM) if:

(1) $\Sigma C^\nu = W$

(2) Capitalist ν chooses $\alpha^\nu \in P^\nu$ to maximize profits at prices p, subject to the capital constraint

$$p\underline{\alpha}^\nu + \alpha_0^\nu \leqq C^\nu$$

(3) Profit-maximizing $\{\alpha^\nu\}$ exist which generate reproducibility:

$$\Sigma\bar{\alpha}^\nu - \Sigma\underline{\alpha}^\nu \geqq \Sigma\alpha_0^\nu b$$

(4) Subject to (1) and (2) above, $(C^1, \ldots, C^N)$ is the distribution of capital values which maximizes joint profits.

(5) $pb = 1$

A QRJPM solution is basically a reproducible solution where feasibility is ignored. (See condition (2) of Definition 1.)

Lemma 3. For any W, a QRJPM solution $(p; C^1, \ldots, C^N)$ exists.

Proof:

Virtually identical to Theorem 2.5, Roemer (1978).

Proof of Theorem 3:

1. Define $W = \Sigma W^{\nu}$. By Lemma 3, a QRJPM solution $(p; C^1, \ldots, C^N)$ exists. Associated with this solution are production points $\{\alpha^{\nu}\}$ which generate reproducibility, according to condition (3) of Definition 3.

2. Let ω be any vector of aggregate endowments such that $\omega \geqq b\Sigma\alpha_0^{\nu} + \Sigma\underline{\alpha}^{\nu}$ and $p\omega = W$. (Such ω exists by Definition 3, since $p(b\Sigma\alpha_0^{\nu} + \underline{\alpha}^{\nu}) \leqq W$.) Decompose ω into $\omega = \Sigma\omega^{\nu}$ in any way so that $p\omega^{\nu} = W^{\nu}$.

3. Let $\omega = \Sigma\omega'^{\nu}$ be any decomposition of ω such that $p\omega'^{\nu} = C^{\nu}$. By choice of ω, it follows that $\{p; C^1, \ldots, C^N\}$ is a JPM competitive equilibrium.

4. Let $D^{\nu} = C^{\nu} - W^{\nu}$. Since any JPM competitive equilibrium induces a competitive equilibrium with finance capital market, it follows that a competitive equilibirum with finance capital market $\{(p,r); D^1, \ldots, D^N\}$ is here induced. Moreover, this equilibrium is a reproducible solution, since the optimal production points satisfy (3) of Definition 3. q.e.d.

4. What Drives Profit-Rate Equalization in the Marxian Model?

It has now been shown that if a finance capital market exists, then Marxian equilibria (reproducible solutions) exist for any given distribution of capital values, and that at these reproducible solutions the following holds: the marginal rate of profit at the chosen production point α^{ν} in the production set P^{ν} is equalized, for all ν. The capital market thus equalizes profit rates for all production sets, and for all capitalists.

As has been shown, the function of the capital market is to allow joint profits to be maximized: in this sense, the capital market distributes available capital optimally from the point of view of capital as a whole. The inefficiencies which may exist without a capital market are of two types, as has been discussed: (1) where capitalists do not all face the same production set; (2) where capitalists face the same production set, but there are non-constant returns to scale. The capital market overcomes the inefficiencies due to (1) and (2), in the precise sense that it finds a joint-profit-maximizing solution in those cases--and, consequently, a solution where profit rates are equalized in the sense discussed.

If neither (1) nor (2) is a problem--that is, if all capitalists face the same conical production set--then a capital market is unnecessary to equalize profit rates across capitalists. At any competitive equilibrium, profit rates for all capitalists will be equal.

Finally, for certain special technologies, we can speak about individual production activities. In the Leontief or von Neumann models, for instance, we can think of the production cones as being generated by various activities. As has been discussed, in an indecomposable Leontief model, profit rates are equalized for all activities by the requirement of reproducibility. In a decomposable Leontief model where all capitalists face the same production cone, or in the von Neumann model, reproducible solutions can exist generating different profit rates in different activities, but only the activities with the maximal profit rate will be operated. Thus, profit rates continue to be equalized for capitalists, and in all production sets P^{ν}, but not for all production activities.

We can thus summarize the mechanisms which equalize profit rates in this way:

-- if there is perfect free entry and constant returns to scale, then profit rates are equalized for all capitalists by individual profit-maximization in the absence of a capital market; nevertheless, different production activities, if they can be defined, may generate different profit rates;

-- if there is imperfect entry or non-constant returns, then profit rates for all capitalists are equalized by individual profit-maximization, and the existence of a capital market;

-- even if a capital market exists, profit rates for all production activities may not be equalized. In the indecomposable Leontief model, the equalization of profit rates across activities is driven by the requirement of reproducibility.

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